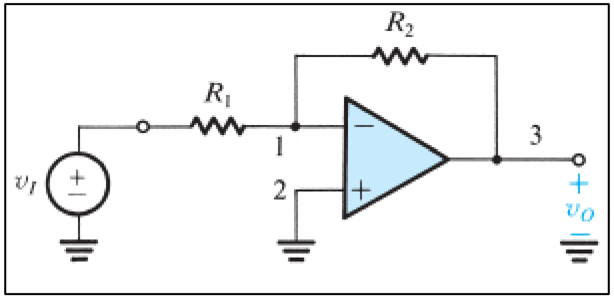
Inverting Amplifier

The Inverting Configuration

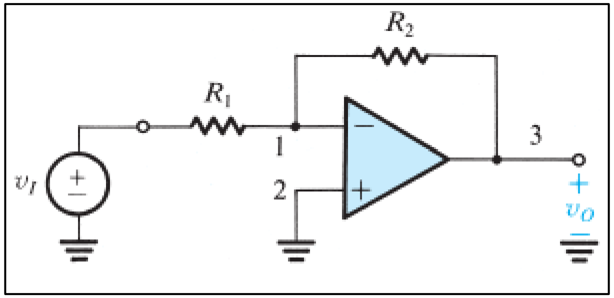
1 **Op amps are not used alone; rather, the op amp is connected to passive components in a feedback circuit**. There are two such **basic circuit configurations** employing an op amp and two resistors: the **inverting configuration** and the **noninverting configuration.**



2 Resistor R_2 is connected from the output terminal of the op amp, terminal 3, *back* to the input terminal, terminal 1.

We speak of R_2 as applying **negative feedback.**

If R_2 were connected between terminals 3 and 2 we would have called this **positive feedback**.

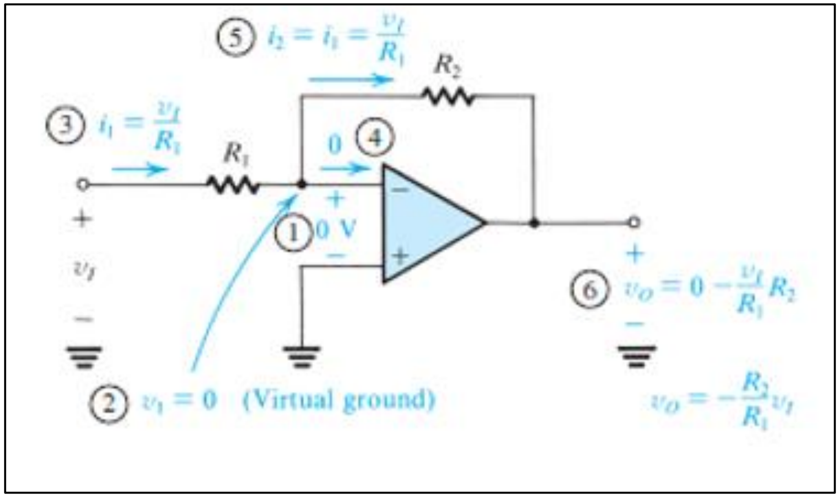


3 We will do so assuming the op amp to be ideal. The gain *A* is very large (ideally infinite).

$$v_O = A(v_2 - v_1) \longrightarrow v_2 - v_1 = \frac{v_O}{A} = 0 \longrightarrow v_2 = v_1$$

4 It follows that the voltage at the inverting input terminal (v_1) is given by $v_1 = v_2$. That is, because the gain *A* approaches infinity, the voltage v_1 approaches and ideally equals v_2. We speak of this as the **two input terminals "tracking each other in potential**." We also speak of a "**virtual short circuit**" that exists between the two input terminals. Here the word *virtual* should be emphasized, and one should *not* make the mistake of physically shorting terminals 1 and 2 together while analyzing a circuit. **A virtual short circuit means that whatever voltage is at 2 will automatically appear at 1 because of the infinite gain *A***. But terminal 2 happens to be connected to ground; thus $v_2 = 0$ and $v_1 = 0$. We speak of terminal 1 as being a **virtual ground**— that is, **having zero voltage but not physically connected to ground.**

$$Ganho = G = \frac{v_o}{v_I}$$



$$i_1 = \frac{v_I - v_1}{R_1} = \frac{v_i - 0}{R_1} = \frac{v_I}{R_1} \qquad (1)$$

Where will this current go? It cannot go into the op amp, since the ideal op amp has an infinite input impedance and hence draws zero current. **It follows that i_1 will have to flow through R_2 to the low-impedance terminal** 3.

