Estruturas Tridimensionais

No espaço tem-se as seguintes estruturas isostáticas:

- vigas poligonais
- grelhas (caso particular)

Relembrando, o equilíbrio estático implica em:

$$\begin{cases} \vec{R} = \vec{0} \Rightarrow \sum_{i=1}^{n} \vec{F}_i = \vec{0} \\ \vec{M}_0 = \vec{0} \Rightarrow \sum_{i=1}^{n} (P_i - O) \wedge \vec{F}_i = \vec{0} \end{cases}$$

onde P_i é o ponto de aplicação da carga $\vec{F}_i$.

Assim:

$$\vec{R} = R_x \vec{i} + R_y \vec{j} + R_z \vec{k} = \vec{0}$$

$$\vec{M}_0 = M_{0,x} \vec{i} + M_{0,y} \vec{j} + M_{0,z} \vec{k} = \vec{0}$$

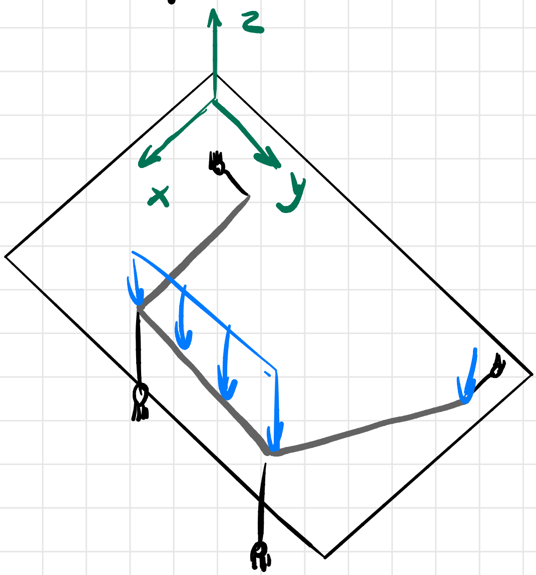
E obtém-se 6 equações de equilíbrio:

$$R_x = 0;\ R_y = 0;\ R_z = 0;\ M_{0,x} = 0;\ M_{0,y} = 0;\ M_{0,z} = 0.$$

Grelhas

As grelhas são um caso particular de vigas poligonais espaciais, sendo que a estrutura está contida no plano, porém o carregamento é perpendicular ao plano.

Por exemplo:

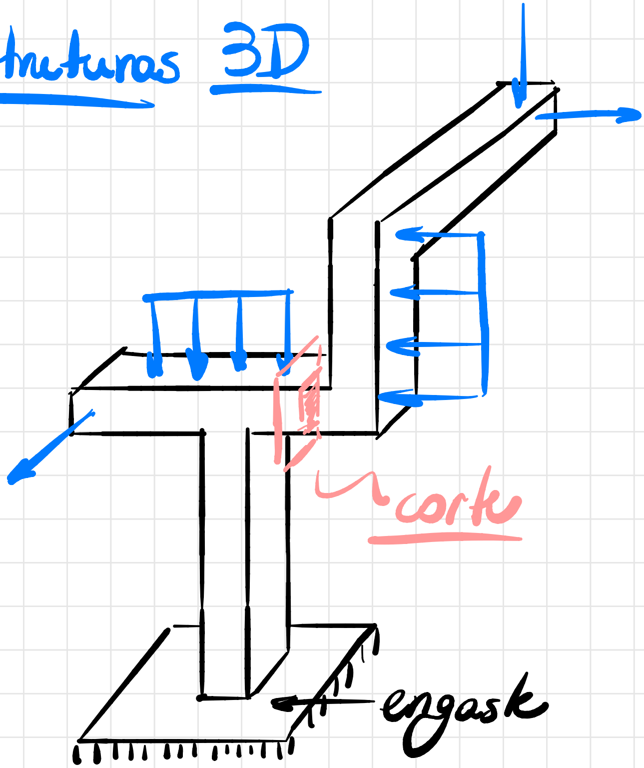


$$\begin{cases} \text{estrutura} \in Oxy \\ \vec{F}_i \perp Oxy, \forall i \end{cases}$$

