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Memory psychophysics

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Summary. The relationship between perceptual and cognitive processes has been a topic of increasing interest. This review focuses on the use of techniques and theory drawn from classical psychophysics and applied to the study of mental representation. Several issues including examination of the functions that relate remembered and perceived magnitude to physical intensity, the relationship of memorial to perceptual functions, the effect of time on the memorial function, considerations in the methodology of memory psychophysics experiments, the level of functional equivalence between memorial and perceptual representation, and the use of psychophysical techniques and theory in the study of visual imagery are addressed. While the data suggest that the relationship between remembered magnitude and physical intensity and between perceived magnitude and physical intensity can be described by power functions, a model capable of accounting for the behavior of the memory magnitude for all stimulus dimensions and at all time intervals has not yet been found. Several unresolved issues and typical difficulties with research in memory psychophysics are also discussed.

Introduction

Psychophysics has commonly been defined as the quantitative study of perception, but this limited definition ignores the use of psychophysical methods in cognitive areas such as learning and memory (Baird & Noma, 1978). Similarly, research on mental representation has often ignored important similarities between perceptual and cognitive processes (Shepard & Podgorny, 1978). This isolationism has been changing of late, as psychophysical techniques developed in the study of perception have recently been applied in the study of mental representation and how representations of objects relate to the objects they represent (e.g., readings in Algom, 1992b). This area of investigation has been referred to as memory psychophysics. This article will review data and models of memory psychophysics, give a critique of the methodology involved in the application of psychophysical techniques to

the study of cognitive representation and give suggestions for methodological improvement, and provide examples of how psychophysical techniques may be used to answer questions about the structure and content of mental representations.

Psychophysical theory

Classical psychophysical theory will be considered briefly in order that the findings from memory psychophysics can be understood better. More detailed presentations of psychophysical theory can be found in Falmagne (1986), Gescheider (1985), and Stevens (1975). Stevens (1957, 1975) described a general psychophysical law that details the form of the relationship between stimulus (physical) intensity and response (perceived) magnitude. This law is of the form

$$P = \lambda S^{\gamma} \tag{1}$$

where P is the perceived magnitude and S the stimulus intensity. Lambda (λ) represents a multiplicative constant determined by the unit of measurement (e.g., meters or feet). The parameter of primary interest is gamma (γ), the exponent of the function and the parameter that determines the overall shape of the function. When γ is less than 1 (as it is for perceived brightness), response magnitude is a negatively accelerated function of stimulus intensity. When γ is equal to 1 (as it is for perceived length), response magnitude and stimulus intensity are linearly related. When γ is greater than 1 (as it is for perceived electric shock), response magnitude is a positively accelerated function of stimulus intensity. Examples of these functions are shown in Figure 1.

In order to determine the exponent of the psychophysical function, perceived magnitude must be measured first. Stevens (1957) suggested that there were four primary methods of directly scaling perceived magnitude: ratio production, ratio estimation, magnitude production, and magnitude estimation. The last of these methods, magnitude estimation, has been the method of choice in the vast majority of experiments in memory psychophysics, and so we shall limit our inquiry to consideration of this method.

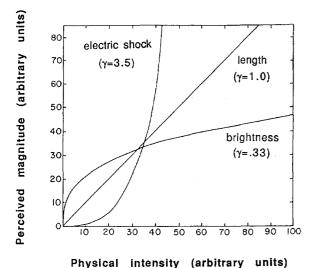


Fig. 1. Psychophysical functions for perceived brightness, length, and electric shock. Along the x axis are plotted units of stimulus intensity, and along the y axis are plotted units of perceived magnitude. The exponent for brightness approximates 0.33, the exponent for length approximates 1, and the exponent for electric shock approximates 3.5 (adapted from Stevens, 1975)

In magnitude estimation, the subject is typically presented with a sequence of stimuli. The stimuli are presented individually, and after the presentation of each particular stimulus, the subject estimates the perceived magnitude of that stimulus. Two types of magnitude estimation should be distinguished: prescribed-modulus and free-modulus. In prescribed-modulus magnitude estimation, the observer's judgments reflect the ratio between some standard (modulus) and some other stimulus. First, the subject is presented with a standard (modulus) and told that the intensity level of the standard reflects a certain value, for example, 10. If the second stimulus in the series is twice as intense (loud, long, etc.) as the modulus, then the subject is told to assign the second stimulus a value twice that of the modulus, that is, 20. If the second stimulus in the series is only half as intense (loud, long, etc.) as the modulus, then the subject should assign the second stimulus a value only half that of the modulus, that is, 5. In free-modulus magnitude estimation, the subject is not given a standard, but is free to assign whatever numbers and to use whatever scale he or she chooses. In both prescribed- and free-modulus forms, however, the ratio between the numbers the subject uses is assumed to reflect the ratio between the perceived magnitudes.

Early ideas of memory psychophysics

Björkman, Lundberg, and Tärnblom (1960) were among the first to point out that the scope of psychophysics could be broadened from immediate perceptual experience to include the study of memory. They suggested, "A generalized psychophysics . . . has to take into account magnitudes of two subjective continua (spaces): the perceptual (immediate experiences) and the memory continuum (past experiences)." They broadened the concept of the psycho-

physical law to include relationships between stimulus intensity, perceived magnitude, and memory magnitude:

$$P = \lambda S^{\gamma} \tag{2}$$

$$M = \pi P^{\beta} \tag{3}$$

$$M = \phi S^{\eta} \tag{4}$$

Equation 2 shows the relationship between perceived magnitude and stimulus intensity and is the same relationship as that proposed by Stevens (see Equation 1 above). Equation 3 shows the relationship between remembered magnitude and perceived magnitude, where M is remembered magnitude, P is perceived magnitude, π is the multiplicative constant, and β is the exponent. Equation 4 shows the relationship between remembered magnitude and stimulus intensity, where M is remembered magnitude, S is stimulus intensity, ϕ is multiplicative constant, and η is the exponent. The parameter of primary interest in all of these equations is the exponent. In Equation 2, γ is a measurable quantity and has been the object of much empirical research over the past several decades (for review, see Stevens, 1975). In Equation 3, however, β is not a directly measurable quantity because we have no way of directly measuring the magnitude of the perceived intensity that is being remembered. In Equation 4, η is a measurable quantity because both M and S can be empirically determined.

Björkman et al. (1960) suggest that any two of Equations 2, 3, and 4 may be regarded as independent, and that the third equation may be rewritten in terms of the other two. This interdependence of Equations 2, 3, and 4 is based upon the proposal that $\eta = \beta * \gamma$. Given this interdependence, once we have measured P (using Equation 2), we can use Equations 3 and 4 to infer β . In this case β represents a theoretical construct, however, and not an empirical measurement. At present, though, there is no strong empirical evidence for constraining η to this extent, and so the precise relationship between η , β , and γ must remain open. The primary concern of memory psychophysics at the present time is the determination of η (i.e., the relationship of stimulus intensities and memory magnitudes) and, as will be discussed below, different theories of memory psychophysics make differing claims concerning the behavior of β and η .

In Björkman et al.'s (1960) experiments, subjects were trained to identify either circles of varying diameters or objects of varying weights. Subjects then judged the ratio between the intensity of a perceived object and the magnitude of a remembered object (named by the experimenter). Scaling of the subjects' responses revealed an η slightly greater than 1 for both stimulus dimensions, suggesting that the magnitude portrayed in the mental representation of size and weight was a slightly positively accelerated function of the physical intensities of those stimulus dimensions. The authors concluded that the power function was an adequate description of the relationship between the memory and the perceptual magnitudes, but despite the similarity of the two exponents, Björkman et al. were hesitant to interpret their data as suggesting a common memory exponent.