$$v_O = v_i - i_1 R_2 = 0 - \frac{v_I R_2}{R_1} \qquad (2)$$

(1) e (2) →

$$G = \frac{v_O}{v_I} = -\frac{R_2}{R_1}$$

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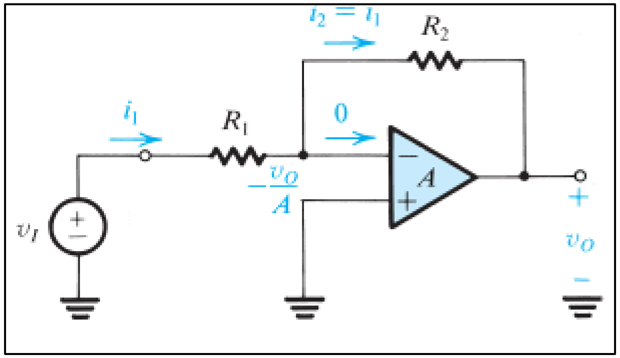
We note that:

5.1 **The fact that the closed-loop gain depends entirely on external passive components (resistors R_1 and R_2) is very significant. It means that we can make the closed-loop gain as accurate as we want by selecting passive components of appropriate accuracy**.

5.2 **We started out with an amplifier having very large gain A, and through applying negative feedback we have obtained a closed-loop gain that is much smaller than A but is stable and predictable. That is, we are trading gain for accuracy**.

Effect of Finite Open-Loop Gain



Analysis of the inverting configuration taking into account the finite open-loop gain of the op amp.

1 If we denote the output voltage v_O then the voltage between the two input terminals of the op amp will be v_o/A. Since the positive input terminal is grounded, the voltage at the negative input terminal must be - v_o/A (out of phase with respect the output). The current i_1 through R_1 can now be found:

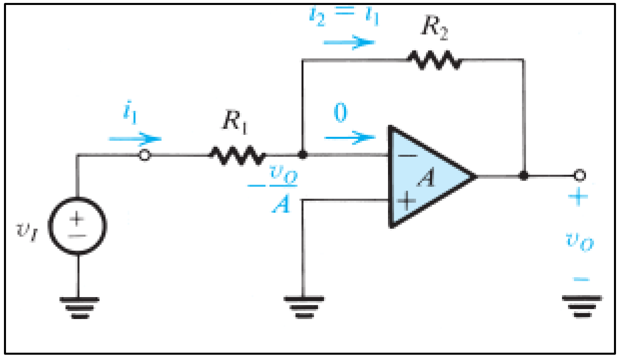
$$i_1 = \frac{v_I - (-v_O/A)}{R_1} = \frac{v_I + v_O/A}{R_1} \qquad (3)$$

2 The infinite input impedance of the op amp forces the current i_1 to flow entirely through R_2. The output voltage v_O can thus be determined from:

$$v_O = -\frac{v_O}{A} - i_1 R_2 = -\frac{v_O}{A} - \left(\frac{v_I + v_O/A}{R_1}\right) R_2 \qquad (4)$$

(3) e (4) →

$$G = -\frac{v_O}{v_I} = \frac{-R_2/R_1}{1 + \frac{(1 + R_2/R_1)}{A}}$$



$$G = -\frac{v_o}{v_i} = \frac{-R_2/R_1}{1+(1+R_2R_1)/A}$$

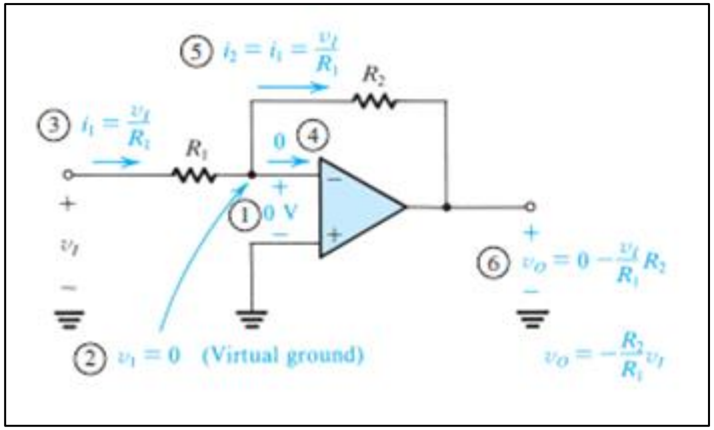
3 We note that:

3.1 As *A* approaches ∞, *G* approaches the ideal value – R_2 /R_1.

3.2 Also, we see that as ***A* approaches ∞, the voltage at the inverting input (–v_o/A) terminal approaches zero**. This is the virtual-ground assumption we used in our earlier analysis when the op amp was assumed to be ideal.

