Computational Science: Modeling and Simulation

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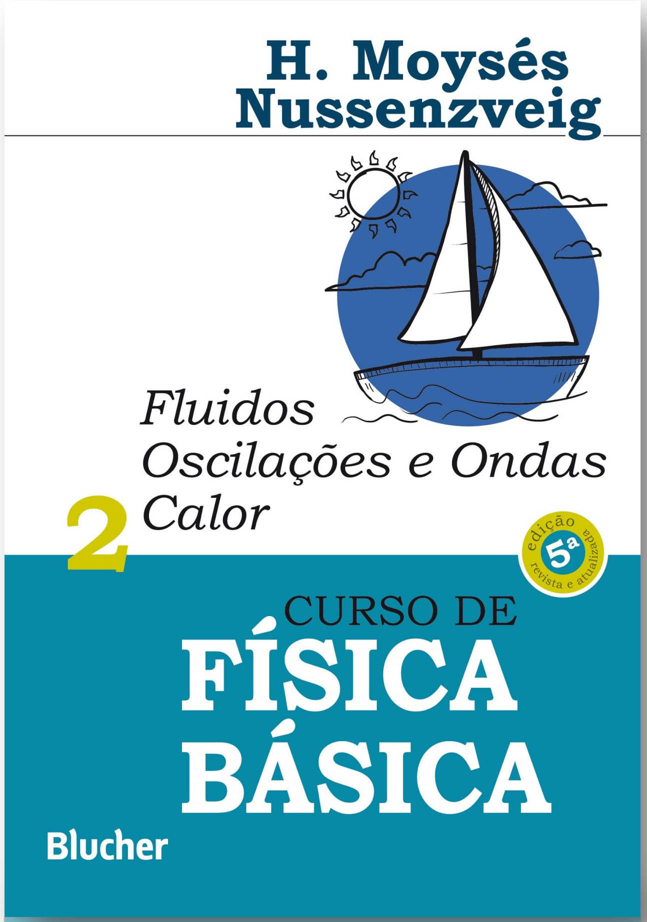
Movimentos oscilatórios

MOVIMENTO HARMÔNICO SIMPLES



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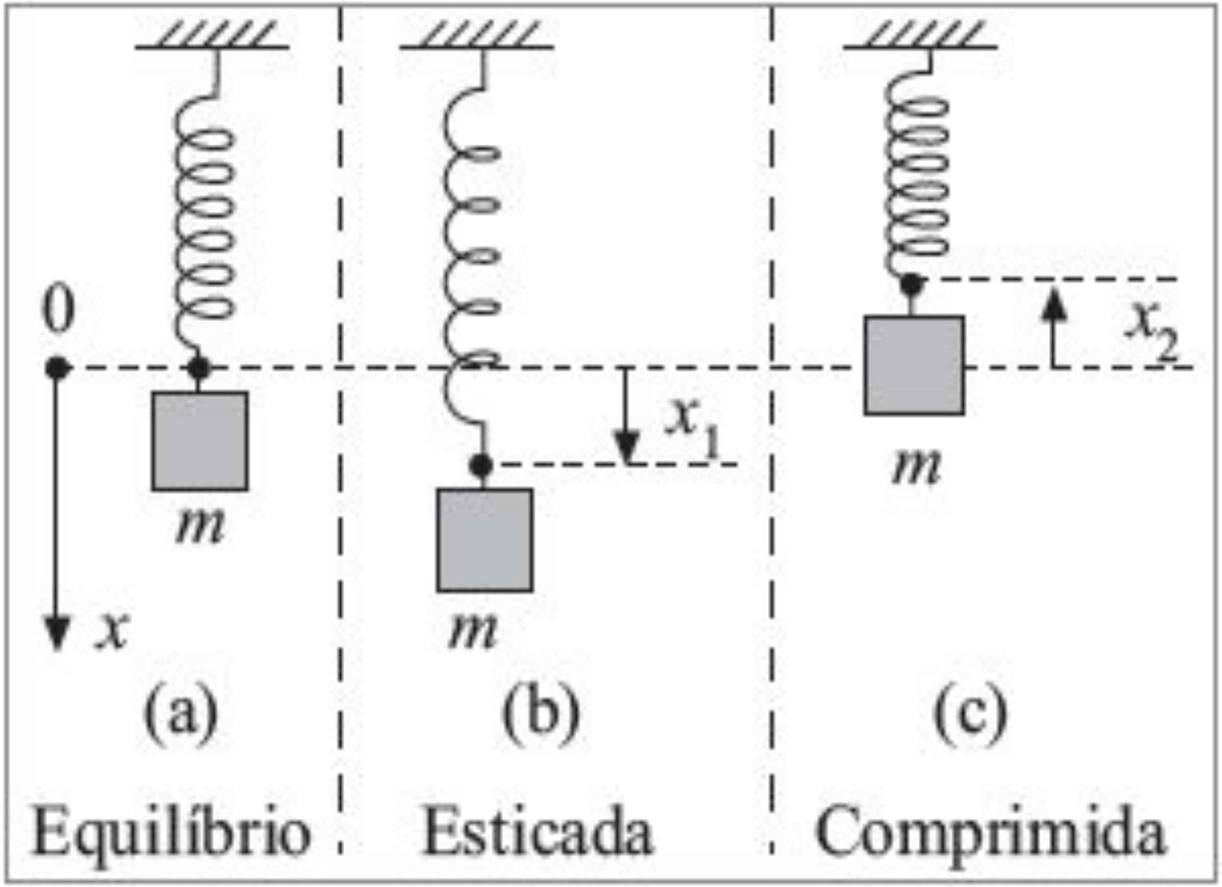

Figura 3.2 Massa e mola.

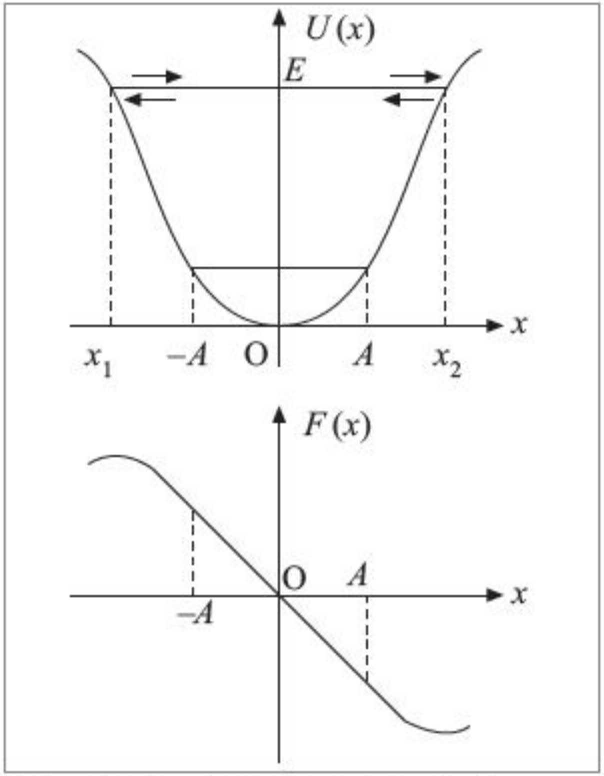

Figura 3.1 Energia potencial *U* e força *F* num movimento oscilatório.

$$F(x) = -\frac{dU}{dx} \qquad (3.1.1)$$

Vimos também que, para *pequenos desvios da posição de equilíbrio estável*, o gráfico de $F(x)$ é aproximadamente linear. Assim, na fig. ao lado, para $-A \leq x \leq A$, temos aproximadamente

$$\boxed{F(x) = -kx} \qquad (3.1.2)$$

$$\boxed{U(x) = \frac{1}{2}kx^2} \qquad (3.1.3)$$

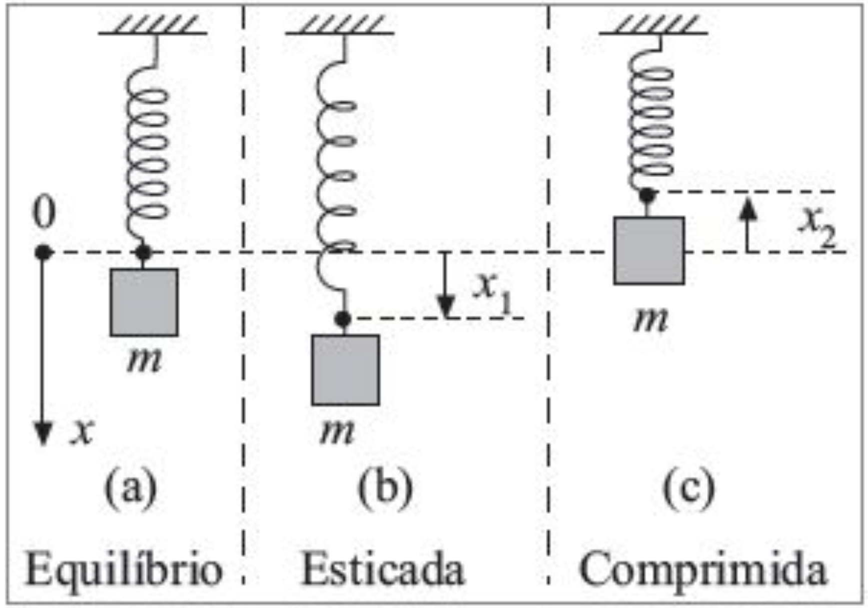

Figura 3.2 Massa e mola.

