Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3210 - Mecânica dos Sólidos I

Aula #10

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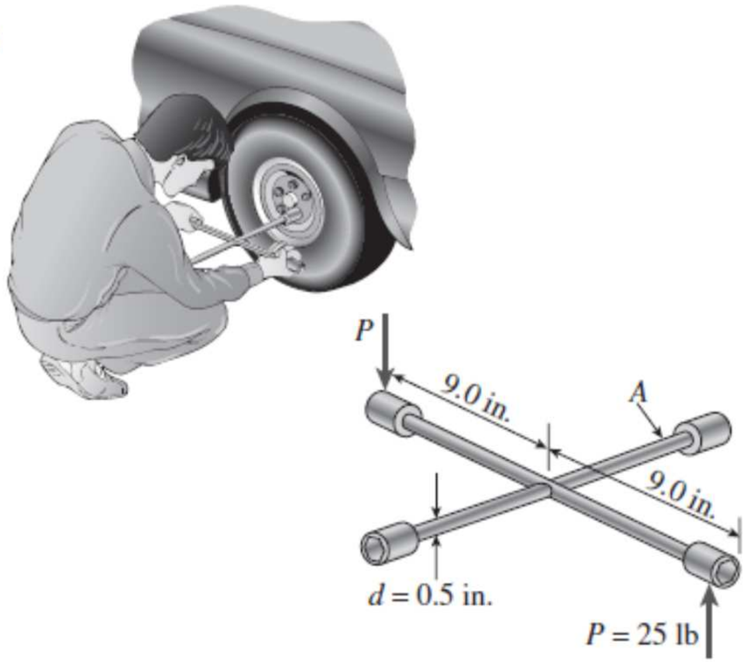
02/05/2023

Problem 3.3-3 While removing a wheel to change a tire, a driver applies forces $P = 25$ lb at the ends of two of the arms of a lug wrench (see figure). The wrench is made of steel with shear modulus of elasticity $G = 11.4 \times 10^6$ psi. Each arm of the wrench is 9.0 in. long and has a solid circular cross section of diameter $d = 0.5$ in.

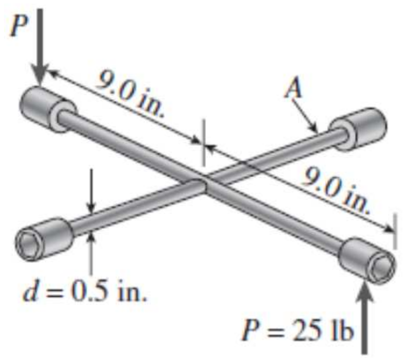
(a) Determine the maximum shear stress in the arm that is turning the lug nut (arm A).
(b) Determine the angle of twist (in degrees) of this same arm.

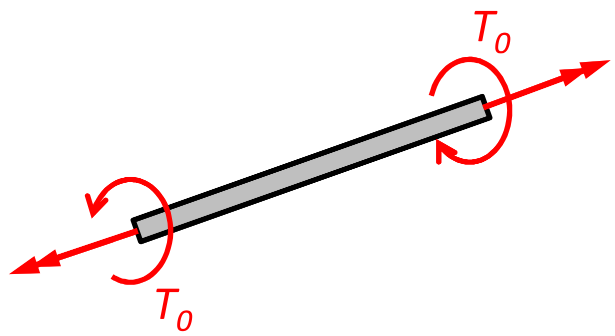


Answers:
$a)\ \tau_{máx} \cong 18{,}33\ ksi$

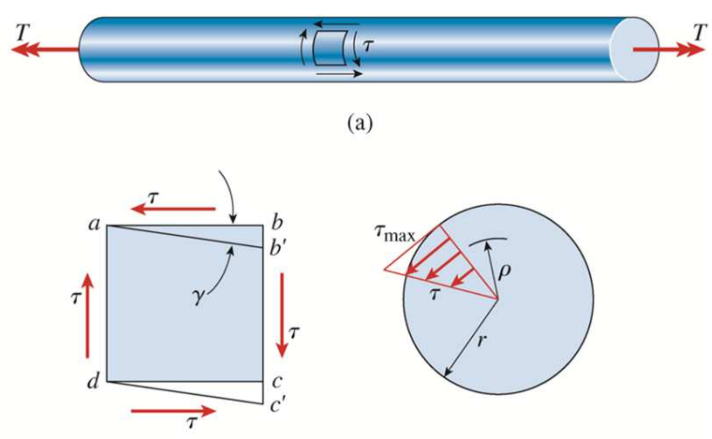
$b)\ \Phi \cong 0{,}0579\ rad \cong 3{,}3°$

