



Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3210 - Mecânica dos Sólidos I

Aula #06

Prof. Dr. Roberto Ramos Jr.

14/04/2023



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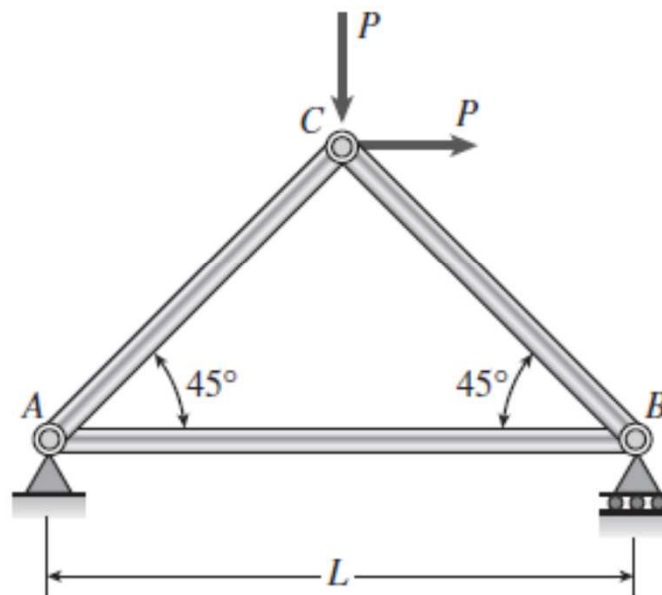
Problem 2.2-8 The three-bar truss ABC shown in the figure has a span $L = 3$ m and is constructed of steel pipes having cross-sectional area $A = 3900 \text{ mm}^2$ and modulus of elasticity $E = 200 \text{ GPa}$. Identical loads P act both vertically and horizontally at joint C , as shown.

- (a) If $P = 650 \text{ kN}$, what is the horizontal displacement of joint B ?
- (b) What is the maximum permissible load value $P_{\max}$ if the displacement of joint B is limited to 1.5 mm ?

Answers:

a) $\delta_B = 2,5 \text{ mm}$

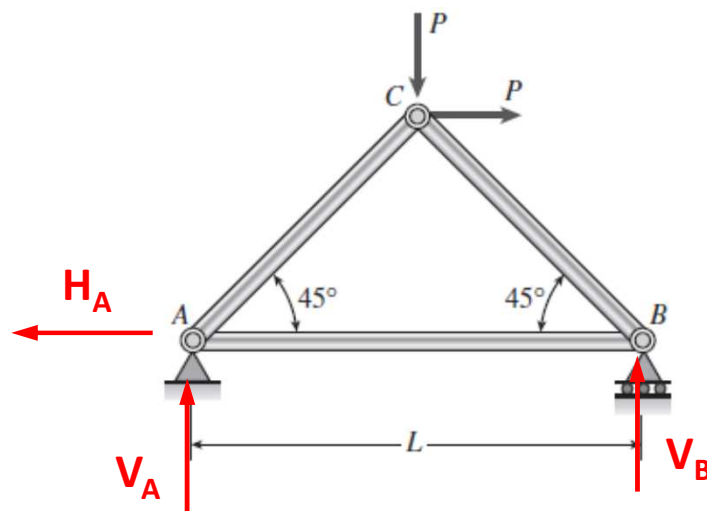
b) $P_{\max} = 390 \text{ kN}$





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Solução:



$$\sum F_x = 0 \Leftrightarrow H_A = P$$

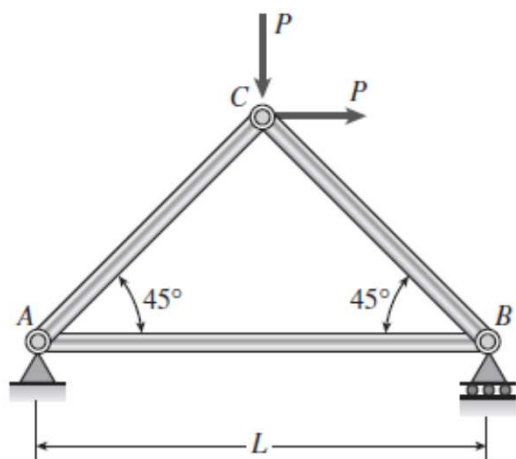
$$\sum M_{\text{pólo } A} = 0 \Leftrightarrow V_B L = P \frac{L}{2} + P \frac{L}{2} \Leftrightarrow V_B = P$$

$$\sum F_y = 0 \Leftrightarrow V_A + V_B = P \Leftrightarrow V_A = 0$$

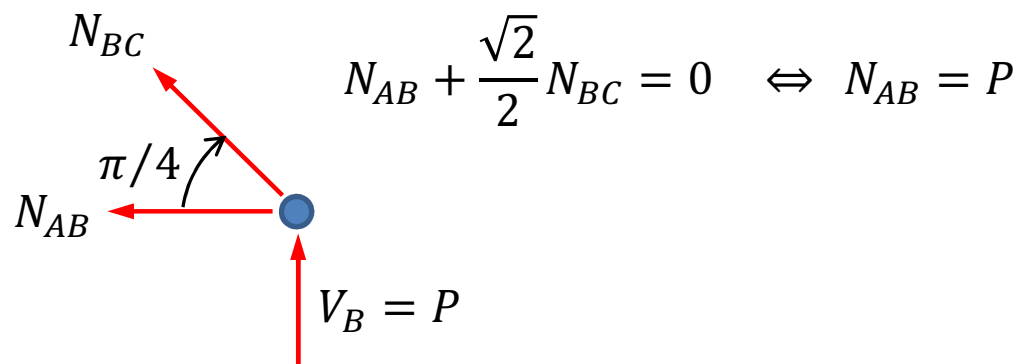


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Equilíbrio do nó B:



$$\frac{\sqrt{2}}{2} N_{BC} + P = 0 \Leftrightarrow N_{BC} = -\sqrt{2}P$$



$$N_{AB} + \frac{\sqrt{2}}{2} N_{BC} = 0 \Leftrightarrow N_{AB} = P$$

Logo:
$$\delta_B = \frac{N_{AB}L}{EA} = \frac{PL}{EA} = \frac{(6,5 \times 10^5)(3000)}{(2 \times 10^5)(3900)} = 2,5 \text{ mm}$$

b) Se o deslocamento horizontal de B está limitado a 1,5 mm, então a máxima força que pode ser aplicada é:

$$P_{\text{máx}} = 1,5\text{mm} \frac{650 \text{ kN}}{2,5\text{mm}} = 390 \text{ kN}$$



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Problem 2.3-12 A prismatic bar AB of length L , cross-sectional area A , modulus of elasticity E , and weight W hangs vertically under its own weight (see figure).

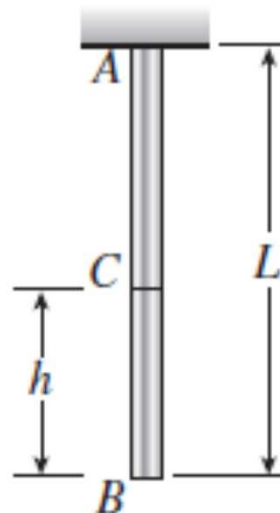
- (a) Derive a formula for the downward displacement δ_C of point C , located at distance h from the lower end of the bar.
- (b) What is the elongation δ_B of the entire bar?
- (c) What is the ratio β of the elongation of the upper half of the bar to the elongation of the lower half of the bar?