A equação de movimento correspondente é

$$m\ddot{x} = F(x) = -kx \tag{3.1.4}$$

ou seja,

$$\boxed{\ddot{x} = \frac{d^2x}{dt^2} = -\omega^2 x} \tag{3.1.5}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \tag{3.1.6}$$

(a) *Soluções*

O movimento de um oscilador harmônico chama-se *movimento harmônico simples* (MHS). Para obter a lei horária do MHS, temos de resolver a equação de movimento (3.1.5) em relação à função incógnita $x(t)$. A (3.1.5) é uma *equação diferencial ordinária* para $x(t)$, porque contém derivadas de x em relação a t. Ela é de 2ª *ordem*, porque a derivada mais elevada que aparece é a 2ª.

Para resolver uma equação diferencial mais geral, como a (3.1.5), dadas as condições iniciais, podemos sempre procurar soluções aproximadas por métodos numéricos. Na prática, isto se faz efetivamente, com o auxílio de computadores. Para Δt suficientemente pequeno, podemos aproximar

$$\frac{d^2x}{dt^2}(t) \approx \frac{1}{\Delta t}\left[\frac{dx}{dt}(t+\Delta t) - \frac{dx}{dt}(t)\right] \qquad (3.2.6)$$

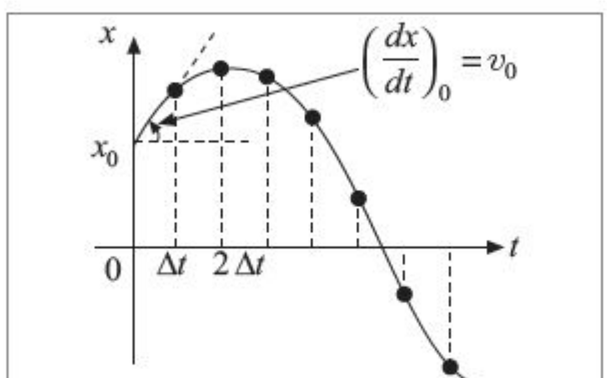

Figura 3.3 Resolução numérica.

A (3.1.5) se escreve

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) = -\omega^2 x \qquad (3.2.10)$$

O gráfico aproximado lembra uma senóide, sugerindo considerar soluções do tipo sen (ct) , cos (ct) onde c é uma constante a ser ajustada . Temos:

$$\frac{d}{dt}[\operatorname{sen}(ct)] = c\cos(ct) \left\{ \frac{d^2}{dt^2}[\operatorname{sen}(ct)] = -c^2 \operatorname{sen}(ct) \right.$$

$$\frac{d}{dt}[\cos(ct)] = -c\operatorname{sen}(ct) \left\{ \frac{d^2}{dt^2}[\cos(ct)] = -c^2 \cos(ct) \right.$$

Logo, tomando $c = \omega$, obtemos as seguintes soluções da (3.1.5):

$$\boxed{\begin{aligned} x_1(t) &= \cos(\omega t) \\ x_2(t) &= \operatorname{sen}(\omega t) \end{aligned}} \qquad (3.2.11)$$

(*b*) *Linearidade e princípio de superposição*

A (3.1.5) é uma equação diferencial *linear,* ou seja, só contém termos *lineares* na função incógnita e suas derivadas : não comparecem termos em x^2 , x^3 , ... , $(dx/dt)^2$, ... , $(d^2x/dt^2)^3$, ... A equação diferencial linear de 2ª ordem mais geral é da forma

$$A \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Cx = F \qquad (3.2.12)$$

onde os coeficientes A , B , C e F não dependem de x, mas poderiam, em geral, depender de t. Na (3.1.5), esses coeficientes são *constantes.* Além disso, a (3.1.5) é uma equação *homogênea,* ou seja, com

$$F = 0 \qquad (3.2.13)$$

Qualquer *equação diferencial linear de* 2ª *ordem homogênea* tem as seguintes propriedades fundamentais, cuja verificação é imediata:

(i) *Se* $x_1(t)$ *e* $x_2(t)$ *são soluções,* $x_1(t) + x_2(t)$ *também é.*

(ii) *Se* $x(t)$ *é solução,* $a\,x(t)$ (a = const.) *também é.*

Note que estes resultados não seriam válidos para equações não-lineares: por exemplo, se o 2º membro da (3.1.5) fosse proporcional a x^2 em lugar de x (verifique!).

Combinando (i) e (ii), vemos que, se $x_1(t)$ e $x_2(t)$ são soluções, qualquer *combinação linear*

$$x(t) = a\,x_1(t) + b\,x_2(t) \tag{3.2.14}$$

onde a e b são constantes arbitrárias, é solução.

Este resultado é uma forma do *princípio de superposição.* Resultados análogos valem para equações diferenciais lineares de ordem qualquer, como é fácil ver.

Uma conseqüência imediata é que, se $x_1(t)$ e $x_2(t)$ são duas soluções *independentes,* ou seja, se $x_2(t)$ não é múltipla de $x_1(t)$, a (3.2.14) é a *solução geral,* pois depende de duas constantes arbitrárias a e b [se $x_2(t)$ fosse múltipla de $x_1(t)$, $x_2(t) = c\,x_1(t)$, a (3.2.14) ficaria : $x(t) = (a + bc)\,x_1(t) = d\,x_1(t)$, ou seja , só teríamos uma constante efetivamente ajustável].

Aplicando a (3.2.14) à (3.2.11), obtemos a forma geral das *oscilações livres do oscilador harmônico :*

$$\boxed{x(t) = a\cos(\omega t) + b\,\mathrm{sen}(\omega t)} \qquad (3.2.15)$$

o que também podemos escrever de outra forma equivalente:

$$\boxed{x(t) = A\cos(\omega t + \varphi)} \qquad (3.2.16)$$

onde as duas constantes arbitrárias passam a ser A e φ.

E fácil obter a relação entre essas duas formas de escrever a solução. Como $\cos(\omega t + \varphi) = \cos(\omega t)\cos\varphi - \operatorname{sen}(\omega t)\operatorname{sen}\varphi$, vem

$$\boxed{a = A\cos\varphi \quad , \quad b = -A\operatorname{sen}\varphi} \qquad (3.2.17)$$

Inversamente, dados a e b, podemos obter A e φ:

$$\boxed{\begin{aligned} &A = \sqrt{a^2 + b^2} \\ &\cos\varphi = \frac{a}{\sqrt{a^2 + b^2}} \quad , \quad \operatorname{sen}\varphi = -\frac{b}{\sqrt{a^2 + b^2}} \end{aligned}} \qquad (3.2.18)$$

o que determina φ a menos de um múltiplo de 2π, ou seja, de forma consistente com a (3.2.16), onde φ só é definido a menos de um múltiplo de 2π.

(*c*) *Interpretação física dos parâmetros*

Pela (3.2.16), vemos que $x\,(t\,)$ oscila entre os valores extremos $-A$ e A (cf. pg. 40). Logo, $A = |\,x\,(\,t\,)\,|_{máx}$ = *amplitude* de oscilação.

Como $\cos\,(\,\omega\, t + \varphi\,)$ é uma função periódica de $\omega\, t$ de período 2π, vemos que o *período* de oscilação é

$$\boxed{\tau = \frac{2\pi}{\omega} = \frac{1}{\nu}} \qquad (3.2.19)$$

onde ν, a *freqüência* de oscilação, se mede em ciclos por segundo ou hertz (Hz). A grandeza $\omega = 2\pi\,\nu$ chama-se *freqüência angular* e se mede em rad/s ou simplesmente s^{-1}. O argumento do coseno na (3.2.16),

$$\boxed{\theta = \omega\, t + \varphi} \qquad (3.2.20)$$

chama-se *fase* do movimento , e φ é a *constante de fase* ou *fase inicial* (valor da fase para $t = 0$).

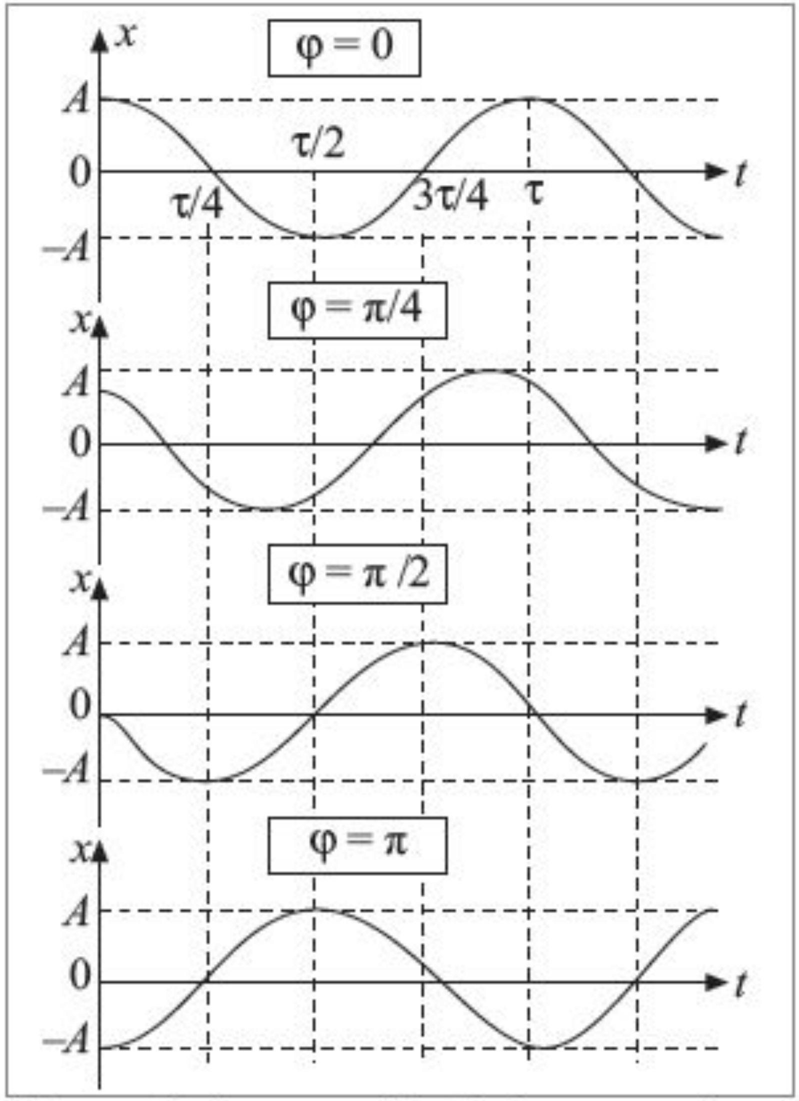

Figura 3.4 Variação da fase inicial.