Solução:





$$T_0 = P \times 2L = 25lbf \times 2 \times 9in = 450\ lbf.in$$



(a) (b) (c)

$$\tau_{máx} = \frac{T}{I_p} r = \frac{T}{(\pi d^4/32)} \frac{d}{2} = \frac{16T}{\pi d^3} = 18{,}33\ ksi$$

$$\Phi = \frac{TL}{GI_p} = \frac{450 \times 9 \times 32}{11{,}4 \times 10^6 \times \pi \times (0{,}5)^4} \cong$$

$$\cong 0{,}0579\ rad \cong 3{,}3°$$

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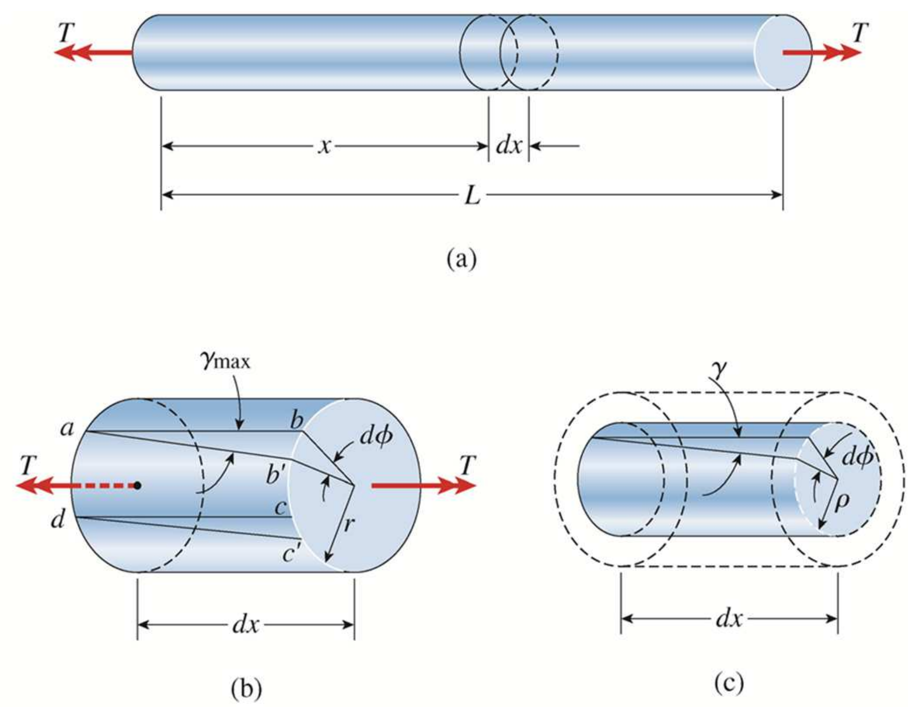
Outra forma de calcular o ângulo de giro:



$$\Phi r = \gamma_{máx} L$$

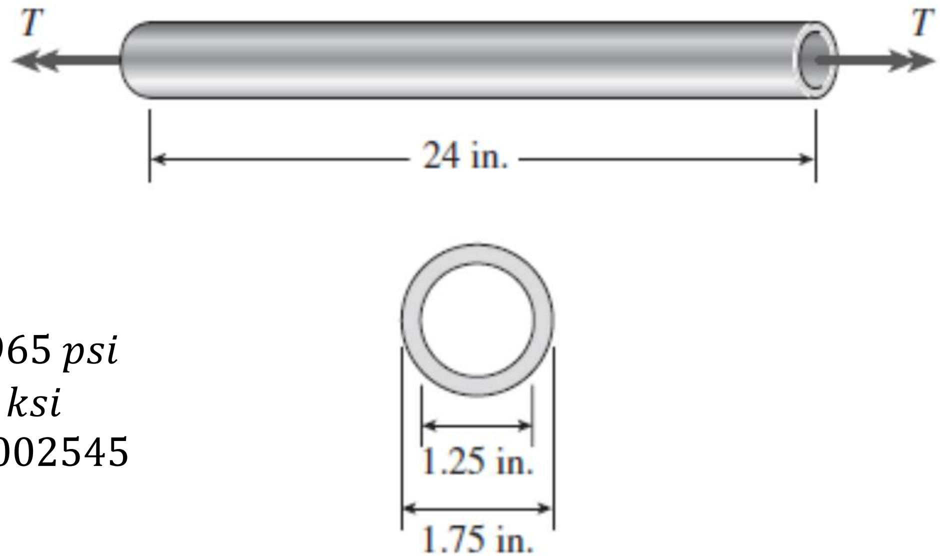
$$\Phi = \frac{\tau_{máx}}{G}\frac{L}{r}$$

