Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3210 - Mecânica dos Sólidos I

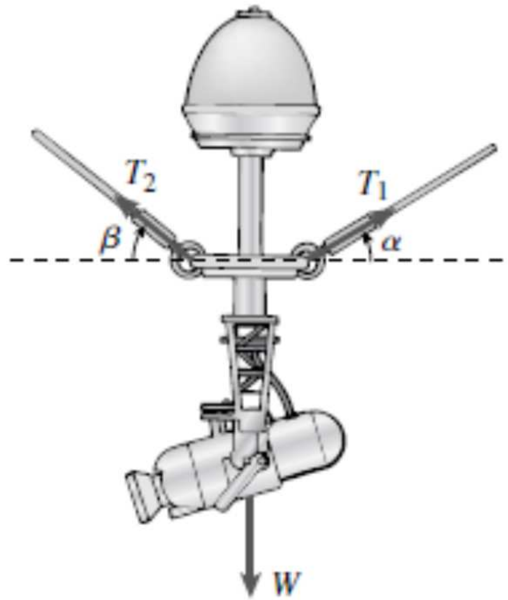
Aula #04

Prof. Dr. Roberto Ramos Jr.

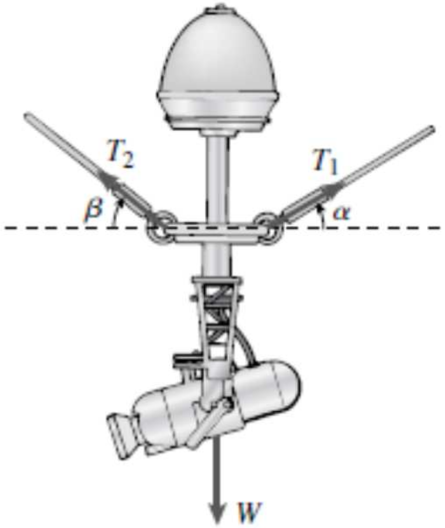
31/03/2023

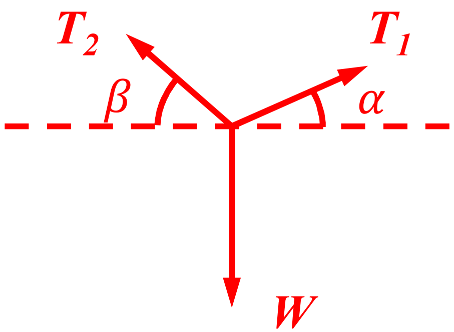
Problem 1.2-7 Two steel wires support a moveable overhead camera weighing W = 25 lb (see figure) used for close-up viewing of field action at sporting events. At some instant, wire 1 is at on angle $\alpha = 20°$ to the horizontal and wire 2 is at an angle $\beta = 48°$. Both wires have a diameter of 30 mils. (Wire diameters are often expressed in mils; one mil equals 0.001 in.)

Determine the tensile stresses σ_1 and σ_2 in the two wires.



Solução:





$$T_1 cos\alpha - T_2 cos\beta = 0$$

$$T_1 sen\alpha + T_2 sen\beta = W$$

$$\begin{bmatrix} cos\alpha & -cos\beta \\ sen\alpha & sen\beta \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ W \end{Bmatrix}$$

$$T_1 = \frac{\begin{Vmatrix} 0 & -cos\beta \\ W & sen\beta \end{Vmatrix}}{\begin{Vmatrix} cos\alpha & -cos\beta \\ sen\alpha & sen\beta \end{Vmatrix}} = \frac{W cos\beta}{sen\beta cos\alpha + sen\alpha cos\beta} = \frac{W cos\beta}{sen(\alpha + \beta)}$$

$$T_2 = \frac{\begin{Vmatrix} cos\alpha & 0 \\ sen\alpha & W \end{Vmatrix}}{\begin{Vmatrix} cos\alpha & -cos\beta \\ sen\alpha & sen\beta \end{Vmatrix}} = \frac{W cos\alpha}{sen\beta cos\alpha + sen\alpha cos\beta} = \frac{W cos\alpha}{sen(\alpha + \beta)}$$