Answers:

$$a) \delta_C = \frac{W}{LEA} \frac{(L^2 - h^2)}{2}$$

$$b) \delta_B = \frac{WL}{2EA}$$

$$c) \beta = 3$$

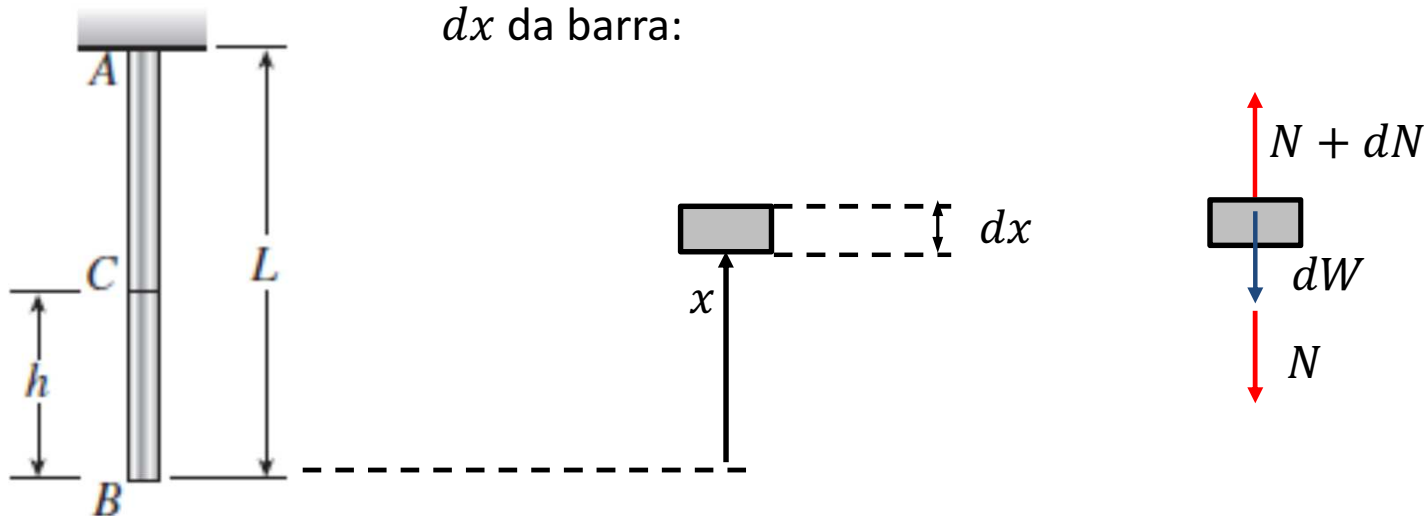




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Solução:

Equilíbrio de um segmento infinitesimal de comprimento dx da barra:



$$N + dN = N + dW \Leftrightarrow dN = dW = \frac{W}{L} dx$$

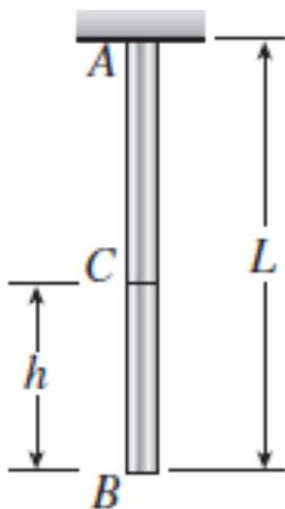
Logo:

$$N(x) = N(0) + \int_0^x \left(\frac{W}{L} \right) dx = \frac{W}{L} x$$



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a) O deslocamento vertical do ponto C corresponde ao alongamento do trecho AC da barra. Assim:



$$\delta_C = \int_h^L \frac{N(x)}{EA} dx = \int_h^L \frac{Wx}{LEA} dx = \frac{W}{LEA} \frac{x^2}{2} \Big|_h^L$$

$$\delta_C = \frac{W}{LEA} \frac{(L^2 - h^2)}{2}$$

Obs:

- 1) Verifique sempre se a equação obtida é dimensionalmente correta!
- 2) Verifique se a equação atende a requisitos óbvios (ex: qual deve ser o deslocamento esperado no ponto A?)