$$\Phi = \frac{18{,}33}{11400} \times \frac{9}{0{,}25} rad$$

$$\Phi = 0{,}0579\, rad \cong 3{,}3°$$

Problem 3.3-7 A circular tube of aluminum is subjected to torsion by torques T applied at the ends (see figure). The bar is 24 in. long, and the inside and outside diameters are 1.25 in. and 1.75 in., respectively. It is determined by measurement that the angle of twist is 4° when the torque is 6200 lb-in.

Calculate the maximum shear stress τ_{max} in the tube, the shear modulus of elasticity G, and the maximum shear strain γ_{max} (in radians).



Answers:
$a)\ \tau_{máx} = 7965\ psi$
$b)\ G = 3129\ ksi$
$c)\ \gamma_{máx} \cong 0{,}002545$

<u>Solução:</u>

$$I_p = \frac{\pi}{32}\left(d_o^4 - d_i^4\right) = \frac{\pi}{32}\left((1{,}75)^4 - (1{,}25)^4\right) \cong 0{,}6811\ in^4$$

$$\tau_{máx} = \frac{T}{I_p}r_o = \frac{6200\ (lbf \times in) \times 0{,}875\ (in)}{0{,}6811(in^4)} = 7965\ psi$$

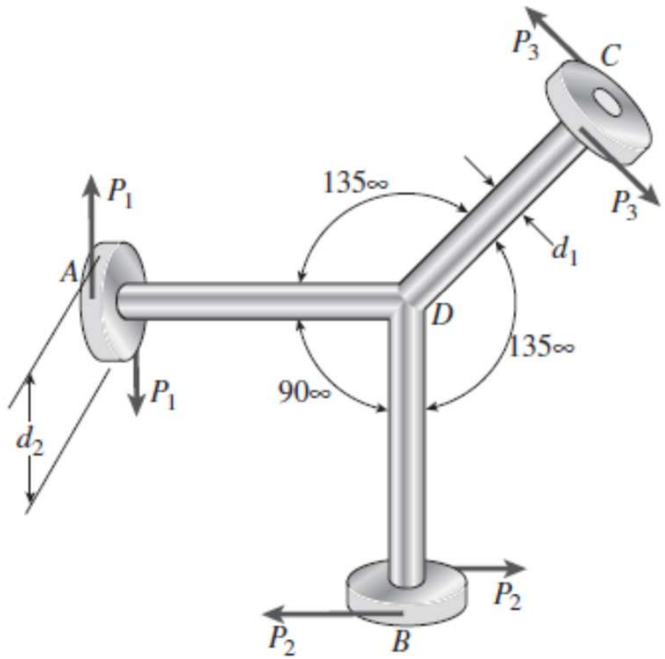
$$\Phi = \frac{TL}{GI_p} \quad \Leftrightarrow \quad G = \frac{TL}{I_p\Phi} = \frac{6200(lbf \times in) \times 24in}{0{,}6811\ in^4\ \times \left(\frac{4\pi}{180}\right) rad} \cong 3129\ ksi$$