Another early study of memory psychophysics was conducted by Moyer (1973), who presented the names of two animals visually and had subjects indicate which name indicated the larger animal. Response times and error rates were inversely related to the difference in size between the animals, a pattern that has since been labeled the symbolic distance effect (Moyer & Bayer, 1976). These findings mirrored those for perceptual comparison of two animals (Johnson, 1939), thus leading Moyer to propose that subjects first converted the animal names to analog representations that preserved animal size and then compared these analogs by making an "internal psychophysical judgment." The longer response times and higher error rates with smaller size differences resulted from a decrease in discriminability, as smaller size differences between animals were represented as smaller differences between the internal analogs. Paivio (1975) extended these findings by also examining animal-object and object-object comparisons and found that Moyer's results applied both within and across categories (but see also Foltz, Poltrock, & Potts, 1984). Importantly, Paivio found that the differences in response time and error rate were related to the size ratios between the two items such that similar psychophysical functions emerged for comparable ratios between members of pairs within relatively small, medium, or large pairs.

Moyer, Bradley, Sorensen, Whiting, and Mansfield (1978) used magnitude estimation to determine the psychophysical functions for both remembered and perceived line length, area, and volume. In their first experiment subjects learned different nonsense syllables associated with different line lengths. Subjects then left and returned 24 hours later, at which time they were put into either a Memory or a Perception condition. Perception subjects were shown each of the lines again and estimated the lengths of the lines, and Memory subjects were given the nonsense-syllable names of the lines and estimated the lengths of the lines from memory. The data of both groups were well described by power functions, $\eta = 0.70 \, (r^2 > .99)$ for the Memory group, and $\gamma = 0.87$ ($r^2 > .99$) for the Perception group. In two additional experiments similar results were found when stimuli consisted of outlines of states of the U.S.A. and subjects estimated area ($\eta = 0.46$, r^2 = .94 for the Memory group; γ = 0.64, r^2 = .98 for the Perception group), and also when stimuli consisted of spherical objects ranging in size from a BB to a beach ball and subjects estimated volume ($\eta = 0.53$, $r^2 > .99$ for the Memory group; $\gamma = 0.73$, $r^2 > .99$ for the Perception group).

The relationship between perceptual magnitude and memory magnitude suggested by Björkman et al. (1960) (Equation 3 above) allows the memory exponent to take on any value. The results of Moyer et al. (1978) suggested that the memory exponent is generally lower than the perceptual exponent. Kerst and Howard (1978) proposed a model limiting the range of values of the memory exponent further. In their experiments, subjects made magnitude judgments of geographical area and interstate distance in the United States. Perception subjects viewed a map of the United States when making their judgments, and Memory subjects viewed a map of the United States immediately prior to making their judgments. Interestingly, the exponent in the Memory group ($\eta = 0.60$, $r^2 = .94$ for area, $\eta = 1.10$,

 r^2 = .92 for distance) was approximately equal to the square of the exponent in the Perception group (γ = 0.79, r^2 = .98 for area, γ = 1.04, r^2 = .96 for distance).

On the basis of this apparent relationship, Kerst and Howard (1978) proposed a more constrained model in which β is equal to γ . In essence, P in Equation 3 is set equal to λS^{γ} from Equation 2 yielding $M = \pi(\lambda S^{\gamma})\beta$. Assuming $\beta = \gamma$ (and ignoring for a moment the multiplicative constants), this is equivalent to $M = (S^{\gamma})^{\gamma}$. Thus, the magnitude portrayed in memory, η , equals the stimulus intensity raised to γ^2 :

$$M = (S^{\gamma})^{\beta} = (S^{\gamma})^{\gamma} = S^{\gamma 2} \tag{5}$$

Kerst and Howard christened this the *reperceptual hypothesis*. An initial power-function transformation operates on the sensory input to produce a perceptual representation (reflecting the power law proposed by Stevens). This value may then be stored in long-term memory. When observers are required later to access or retrieve this magnitude and make psychophysical judgments about it, the power-function transformation is applied to the stored representation. Retrieving the magnitude of a stimulus from memory is thus equivalent to a reperception of the original stimulus.

Perceived, remembered, and inferred distance and area

The majority of studies in memory psychophysics have involved scaling of distance or area, so we shall now focus on these dimensions (see Algom, 1992a, for a review of studies examining other dimensions). In most of the experiments that have examined remembered distance, distance was considered as distance on a map. Such map distance should perhaps be more properly considered length rather than distance because the dimension is portrayed by the length of lines on a map and not by the distance of the map from the observer. In one study that utilized true distance rather than length, Bradley and Vido (1984) had subjects learn the names and locations of 15 objects while the subjects viewed the objects from the top of a small mountain. The distances of the objects ranged from 20 feet to 14.28 miles. Twenty-four hours later Perception subjects were taken back up the mountain to judge the distances while viewing the landscape, and Memory subjects were taken into a laboratory to judge the distances while visualizing the landscape from memory. The Memory exponent ($\eta = 0.60$, $r^2 = .97$) was smaller than the Perception exponent ($\gamma = 0.81$, $r^2 = .99$) and appeared to approximate the square of the Perception exponent. Subjects were then asked to draw maps, but the exponents obtained from the maps for both groups were very similar ($\eta = 0.483$, $\gamma = 0.514$).

Kerst, Howard, and Gugerty (1987) used a map of a fictitious college and obtained distance judgments, using both magnitude estimation and map sketching. Subjects made judgments in either a Perception, Immediate-memory, or Delayed-memory condition. All groups began with a 10-minute map-study period. After the study period, the map was then placed on a stand within the subject's clear view for Perception subjects or removed for Immediate-memory subjects. In the Delayed-memory condition, subjects studied the map and then left, returning 24 hours later

to make their distance judgments. Subjects were presented with the names of two buildings and judged the "Straightline or direct distance between them relative to the scale of the map." Perception judgments were reliably larger and more accurate ($\gamma = 1.09$, $r^2 = .98$) than either the Immediate-memory ($\eta = 0.77$, $r^2 = .83$) or Delayed-memory $(\eta = 0.66, r^2 = .77)$ judgments in both the distance estimation and map-sketching conditions. These results do not support the reperceptual hypothesis because the square of 1.09 does not equal 0.77; in fact, the trends are in the direction opposite to that predicted by the reperceptual hypothesis. The interesting comparison, and one that Kerst et al. (1987) failed to follow up, is between the Immediateand the Delayed-memory groups. The decrease in n between Immediate- and Delayed-memory groups suggests that η , at least for two-dimensional distance on a map, may change over time.

DaSilva, Ruiz, and Marques (1987) examined inferred, remembered, and perceived geographical map distance. Students in Brazil gave magnitude estimates of the distances between the capitals of Brazilian states. There were two sessions, one a replication of the other, which occurred 1 month apart. Subjects made their judgments in either a Perception, a Memory, or an Inferred condition. Perception subjects made judgments while viewing a map, Memory subjects made judgments based on memory of a map they studied immediately prior to testing, and Inferred subjects made judgments based on their general geographic knowledge. The exponents were relatively constant from the first to the second session ($\gamma = 1.05$, 1.06 for Perception; $\eta = 0.86, 0.72$ for Memory; and $\eta = 0.76, 0.68$ for Inferred). Consistently with Kerst et al. (1987), but not supportive of Kest and Howard (1978), η for remembered distance did not equal the square of γ for perceived distance. In a similar study, DaSilva, Margues, and Ruiz (1987) examined inferred, remembered, and perceived area. Students in Brazil made magnitude estimates of areas of Brazilian states. The Memory exponent ($\eta = 0.63$) approximated the square of the Perception exponent ($\gamma = 0.79$). As with distance, for area the Inferred exponent ($\eta = 0.40$) was less than the Perception or Memory exponents. A second experiment replicated these results with a 1-month interval between two sessions ($\gamma = 0.84, 0.81$ for Perception; $\eta = 0.60, 0.64$ for Memory; and $\eta = 0.51$, 0.49 for Inferred). These results for area are consistent with the reperceptual hypothesis.