3.3 Finally, note that the gain equation in fact indicates that **to minimize the dependence of the closed-loop gain *G* on the value of the open-loop gain *A*, we should make $(1 + R_2R_1)$ <<A**.

Input Resistance



$$R_i = \frac{v_I}{i_I} = \frac{v_I}{v_I / R_1}$$

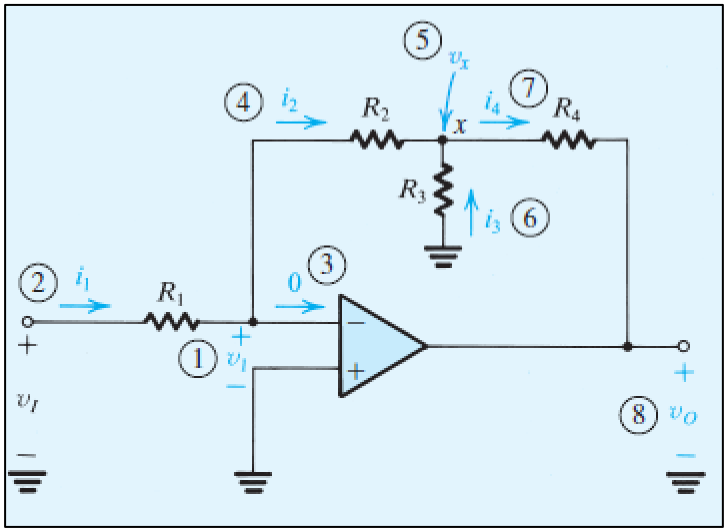
$$\rightarrow \quad R_i = R_1$$

1 **Amplifier input resistance forms a voltage divider with the resistance of the source that feeds the amplifier. Thus, to avoid the loss of signal strength, voltage amplifiers are required to have high input resistance**.

In the case of the inverting op-amp configuration we are studying, to make R_i high we should select a high value for R_1. However, if the required gain is also high, then R_2 could become impractically large (e.g., greater than a few megohms).

We may conclude that the inverting configuration suffers from a low input resistance.

A solution to problem of low input resistance is discussed in this exercise !

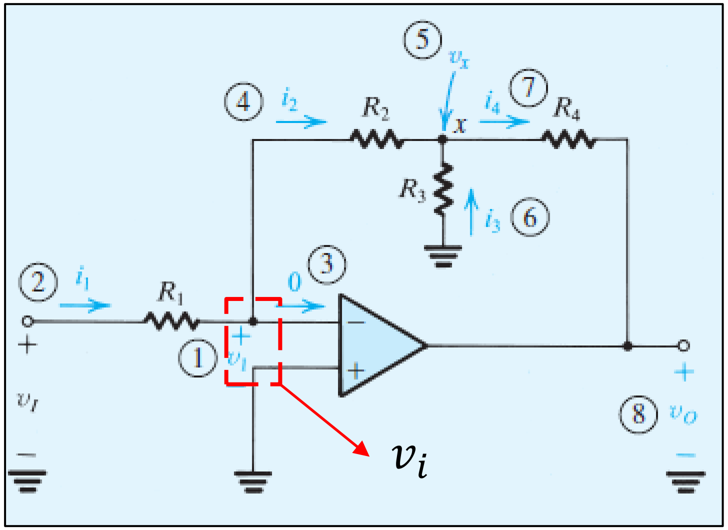


Assuming the op amp to be ideal, derive an expression for the closed-loop gain of the circuit shown.

Use this circuit to design an inverting amplifier with a gain of 100 and an input resistance of 1 MΩ.

Assume that for practical reasons it is required not to use resistors greater than 1 MΩ.