$$\gamma_{máx} = \frac{\tau_{máx}}{G} = \frac{7965}{3129000} \cong 0{,}002545$$

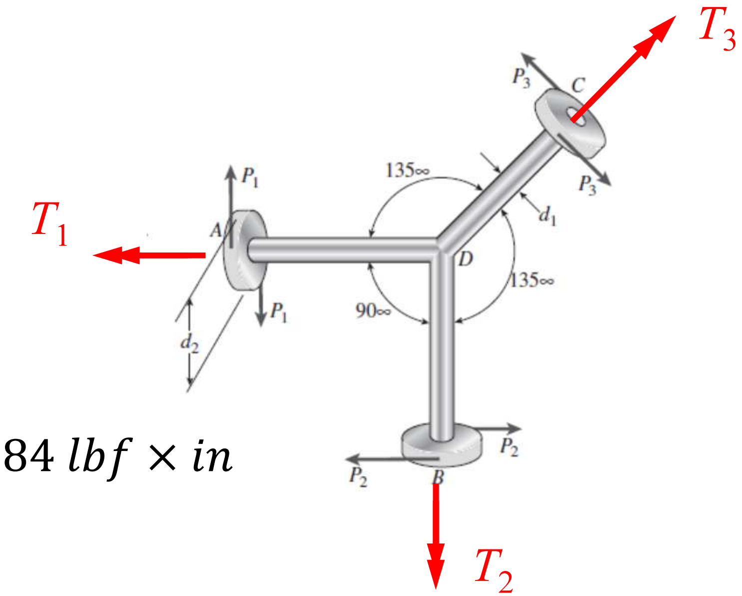
Problem 3.3-9 Three identical circular disks A, B, and C are welded to the ends of three identical solid circular bars (see figure). The bars lie in a common plane and the disks lie in planes perpendicular to the axes of the bars. The bars are welded at their intersection D to form a rigid connection. Each bar has diameter $d_1 = 0.5$ in. and each disk has diameter $d_2 = 3.0$ in.

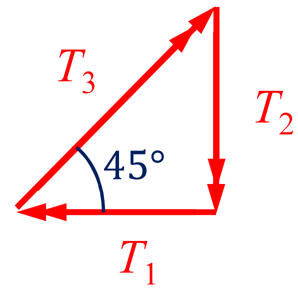
Forces P_1, P_2, and P_3 act on disks A, B, and C, respectively, thus subjecting the bars to torsion. If $P_1 = 28$ lb, what is the maximum shear stress τ_{max} in any of the three bars?

Answer:
$\tau_{máx} \cong 4840\, psi$



Solução:





$$T_1 = P_1 d_2 = 28 \times 3 = 84\ lbf \times in$$

Equilíbrio de momentos:

$$\begin{cases} T_3\, cos(45°) = T_1 \\ T_3\, sen(45°) = T_2 \end{cases} \Rightarrow \begin{cases} T_2 = T_1 = 84\ lbf \times in \\ T_3 = \sqrt{2}T_1 = 118{,}8\ lbf \times in \end{cases}$$

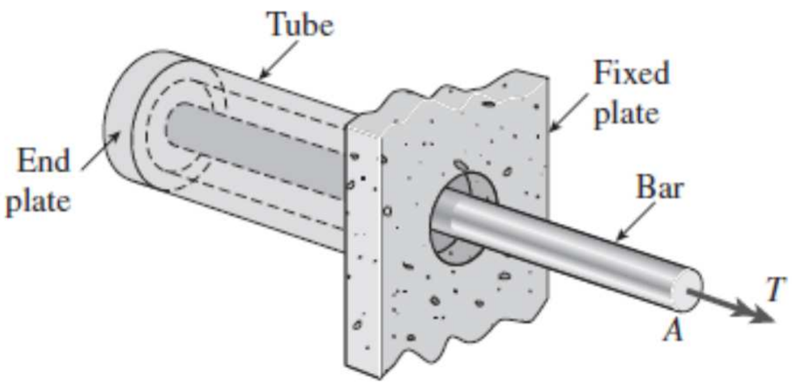
A máxima tensão de cisalhamento será, portanto, devida ao torque T_3 e vale:

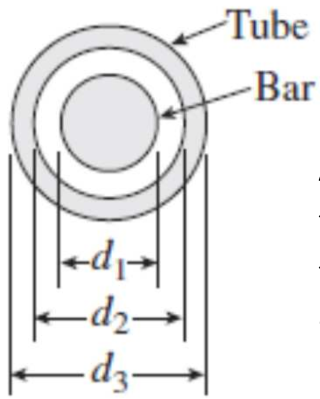
$$\tau_{máx} = \frac{T_3}{I_p} r = \frac{16T_3}{\pi d_1^3} = \frac{16 \times 118{,}8\ (lbf \times in)}{\pi (0{,}5\ in)^3} \cong 4840\ psi$$

Problem 3.4-2 A circular tube of outer diameter $d_3 = 70$ mm and inner diameter $d_2 = 60$ mm is welded at the right-hand end to a fixed plate and at the left-hand end to a rigid end plate (see figure). A solid circular bar of diameter $d_1 = 40$ mm is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the rigid end plate.