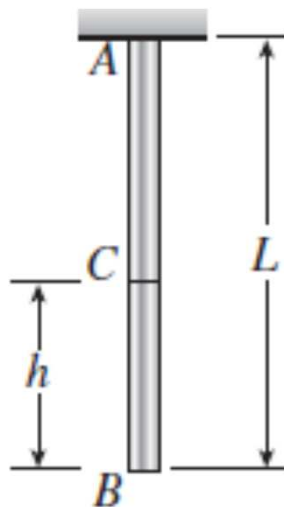


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b) O alongamento total da barra corresponde ao deslocamento vertical do ponto B que pode ser calculado tomando $h = 0$. Logo:

$$\delta_B = \frac{WL}{2EA}$$

c) Pelo enunciado a razão β é dada por: $\beta = \frac{\Delta L_{AC}}{\Delta L_{CB}}$ (...para $h = L/2$)



$$\Delta L_{AC} = \frac{W}{LEA} \frac{(L^2 - (L/2)^2)}{2} = \frac{3WL}{8EA}$$

$$\Delta L_{CB} = \delta_B - \Delta L_{AC} = \frac{WL}{2EA} - \frac{3WL}{8EA} = \frac{1WL}{8EA}$$

Logo: $\beta = 3$



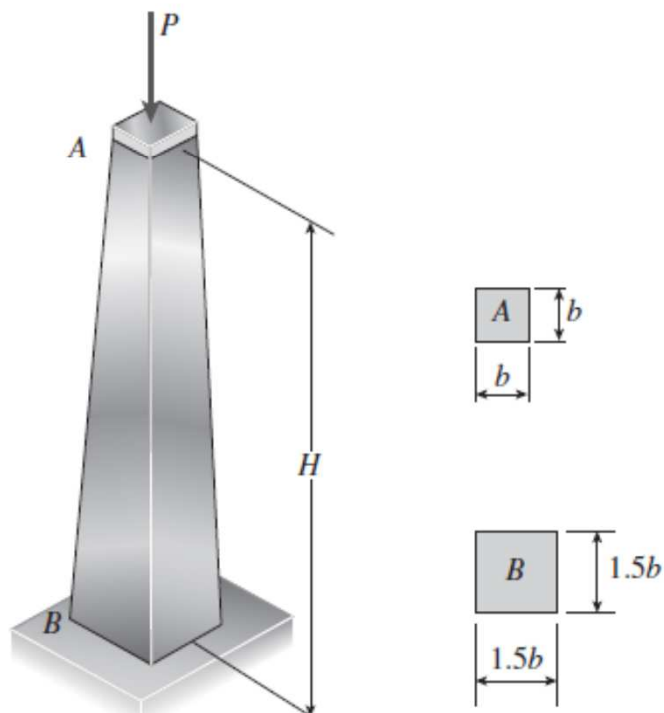
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Problem 2.3-14 A post AB supporting equipment in a laboratory is tapered uniformly throughout its height H (see figure). The cross sections of the post are square, with dimensions $b \times b$ at the top and $1.5b \times 1.5b$ at the base.

Derive a formula for the shortening δ of the post due to the compressive load P acting at the top. (Assume that the angle of taper is small and disregard the weight of the post itself.)

Answer:

$$\delta = -\frac{2PH}{3Eb^2}$$





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Solução:

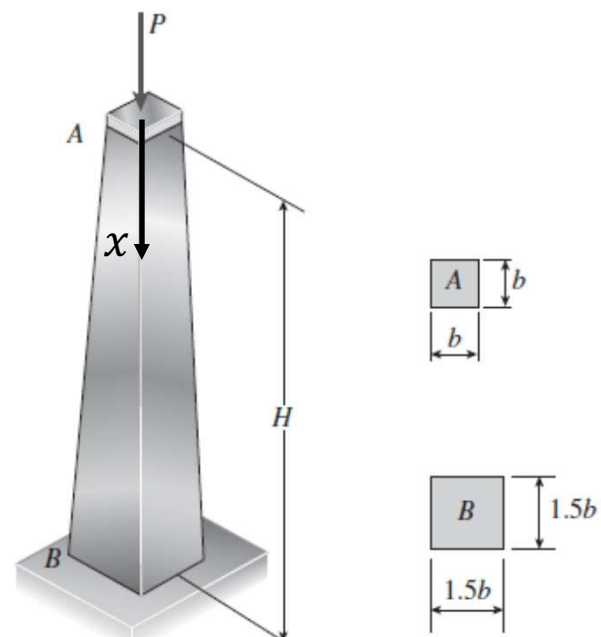
Temos um problema de compressão não uniforme (por conta da variação das seções transversais). O encurtamento total da barra é dado por:

$$\delta = \int_0^H \frac{N(x)}{EA(x)} dx$$

Onde:

$$N(x) = -P$$

$$A(x) = (b(x))^2 = \left(b \left(1 + \frac{x}{2H} \right) \right)^2$$





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Chamando: $\eta = 1 + \frac{x}{2H}$ Teremos: $d\eta = \frac{dx}{2H} \Leftrightarrow dx = 2Hd\eta$

Extremos de integração:
$$\left[\begin{array}{l} x = 0 \Rightarrow \eta = 1 \\ x = H \Rightarrow \eta = 1,5 \end{array} \right.$$

Logo:
$$\delta = - \int_0^H \frac{P}{E \left(b \left(1 + \frac{x}{2H} \right) \right)^2} dx = - \int_1^{3/2} \frac{2HP}{Eb^2\eta^2} d\eta = \frac{2PH}{Eb^2} \left(\frac{1}{\eta} \right) \Big|_1^{3/2}$$

$$\delta = \frac{2PH}{Eb^2} \left(\frac{2}{3} - 1 \right) = -\frac{2PH}{3Eb^2}$$



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Problem 2.4-7 The axially loaded bar $ABCD$ shown in the figure is held between rigid supports. The bar has cross-sectional area A_1 from A to C and $2A_1$ from C to D .

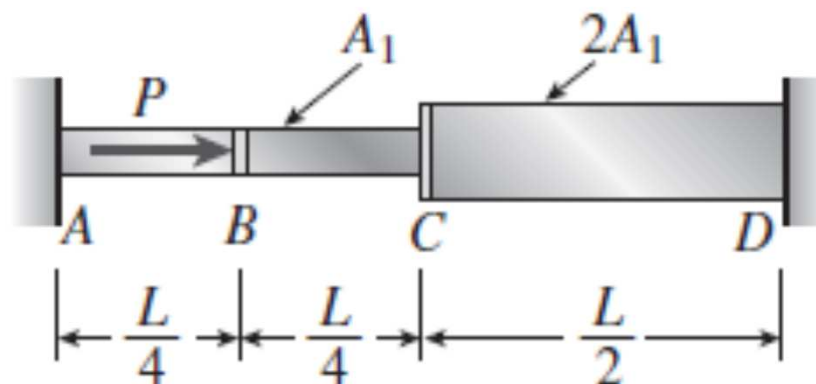
- (a) Derive formulas for the reactions R_A and R_D at the ends of the bar.
- (b) Determine the displacements δ_B and δ_C at points B and C , respectively.
- (c) Draw a diagram in which the abscissa is the distance from the left-hand support to any point in the bar and the ordinate is the horizontal displacement δ at that point.