$$Ganho = G = \frac{v_S}{v_I}$$



1 The analysis begins at the inverting input terminal of the op amp, where the voltage is:

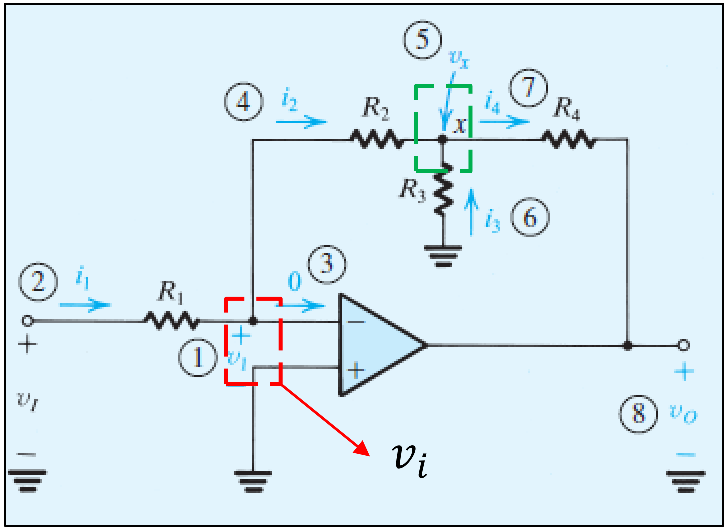
$$v_i = 0 \quad \text{(virtual ground)}$$

2 Knowing v_i we can determine the current i_1 as follows:

$$i_1 = \frac{v_I - v_i}{R_1} = \frac{v_I - 0}{R_1} = \frac{v_I}{R_1}$$

3 Since zero current flows into the inverting input terminal, all of i_1 will flow through R_2, and thus:

$$i_2 = i_1 = \frac{v_I}{R_1}$$



4 Now we can determine the voltage at **node *x***:

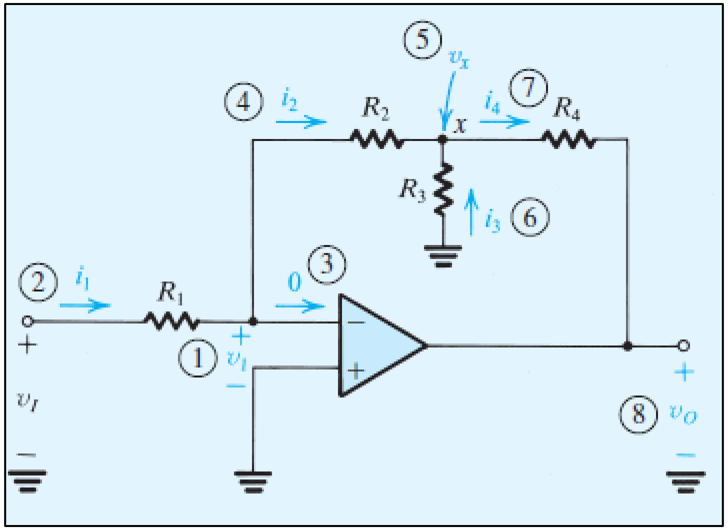
$$v_x = v_i - i_2 R_2 = 0 - \frac{v_I}{R_1} R_2 = \frac{v_I}{R_1} R_2$$

5 This in turn enables us to find the current i_3:

$$i_3 = \frac{0 - v_x}{R_3} = \frac{R_2}{R_1 R_3} v_I$$

6 Next, a node equation at x yields i_4:

$$i_4 = i_2 + i_3 = \frac{v_I}{R_1} + \frac{R_2}{R_1 R_3} v_I$$



7 Finally, we can determine v_O:

$$v_0 = v_x - i_4 R_4 = -\frac{R_2}{R_1} v_I - \left(\frac{v_I}{R_1} + \frac{R_2}{R_1 R_3} v_I\right) R_4$$

$$\frac{v_0}{v_I} = -\left[\frac{R_2}{R_1} + \frac{R_4}{R_1}\left(1 + \frac{R_2}{R_3}\right)\right]$$

$$G = \frac{v_0}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

$$G = \frac{v_O}{v_I} = -\frac{R_2}{R_1}\left(1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right)$$

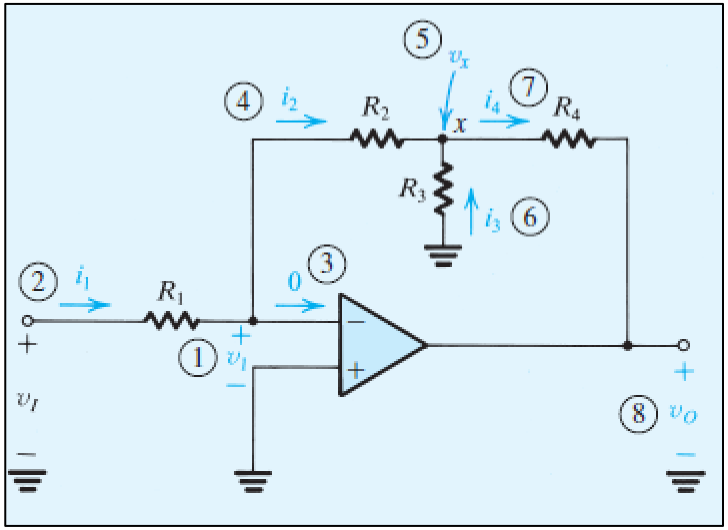
8 Now, since an input resistance of 1 MΩ is required, we select R_1 = 1 MΩ. Then, with the limitation of using resistors no greater than 1 MΩ, the maximum value possible for the first factor in the gain expression is 1 and is obtained by selecting R_2 = 1 MΩ.