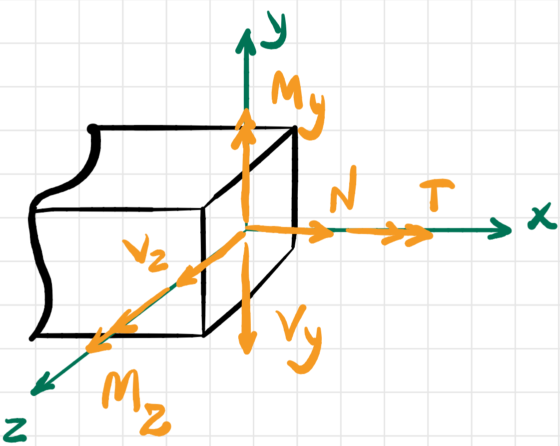
Dessa forma as seguintes equações são trivialmente atendidas:

$$R_x = 0; \quad R_y = 0; \quad M_z = 0.$$

Estruturas 3D

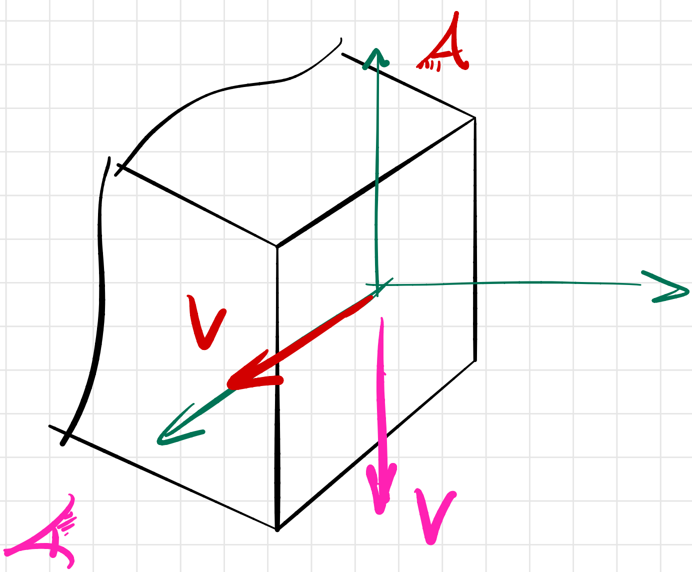


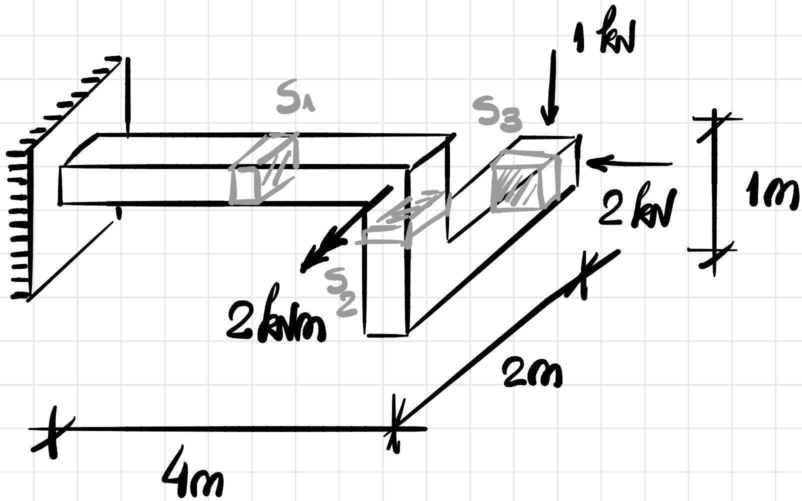
Fazendo um corte:

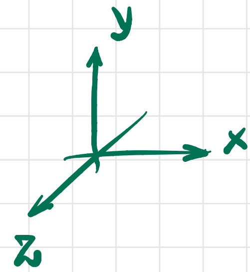


Convenção de Sinais

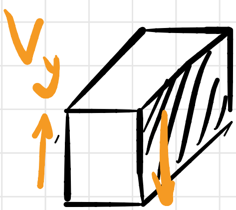
- $N > 0$: saindo da seção
- $T > 0$: saindo da seção
- M_y, M_z: desenhar do lado tracionado
- V_y, V_z: são positivas quando provocam um giro na seção transversal no sentido horário quando se está olhando no sentido contrário do eixo.