The bar is 1.0 m long and the tube is half as long as the bar. A torque $T = 1000$ N · m acts at end A of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity $G = 27$ GPa.

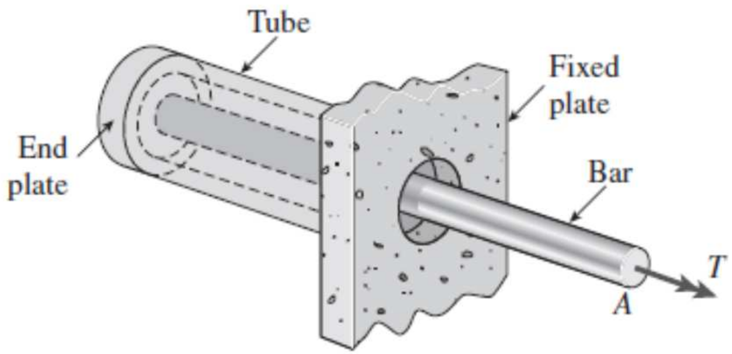
(a) Determine the maximum shear stresses in both the bar and tube.
(b) Determine the angle of twist (in degrees) at end A of the bar.

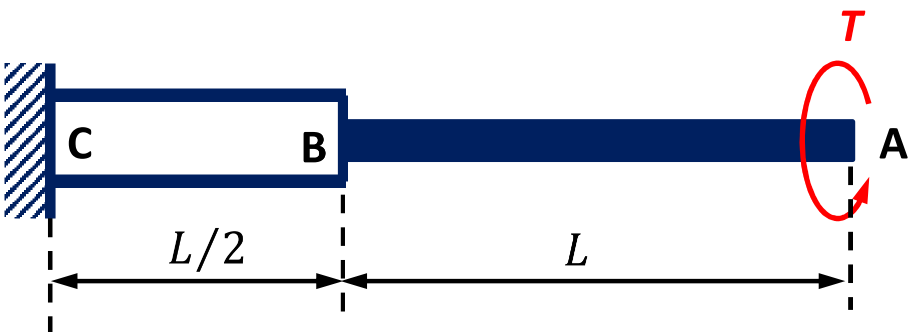




Answers:
$\tau_{máx} \cong 79{,}6\ MPa$ (in the bar)
$\tau_{máx} \cong 32{,}26\ MPa$ (in the tube)
$\Phi_{A,C} \cong 0{,}164\ rad \cong 9{,}4°$

Solução:





O momento de torção é o mesmo tanto na barra AB quanto no tubo BC. Assim:

Na barra AB:

$$\tau_{máx} = \frac{16T}{\pi d_1^3} =$$

$$= \frac{16 \times 10^6 Nmm}{\pi (40mm)^3} \cong 79{,}6\ MPa$$

No tubo BC:

$$\tau_{máx} = \frac{16Td_3}{\pi(d_3^4 - d_2^4)} = \frac{16 \times 10^6 Nmm \times 70mm}{\pi(70^4 - 60^4)^4 mm^4} \cong 32{,}26\, MPa$$

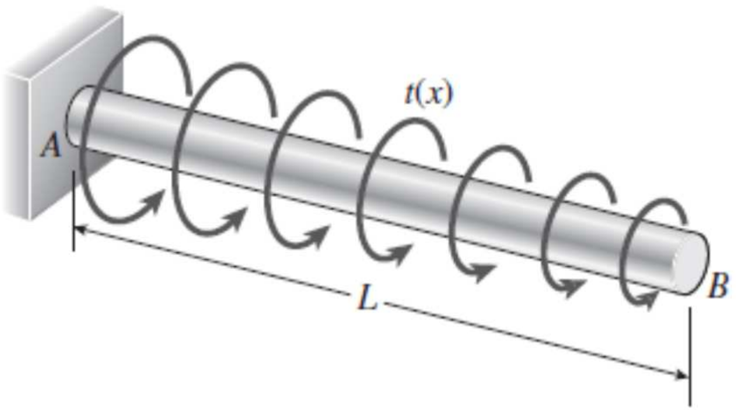
O ângulo de giro da seção A (relativamente à seção C, engastada) é:

$$\Phi_{A,C} = \Phi_{A,B} + \Phi_{B,C} = \frac{TL}{GI_{p,AB}} + \frac{T\,L/2}{GI_{p,BC}} = \frac{TL}{G}\left(\frac{32}{\pi d_1^4} + \frac{16}{\pi(d_3^4 - d_2^4)}\right)$$

$$\Phi_{A,C} \cong 0{,}147\, rad + 0{,}017\, rad = 0{,}164\, rad \cong 9{,}4°$$

Problem 3.4-17 A prismatic bar *AB* of solid circular cross section (diameter *d*) is loaded by a distributed torque (see figure). The intensity of the torque, that is, the torque per unit distance, is denoted $t(x)$ and varies linearly from a maximum value t_A at end *A* to zero at end *B*. Also, the length of the bar is *L* and the shear modulus of elasticity of the material is *G*.

(a) Determine the maximum shear stress τ_{max} in the bar.
(b) Determine the angle of twist ϕ between the ends of the bar.



Answers:

a) $\tau_{máx} = \dfrac{8 t_A L}{\pi d^3}$

b) $\Phi_{B,A} = \dfrac{16}{3\pi} \dfrac{t_A L^2}{G d^4}$

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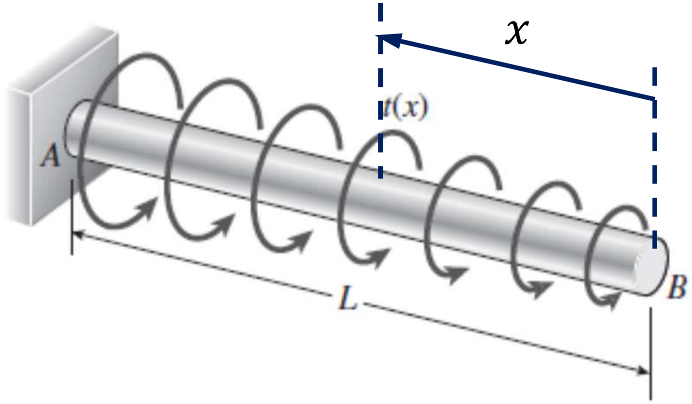
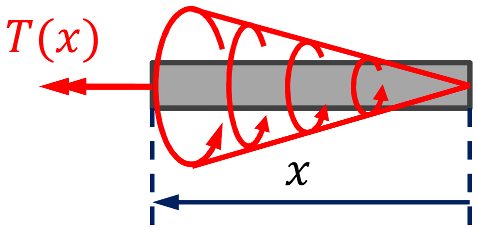
Solução:



$$t(x) = \frac{t_A x}{L} \qquad (0 \leq x \leq L)$$

Obs: $[t] = [t_A] = \frac{Nm}{m} = N$

Torque (interno) na seção definida pela variável x:



$$T(x) = \int_0^x t(x)dx = \int_0^x \frac{t_A x}{L} dx = \frac{t_A x^2}{2L}$$

$$(0 \leq x \leq L)$$

O maior momento de torção ocorrerá na seção A e vale:

$$T_{máx} = T_A = T(L) = \frac{t_A L^2}{2L} = \frac{t_A L}{2}$$

Obs: Note que a dimensão do torque interno está correta: $[T] = Nm$

A máxima tensão cisalhante em A será então:

$$\tau_{máx} = \frac{16T_{máx}}{\pi d^3} = \frac{16}{\pi d^3}\frac{t_A L}{2} = \frac{8t_A L}{\pi d^3}$$

O ângulo de giro da seção B com relação à seção A será:

$$\Phi_{B,A} = \int_0^L \frac{T(x)dx}{GI_p} = \frac{1}{GI_p}\int_0^L \frac{t_A x^2}{2L}dx = \frac{32}{G\pi d^4}\frac{t_A L^2}{6} = \frac{16}{3\pi}\frac{t_A L^2}{G d^4}$$

Obs: Note que o ângulo de giro está corretamente expresso em radianos.