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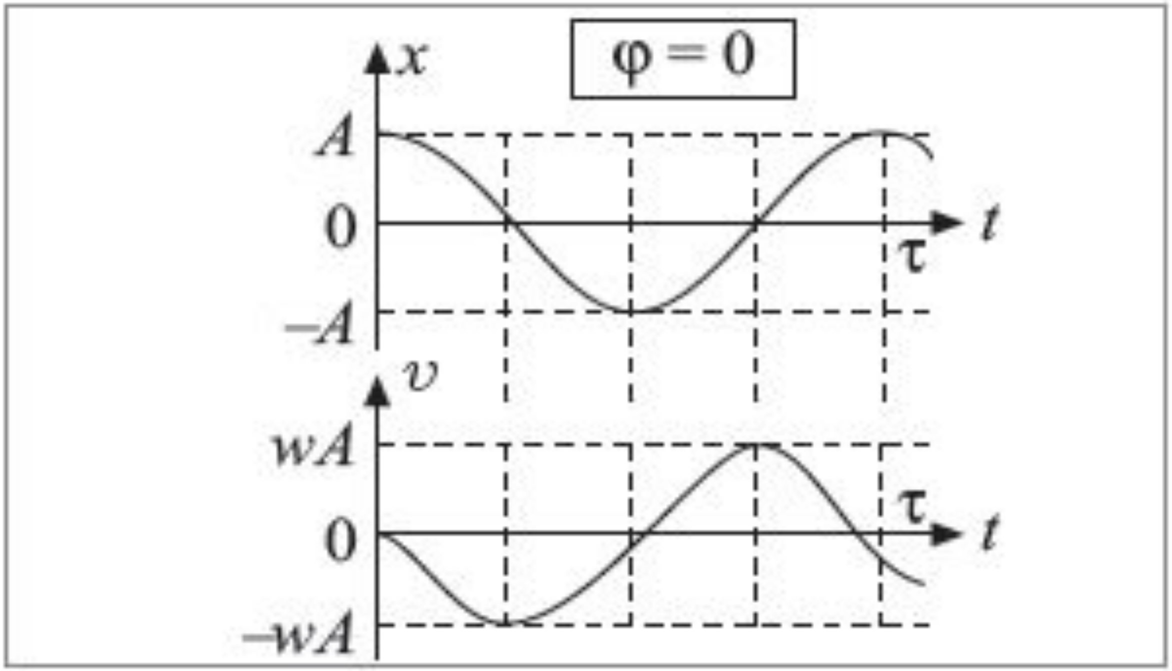


Figura 3.5 Deslocamento e velocidade.

Jupyter Notebooks e Dados

Exercício do laboratório - SHO

Exercício do laboratório - SHO .py

Material complementar

OSCILLATORY MOTIONS

Simple Harmonic Motion

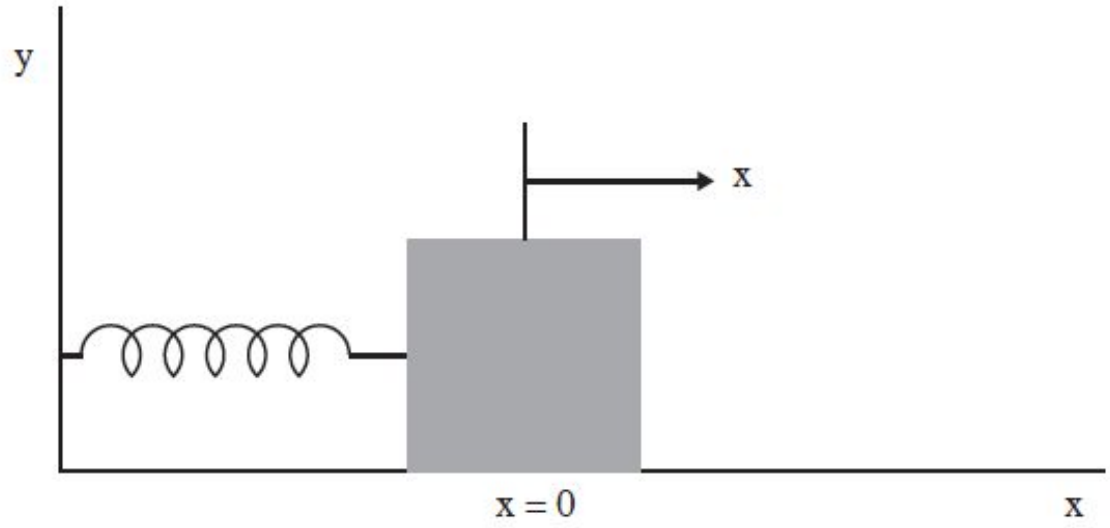


Figure 4.1: A one-dimensional harmonic oscillator. The block slides horizontally on the frictionless surface.

(*e*) *Energia do oscilador*

Pela (3.2.23), a energia cinética do oscilador no instante t é

$$\boxed{T(t) = \frac{1}{2}\, m\dot{x}^2(t) = \frac{1}{2}\, m\omega^2 A^2 \operatorname{sen}^2(\omega t + \varphi)} \tag{3.2.28}$$

e, pelas (3.1.3) e (3.2.16), a energia potencial é

$$\boxed{U(t) = \frac{1}{2}\, kx^2 = \frac{1}{2}\, m\omega^2 x^2 = \frac{1}{2}\, m\omega^2 A^2 \cos^2(\omega t + \varphi)} \tag{3.2.29}$$

onde utilizamos também a (3.2.21).

Somando membro a membro, obtemos a energia total E, que se conserva:

$$\boxed{\frac{1}{2}\, m\dot{x}^2 + \frac{1}{2}\, kx^2 = E = \frac{1}{2}\, m\omega^2 A^2 = \text{const.}} \tag{3.2.30}$$

Vemos que a energia total é proporcional ao quadrado da amplitude, e também ao quadrado da freqüência.

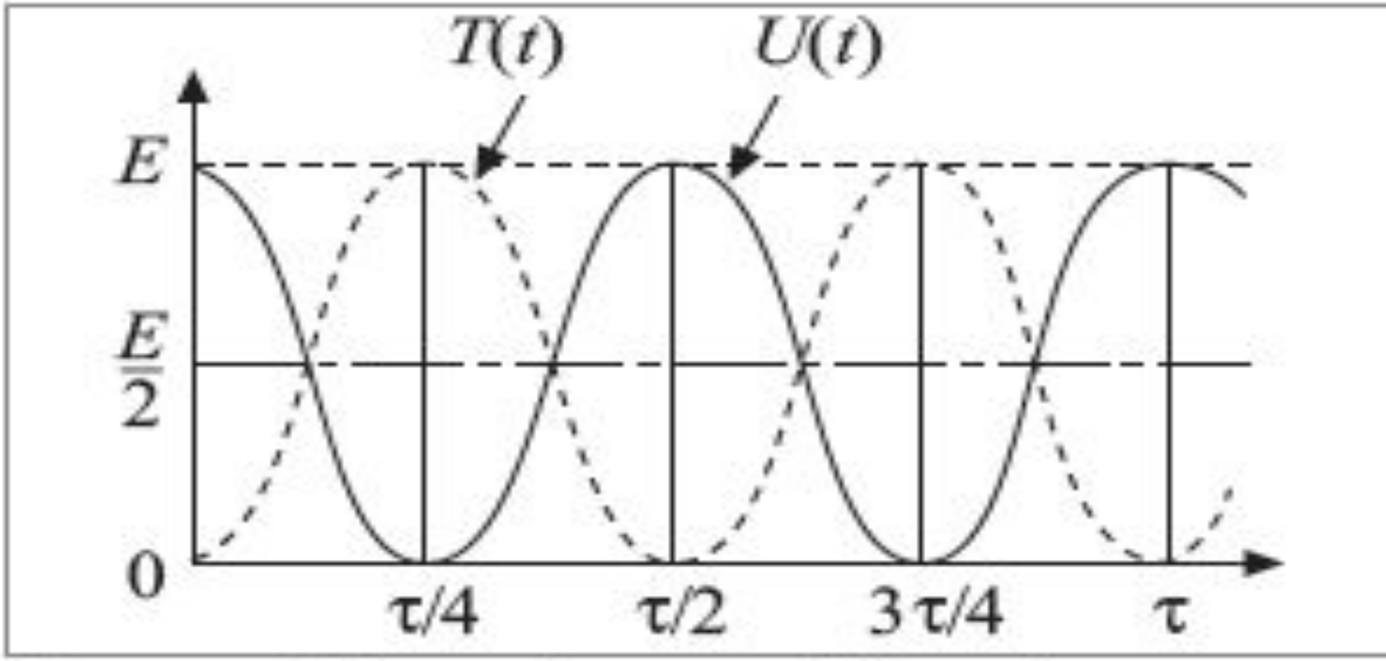


Figura 3.6 Valores médios de T e U.