The implications of inferred distance and area for memory psychophysics are not clear, nor is it clear how theories of memory psychophysics should be constrained, if at all, by findings in such inferred conditions. Studies using an inferred condition have considered responses of remembered distance and area to be based on general geographic knowledge, but the form of this general knowledge is rarely controlled or examined. Inferred conditions are operationally distinguished from memory or perception conditions by the subjects not being shown the stimuli at any time. Unfortunately, the origin of inferred knowledge is not known. Inferred responses could come from semantic memory (e.g., subjects were previously taught that City A is x number of miles or kilometers from City B, State C is y square miles or square kilometers in area, etc.), and in this case subjects' responses would be based on an abstract knowledge structure. Alternatively, subjects may visualize maps or other stimuli they have seen and attempt to read distance or other metric information off of these images. In any event, the unconstrained nature of inferred distance renders it useless in delimiting a psychophysical theory of mental representation.

The experiments involving remembered distance and area tested at a single delay (although with differing delays across studies) are summarized in Table 1. For some dimensions, such as area, it appears that the relationship between perceived and remembered extent is adequately described by the reperceptual hypothesis, at least for the brief memory delay between the removal of a map and the judging of that extent. An alternative hypothesis, the gradual-transformation model, suggested by the lower exponent obtained by the delayed-memory group of Kerst et al. (1987) is that η decreases as time from the percept increases. While the results of DaSilva and Fukusima (1986) show that γ is relatively constant in perception, the stability of β or η in memory has not yet been addressed. The difference in η between the Immediate-memory and the Delayed-memory conditions is very suggestive; it may be that the relationship between γ and η found by Kerst and Howard (1978) is simply an artifact of the time interval between their Perception and Memory conditions. If η were tracked over a longer range of time, then its true relationship to y might be revealed better. For other dimensions, such as distance, the case is not so clear. The only study using distance in three dimensions (depth) reports data

Table 1. Exponents for perceived and remembered distance and area

	Distance		Area	
	Perception	Memory	Perception	Memory
Moyer et al. (1978)	0.87	0.70	0.64	0.46
Kerst & Howard (1978)	1.04	1.10	0.79	0.60
Bradley & Vido (1984)	0.81	0.60	*	*
Algom, Wolf, & Bergman (1985)	*	*	0.76	0.66
DaSilva, Ruiz, & Margues (1987), Session 1	1.05	0.86	*	*
DaSilva, Ruiz, & Marques (1987), Session 2	1.06	0.72	*	*
DaSilva, Marques, & Ruiz (1987), Exp. 1	*	*	0.79	0.63
DaSilva, Marques, & Ruiz (1987), Exp. 2, Ses. 1	*	*	0.84	0.60
DaSilva, Marques, & Ruiz (1987), Exp. 2, Ses. 2	*	*	0.81	0.64

Note. An asterisk is used when that condition was not included within an experiment

consistent with the reperceptual hypothesis (Bradley & Vido, 1984), while other studies using a representation of distance (i.e., length) reveal data both consistent (Kerst & Howard, 1978; Moyer et al., 1978) and inconsistent (Da-Silva, Ruiz, & Marques, 1987; Kerst et al. 1987) with the reperceptual hypothesis.

Temporal aspects of remembered distance and area

The Kerst et al. (1987) experiment is consistent with the idea that the memory exponent for distance may decline as the time from exposure to the perceived stimulus increases, as the exponent at 1 week was lower than the exponent after only 1 day. Further evidence that the memory exponent may decline with the passage of time is found in a study by Kemp (1988) examining the memory exponent for area. In his first experiment, Kemp included a perception group, three different memory groups, and a control group. Perception subjects looked at maps of European countries when they made their judgments of the areas of those countries. Memory subjects were given the same maps and allowed to study them for 10 minutes; the maps were then removed and subjects made their area judgments after either 2 minutes, 90 minutes, or 1 week had elapsed. The control subjects were similar to the Inferred groups used by DaSilva and his colleagues and made judgments without having viewed the maps.

The exponents Kemp (1988) obtained for the perceptual group, three memory groups, and control group were 0.82, 0.67, 0.65, 0.54, and 0.43, respectively. Significant differences between the memory exponents and the perception exponent and between the memory exponents and the control exponent were found. The point of note in the data is that there is a trend for the memory exponent to decline as the time from the initial perception increases. A similar pattern was found when the stimulus map was switched to a configuration of yellow on a red background. Kemp interpreted the data as suggesting that two factors may possibly influence the level of the memory exponents. One factor is uncertainty, whose presence is reflected in slight declines in the correlation coefficients describing the fit of the power function across the memory groups. Another factor is a gradual transformation of the map in memory. Such a transformation is different from that proposed by the reperceptual hypothesis, as implicit in the reperceptual hypothesis is the idea that the memory is veridical until recalled, and then once recalled, it is scaled again according to y.

As is summarized in Table 2, the Kerst et al. (1987) and Kemp (1988) studies suggest that the memory exponents for distance and area may decline with the passage of time. This possibility was systematically evaluated in Hubbard (1988) when the psychophysical functions for distance, height, and area were measured at eight temporal intervals: Perception, Immediate-, 6-Hour, 12-Hour, 1-Day, 2-Day, 1-Week, and 2-Week memory. All subjects learned to associate examples of each dimension with letter names and then made estimates of the magnitude of each stimulus when the experimenter gave the letter name associated with each stimulus. The mean exponent for each dimension at

Table 2. Exponents for perceived and remembered magnitude at multiple delays

	Condition	Dimension	Exponent
Kerst, Howard, & Gugerty (1987)	Perception	Distance	1.09
Kerst, Howard, & Gugerty (1987)	Imm. Memory	Distance	0.77
Kerst, Howard, & Gugerty (1987)	1-week Memory	Distance	0.66
Kemp (1988)	Perception	Area	0.82
Kemp (1988)	2-minute Memory	Area	0.67
Kemp (1988)	90-minute Memory	Area	0.65
Kemp (1988)	1-week Memory	Area	0.54

Table 3. Exponents for perceived and remembered distance, height and area in Hubbard (1988)

Condition	Distance	Height	Area
Perception	0.98 ± 0.26	1.09 ± 0.22	0.85 ± 0.21
Immediate Memory	0.91 ± 0.17	0.97 ± 0.15	0.82 ± 0.24
6-hour Memory	0.91 ± 0.16	0.89 ± 0.16	0.72 ± 0.12
12-hour Memory	0.94 ± 0.17	0.94 ± 0.13	0.76 ± 0.13
1-day Memory	0.97 ± 0.28	0.95 ± 0.08	0.75 ± 0.12
2-day Memory	0.96 ± 0.14	0.94 ± 0.17	0.79 ± 0.25
1-week Memory	0.87 ± 0.15	0.80 ± 0.14	0.75 ± 0.18
2-week Memory	0.89 ± 0.18	0.79 ± 0.13	0.74 ± 0.20

Note. The first number in columns 2-4 represents the mean exponent for that function and the second number represents the standard deviation of that mean exponent

each delay is shown in Table 3. The values for γ are similar to those reported in the psychophysical literature (e.g., Baird, 1970; Teghtsoonian, 1965; Weist & Bell, 1985). The Time effect is significant, but is driven primarily by the presence of γ . When only ηs are examined, neither the Time effect nor the Time \times Dimension interaction attains significance. Nonetheless, there is a trend for η to decline with time, similar to the trend in Kemp (1988) and Kerst et al. (1987). Furthermore, this pattern is found both when γ is approximately equal to 1 (e.g., distance) and when γ is greater than 1 (e.g., height) – trends not consistent with the reperceptual hypothesis.