To obtain a gain of −100, R_3 and R_4 must be selected so that the second factor in the gain expression is 100.

If we select the maximum allowed (in this example) value of 1 MΩ for R_4, then the required value of R_3 can be calculated to be 10.2 kΩ.

Thus this circuit utilizes three 1MΩ resistors and a 10.2kΩ resistor.

In comparison, if the inverting configuration were used with R_1 = 1 MΩ we would have required a feedback resistor of 100 MΩ, an impractically large value !

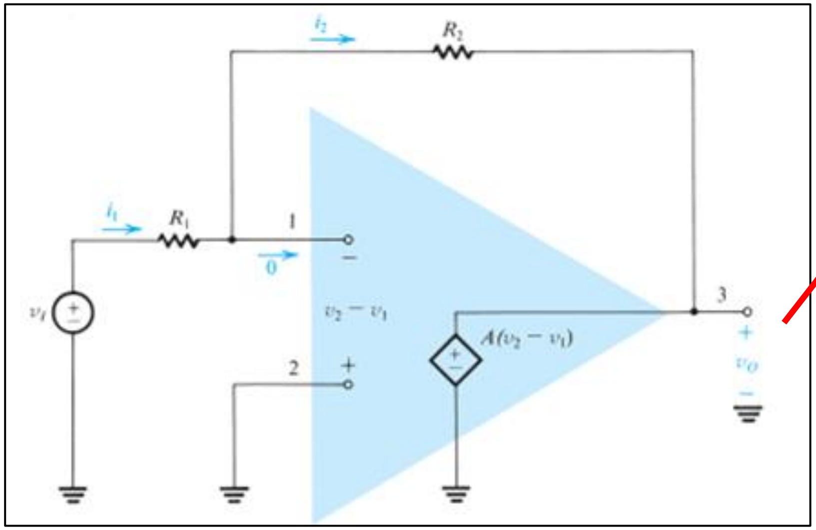


$$\begin{cases} i_3 R_3 = R_2 i_I \\ i_2 = i_I \\ i_4 = i_2 + i_3 \end{cases}$$

$$i_4 = \left(1 + \frac{R_2}{R_3}\right) i_I$$

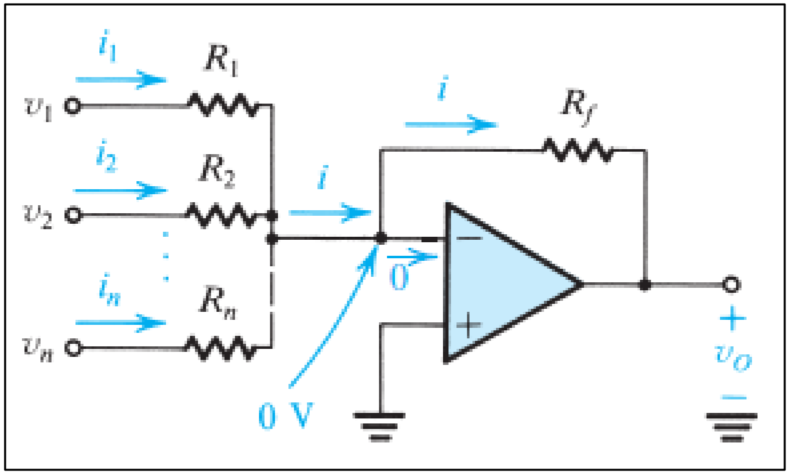
I_4 não depende de R_4 !

Output Resistance



Since the output of the inverting configuration is taken at the terminals of the **ideal voltage source** $\boldsymbol{A(v_2 - v_1)}$, it follows that the output resistance of the closed-loop amplifier is zero.