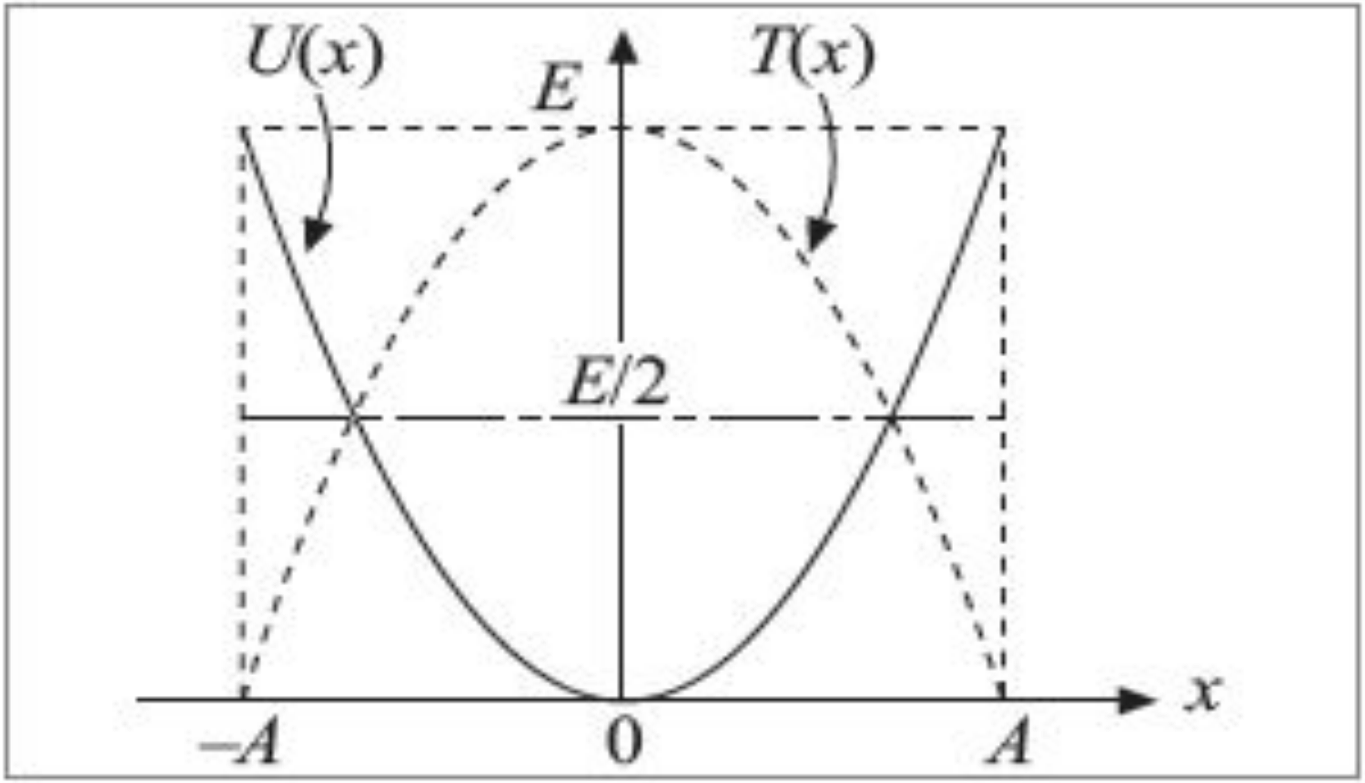
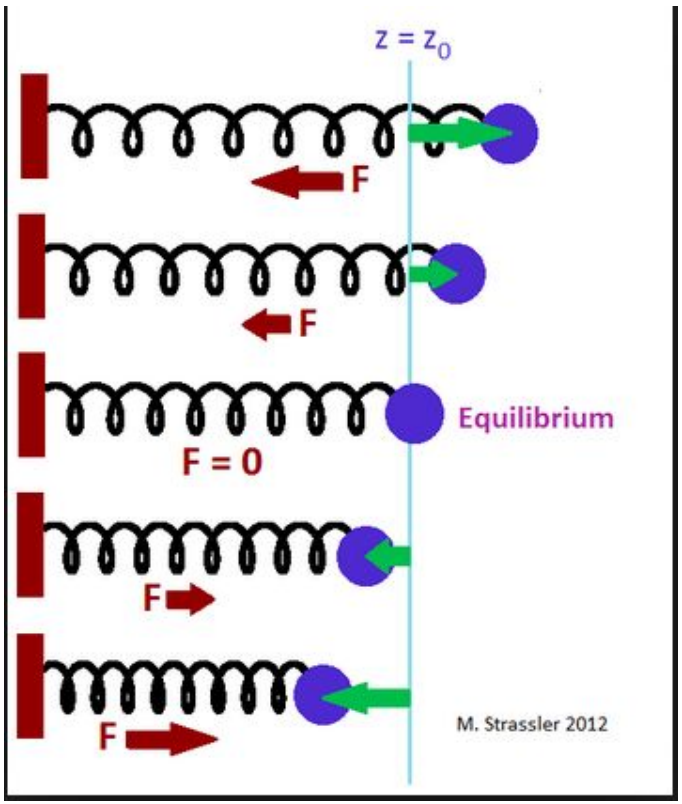


Figura 3.7 *U* e *T* em função de *x*.

It is easy to create additional models for other kinds of motion. Cut and paste the code in the `FallingBall` into a new file named SHO.java and change the code to solve the following two first-order differential equations for a ball attached to a spring:

$$\frac{dx}{dt} = v \tag{2.11a}$$

$$\frac{dv}{dt} = -\frac{k}{m}x, \tag{2.11b}$$



```java
  double x, v, t;                     // the dynamical variables
  double dt;
  double k = 1.0;                     // spring constant
  double omega0 = Math.sqrt(k); // assume unit mass

  public SHO() {                      // constructor
     System.out.println("A new harmonic oscillator object is created.");
  }

  public void step() {
     // modified Euler algorithm
     v = v-k*x*dt;
     x = x+v*dt; // note that updated v is used
     t = t+dt;
  }

  public double analyticPosition(double y0, double v0) {
     return y0*Math.cos(omega0*t)+v0/omega0*Math.sin(omega0*t);
  }

  public double analyticVelocity(double y0, double v0) {
     return -y0*omega0*Math.sin(omega0*t)+v0*Math.cos(omega0*t);
  }
}
```

Exercise 2.9. Simple harmonic oscillator

a. Explain how the implementation of the Euler algorithm in the `step` method of class `SHO` differs from what we did previously.

b. The general form of the analytical solution of (2.11) can be expressed as

$$y(t) = A\cos\omega_0 t + B\sin\omega_0 t, \tag{2.12}$$

where $\omega_0^2 = k/m$. What is the form of $v(t)$? Show that (2.12) satisfies (2.11) with $A = y(t = 0)$ and $B = v(t = 0)/\omega_0$. These analytical solutions are used in class `SHO`.

c. Write a target class called `SHOApp` that creates an `SHO` object and solves (2.11). Start the ball with displacements of $x = 1$, $x = 2$, and $x = 4$. Is the time it takes for the ball to reach $x = 0$ always the same?

Exercise 2.12. Extending classes

a. Extend the **FallingParticle** and **SHOParticle** classes and give them names such as **FallingParticleEC** and **SHOParticleEC**, respectively. These subclasses should redefine the **step** method so that it first calculates the new velocity and then calculates the new position using the new velocity, that is,

```
public void step() {
    v = v - g*dt;          // falling ball
    y = y + v*dt;f
    t = t + dt;
}
```

```
public void step() {
    v = v - k*x*dt;        // harmonic oscillator
    x = x + v*dt;
    t = t + dt;
}
```

Methods can be redefined (overloaded) in the subclass by writing a new method in the subclass definition with the same name and parameter list as the super class definition.

b. Confirm that your new **step** method is executed instead of the one in the superclass.

c. The algorithm that is implemented in the redefined step method is known as the *Euler-Cromer* algorithm. Compare the accuracy of this algorithm to the original Euler algorithm for both the falling particle and the harmonic oscillator. We will explore the Euler-Cromer algorithm in more detail in Problem 4.1.

Simple Harmonic Motion

$$F = -kx.$$

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x,$$

the angular frequency ω_0 is defined by

$$\omega_0{}^2 = \frac{k}{m}.$$

Amplitude

$$x(t) = A\cos(\omega_0 t + \delta),$$

Phase

Simple Harmonic Motion

$$x(t+T) = x(t).$$

Period T

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{k/m}}.$$

The frequency ν of the motion is the number of cycles per second and is given by $\nu = 1/T$.

Simple Harmonic Motion

Although the position and velocity of the oscillator are continuously changing, the total energy E remains constant and is given by

Kinectic energy

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2.$$

Total Energy

Potential energy

Problem 4.1. Energy conservation

a. Use the Euler **ODESolver** to solve the dynamical equations for a simple harmonic oscillator by extending **AbstractSimulation** and implementing the **doStep** method. (See Section 4.2 for an example of such a program for the pendulum.) Have your program plot $\Delta E_n = E_n - E_0$, where E_0 is the initial energy and E_n is the total energy at time $t_n = t_0 + n\Delta t$. (It is necessary only to consider the energy per unit mass.) Plot the difference ΔE_n as a function of t_n for several cycles for a given value of Δt. Choose $x(t = 0) = 1$, $v(t = 0) = 0$ and ${\omega_0}^2 = k/m = 9$ and start with $\Delta t = 0.05$. Is the difference ΔE_n uniformly small throughout the cycle? Does ΔE_n drift, that is, become bigger with time? What is the optimum choice of Δt?

b. Implement the Euler-Cromer algorithm by writing an Euler-Cromer ODESolver and answer the same questions as in part (a).

c. Modify your program so that the Euler-Richardson or Verlet algorithms are used and answer the same questions as in part (a). (The Verlet algorithm is discussed in Appendix 3.)