The data from Hubbard (1988) and Kemp (1988) question the broad validity of the reperceptual model on two counts: (a) the memory exponent appears, albeit nonsignificantly, to decline systematically over time (up to a 1-week delay), suggesting that the apparent relationship between perception and memory exponents found previously may have been an artifact of the particular delays used; (b) the reperceptual hypothesis predicts that dimensions with perceptual exponents equal to 1 should yield memory exponents of 1, and that dimensions with perceptual exponents larger than 1 should yield memory exponents larger than the perceptual exponents. As shown in Tables 2 and 3, this was not found. Unless it is posited that the reperceptual hypothesis holds only within limited boundary conditions (e.g., immediate memory, the presence of substantial retrieval cues, etc.), the data are not consistent with the reperceptual notion.

In Hubbard's first experiment (1988), free-modulus instructions were used, that is, subjects were free to select their own modulus, and therefore the scale chosen by each subject was arbitrary. In Kemp (1988), prescribed-modulus instructions were used (UK was 100 units). The precise form of the instructions is interesting because in a partial replication of Hubbard (1988) a very specific form of prescribed-modulus instructions was used; subjects estimated distance in feet, height in inches, and area in square inches. With specification of a familiar unit as the response scale, the apparent decline witnessed previously did not occur; no difference between perception and 1-week memory conditions was found. Since instructions were not given until the time of retrieval, the form of the instructions obviously cannot have affected encoding or retention. The apparent declines seen earlier may have resulted from a response bias because apparently the information for an accurate (i.e., equivalent to perception) judgment can be retrieved (or reconstructed) if subjects are told the appropriate units in which to respond. Somehow the instructions to use particular units appears to make additional information more available to the subjects. This issue is currently under further investigation.

Although data from several investigators (e.g., Hubbard, 1988; Kemp, 1988; Kerst et al., 1987) are consistent with the notion that the memory exponent may decline slightly with time, the data do not rule out an obvious confound: the longer the retention interval, the more opportunities there are for rehearsal of the stimulus. If any type of reperceptual mechanism scales the remembered intensity upon retrieval and that new intensity value is subsequently encoded into memory, replacing the original value, then we would expect remembered intensity to change as a function of the number of times the stimulus was recalled and rehearsed. If, however, the remembered value is scaled upon retrieval, but does not overwrite the original memory, then no effect of number of rehearsals should be seen.

The role of rehearsal was examined in Hubbard (1988) by comparing the memory exponents of subjects who rehearsed the stimuli nonverbally (i.e., by repeatedly visualizing the stimuli) with the memory exponents of subjects who did not rehearse the stimuli. After subjects had learned the letter names of the stimuli, they were taken from the experimental area to a different room. Rehearsal subjects were asked repeatedly to form visual images of how the objects looked. Distractor subjects were given a verbal task so that they would not be able to rehearse the objects. They were asked to write down as many words as they could that could be formed out of the letters of either the word PARTICIPATION or the word ENCYCLOPEDIA. For both Rehearsal and Distractor groups, the experimenter returned after 20 minutes had elapsed and subjects then made their memory-magnitude judgments. There were no significant differences between memory exponents in the two groups, nor did the number of derived words systematically correlate with any of the memory exponents for the distractor group.

In a related experiment, Chew and Richardson (1980) allowed subjects to study a map before collecting either perceptual or immediate-memory judgments of the relative sizes of European, African, and Asian nations. The ex-

ponents obtained were consistent with those predicted by the reperceptual hypothesis, leading Chew and Richardson to suggest that although uncertainty and forgetting may affect the overall performance (i.e., the exponent), these two factors do not affect the relationship postulated between the exponents obtained in perceptual and memorial judgments. These conclusions are untenable, however, because uncertainty or forgetting could not have taken place. Subjects were allowed to study the map as long as they needed, and were then tested in either a perceptual or an immediate-memory condition. There was no opportunity or time in which any significant forgetting could occur. In order to draw any conclusions about the effects of forgetting or uncertainty on the memory exponents, Chew and Richardson needed to examine at least two memory conditions, one of which should have been designed to induce (or at least allow for) forgetting.

To this point we have examined two models for the memory exponents; the reperceptual hypothesis and a gradual-transformation hypothesis. The reperceptual model suggests that a stimulus is scaled both upon perception and again upon subsequent recall; furthermore, the scaling factor is the same for both perception and memory. The reperceptual hypothesis implies that no additional scaling or change occurs to the remembered magnitude during the time interval in which the information is stored in memory but is not accessed. The gradual-transformation model, on the other hand, does not suggest that the stimulus is rescaled upon recall, but it does suggest that once the stimulus is stored in memory there may be a gradual change (typically a decrease) in the level of the remembered magnitude.

A third model, the uncertainty model, also suggests that the representation may undergo change during the period of storage, but the nature of the change is different from that specified by the reperceptual and gradual-transformation models. Unlike the reperceptual model, the uncertainty model does not specify the scaling (if any) that is applied to the stimulus upon retrieval and the uncertainty model also suggests that changes in the representation occur during the period of storage. In the gradual-transformation model, all of the stimuli in a given set are transformed in a similar fashion (e.g., remembered as a little smaller or further). Presumably all stimuli would be transformed in the same direction and relative differences between the exemplars would be maintained. In the uncertainty model, however, all of the stimuli in a given set need not be transformed in a similar fashion, and the relative differences between the stimuli need not be maintained. The effect of these different types of transformation operating upon the stimuli would serve to increase the uncertainty by diminishing (e.g., decreasing or blurring) the differences between the stimuli.

The uncertainty model takes two forms. The first form suggests that the response range decreases over time and the second form suggests that the stimulus range increases over time. The effect of both of these changes in range would be to lower the exponent. In the first case, reduction in the response range is consistent with the idea that a regression effect of sorts occurs in memory (i.e., smaller objects would be remembered as larger and larger objects would be remembered as smaller). If this were so, then the

perception and memory functions should pivot around a point in the approximate center of the stimulus intensity range. However, published data (e.g., Moyer et al., 1978) seem to suggest a pivot point near one of the ends of the stimulus scale rather than in the middle of the stimulus scale, a pattern inconsistent with regression. There is a bit more support for the second form of the uncertainty model, as Teghtsoonian (1971, 1973) has presented evidence that stimulus range and size of the perceptual exponent are significantly negatively correlated; increases in stimulus range result in lower values of perceptual exponents. The decline in memory exponents may thus be due to this more general psychophysical principle rather than to any property of memory representations per se (see also Algom, Wolf, & Bergman, 1985). Uncertainty seems intuitively to be related to the complexity of the stimulus as well as to the stimulus intensity range, so we shall now consider memory exponents of more complex multidimensional stimuli.

Integration of stimulus dimensions in memory psychophysics

While few studies have systematically examined the behavior of memory exponents over time for a single stimulus dimension, even fewer studies have examined the behaviors of memory exponents for several different stimulus dimensions over time. This is an interesting issue, however, inasmuch as different dimensions can be made either dependent or independent of each other in a given set of stimuli and most of our real-world experiences involve multidimensional stimuli. As an example, consider the following sort of experiment: subjects estimate from memory the heights, widths, and areas of previously learned square and rectangular stimuli. To an arbitrary extent, height and width can be made independent of each other, but area is determined by both height and width, specifically Area = Height \times Width. The question then becomes: how do the representations of height and width combine to produce a perceptual (or a memorial) representation of area?