The Weighted Summer



$$i_1 = \frac{v_1}{R_1}$$

$$i_2 = \frac{v_2}{R_2}$$

$$\ldots\ i_n = \frac{v_n}{R_n}$$

$$i = i_1 + i_2\ \ldots .\ i_n$$

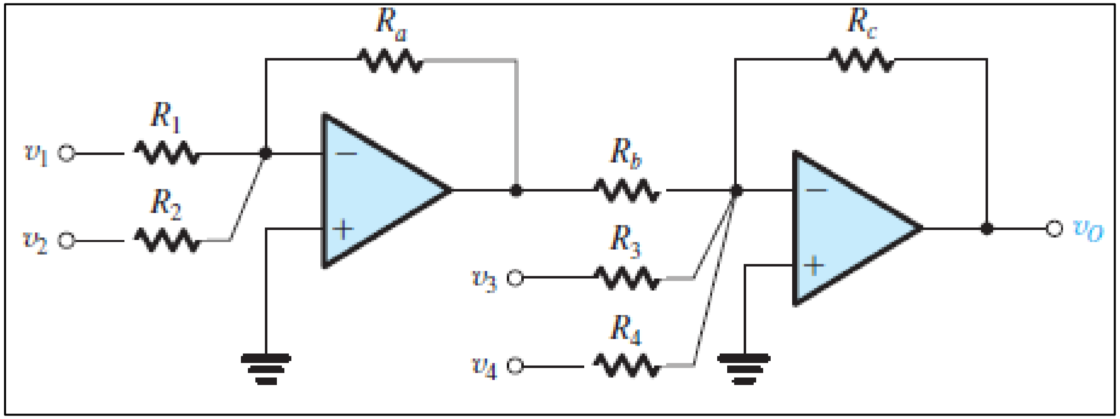
$$v_o = 0 - \mathrm{i}R_f$$

$$\rightarrow\ v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \cdots \frac{R_f}{R_n}v_n\right)$$

1 That is, the output voltage is a weighted sum of the input signals v_1, v_2, . . . , v_n. This circuit is therefore called a **weighted summer**. Note that each summing coefficient may be independently adjusted by adjusting the corresponding “feed-in” resistor (R_1 to R_n). This nice property, which greatly simplifies circuit adjustment, is a direct consequence of the virtual ground that exists at the inverting op-amp terminal.

2

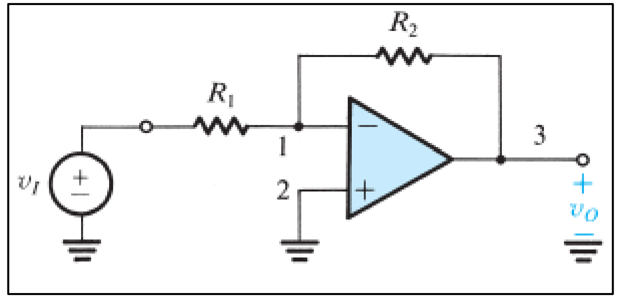
In the weighted summer saw all the summing coefficients must be of the same sign. The need occasionally arises for summing signals with opposite signs. Such a function can be implemented, however, using two op amps as shown below. Assuming ideal op amps, it can be easily shown that the output voltage is given by:



$$v_o = v_1\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) + v_2\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) - v_3\left(\frac{R_c}{R_3}\right) - v_4\left(\frac{R_c}{R_4}\right)$$

Weighted summers are utilized in a variety of applications including in the design of audio systems where they can be used in mixing signals originating from different musical instruments !