Problem 4.4. Superposition of waves

a. Write a program to plot $A\sin(kx+\omega t)$ from $x = x_{\text{min}}$ to $x = x_{\text{max}}$ as a function of t. (Implement an **AbstractSimulation** rather than an **AbstractCalculation**.) For simplicity, take $A = 1$, $\omega = 2\pi$, and $k = 2\pi/\lambda$, and $\lambda = 2$.

b. Modify your program so that it plots the sum of $y_1 = \sin(kx - \omega t)$ and $y_2 = \sin(kx + \omega t)$. The quantity $y_1 + y_2$ corresponds to the superposition of two waves. Choose $\lambda = 2$ and $\omega = 2\pi$. What kind of a wave do you obtain?

c. Use your program to demonstrate beats by plotting $y_1 + y_2$ as a function of time in the range $x_{\text{min}} = -10$ and $x_{\text{max}} = 10$. Determine the beat frequency for each of the following superpositions: $y_1(x,t) = \sin[8.4(x - 1.1t)]$, $y_2(x,t) = \sin[8.0(x - 1.1t)]$; $y_1(x,t) = \sin[8.4(x - 1.2t)]$, $y_2(x,t) = \sin[8.0(x - 1.0t)]$; and $y_1(x,t) = \sin[8.4(x - 1.0t)]$, $y_2(x,t) = \sin[8.0(x - 1.2t)]$. What difference do you observe between these superpositions?

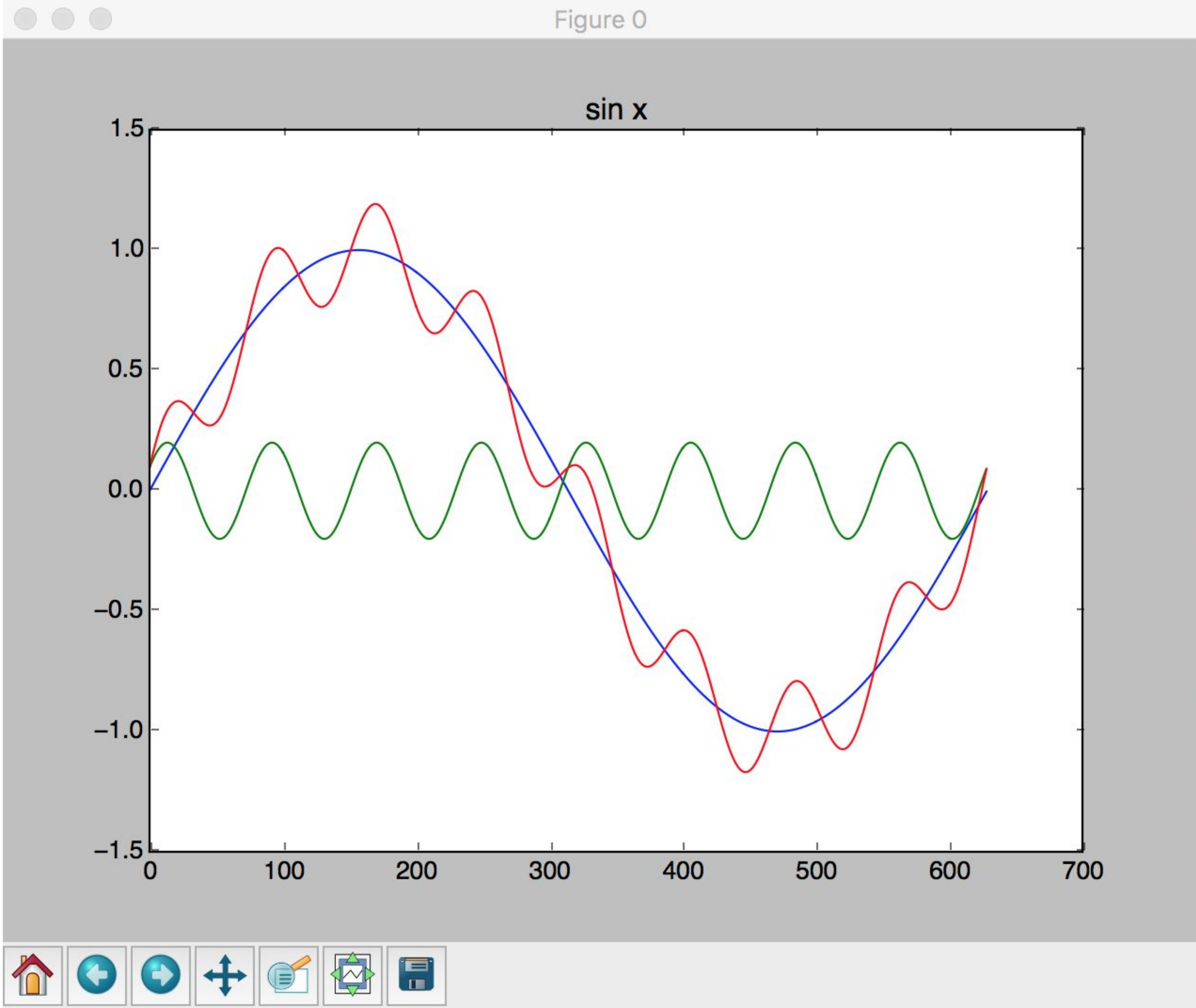
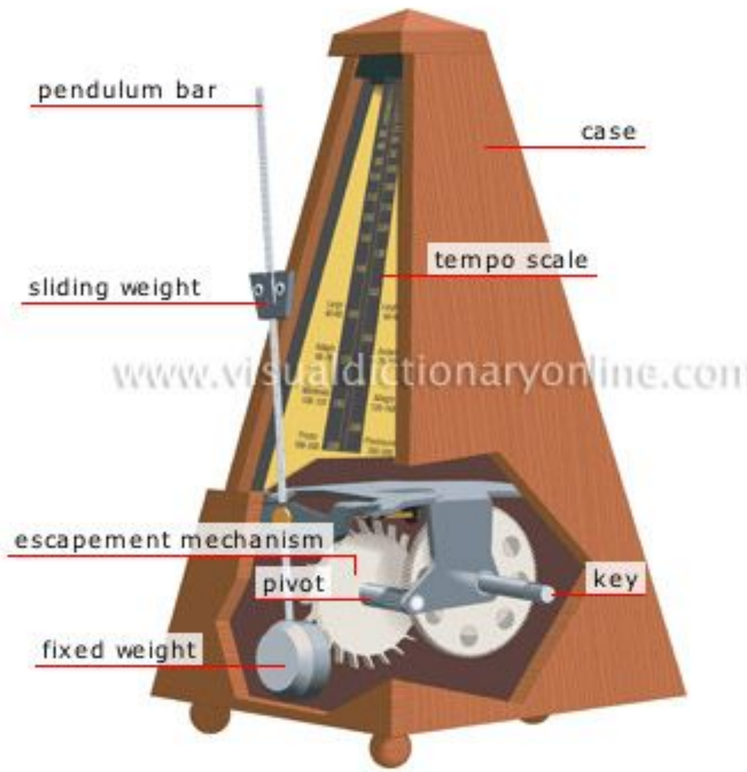


Figure 0
sin x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500
600
700

PENDULUM

PENDULUM - aplicações

https://sites.google.com/site/swegpendulums/_/rsrc/1426700592563/applications/Old_Pendulum_clock.jpg