The first experiment in Hubbard (1988) reported above suggests that height may decline at a different rate than area, yet changes in perceived or remembered area must be determined to some extent by changes in perceived or remembered height and width. If the representations of different dimensions changed at different rates, then after some amount of time the intensities portrayed by some dimensions (e.g., height and width), when multiplied together, might not equal the intensity portrayed by another dimension (e.g., area) of which they are factors. This sort of disassociation of the dimensions or elements of a stimulus display could result in a noneuclidean representation of the stimulus. Indeed, such noneuclidean representations have already been reported in the cognitive mapping literature (e.g., Baird, Wagner, & Noma, 1982; Moar & Bower, 1983). One outcome of such a differential change would be that after extensive periods of time have passed, and each of the dimensions changed according to its own particular decay function, then an object created by the sampling of remembered values along each of the encoded dimensions would not in fact remotely resemble the original object at all!

This sort of approach suggests that subjects maintain separate memory traces for each of the stimulus dimensions and that they retrieve these traces when asked to remember stimulus magnitudes. While this is perhaps a reasonable suggestion for the scaling of displays involving only one stimulus factor, it may not be so reasonable with more complex displays. For example, Algom and his colleagues (Algom et al., 1985; Wolf & Algom, 1987) provided substantial evidence that when subjects provide magnitude judgments for both perceived and remembered area, their judgments correspond closely to those predicted by a multiplicative strategy whereby subjects determine area by multiplying height and width. Algom et al. (1985) presented a series of experiments that examined the magnitudes of perceived and recalled length and area. In one experiment, subjects first learned nonsense-syllable names for rectangles of several different sizes. The rectangular stimuli had been created by factorially combining four different heights and widths. As might be expected by now, the exponents for the Memory group ($\eta = 0.66$, $r^2 = .98$) were lower than those for the Perception group ($\gamma = 0.76$, r^2 = .98). The size estimates for both groups also showed a strong Height × Width interaction, indicating the use of a multiplying rule by the adults in their subject pool (for developmental aspects of the multiplicative rule for remembered and perceived area, see Wolf & Algom, 1987).

In further experiments in Algom et al. (1985) the experimental tasks required combining both perceptual and memorial components within the same scaling task. For example, in one experiment subjects learned nonsensesyllable names for each of several vertical lines. Subjects were then shown a horizontal line, given the name of a previously studied vertical line, and asked what the area of a rectangle formed by the perceived horizontal line and the imagined vertical line would be. The psychophysical exponent obtained under these conditions was closer to the value of the memory exponent, despite the fact that one of the dimensions was perceived, leading Algom et al. to conclude that memorial values dominate over perceptual values whenever perceptual and memorial information is combined in a single judgment. Again, the Height × Width interaction was found, indicating the subjects' use of a multiplicative strategy. Furthermore, similarity of the data for both memory and perception conditions in all of their experiments led Algom et al. to conclude that subjects used the same "cognitive algebra" for both conditions.

The evidence that subjects may use a multiplicative strategy suggests that psychophysical scaling of complex multidimensional stimuli may be difficult for classical psychophysics as well as for memory psychophysics. It may well be the case that scaling techniques such as magnitude estimation become increasingly ineffective as stimulus dimensionality increases. Thus, it is not clear how psychophysical scaling can further resolve the questions of whether the mental representations of normal euclidean objects become increasingly noneuclidean with the passage of time and whether the differential changing of the exponents of each of the portrayed dimensions is a valid finding. Given that most objects encountered outside of the

psychological laboratory consist of many dimensions, it is not clear how to proceed.

In Algom et al.'s (1985) experiments the perceptual and memorial representations were of stimuli in the same modality, that is, visual lines of varying length. There is a wealth of data in classical psychophysics describing how subjects are able to equate and scale stimuli that are from two different modalities. Could subjects also complete a scaling task in which the magnitudes for at least one of the modalities are drawn from memory? Hubbard (1993) reported two cross-modality scaling experiments using brightness and loudness. In both experiments subjects learned nonsense-syllable names for each of five intensity levels for each dimension. In one experiment, one stimulus was perceived and subjects estimated the magnitude that a stimulus in the second dimension would have to possess in order to match the magnitude of the first stimulus, and in the second experiment the nonsense-syllable name of a stimulus in one dimension was given and subjects had to imagine that stimulus and then estimate the magnitude that a stimulus in the second dimension would have to possess in order to match the magnitude portrayed in their image of the stimulus in the first dimension. Subjects also completed a remembered magnitude task in which they were told the nonsense-syllable name of each stimulus and asked to estimate that stimulus magnitude.

When remembered brightness was cross-modally matched against perceived loudness, the resultant power function closely resembled that traditionally found for perception of the perceived loudness ($\eta = 0.41$); similarly, when remembered loudness was cross-modally matched against perceived brightness, the resultant power function closely resembled that traditionally found for perceived brightness ($\eta = 0.30$). These values were significantly different from the values predicted from the calculation of the ratio of the perceptual exponents for the two dimensions, however. When remembered brightness was crossmodally matched against remembered loudness, the resultant power function closely resembled that traditionally found for perceived loudness ($\eta = 0.46$); similarly, when remembered loudness was cross-modally matched against remembered brightness, the resultant power function closely resembled that traditionally found for perceived brightness ($\eta = 0.31$). In neither cross-modal case did the exponents obtain the predicted values; instead, the value of the exponent seemed to be similar to the value of the exponent of the function relating perceived magnitude to stimulus intensity for the stimulus dimension functioning as the independent variable. Additionally, exponents in the remembered magnitude task did not differ significantly from exponents from a control group judging perceived intensity, offering further evidence that the reperceptual hypothesis does not offer an adequate explanation of the value of remembered magnitude.

Problems confronting research in memory psychophysics

Despite the apparent success of many studies in documenting the existence of psychophysical-type functions relating physical intensity and remembered magnitude of stimuli, there are a number of difficulties that must be addressed. The first involves the conventions used in naming the stimuli to be remembered. In many studies (e.g., Kerst & Howard, 1978; Moyer et al., 1978; DaSilva et al., 1987) the stimuli involved the names of countries, states, or objects with which the subjects would have been familiar with prior to the experiment; in fact, this familiarity was relied upon in several instances. If the stimuli are drawn from such naturally occurring examples, there is a danger that the names of the stimuli may include sufficient ordinal information as to render precise (nonverbal) memory for stimuli along the judged dimension unnecessary. For example, geographically knowledgeable American subjects should know that Texas is larger than Colorado, Colorado is larger than Connecticut, and Connecticut is larger than Rhode Island. Such abstract or semantic ordinal knowledge alone might be sufficient to allow the subject to generate an increasing monotonic function with a large r^2 value (see Parker, Casey, Ziriax, & Silberberg, 1988) without ever truly tapping memories for the perceptual qualities the experimenter is trying to scale.

It might be possible to avoid this difficulty by creating artificial stimuli that are referred to by nonsense-syllable names (or any other sort of name that does not preserve ordinality). However, the investigator who attempts this course runs into another difficulty: that of memory limitations. Experience in my laboratory (unpublished data) has shown that with artificial stimuli, subjects are often not able to remember more than about 5-7 different items, a result consistent with what we know about the limitations of human information processing (e.g., Miller, 1956). Unfortunately, this is a small number of points for determining a function, and is seen by some as inadequate for the fitting of a function and specification of the exponent. The easiest way around this difficulty is to use stimuli that are already well learned, of which the subject knows many examples, but this leads back to the difficulties discussed in the last paragraph.

Even if artificial stimuli and artificial names are used, however, there still remains the possibility of judgment and verbal encoding of judged values along the dimension(s) of interest by the subject during the phase in which he or she is learning (and thus perceiving) the stimuli. During the subsequent memory phase all that the subject would have to do would be merely to recall the verbal estimate from the prior learning phase and report that value. If this occurs, the subject would not even have to recall the stimulus at all; instead, the subject would recall the content or value of a judgment made during the previous learning phase. This problem may be especially acute if artificial stimuli are used, as the dimension of interest may be the only dimension along which the stimuli differ and hence the purpose of the experiment may become transparent to the subject. If, however, the memory exponent behaves in a systematic fashion across groups this criticism loses some of its impact, as subjects would have no way of knowing the exponents describing the performance of other groups and hence no way of altering their estimates so as to produce exponents of the appropriate relative value.