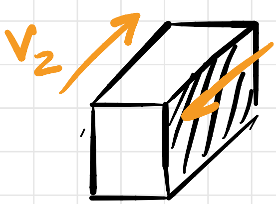




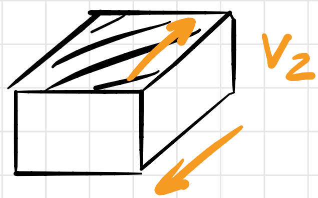


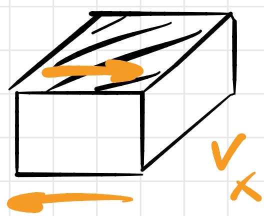
Seção S_1:



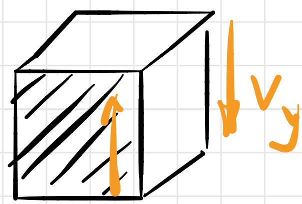


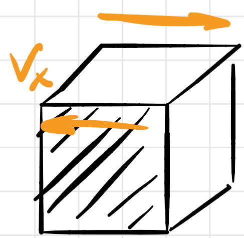
Seção S_2:





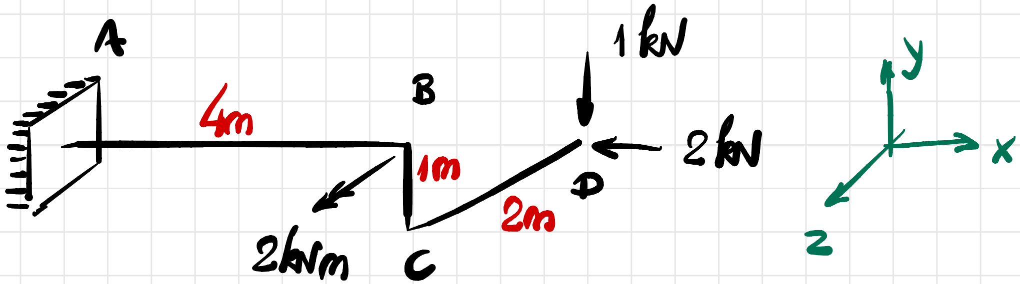
Seção S_3:



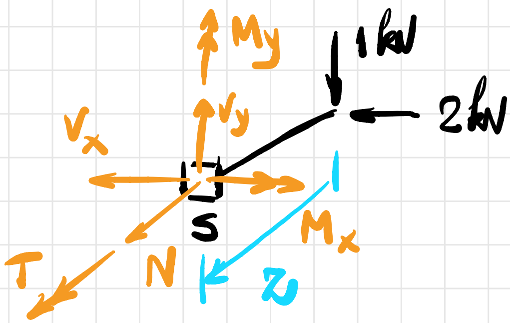


Voltando ao exemplo para obter os diagramas.

Representando a estrutura usando vigas:



Fazendo um corte na barra CD:



$$\sum F_x = 0: \quad V_x + 2 = 0 \Rightarrow \boxed{V_x = -2\,kN}$$

$$\sum F_y = 0: \quad V_y - 1 = 0 \Rightarrow \boxed{V_y = 1\,kN}$$

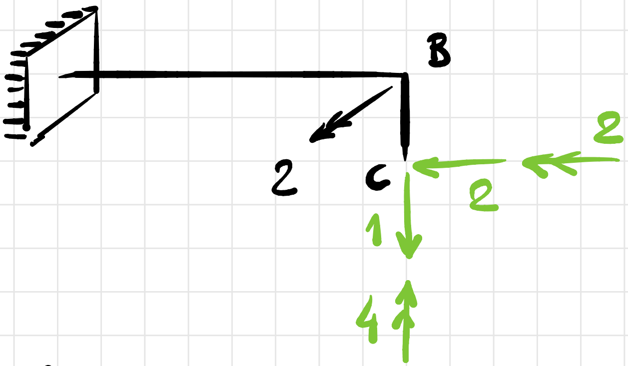
$$\sum F_z = 0: \quad \boxed{N = 0}$$

$$\sum M_{x,S} = 0: \quad M_x - 1.z = 0 \Rightarrow \boxed{M_x = z}$$ ⊕ em cima

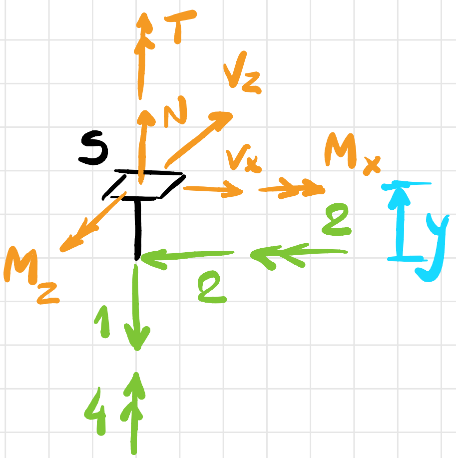
$$\sum M_{y,S} = 0: \quad M_y + 2.z = 0 \Rightarrow \boxed{M_y = -2z}$$ ⊕ esquerda

$$\sum M_{z,S} = 0: \quad \boxed{T = 0}$$

Fazendo o transporte para C:



E fazendo um corte na barra BC:



$$\sum F_x = 0: \; V_x - 2 = 0$$

$$\boxed{V_x = 2 \text{ kN}}$$

$$\sum F_y = 0: \; N - 1 = 0$$

$$\boxed{N = 1 \text{ kN}}$$

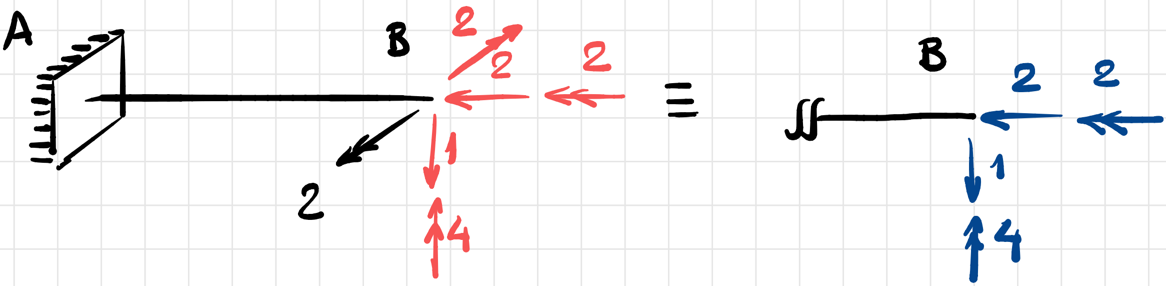
$$\sum F_z = 0: \; \boxed{V_z = 0}$$

$$\sum M_{x,s} = 0: \; M_x - 2 = 0 \Rightarrow \boxed{M_x = 2 \text{ kNm}}$$ ⊕ atrás

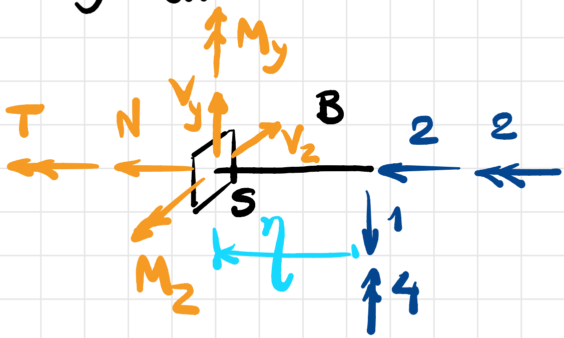
$$\sum M_{y,s} = 0: \; T + 4 = 0 \Rightarrow \boxed{T = -4 \text{ kNm}}$$

$$\sum M_{z,s} = 0: \; M_z - 2y = 0 \Rightarrow \boxed{M_z = 2y}$$ ⊕ direita

Fazendo o transporte para B:



E fazendo um corte na barra AB:



$$\sum F_x = 0: -N - 2 = 0 \Rightarrow \boxed{N = -2\ kN}$$

$$\sum F_y = 0: V_y - 1 = 0 \Rightarrow \boxed{V_y = 1\ kN}$$

$$\sum F_z = 0: \boxed{V_z = 0}$$

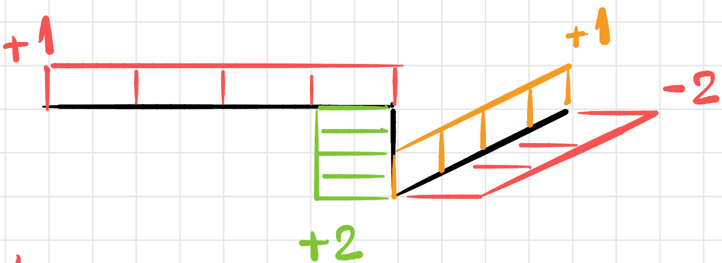
$$\sum M_{x,s} = 0: -T - 2 = 0 \Rightarrow \boxed{T = -2\ kNm}$$

$$\sum M_{y,s} = 0: M_y + 4 = 0 \Rightarrow \boxed{M_y = -4\ kNm}$$ (+) atrás

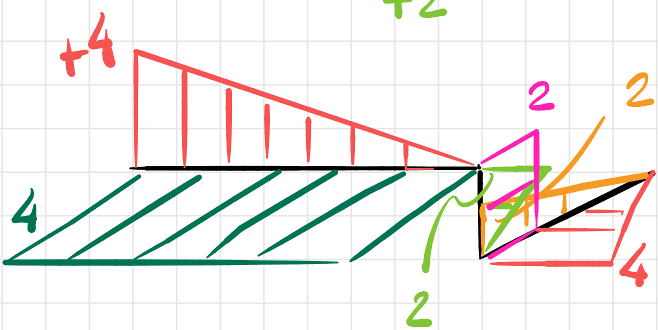
$$\sum M_{z,s} = 0: M_z - 1\eta = 0 \Rightarrow \boxed{M_z = \eta}$$ (+) em cima

Diagramas:

N [kN]

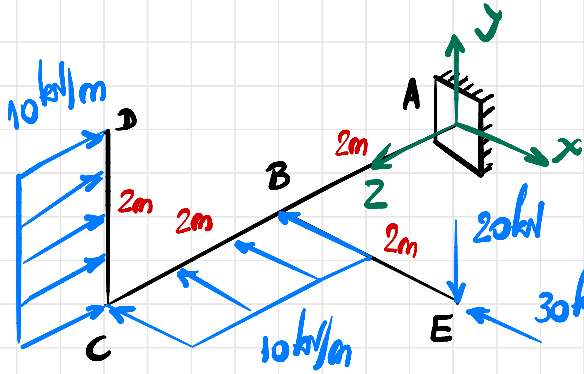


V [kN]

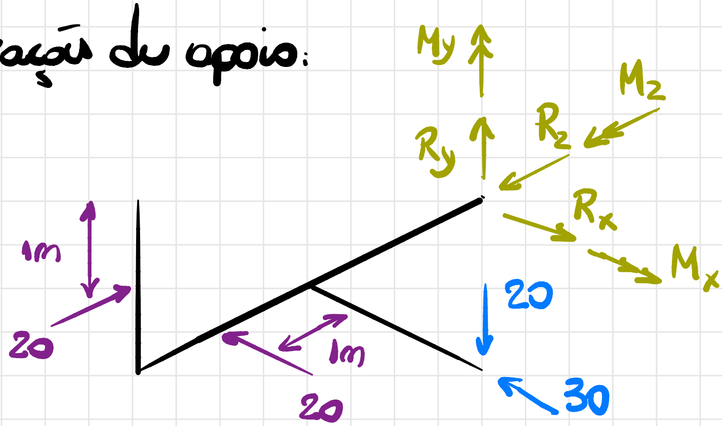


M [kNm]