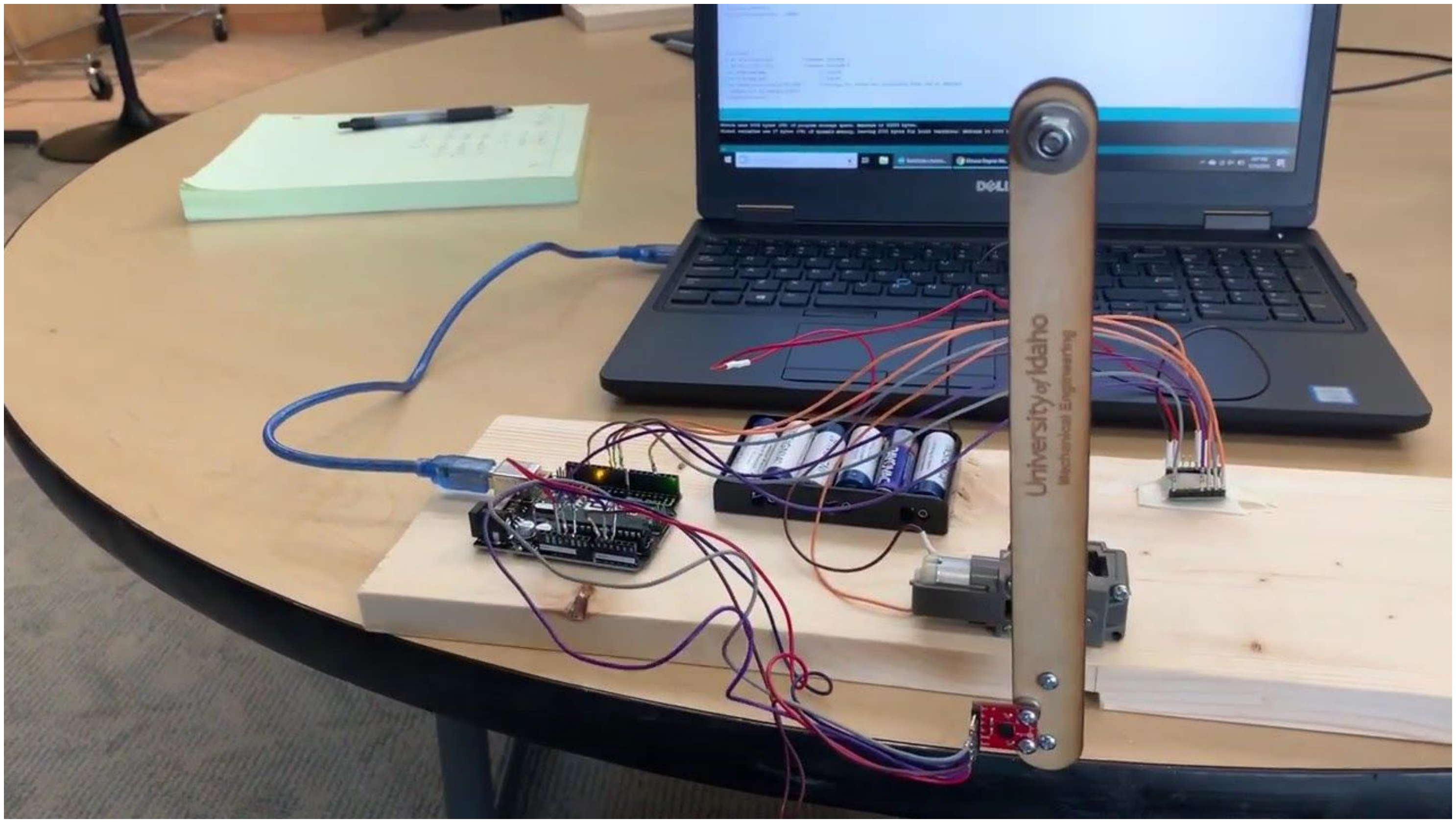


http://www.visualdictionaryonline.com/images/arts-architecture/music/musical-accessories/metronome_2.jpg

https://cosmolearning.org/images_dir/education/photos/487.jpg

Temporizador

PENDULUM - aplicações



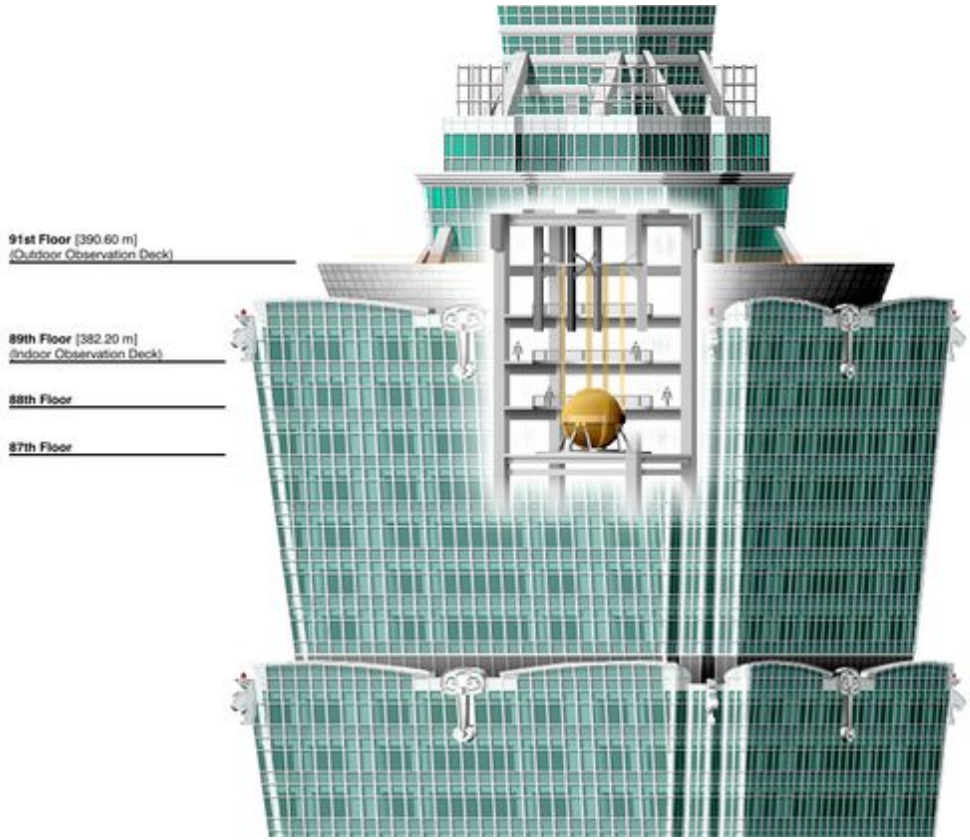
Pêndulo invertido

PENDULUM - aplicações

https://www.researchgate.net/publication/330825329_Stability_and_Control_of_an_Inverted_Pendulum_Motion/figures?lo=1

Pêndulo invertido

PENDULUM - aplicações



https://practical.engineering/blog/2016/2/14/tuned-mass-dampers-in-skyscrapers

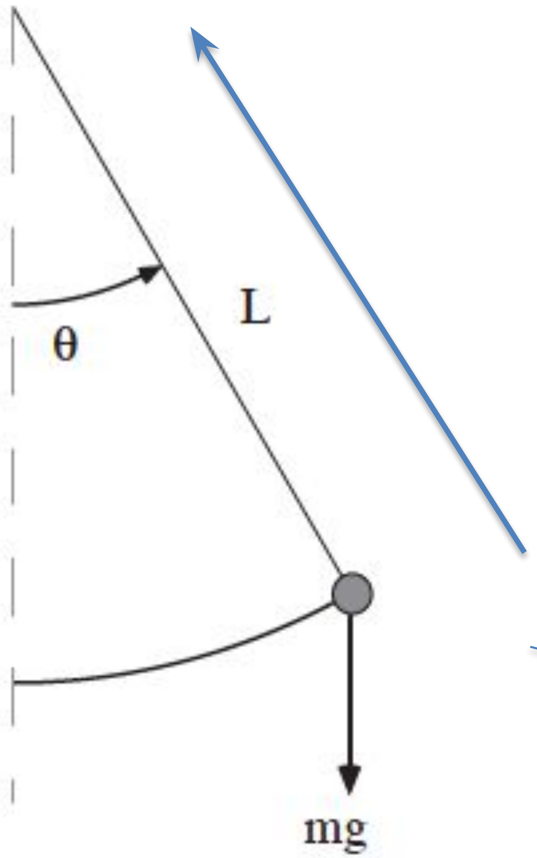
Freio pêndulo

Foucault Pendulum

Foucault Pendulum

https://www.youtube.com/watch?v=aMxLVDuf4VY

Pendulum



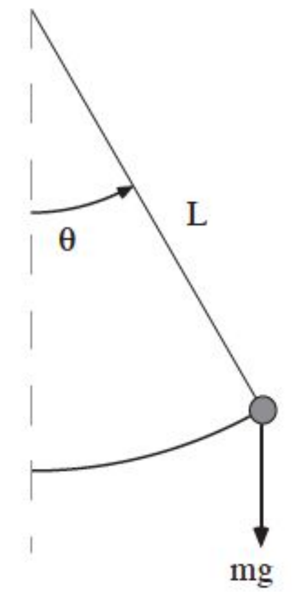
$$v = L\frac{d\theta}{dt}$$

$$a = L\frac{d^2\theta}{dt^2}.$$

Forces: g and rod L

Pendulum

$$v = L\frac{d\theta}{dt}$$
$$a = L\frac{d^2\theta}{dt^2}.$$



$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta,$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta.$$

Non-linear equations which seldom have simple analytical solutions

Pendulum

$$v = L\frac{d\theta}{dt}$$

$$a = L\frac{d^2\theta}{dt^2}.$$

$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta,$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta.$$

For small amplitudes: $\sin\theta \approx \theta,$

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta. \qquad (\theta \ll 1)$$

Pendulum

Compare!