Logo: $$T_1 = \frac{W cos\beta}{sen(\alpha + \beta)} = \frac{25\cos(48°)}{sen(68°)} lbf \cong 18{,}04\ lbf$$

$$T_2 = \frac{W cos\alpha}{sen(\alpha + \beta)} = \frac{25\cos(20°)}{sen(68°)} lbf \cong 25{,}34\ lbf$$

Tensão no fio 1:

$$\sigma_1 = \frac{T_1}{A_1} = \frac{4T_1}{\pi d^2} = \frac{4 \times 18{,}04}{\pi(0{,}030)^2} psi = 25521\ psi \cong 25{,}52\ ksi \qquad (\cong 176 MPa)$$

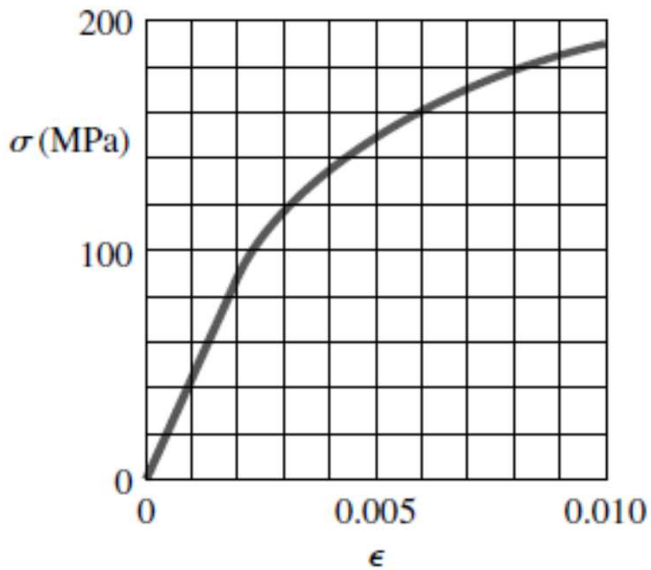
Tensão no fio 2:

$$\sigma_2 = \frac{T_2}{A_2} = \frac{4T_2}{\pi d^2} = \frac{4 \times 25{,}34}{\pi(0{,}030)^2} psi = 35849\ psi \cong 35{,}85\ ksi \qquad (\cong 247 MPa)$$

Obs: $1\ ksi \cong 6{,}895\ MPa$

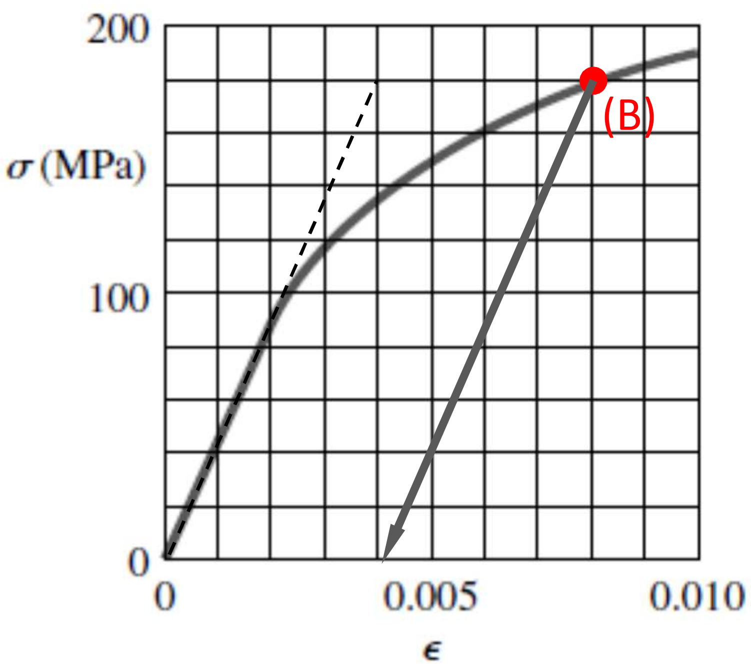
Problem 1.4-4 A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint*: Use the concepts illustrated in Figs. 1-18b and 1-19.)