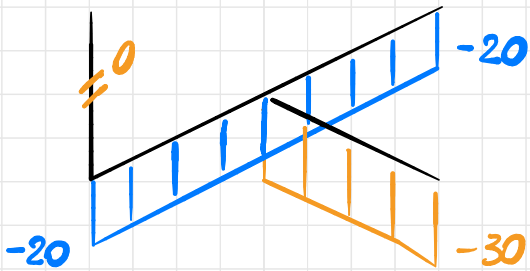
T [kNm]



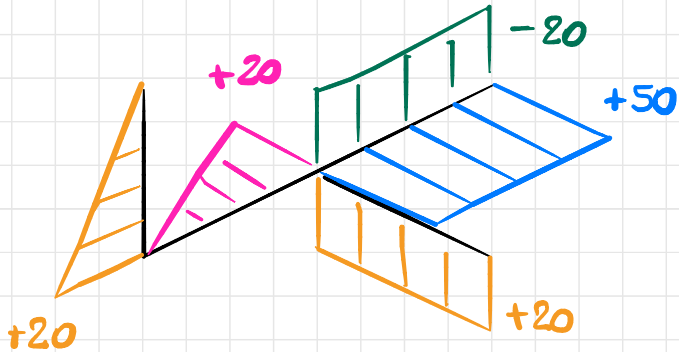
Reações do apoio:



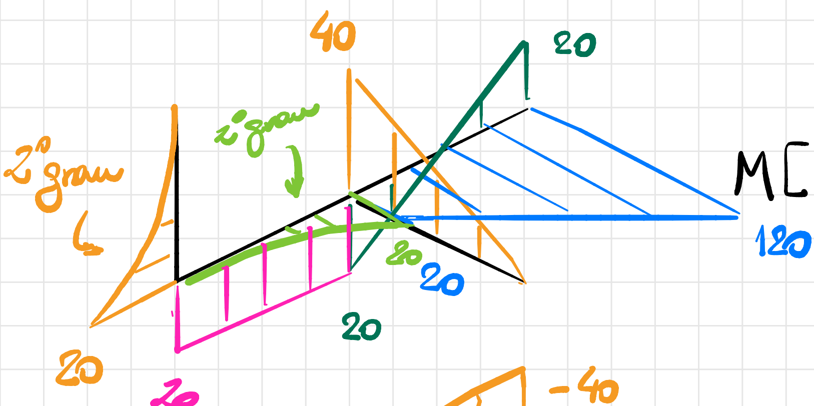
$$
\begin{cases}
\sum F_x = 0: R_x - 20 - 30 = 0 \Rightarrow \boxed{R_x = 50\,kN} \\
\sum F_y = 0: R_y - 20 = 0 \Rightarrow \boxed{R_y = 20\,kN} \\
\sum F_z = 0: R_z - 20 = 0 \Rightarrow \boxed{R_z = 20\,kN} \\
\sum M_{x,A} = 0: M_x + 20.2 - 20.1 = 0 \Rightarrow \boxed{M_x = -20\,kNm} \\
\sum M_{y,A} = 0: M_y - 30.2 - 20.3 = 0 \Rightarrow \boxed{M_y = 120\,kNm} \\
\sum M_{z,A} = 0: M_z - 20.2 = 0 \Rightarrow \boxed{M_z = 40\,kNm}
\end{cases}
$$



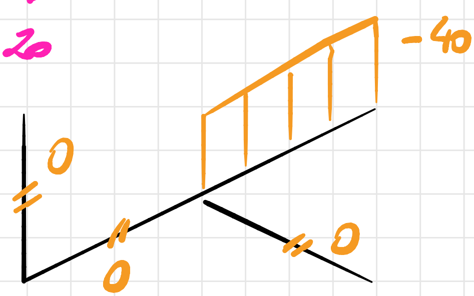
N[kN]



V[kN]

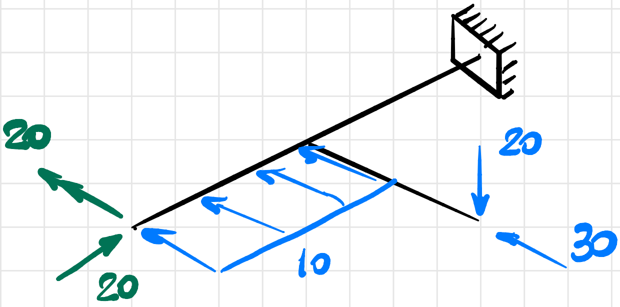


M[kNm]