Answers:

a) $R_A = \frac{2P}{3} \quad (\leftarrow)$
 $R_D = \frac{P}{3} \quad (\leftarrow)$

b) $\delta_B = \frac{1}{6} \frac{PL}{EA_1} \quad (\rightarrow)$

$\delta_C = \frac{1}{12} \frac{PL}{EA_1} \quad (\rightarrow)$

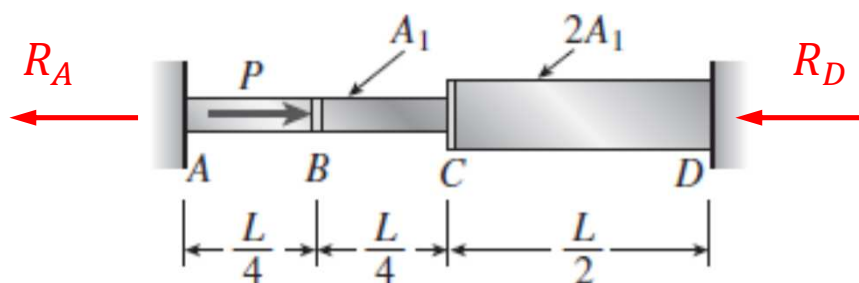




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Solução:

1. D.C.L. e classificação da estrutura (isostática ou hiperestática)



Equilíbrio de forças na direção horizontal: $R_A + R_D = P$

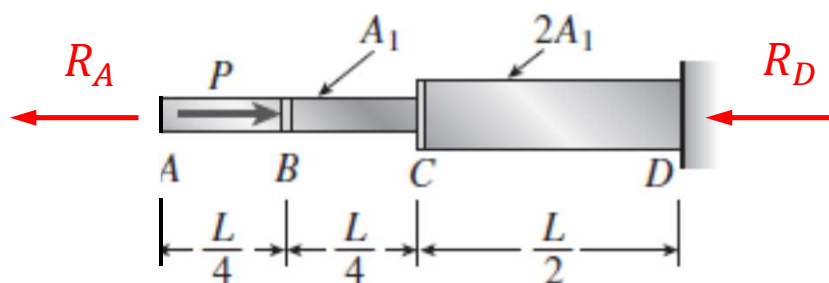
Grau de hiperestaticidade estrutural:

$$g = 2 - 1 = 1$$



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2. Escolha da E.I.F. (e consequentemente da incógnita hiperestática):

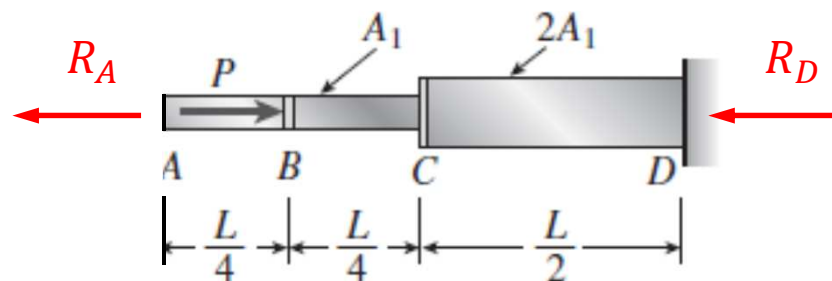


3. Determinação do deslocamento do ponto A (que equivale ao cálculo do alongamento de toda a barra):

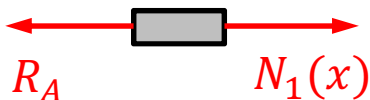
$$\delta_A = \delta = \sum_{i=1}^3 \delta_i = \sum_{i=1}^3 \frac{N_i L_i}{E A_i}$$

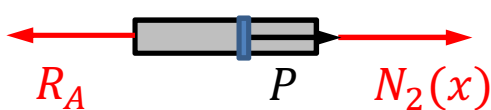


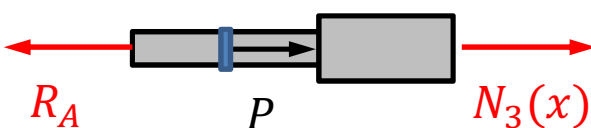
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Esforços internos em cada segmento:

Segmento 1 (AB):  $N_1(x) = R_A \quad \left(0 \leq x < \frac{L}{4}\right)$

Segmento 2 (BC):  $N_2(x) = R_A - P \quad \left(\frac{L}{4} < x \leq \frac{L}{2}\right)$

Segmento 3 (CD):  $N_3(x) = R_A - P \quad \left(\frac{L}{2} < x \leq L\right)$



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$$\delta_A = \sum_{i=1}^3 \frac{N_i L_i}{EA_i} = \frac{R_A(L/4)}{EA_1} + \frac{(R_A - P)(L/4)}{EA_1} + \frac{(R_A - P)(L/2)}{E(2A_1)}$$

$$\delta_A = \frac{R_A L}{EA_1} \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) - \frac{PL}{EA_1} \left(\frac{1}{4} + \frac{1}{4} \right) = \frac{3 R_A L}{4 EA_1} - \frac{1 PL}{2 EA_1}$$

4. Compatibilização de deslocamentos:

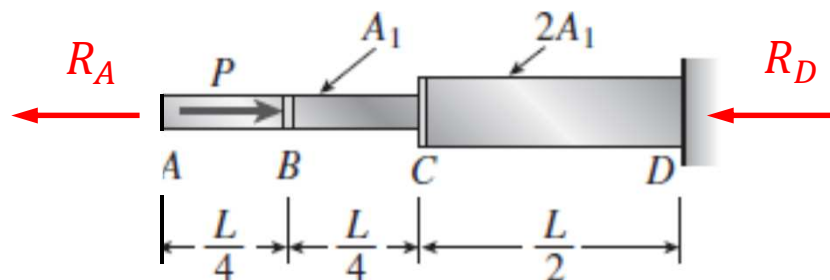
$$\delta_A = \frac{3 R_A L}{4 EA_1} - \frac{1 PL}{2 EA_1} = 0 \quad \longleftrightarrow \quad R_A = \frac{2}{3} P$$

Como: $R_A + R_D = P$ Teremos: $R_D = \frac{1}{3} P$



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b) Deslocamentos em B e em C:



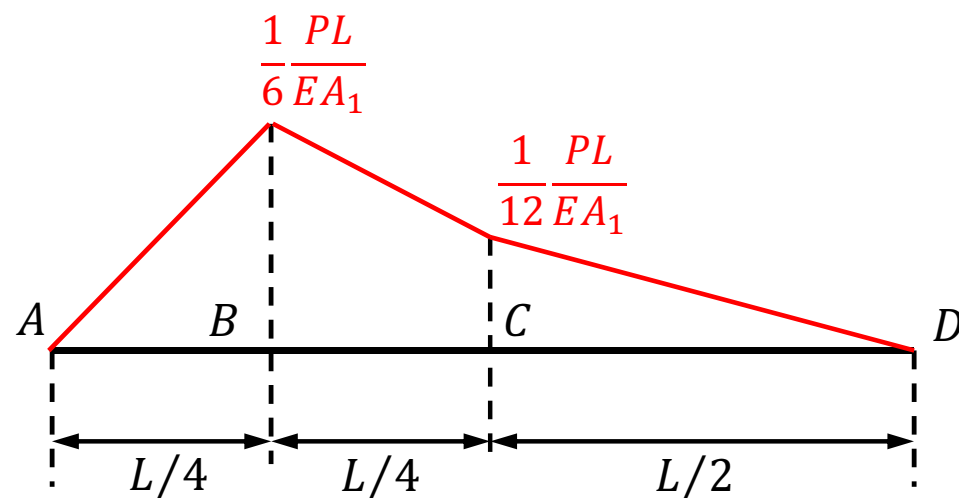
$$\delta_B = \delta_1 = \frac{R_A(L/4)}{EA_1} = \frac{2P}{3} \left(\frac{L}{4EA_1} \right) = \frac{1}{6} \frac{PL}{EA_1}$$

$$\delta_C = \delta_1 + \delta_2 = \frac{R_A(L/4)}{EA_1} + \frac{(R_A - P)(L/4)}{EA_1} = \frac{1}{6} \frac{PL}{EA_1} - \frac{1}{12} \frac{PL}{EA_1} = \frac{1}{12} \frac{PL}{EA_1}$$