Solução: $\varepsilon_B = \dfrac{\Delta L}{L_0} = \dfrac{6mm}{750mm} = 0{,}008$ $\Rightarrow$ $\sigma_B = 180\ MPa$

$$E \cong \frac{180\ MPa}{0{,}004} = 45\ GPa$$ (vide linha tracejada)



$$\varepsilon_{res} = \varepsilon_B - \frac{\sigma_B}{E} \cong 0{,}008 - \frac{180}{45000} = 0{,}004$$

Alongamento final (permanente):

$$\Delta L_{res} = \varepsilon_{res} L_0 \cong 0{,}004 \times 750mm = 3\ mm$$

Novo limite de proporcionalidade (após a barra ser recarregada):

$$\sigma_p' = \sigma_B = 180\ MPa$$

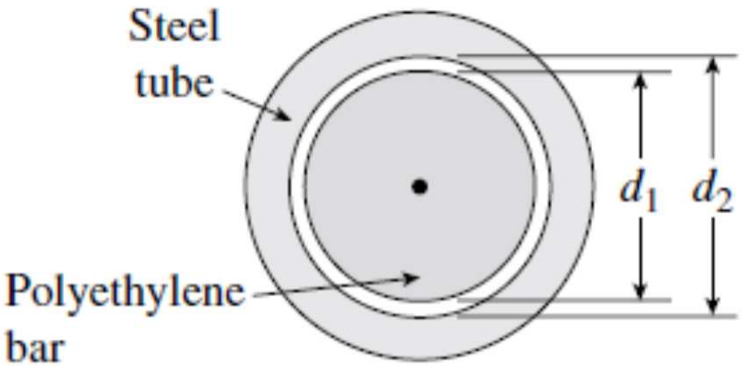
Problem 1.5-3 A polyethylene bar having diameter $d_1 = 4.0$ in. is placed inside a steel tube having inner diameter $d_2 = 4.01$ in. (see figure). The polyethylene bar is then compressed by an axial force P.

At what value of the force P will the space between the nylon bar and the steel tube be closed? (For nylon, assume $E = 400$ ksi and $\nu = 0.4$.)

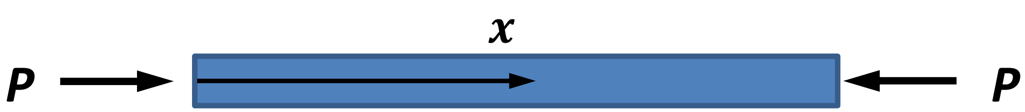
PE

200 ksi

PE



Solução:



Tensão longitudinal na barra: $\sigma_x = -\dfrac{P}{A} = -\dfrac{4P}{\pi d^2}$

Alongamento longitudinal da barra: $\varepsilon_x = \dfrac{\sigma_x}{E} = -\dfrac{4P}{\pi d^2 E}$

Alongamentos transversais da barra: $\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = \dfrac{4\nu P}{\pi d^2 E}$

Também:

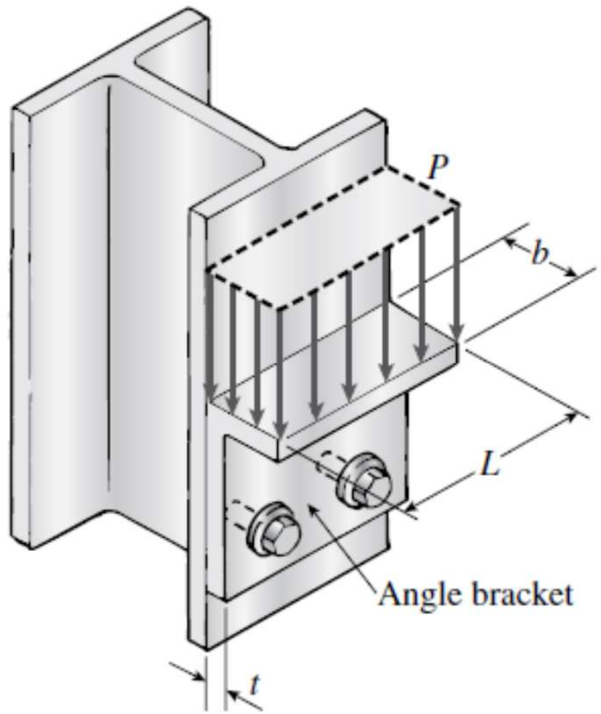
$$\varepsilon_y = \varepsilon_z = \frac{\Delta d}{d_0} = \frac{d_2 - d_1}{d_1} = \frac{0{,}01\ in}{4{,}0\ in} = 0{,}0025$$