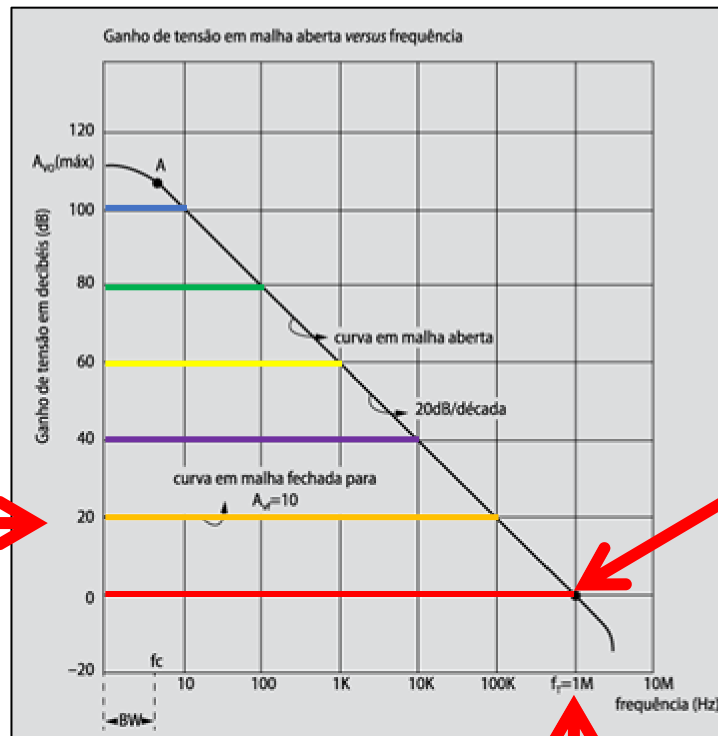
Frequency Response



$$PGL = A_{vf} \times BW = f_T$$

$$A_{vf} = 10(20dB)$$

$$BW = \frac{1MHz}{10} = 100KHz$$



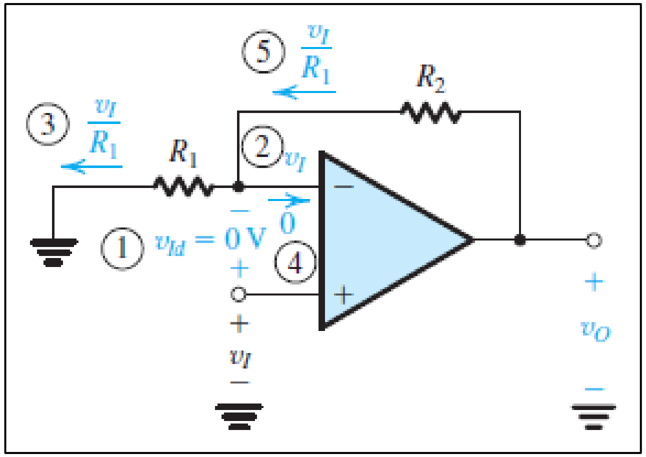
LM741 | f_T = 1MHz

LF351 | f_T = 4MHz

Noninverting Amplifier

The Noninverting Configuration

Here the input signal v_{Id} is applied directly to the positive input terminal of the op amp while one terminal of R_1 is connected to ground.



$$v_{Id} = \frac{v_O}{A} = 0 \qquad (5)$$

$$v_O = v_I + \left(\frac{v_I}{R_1}\right) R_2 \qquad (6)$$

(5) e (6) →

$$G = \frac{v_O}{v_I} = 1 + \frac{R_2}{R_1}$$

Effect of Finite Open-Loop Gain

it can be shown that the closed-loop gain of the noninverting amplifier circuit is given by

$$G = \frac{v_o}{v_i} = \frac{1+(R_2/R_1)}{1+(1+R_2/R_1)/A}$$

If $A >> 1+\frac{R_2}{R_1}$ $\longrightarrow$ $G = 1 + \frac{R_2}{R_1}$

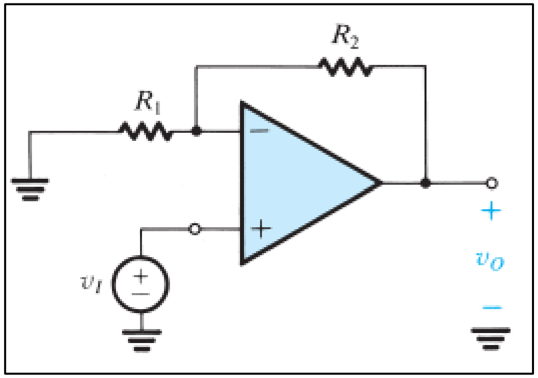
Input and Output Resistance

The gain of the noninverting configuration is positive—hence the name noninverting.

The input impedance of this closed-loop amplifier is ideally infinite, since no current flows into the positive input terminal of the op amp.

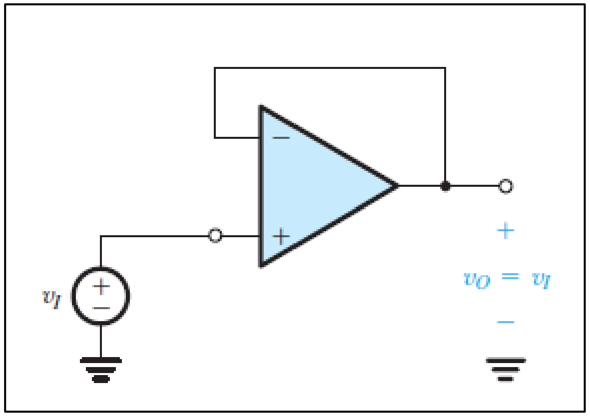
The output of the noninverting amplifier is taken at the terminals of the ideal voltage source $A(v_2 - v_1)$, thus the **output resistance of the noninverting configuration is zero**.