A potentially better way of dealing with the verballabeling difficulty would be to use stimuli that subjects cannot easily label verbally. There are a number of dimensions in which subjects might not be able to easily label verbally the intensity (or other potentially important characteristics) of the stimuli. For example, Lyman (cited in Intons-Peterson & McDaniel, 1991) presented subjects with either samples of various odors (perception) or the names of various odorants (imagery) and had subjects judge the similarity of the different stimulus pairs. Multidimensional scaling of the judgments revealed that the plot for judgments in the imagery condition was very similar to that in the perception condition. As subjects may not have been as well practiced in labeling odor quality as in labeling dimensions such as visual size and distance, it is reasonable to assume that studies such as Lyman's that use non-labelable stimuli are not as susceptible to a verbal-labeling strategy on the part of subjects and thus offer a purer estimation of memory magnitude or quality than would studies which allowed verbal mediation of memory.

Another difficulty for memory psychophysics concerns a problem that also plagues classical perceptual psychophysics. Is the subject remembering (perceiving) the reported intensity, or is he or she calculating the reported intensity? Evidence that subjects may calculate intensity, especially for stimuli of 2 or 3 dimensions, has been presented in both classical (Teghtsoonian, 1965; Butler & Overshiner, 1983) and memory (Algom et al., 1985) psychophysics. If subjects resort to such strategies, it becomes extremely difficult to know with any certainty that remembered or perceived magnitude is in fact what is being measured. One strategy for dealing with this problem has been to use stimuli that are highly irregular in form, as this irregularity makes simple calculation more difficult. Related to the issue of calculation is the possible effect of different types of instruction on perceived and remembered magnitude. The form of instruction has been found to affect perceived magnitude for area (Teghtsoonian, 1965) and size (Gilinsky, 1955); whether or not such instruction effects can be found in memory psychophysics has not yet been reported.

The problems discussed above are not easily correctable, and some of them may require breakthroughs in methodology before they can be successfully dealt with. One problem that is easily correctable, however, involves the possibility of memory confusions confounding the reported functions. In the majority of the published reports, after subjects have given their judgment, their memory is not tested further (or at least, such further testing is not usually reported). This represents a serious problem, however, as it allows the possibility that estimates given by subjects were based not upon the particular named member of the stimulus set, but upon some other member. For example, when the experimenter asked for the area of "A", the subject might have remembered "B," confused "B" with "A," and reported on "B." The experimenter then recorded this remembered magnitude as that of A when the subject was really remembering B.

One step that would at least limit the opportunity for these types of confusions is to test subjects after they have given their memory judgments. Such testing might involve merely showing the subjects each stimulus, one at a time, and asking them to identify that stimulus. Success at this post-estimation identification task does not guarantee a lack of memory confusions during the estimation phase of the experiment, of course, but difficulties at this point would certainly suggest that confusions during the estimation phase for that particular subject were considerably more likely than might otherwise have been thought to be the case. Methodological prudence would demand that the data from subjects who failed this final test be treated with some skepticism and not averaged in with the data of subjects who did not give such evidence of identification confusion; the elimination of this source of noise might clear up the picture considerably. Unfortunately, there remains the possibility that subjects may not be able to properly identify stimuli precisely because of the changes in their memory for those stimuli that the experimenter is trying to assess, and so elimination of subjects who do not properly identify each of the stimuli after the judgment phase may bias the results away from the detection of actual changes in the representation.

Given all of the difficulties inherent in this line of research, is it realistically possible to do memory psychophysics? Yes, but the issues raised here are ones that need to be considered by future researchers. Stimuli should be chosen such that subjects are not able to label values verbally along the stimulus dimension that is being scaled. The names used to refer to the individual stimuli should not contain implicit cues that could allow penetration of ordinal or other background knowledge. Stimuli should be chosen such that the intensity or the quality being measured should not be easily calculable (e.g., studies of visual size should use stimuli that are irregularly rather than regularly shaped e.g., Attneave figures). Assessment of whether the subject knows the stimuli being scaled should occur both before and after subjects' estimations are collected. If subjects learn artificial stimuli, then a criterion for learning (many published studies have used 300% overlearning) should be adhered to; if naturalistic stimuli are used, then knowledge of the domain in question should be demonstrated (e.g., one might not want to scale remembered areas and distances of political states using data from students who do not know geography). In all cases, subjects need also to be tested after their estimates have been collected. One way to assess subjects' post-estimation knowledge is by testing their recognition of the stimuli; when each stimulus is produced subjects should be able to identify that stimulus correctly. The data of subjects who fail this post-estimate recognition test should not be included in any subsequent analysis (or should be analyzed separately from those of other subjects), as it would not be clear that their estimates reflected sufficiently accurate knowledge of the stimulus dimension being measured. While the questions of memory psychophysics are important ones, and deserving of research, these concerns regarding methodology have not always been fully satisfied.

Psychophysical methods have also been used in studies that are not explicitly identified with scaling or otherwise quantifying remembered magnitude. As an example of magnitude estimation applied in this fashion, we shall next consider an area of cognitive research that has traditionally not been considered psychophysical in method – the study of mental imagery. Although some of the studies reviewed

earlier explicitly instructed subjects to base their memory judgments on a vivid image of the way the stimulus appeared, much of the research involving imagery has not been cast in psychophysical terms. We shall concentrate on studies that examine the portrayal of distance in visual imagery and shall also touch upon the notion of functional equivalence of images and percepts.

Studies of imagery as memory psychophysics

Kosslyn (1978, 1980) examined what he termed the overflow point in visual images. The overflow point was that subjective distance at which an imaged object grew too large to be imaged ("seen") all at once, in a single glance of the "mind's eye." Kosslyn's basic technique involved giving the subjects the name and size of some object (various experiments used animals, featureless black rectangles, and other sorts of miscellaneous stimuli) and having subjects form an image of what that object looked like. Subjects were instructed to imagine the object as if it were seen at a distance and at a small subjective size1. After subjects had formed an image of the object, they imagined approaching the object until they reached the point at which the imaged object occupied the whole of their imaginal visual field; subjects then estimated the distance away from them that the object was portrayed to be. Kosslyn found a linear relationship between the distance at which an object overflowed the imaged visual field and the stated size of the referent object along its largest axis, and from this he concluded that the "mind's eye" has a maximal extent (maximal visual angle) that is relatively constant. By collecting judgments of the distance at which various objects overflowed the imaginal visual field, Kosslyn was using a variant of magnitude estimation in which the stimulus was the visual image of the object and the magnitude was the distance away from the subject that the object in the image was portrayed to be. The point of note is that his use of a psychophysical scaling technique to examine a cognitive construct produced new data that led to theoretical breakthroughs that might not have been otherwise attainable.

Kosslyn's work on the overflow concept has been followed up by Hubbard and Baird (1988, 1993; Hubbard, Kall, & Baird, 1989; Baird & Hubbard, 1992) who examined whether magnitude estimation could reveal other characteristics of how distance was portrayed in visual images. Hubbard et al. (1989) looked at the first-sight distance, that is, the distance at which an object is initially imaged², by having subjects estimate the distance that was portrayed in the initial untransformed image of an object, and they found that the relationship between object size and imaged distance was a power function with an exponent substantially less than 1. While substantial metric information was indeed present in an untransformed image, that information was not by itself sufficient to produce a linear function, and hence the linear-overflow function that Kosslyn reported must have been due to some other factor (e.g., properties of the imagery medium).