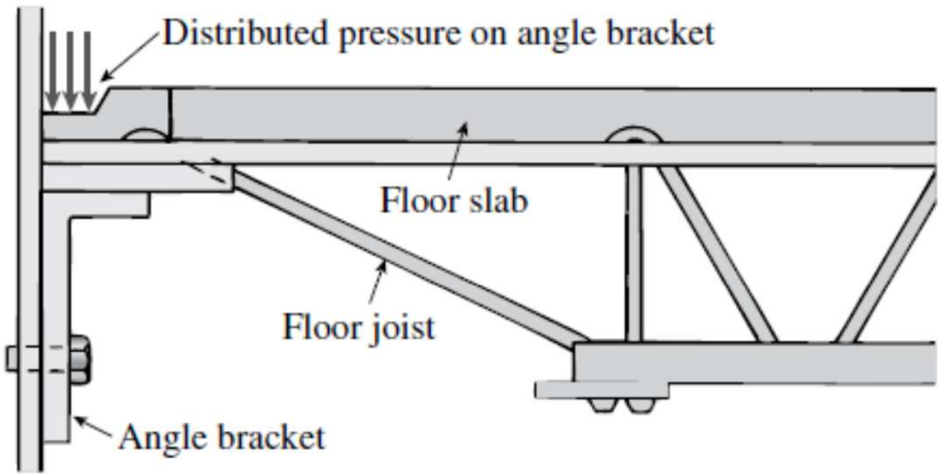
$$\frac{4\nu P}{\pi d^2 E} = \varepsilon_y \quad \Leftrightarrow \quad P = \frac{\pi d^2 E \varepsilon_y}{4\nu} = \frac{\pi (4)^2 \times 2 \times 10^5 \times 0{,}0025}{4 \times 0{,}4} = 15708\ lbf$$

$$P = 15{,}71\ kips$$

Problem 1.6-1 An angle bracket having thickness $t = 0.75$ in. is attached to the flange of a column by two 5/8-inch diameter bolts (see figure). A uniformly distributed load from a floor joist acts on the top face of the bracket with a pressure $p = 275$ psi. The top face of the bracket has length $L = 8$ in. and width $b = 3.0$ in.

Determine the average bearing pressure σ_b between the angle bracket and the bolts and the average shear stress τ_{aver} in the bolts. (Disregard friction between the bracket and the column.)





Solução: A força total aplicada sobre o topo do perfil cantoneira é:

$$F = p(bL) = 275 \times (3 \times 8) = 6600\ lbf$$

Considerando que esta força seja dividida igualmente entre os dois parafusos (hipótese), teremos:

Tensão de esmagamento entre o perfil e os parafusos (*average bearing pressure*):

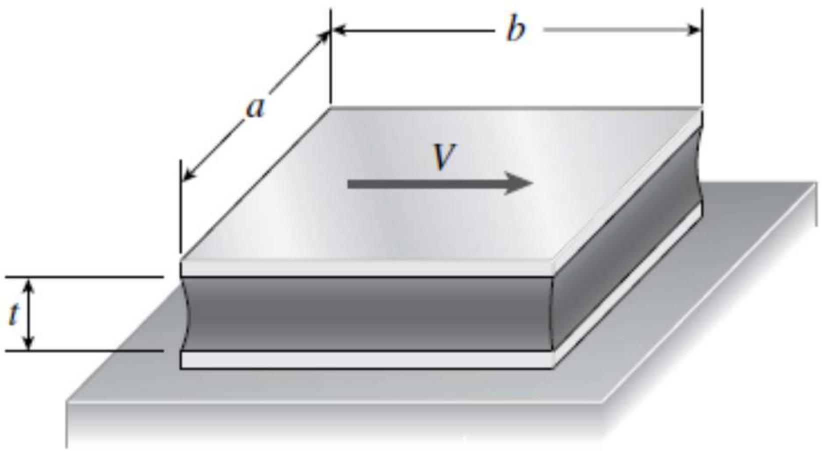
$$\sigma_b = \frac{F}{2(td)} = \frac{6600\ lbf}{2(0{,}75 \times 0{,}625)in^2} = 7040\ psi = 7{,}04\ ksi$$

Tensão de cisalhamento média nos parafusos (*average shear stress*):