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x,$$

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta. \qquad (\theta \ll 1)$$

$$T = 2\pi\sqrt{\frac{L}{g}}. \qquad \text{(small amplitude oscillations)}$$

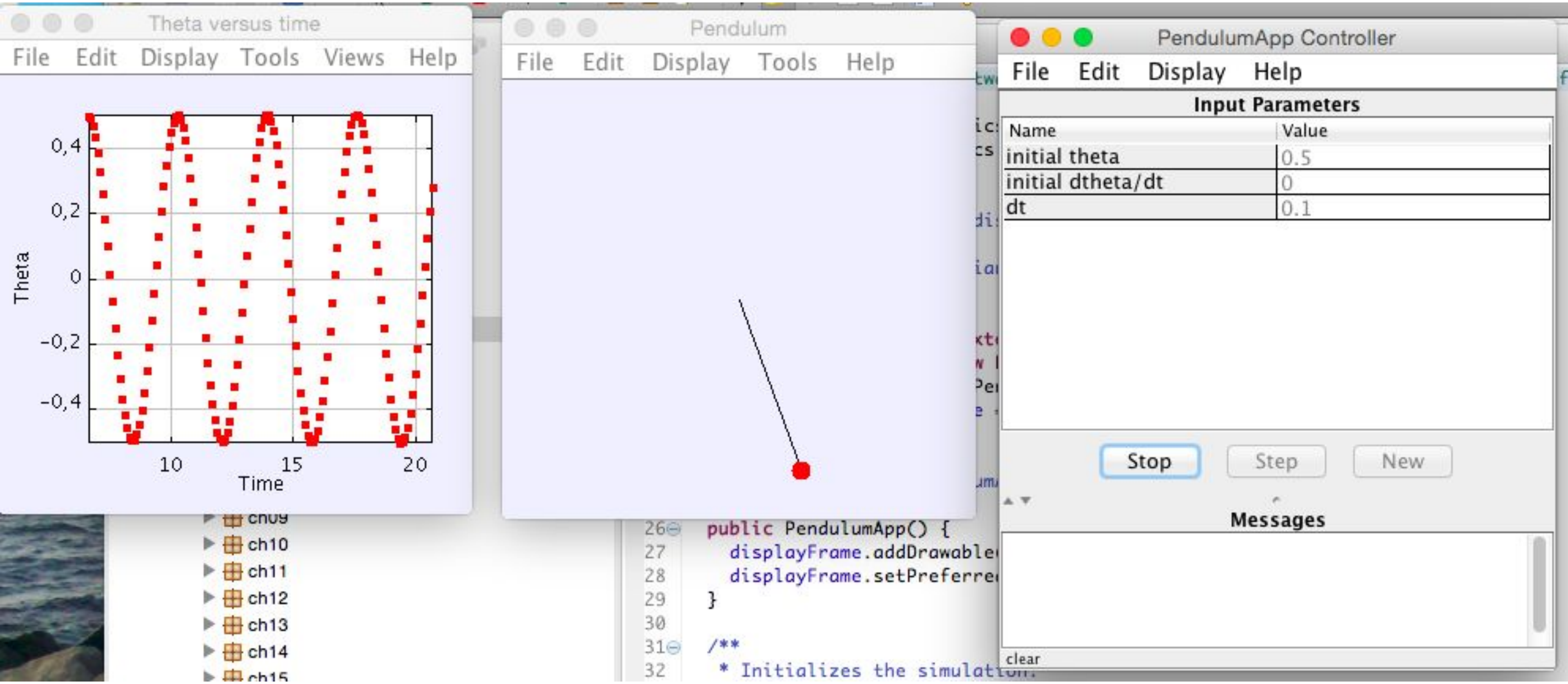
One way to understand the motion of a pendulum with large oscillations is to solve (4.11) numerically. Because we know that the numerical solutions must be consistent with conservation of energy, we derive the form of the total energy here. The potential energy can be found from the following considerations. If the rod is deflected by the angle θ, then the bob is raised by the distance $h = L - L\cos\theta$ (see Figure 4.2). Hence, the potential energy of the bob in the gravitational field of the earth is

$$U = mgh = mgL(1 - \cos\theta), \tag{4.14}$$

where the zero of the potential energy corresponds to $\theta = 0$. Because the kinetic energy of the pendulum is $\frac{1}{2}mv^2 = \frac{1}{2}mL^2(d\theta/dt)^2$, the total energy E of the pendulum is

$$E = \frac{1}{2}mL^2\left(\frac{d\theta}{dt}\right)^2 + mgL(1 - \cos\theta). \tag{4.15}$$

pendulumApp



Damped Harmonic Oscillator

Damped coefficient

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x - \gamma \frac{dx}{dt}.$$

Problem 4.6. Damped linear oscillator

a. Incorporate the effects of damping into your harmonic oscillator simulation and plot the time dependence of the position and the velocity. Describe the qualitative behavior of $x(t)$ and $v(t)$ for $\omega_0 = 3$ and $\gamma = 0.5$ with $x(t = 0) = 1$, $v(t = 0) = 0$.

b. The period of the motion is the time between successive maxima of $x(t)$. Compute the period and corresponding angular frequency and compare their values to the undamped case. Is the period longer or shorter? Make additional runs for $\gamma = 1$, 2, and 3. Does the period increase or decrease with greater damping? Why?

c. The amplitude is the maximum value of x during one cycle. Compute the *relaxation time* τ, the time it takes for the amplitude of an oscillation to decrease by $1/e \approx 0.37$ from its maximum value. Is the value of τ constant throughout the motion? Compute τ for the values of γ considered in part (b) and discuss the qualitative dependence of τ on γ.

d. Plot the total energy as a function of time for the values of γ considered in part (b). If the decrease in energy is not monotonic, explain.

e. Compute the time dependence of $x(t)$ and $v(t)$ for $\gamma = 4$, 5, 6, 7, and 8. Is the motion oscillatory for all γ? How can you characterize the decay? For fixed ω_0, the oscillator is said to be *critically damped* at the smallest value of γ for which the decay to equilibrium is monotonic. For what value of γ does critical damping occur for $\omega_0 = 4$ and $\omega_0 = 2$? For each value of ω_0, compute the value of γ for which the system approaches equilibrium most quickly.

Response to External Forces

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x - \gamma v + \frac{1}{m}F(t).$$

$$\frac{1}{m}F(t) = A_0 \cos \omega t,$$

External force

Problem 4.8. Response of a driven damped linear oscillator

a. Modify your simple harmonic oscillator program so that an external force of the form (4.18) is included. Add this force to the class that encapsulates the equations of motion without changing the target class. The angular frequency of the driving force should be added as an input parameter.

b. Choose $\omega_0 = 3$, $\gamma = 0.5$, $\omega = 2$ and the amplitude of the external force $A_0 = 1$ for all runs unless otherwise stated. For these values of ω_0 and γ, the dynamical behavior in the absence of an external force corresponds to an underdamped oscillator. Plot $x(t)$ versus t in the presence of the external force with the initial condition, $x(t = 0) = 1, v(t = 0) = 0$. How does the qualitative behavior of $x(t)$ differ from the nonperturbed case? What is the period and angular frequency of $x(t)$ after several oscillations? Repeat the same observations for $x(t)$ with $x(t = 0) = 0, v(t = 0) = 1$. Identify a transient part of $x(t)$ that depends on the initial conditions and decays in time, and a steady state part that dominates at longer times and is independent of the initial conditions.

ELECTRIC CIRCUIT OSCILLATIONS

element	voltage drop	symbol	units
resistor	$V_{\mathrm{R}} = IR$	resistance R	ohms (Ω)
capacitor	$V_{\mathrm{C}} = Q/C$	capacitance C	farads (F)
inductor	$V_{\mathrm{L}} = L\, dI/dt$	inductance L	henries (H)

Table 4.1: The voltage drops across the basic electrical circuit elements. Q is the charge (coulombs) on one plate of the capacitor, and I is the current (amperes).

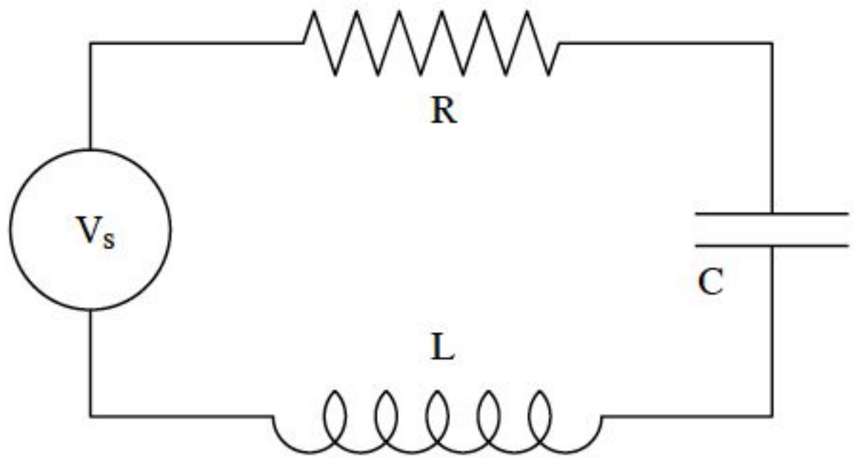


Figure 4.5: A simple series RLC circuit with a voltage source V_s.

$$V_{\mathrm{L}} + V_{\mathrm{R}} + V_{\mathrm{C}} = V_s(t).$$

$$I = dQ/dt.$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_s(t),$$

Electric circuit	Mechanical system
charge Q	displacement x
current $I = dQ/dt$	velocity $v = dx/dt$
voltage drop	force
inductance L	mass m
inverse capacitance $1/C$	spring constant k
resistance R	damping γ

Table 4.2: Analogies between electrical parameters and mechanical parameters.

RC model

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_s(t),$$

$$\frac{d^2x}{dt^2} = -{\omega_0}^2 x - \gamma v + \frac{1}{m}F(t).$$

Damped Harmonic Oscillator with External Force

Exercício: implemente o modelo abaixo usando Euler.

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_s(t),$$

Electric circuit	Mechanical system
charge Q	displacement x
current $I = dQ/dt$	velocity $v = dx/dt$
voltage drop	force
inductance L	mass m
inverse capacitance $1/C$	spring constant k
resistance R	damping γ

Table 4.2: Analogies between electrical parameters and mechanical parameters.