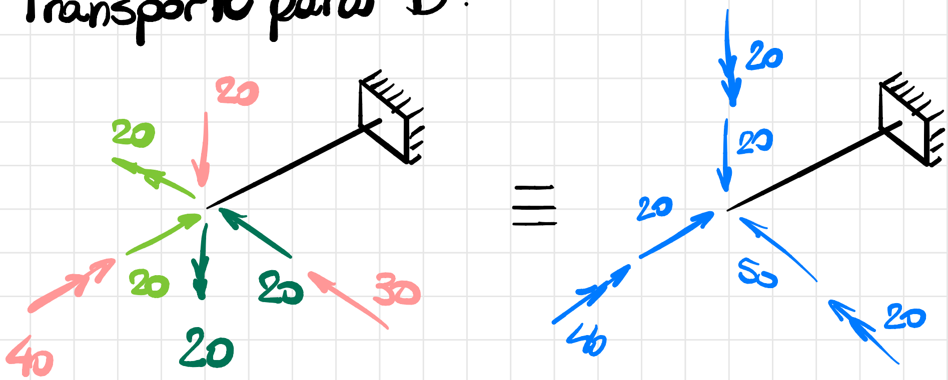


T[kNm]

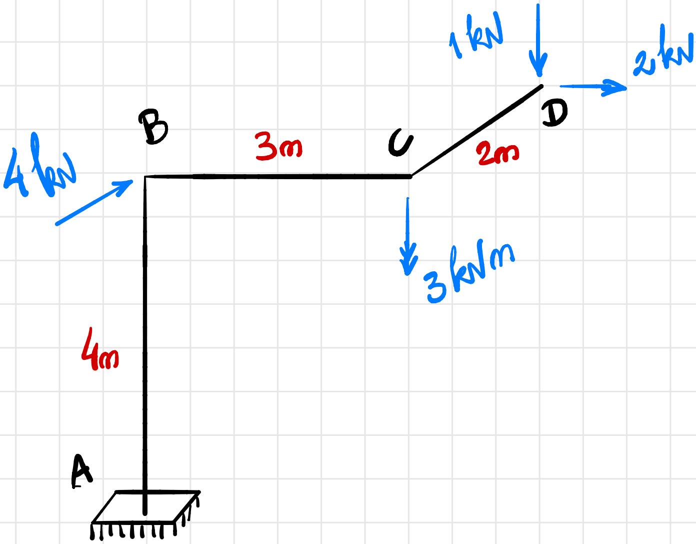
Transporte para C:

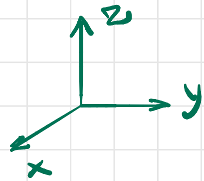


Transporte para B:

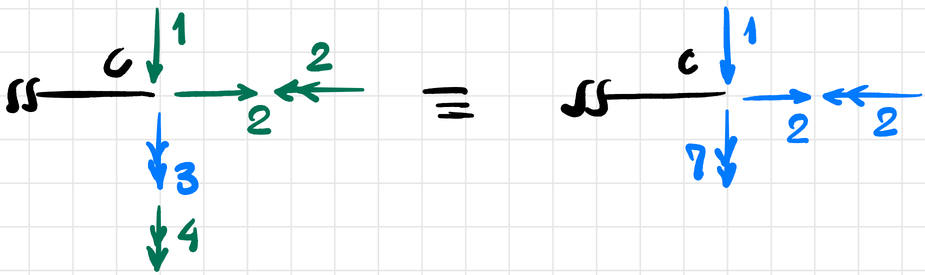


* Os diagramas da barra AB pode ser calculado por esse transporte. Feito via reações, as forças e momentos acima servem como verificação.