Hubbard and Baird (1988) then reversed part of the procedure used by Kosslyn – instead of having subjects mentally approach the object portrayed in their images, they had subjects mentally back away from the objects.

Subjects stopped backing away when they reached that point at which the object had grown so small that it was just barely identifiable. Hubbard and Baird named this the vanishing-point distance. It had been predicted that the distance estimates given by the subjects of the vanishing-point should have resulted in a linear function between estimated vanishing-point distance and stated object size; instead, a power function with an exponent substantially less than 1 was found, thus calling into question theoretical assertions dealing with the minimum resolution (i. e., grain) available in visual images.

Hubbard and Baird (1993) have further demonstrated the usefulness of psychophysical tools in the study of the portrayal of distance in visual imagery. The vanishing-point can be considered a type of threshold; distances closer than the vanishing-point distance are perceivable (can be portrayed) within the image, while distances greater than the vanishing-point distance are not perceivable (cannot be portrayed) within the image. Classical psychophysical theory has noted the presence of two types of bias that arise when any type of threshold is measured - errors of habituation and errors of expectation or anticipation³. One of the traditional ways of coping with errors of anticipation or habituation is to measure the threshold from both directions, with stimuli beginning at subthreshold intensities and increasing in intensity, and with stimuli beginning at suprathreshold intensities and decreasing in intensity, and take an average value. Another way of coping with these biases is to use catch trials (i.e., trials in which a stimulus is

¹ By subjective size is meant the relative visual angle that the imaged object would subsume, a notion similar to the idea of proximal size. To make this notion clearer, consider that an object of constant size will occupy different visual angles, depending on its distance from the observer. All other things being equal, the object will occupy a larger visual angle (a larger proportion of the subject's visual field, i.e., greater subjective size) with closer distances and a smaller visual angle with further distances (a smaller proportion of the subject's visual field, i.e., smaller subjective size). Similarly, a given visual angle may be occupied by either a small object relatively close to the observer or a large object relatively far from the observer. This relationship between the size of the object, the distance of the object from the observer, and the visual angle subsumed by that object has been referred to as the size-distance invariance hypothesis (see Epstein, Park, & Casey, 1961; Sedgwick, 1986).

² One semantic issue should be clarified here, the idea of imaged distance. "Imaged distance" in this context refers to the distance to an object that is portrayed in the image; it does not refer to any sort of attempt to project the image outward into the surrounding environment some arbitrary distance away from the imager.

³ The nature of these biases will be clearer in the context of an example, so consider the following threshold experiment. The subjects hear a series of tones of decreasing intensity and respond "yes" as long as a tone is perceived and "no" when the tones are no longer perceived. Then a series of tones begins at subthreshold levels and gradually increases in loudness, and subjects say "no" when they do not hear the tones and "yes" when they begin to hear the tones. In the decreasing intensity series, an error of habituation involves a subject continuing to say "yes" after the tone has grown too faint to be heard, and an error of anticipation involves a subject saying "no" while the tone can still be faintly heard. In the increasing intensity series, an error of habituation involves a subject continuing to say "no" after the tone can be faintly heard, and an error of anticipation involves the subject saying "yes" before the tone is really heard.

not actually presented), and a third way of coping with these biases is to use a signal detection analysis which allows separation of sensitivity and criterion-bias factors.

There is a possibility that the measurement of vanishingpoint, a type of threshold, in visual imagery may be biased by either errors of anticipation or errors of habituation. Neither catch trials nor signal-detection methods have yet been adapted for use within a memory psychophysics framework, so Hubbard and Baird (1993) measured imaged vanishing-point from both suprathreshold (closer) and subthreshold (further) distances. For the suprathreshold condition, subjects formed an image of the object and moved away from that object. For the subthreshold condition, subjects visualized a wide open plain. They looked in the direction of where they knew the object to be, but at this initial stage they were too far away from the object for it to be visible in their image. They imagined moving in the direction in which they knew the object to be until they had approached close enough for the object to be just barely visible. There were no significant differences between the suprathreshold and the subthreshold exponents; each was approximately equal to 0.7, values very close to those found in the earlier Hubbard and Baird (1988) study of vanishing-point distance. Again, the point of note in these studies of visual imagery is that the use of techniques borrowed from classical psychophysics allowed proposal and examination of questions that might not have been possible otherwise.

Hubbard and Baird (1993) also examined the effect of clutter on the portraval of distances in visual imagery. On each trial subjects formed an image of a square of a different size and complexity level. The square was located on top of a railroad flatcar, and the flatcar was on railroad tracks that extended in a straight line from the observer's viewpoint all the way out to the horizon. One group of subjects, the Clear group, imaged each square and the accompanying railroad equipment with a wide-open plain on the left and right sides of the track all the way out to the horizon. A second group of subjects, the Clutter group, imaged each square and the accompanying railroad equipment with numerous buildings and warehouses on the left and right sides of the track all the way out to the horizon. When clutter in the form of the additional imaged objects (i.e., buildings and warehouses) located to either side of the primary imaged object was added to the image, the exponent of the vanishing-point function (y) increased slightly and the multiplicative constant (λ) decreased. The presence of only a small amount of clutter seemed necessary, as data from various Partialclutter groups more closely resembled the data from the Clutter group than data from the Clear group. These effects of clutter on the portrayal of distance in visual images are consistent with studies in cognitive psychology suggesting that filled (i.e., cluttered) spaces seem larger or longer than unfilled spaces (e.g., Luria, Kinney, & Weisman, 1967; Pressey, 1974; Thorndyke, 1981).

Functional equivalence of perception and imagery

The phenomenal similarity of perception and imagery and the successful use of classical psychophysical techniques to investigate properties of images is consistent with the hypothesis of functional equivalence, that is, the notion that there are similarities in the cognitive structures, mechanisms, or processes that are invoked by subjects when those subjects engage in either perceptual or cognitive tasks. The functional equivalence hypothesis is typically discussed within the context of imagery and perception (e.g., Finke & Shepard, 1986; Hubbard & Stoeckig, 1992), and moreover, experience in my lab has shown that subjects in memory psychophysics experiments often report an image-like experience even when the instructions do not mention imagery. The strong form of the functional equivalence hypothesis, that perceptions and constructed images actually use the same structures and processes, has been attractive to many theorists (e.g., Finke, 1989; Finke & Shepard, 1986; Kosslyn, 1987), but has not been universally accepted (e.g., Chambers & Reisberg, 1985; Pylyshyn, 1981, 1984).

The clearest example of a strong form of functional equivalence may be found in Kosslyn's (1980, 1981) theory of imagery in which a percept or an image is constructed in a visual buffer and evaluated or analyzed using any of a number of processes. These processes operate in a similar fashion regardless of whether the stimulus portrayed in the buffer arises from long-term memory (imagery) or impinges through the sense channels (perception). Finke (1980) restricts the notion of functional equivalence somewhat by suggesting that the degree of equivalence is dependent upon the level of processing involved, so that higher-level processing (e.g., perceiving/imaging movement) results in more overlap, that is, more equivalence, than lower-level processing (e.g., perceiving/imaging color). Algom et al. (1985) address the notion of functional equivalence from a slightly different tack, as they suggest that judgments of perceived and remembered quantities are computed in the same fashion. Use of the same cognitive algebra, as it were, is, of course, tantamount to a functional equivalence. Numerous neurological investigations also support a degree of functional equivalence between imagery and perception (e.g., Farah, 1988; Farah, Weisberg, Monheit, & Péronnet, 1989; Farah, Péronnet, Gonon, & Giard, 1988; Richardson, 1991).