The Voltage Follower



$$G = 1 + \frac{R_2}{R_1}$$

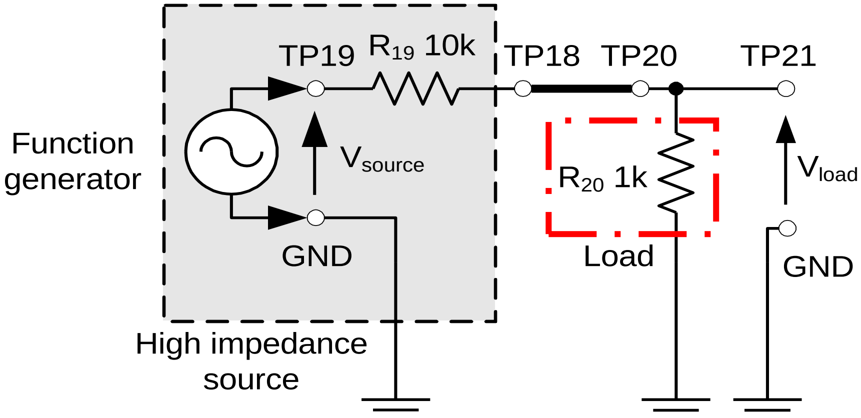
$R_2 = 0$ and $R_1 = \infty$ →



→ $G = 1$

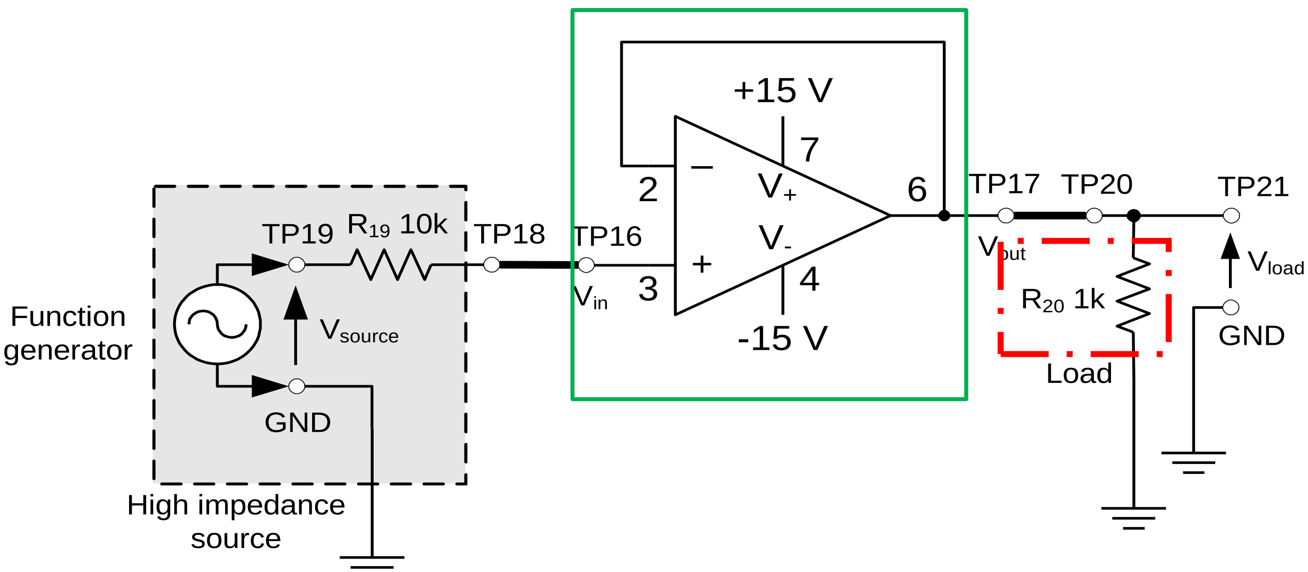
There are situations in which one is interested not in voltage gain but only in a significant power gain. For instance, **the source signal can have a respectable voltage but a source resistance that is much greater than the load resistance. Connecting the source directly to the load would result in significant signal attenuation**. In such a case, one requires an amplifier with a high input resistance (much greater than the source resistance) and a low output resistance (much smaller than the load resistance) but with a modest voltage gain (or even unity gain). Such an amplifier is referred to as a **buffer amplifier**.

Loading Effect - Op Amp Isolador



$$V_{load} = \frac{V_{source}}{10k + 1K} . 1k$$

The resistive divider with 10kΩ e 1KΩ significantly reduces the voltage on the 1KΩ load !



$$V_{load} = V_{source}$$