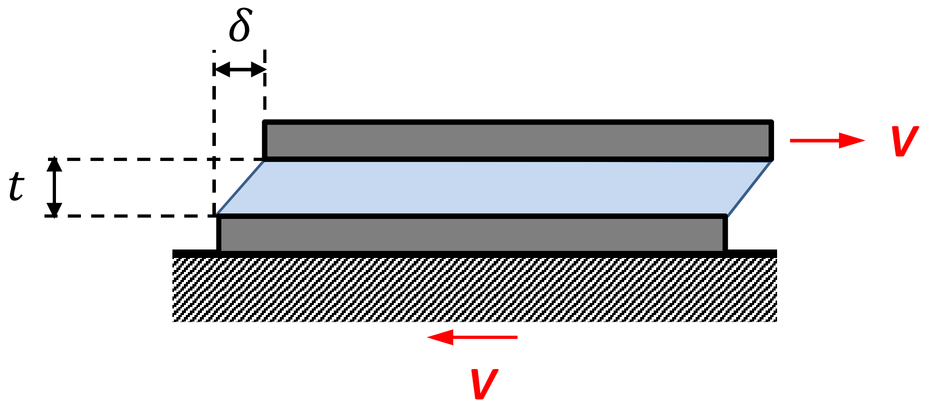
$$\bar{\tau} = \frac{F}{2(\pi d^2/4)} = \frac{2 \times 6600\ lbf}{\pi(0{,}625)^2 in^2} = 10756\ psi \cong 10{,}76\ ksi$$

Problem 1.6-8 An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force V during a static loading test (see figure). The pad has dimensions $a = 125$ mm and $b = 240$ mm, and the elastomer has thickness $t = 50$ mm. When the force V equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity G of the chloroprene?



Solução:



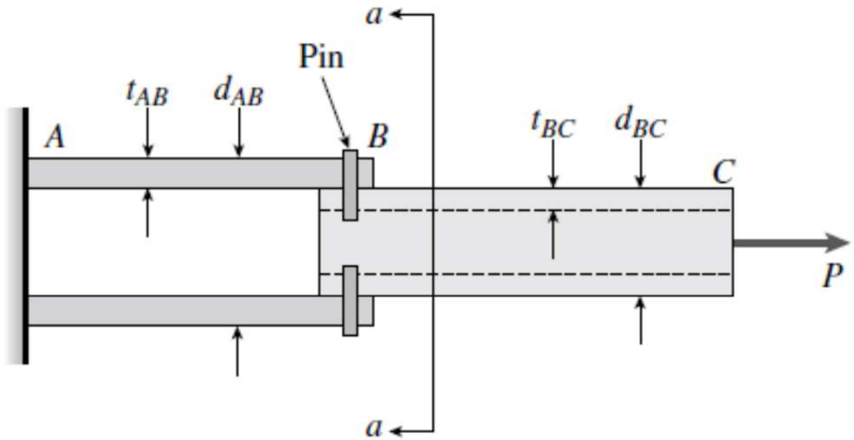
$$\gamma = arctan\left(\frac{\delta}{t}\right) = arctan\left(\frac{8}{50}\right) = 0{,}158655\, rad$$

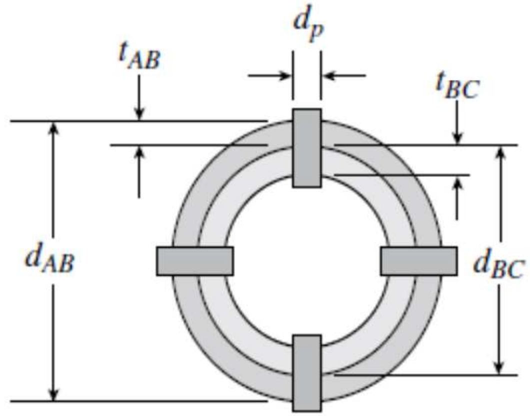
$$\tau = \frac{V}{A} = \frac{V}{ab} = \frac{12000\ N}{(125 \times 240)mm^2} = 0{,}4MPa$$

$$\tau = G\gamma \iff G = \frac{\tau}{\gamma} = \frac{0{,}4}{0{,}158655} MPa = 2{,}52\ MPa$$