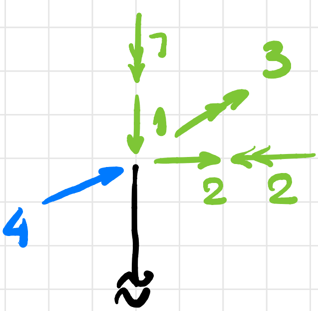


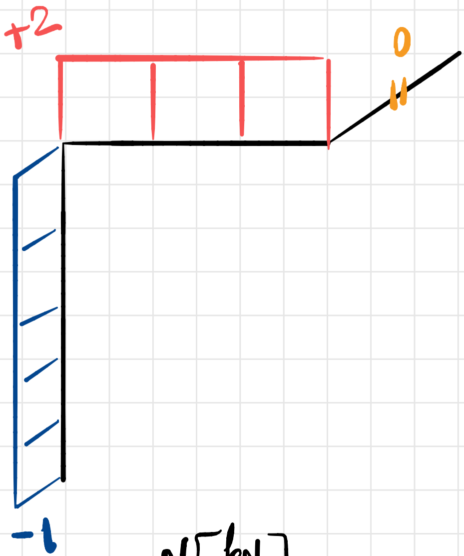


Transporte para C:

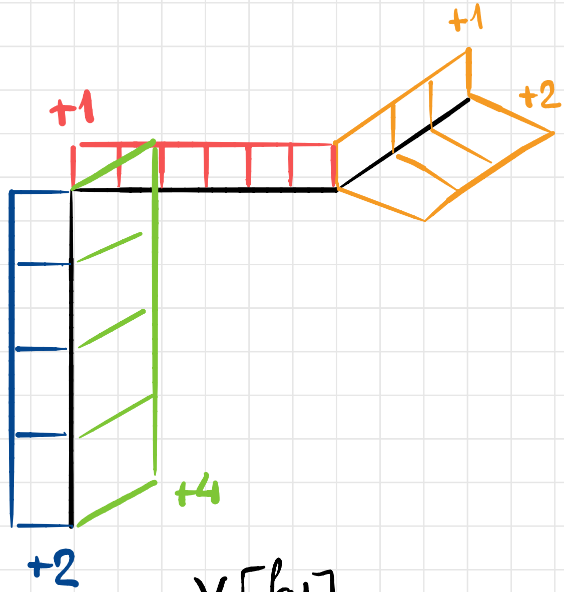


Transporte para B:

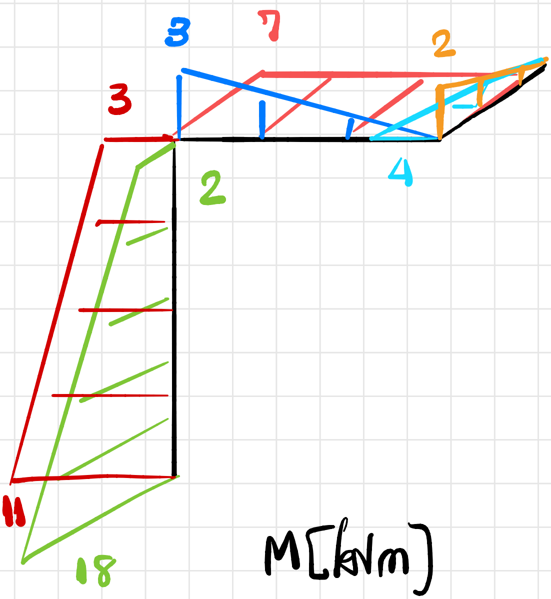




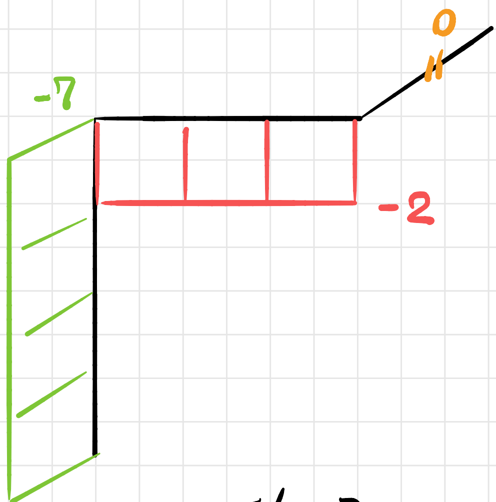
N[kN]



V[kN]



M[kNm]



T[kNm]