The equivalence between imagery and perception cannot be a total equivalence, however, if for no other reason than the fact that we can distinguish between imagery and perception under most waking circumstances (but see Johnson, 1988; Johnson & Raye, 1981). A partial, rather than a total, equivalence is also reflected by the fact that the exponents for memory and perception tasks were generally different. Had equivalence been total, identical functions (including identical exponents) would have been found. Additionally, studies examining subjects' ability to detect subparts of figures (Hinton, 1979; Reed, 1974; Reed & Johnsen, 1975) or alternative interpretations of ambiguous stimuli (Chambers & Reisberg, 1985; Reisberg & Chambers, 1991) show that subjects perform worse with imaginal than with perceptual stimuli, a pattern consistent with the notion that imagery and perception might be processed in at least partially different pathways.

Intons-Peterson and McDaniel (1991) suggest that the extent to which imagery appears to resemble perception is related to the familiarity of the stimuli; specifically, when a

task is familiar, that task may activate and utilize knowledge of the subjects, but when a task is unfamiliar and unlikely to elicit real world knowledge, then imagery more closely approximates perception. The literature on image priming is consistent with this, as Hubbard and Stoeckig (1988) found that images of musical tones and chords could successfully prime subsequently perceived harmonically related musical tones and chords (i.e., images were functionally equivalent to percepts), but Stadler and McDaniel (1990) found that judgments concerning characteristics of imaged letters did not prime subsequent judgments on perceived letters (i.e., images were not functionally equivalent to percepts). Attainment of functional equivalence in musical priming by Hubbard and Stoeckig and the failure to obtain functional equivalence in letter priming by Stadler and McDaniel may reflect the fact that musical stimuli were less familiar (or less verbally available) than the letter stimuli (although see Hubbard & Stoeckig, 1988, for a discussion of how real-world knowledge may influence their result). Intons-Peterson and McDaniel (1991) also argue that the extra knowledge available with images of familiar stimuli may enrich the image and modify it in crucial ways. These knowledge-weighted images differ from images that do not utilize such knowledge (Intons-Peterson & Roskos-Ewoldsen, 1989), but it is not clear that such ancillary information should be counted as evidence against functional equivalence because much of perception may also be facilitated by similar penetration of background knowledge (e. g., see Rock, 1983).

What can studies using a memory psychophysical methodology contribute to the discussion of whether a functional equivalence exists between perceptual representations and cognitive representations? Scaling of the relationships between remembered magnitude and physical intensity and between perceived magnitude and physical intensity both result in power functions. The similarity in the forms of the functions offers some limited support for the notion of a partial functional equivalence, although it certainly does not prove such an equivalence. The most that we may infer is that memorial input is processed in a way similar to the processing of perceptual input; the essentially correlational nature of the comparisons of imagery and perceptual performance do not yet allow us to reach the strong causal conclusions that memorial input is processed in the same ways or structures as perceptual input (see also Farah, 1985). While it is possible that different processes operate in imagery and in perception and that those different processes produce similar functions, nonetheless, it is more parsimonious to accept the notion of a partial functional equivalence at some level because it is the simplest explanation to account for the majority of the data (Finke & Shepard, 1986).

Conclusions and future directions

We have reviewed studies dealing with several aspects of research in memory psychophysics. Most of the research has involved the simple scaling of remembered magnitude for isolated dimensions (typically area or distance) after one or two durations of time (typically immediately or after 24 hours). While the data at this time suggest that the power function provides a valid description of remembered magnitude, no single systematic rule for predicting the precise value of the exponent of the power function for each stimulus dimension and at each delay has been determined. In terms of trends, the most that can be said is that for a limited number of dimensions the memory exponent is generally less than, or equal to, the perception exponent. Of course, the bulk of the existent data uses stimulus dimensions with exponents less than, or equal to, 1; the data from memory scaling of dimensions with exponents greater than 1 may indeed show this trend to be an incomplete picture.

The data as a whole suggest that the reperceptual hypothesis is not a valid model for the scaling of remembered magnitude. Although a selected subset of the data is indeed consistent with the reperceptual model, the larger picture shows that this model is insufficient. For example, in Ward (1987) the exponents for remembered loudness were different from those predicted by the reperceptual hypothesis, and in Hubbard (1993) the exponents for remembered brightness and remembered loudness were significantly different from values predicted by the reperceptual hypothesis. The few tests that have been conducted with electric shock and other dimensions with y greater than 1 have also yielded results inconsistent with the reperceptual hypothesis (Algom, 1992a). Nor does the reperceptual hypothesis include any parameter reflecting the time elapsed since the stimulus was perceived, but several studies (e.g., Hubbard, 1988; Kemp, 1988; Kerst et al., 1987) suggest that this parameter may be an important one. The exponents of the functions relating remembered magnitude to physical intensity do not support a simple reperception of the intensity of the physical stimuli (or its logical equivalent – a perception of the mental representation in memory), but neither the gradual-transformation hypothesis nor the uncertainty hypothesis has garnered exclusive or overwhelming support, either.

The studies that support the importance of a temporal parameter are generally consistent with both the gradualtransformation and the uncertainty hypotheses. In the gradual-transformation notion, the precise nature of what it is that gets transformed is far from clear. One possibility is that the mental representation itself changes; another logical possibility is that a memory-scaling factor (β in Equation 3) changes over time, but that the memory representation itself remains unchanged. When the intensity is retrieved, it is scaled by the memory-scaling factor and this results in the difference between remembered and perceived magnitude. Another possibility is that the difference in memory and perception exponents results from greater levels of uncertainty that are assumed to be present in the memory conditions. The greater levels of uncertainty would result in lowered exponents either by a decrease in the response range or by an increase in the stimulus range. The existent data do not support one single theory; rather, the data suggest that many factors may influence the level of the memory exponent. The relationships among these factors, indeed what elements we should even consider as factors, are not yet clear.

The application of psychophysical theory to the study of mental representation and memory can extend beyond the mere scaling of remembered magnitude. By conceptualizing imaged vanishing-point distance as a type of threshold, a new literature within classical psychophysics was made relevant to the study of cognitive representation (Hubbard & Baird, 1993), helping to reveal data about how distance is portraved in visual images. We should be cautioned, however, that the runaway application of psychophysical techniques and theories to all questions concerning cognitive representation would be foolhardy. For example, some perceived dimensions, such as pitch, are not well treated by psychophysical means. The mel scale of pitch (derived from psychophysical scaling, see Stevens & Volkmann, 1940) assumes that pitch is a unitary dimension, but several theorists (e.g., Krumhansl & Shepard, 1979; Shepard, 1982a, 1982b) argue that the representation of pitch is multidimensional. Thus, we would not expect unidimensional psychophysical scaling techniques to be appropriate for studies of the representation of pitch in memory (see Hubbard & Stoeckig, 1992). As a working general rule, it is suggested that if the methods of classical psychophysics are appropriate for the study of a perceived dimension, it may be appropriate to adapt those techniques to studies of the analogous mental continua. If, however, the methods of classical psychophysics are clearly not appropriate for study of a perceived dimension, it is probably inappropriate to adapt those techniques to studies of the analogous mental continua.

The hypothesis of Björkman et al. (1960), that psychophysics can be extended into the realm of memory, has been borne out. Systematic relationships between stimulus intensity and remembered magnitude have been found, and to paraphrase Shepard and Podgorny (1978), the cognitive processes of memory psychophysics certainly do appear at least partially to resemble the perceptual processes of classical psychophysics. The adaptation of scaling methods from classical psychophysics to memory psychophysics for some types of stimuli suggests that cognitive information and perceptual information can interact in a common framework within the same task. It remains for future research to specify more completely the asymmetry between classical and memory psychophysics. Studies in memory psychophysics may not only tell us about the range and usefulness of psychophysical theory, they may also tell us about the cognitive representations underlying the recreation of perceptual experience.

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