Problem 1.7-4 Two steel tubes are joined at B by four pins (d_p = 11 mm), as shown in the cross section a–a in the figure. The outer diameters of the tubes are d_{AB} = 40 mm and d_{BC} = 28 mm. The wall thicknesses are t_{AB} = 6 mm and t_{BC} = 7 mm. The yield stress in tension for the steel is σ_Y = 200 MPa and the ultimate stress in *tension* is σ_U = 340 MPa. The corresponding yield and ultimate values in *shear* for the pin are 80 MPa and 140 MPa, respectively. Finally, the yield and ultimate values in *bearing* between the pins and the tubes are 260 MPa and 450 MPa, respectively. Assume that the factors of safety with respect to yield stress and ultimate stress are 4 and 5, respectively.

(a) Calculate the allowable tensile force P_{allow} considering tension in the tubes.
(b Recompute P_{allow} for shear in the pins.
(c) Finally, recompute P_{allow} for bearing between the pins and the tubes. Which is the controlling value of P?





Solução:

a) Considerando o material dos tubos, temos pelos dados do problema:

$$n_Y = \frac{\sigma_Y}{\sigma_{ad}} = 4 \iff \sigma_{ad} = \frac{\sigma_Y}{4} = \frac{200}{4} = 50\ MPa$$

$$n_U = \frac{\sigma_U}{\sigma_{ad}} = 5 \iff \sigma_{ad} = \frac{\sigma_U}{5} = \frac{340}{5} = 68\ MPa$$

$$\Rightarrow \sigma_{ad} = 50\ MPa$$

Assim, a força de tração admissível considerando a tração nos tubos é tal que:

$$\sigma_{AB} = \frac{P}{\pi(20^2 - 14^2) - 4(11 \times 6)} = \frac{P}{377\ mm^2} \leq 50\ MPa \iff P \leq 18{,}84\ kN$$

$$\sigma_{BC} = \frac{P}{\pi(14^2 - 7^2) - 4(11 \times 7)} = \frac{P}{153{,}8\ mm^2} \leq 50\ MPa \iff P \leq 7{,}69\ kN$$

$$P_{ad} = 7{,}69\ kN$$

b) Considerando o cisalhamento dos pinos:

$$\left.\begin{aligned} n_Y &= \frac{\tau_Y}{\tau_{ad}} = 4 \iff \tau_{ad} = \frac{\tau_Y}{4} = \frac{80}{4} = 20\ MPa \\ n_U &= \frac{\tau_U}{\tau_{ad}} = 5 \iff \tau_{ad} = \frac{\tau_U}{5} = \frac{140}{5} = 28\ MPa \end{aligned}\right] \Rightarrow \tau_{ad} = 20\ MPa$$

Assim, a força admissível considerando o cisalhamento dos pinos é tal que:

$$\bar{\tau} = \frac{P}{4(\pi(11)^2/4)} = \frac{P}{380\ mm^2} \leq 20\ MPa \iff P \leq 7{,}60\ kN$$

$$P_{ad} = 7{,}60\ kN$$

c) Considerando o esmagamento da parede entre tubos e pinos:

$$\left.\begin{array}{l} n_Y = \dfrac{\sigma_Y}{\sigma_{ad}} = 4 \;\Leftrightarrow\; \sigma_{ad} = \dfrac{\sigma_Y}{4} = \dfrac{260}{4} = 65\ MPa \\ n_U = \dfrac{\sigma_U}{\sigma_{ad}} = 5 \;\Leftrightarrow\; \sigma_{ad} = \dfrac{\sigma_U}{5} = \dfrac{450}{5} = 90\ MPa \end{array}\right] \Rightarrow \tau_{ad} = 65\ MPa$$

Assim, a força admissível considerando o esmagamento da parede é tal que:

$$\sigma_{b,AB} = \frac{P}{4(11 \times 6)} = \frac{P}{264\ mm^2} \leq 65\ MPa \;\Leftrightarrow\; P \leq 17{,}16\ kN$$

$$\sigma_{b,BC} = \frac{P}{4(11 \times 7)} = \frac{P}{308\ mm^2} \leq 65\ MPa \;\Leftrightarrow\; P \leq 20{,}02\ kN$$

$$P_{ad} = 17{,}16\ kN$$

Considerando os três modos de falha:

$$P_{ad} = 7{,}60\ kN$$