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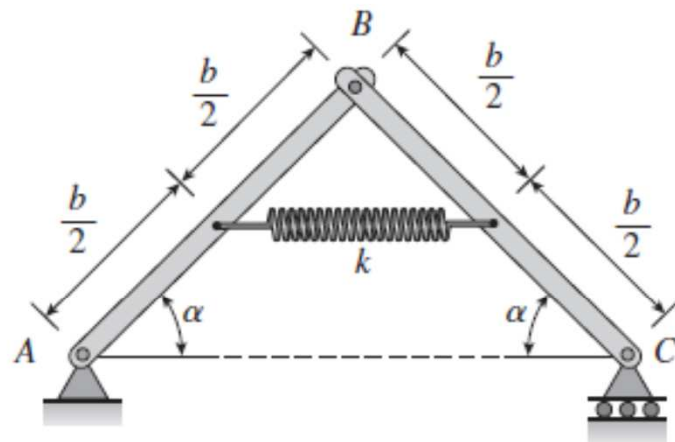
c) Diagrama de deslocamentos:





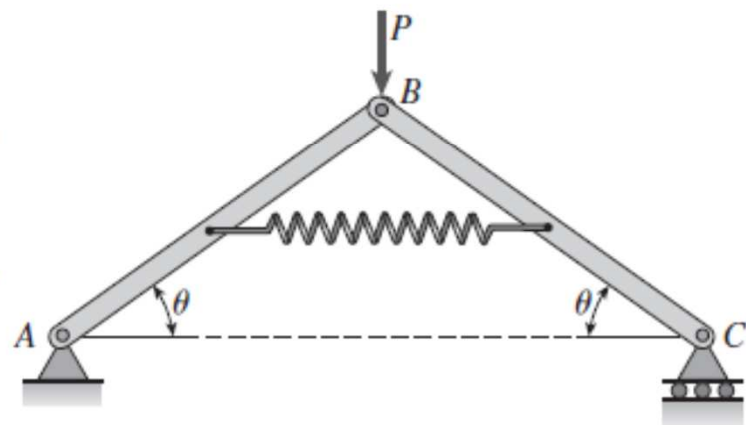
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Problem 2.2-13 A framework ABC consists of two rigid bars AB and BC , each having length b (see the first part of the figure). The bars have pin connections at A , B , and C and are joined by a spring of stiffness k . The spring is attached at the midpoints of the bars. The framework has pin support at A and a roller support at C , and the bars are at an angle α to the horizontal.



When a vertical load P is applied at joint B (see the second part of the figure) the roller support C moves to the right, the spring is stretched, and the angle of the bars decreases from α to the angle θ .

Determine the angle θ and the increase δ in the distance between points A and C . (Use the following data; $b = 8.0$ in., $k = 16$ lb/in., $\alpha = 45^\circ$, and $P = 10$ lb.)





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Problem 2.4-12 A rigid bar of weight $W = 800$ N hangs from three equally spaced vertical wires (length $L = 150$ mm, spacing $a = 50$ mm): two of steel and one of aluminum. The wires also support a load P acting on the bar. The diameter of the steel wires is $d_s = 2$ mm, and the diameter of the aluminum wire is $d_a = 4$ mm. Assume $E_s = 210$ GPa and $E_a = 70$ GPa.

- (a) What load P_{allow} can be supported *at the midpoint of the bar* ($x = a$) if the allowable stress in the steel wires is 220 MPa and in the aluminum wire is 80 MPa? (See figure part a.)
- (b) What is P_{allow} if the load is positioned at $x = a/2$? (See figure part a.)
- (c) Repeat (b) above if the second and third wires are *switched* as shown in figure part b.