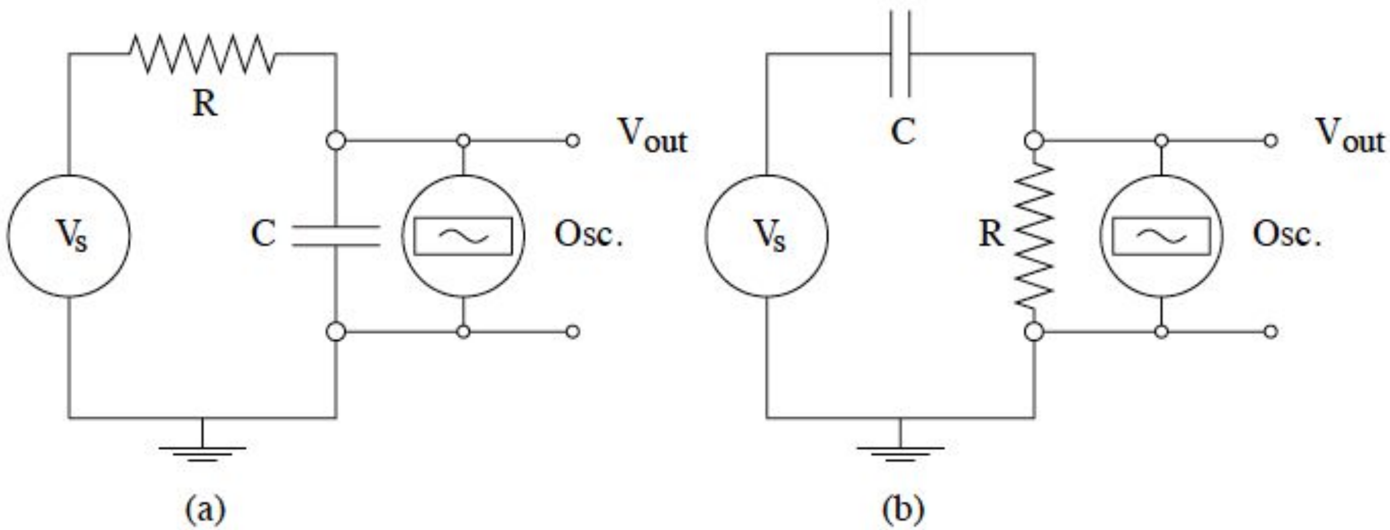
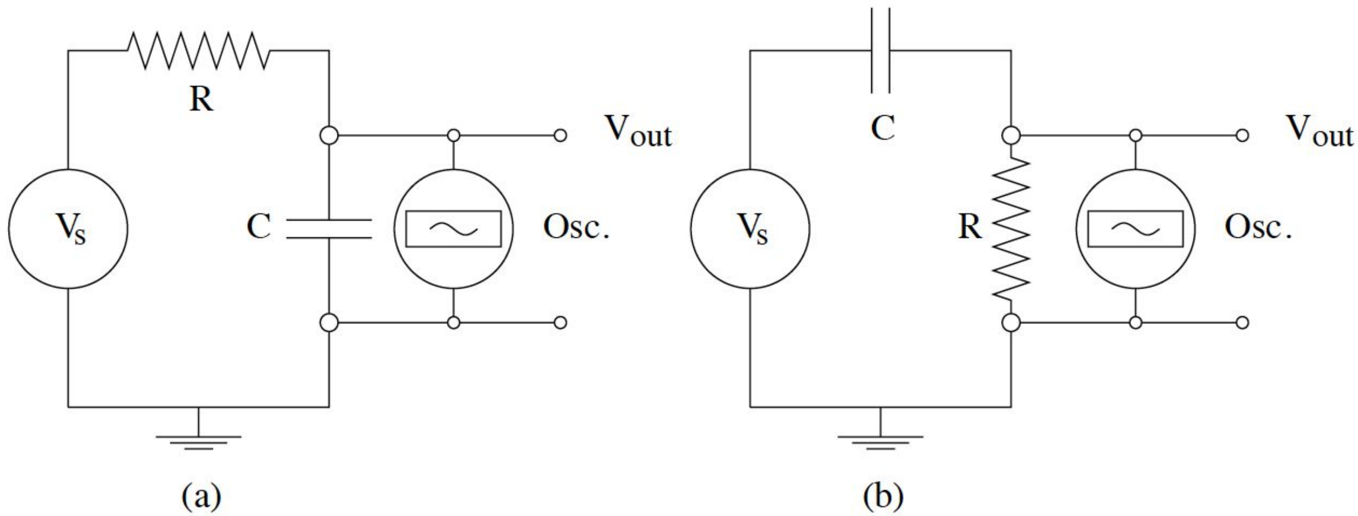


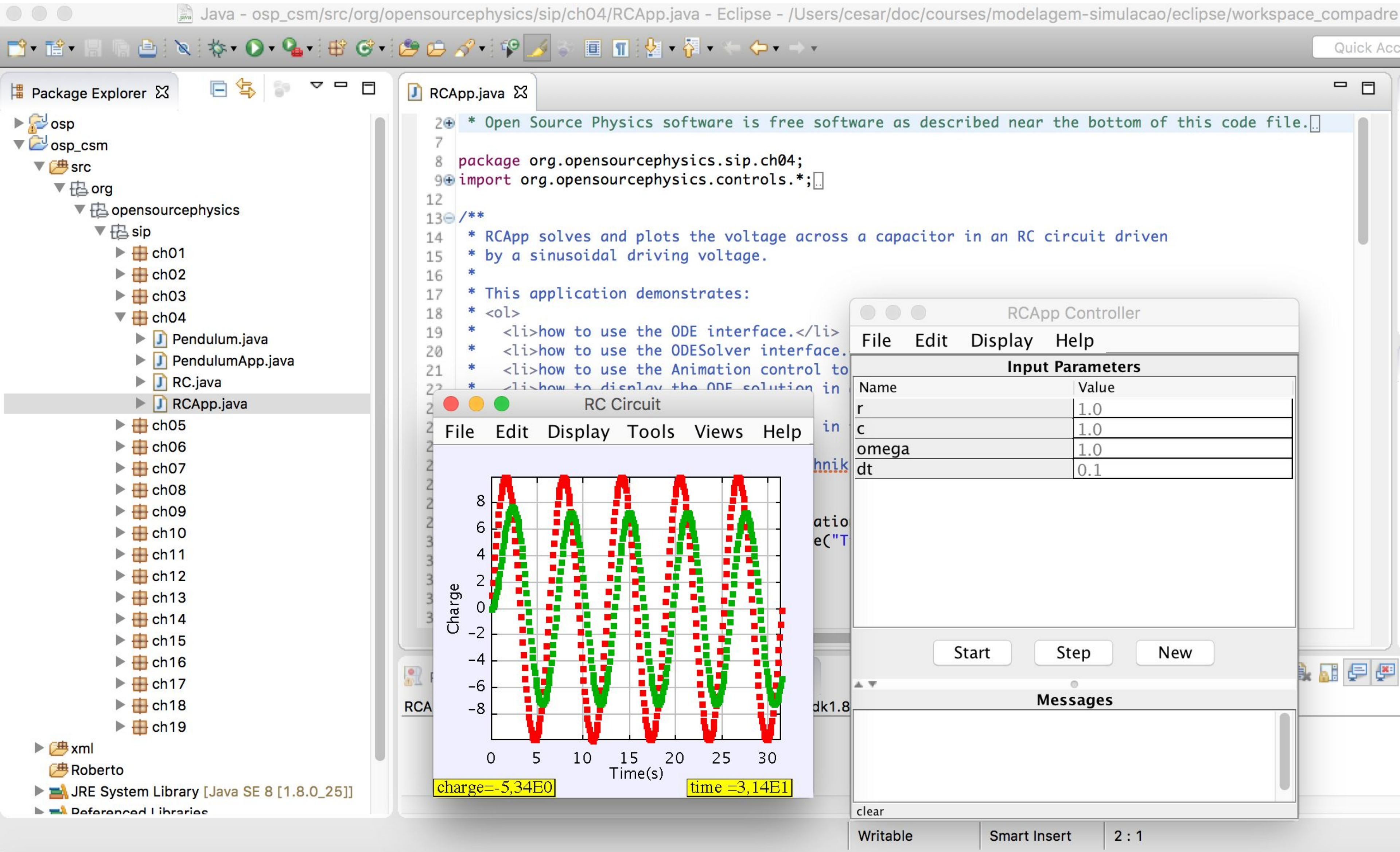
Figure 4.6: Examples of RC circuits used as low and high pass filters. Which circuit is which?



$$RI(t) = R\frac{dQ}{dt} = V_s(t) - \frac{Q}{C}.$$

```java
  /**
   * Get the source voltage at given time.
   * @param t
   * @return
   */
  public double getSourceVoltage(double t) {
    return 10*Math.sin(omega*t);
  }

  /**
   * Get the rate array. Implementation of ODE interface.
   * This method may be invoked many times with different intermediate states
   * as an ODESolver is carrying out the solution.
   *
   * @param state the state array
   * @param rate the rate array
   */
  public void getRate(double[] state, double[] rate) {
    rate[0] = (-state[0]/r/c)+(getSourceVoltage(state[1])/r); // dQ/dt
    rate[1] = 1;                                              // dt/dt = 1
  }
}
```


Java - osp_csm/src/org/opensourcephysics/sip/ch04/RCApp.java - Eclipse - /Users/cesar/doc/courses/modelagem-simulacao/eclipse/workspace_compadre
Package Explorer
RCApp.java
* Open Source Physics software is free software as described near the bottom of this code file.
package org.opensourcephysics.sip.ch04;
import org.opensourcephysics.controls.*;
/**
* RCApp solves and plots the voltage across a capacitor in an RC circuit driven
* by a sinusoidal driving voltage.
* This application demonstrates:
RC Circuit
File Edit Display Tools Views Help
Charge
Time(s)
charge=-5,34E0
time =3,14E1
RCApp Controller
File Edit Display Help
Input Parameters
Name Value
r 1.0
c 1.0
omega 1.0
dt 0.1
Start Step New
Messages
clear
Writable Smart Insert 2 : 1

4.6 Accuracy and Stability

Project 4.18. Nerve impulses

In 1952 Hodgkin and Huxley developed a model of nerve impulses to understand the nerve membrane potential of a giant squid nerve cell. The equations they developed are known as the Hodgkin-Huxley equations. The idea is that a membrane can be treated as a capacitor where $CV = q$ and thus the time rate of change of the membrane potential V is proportional to the current, dq/dt, flowing through the membrane. This current is due to the pumping of sodium and potassium ions through the membrane, a leakage current, and an external current stimulus. The model is capable of producing single nerve impulses, trains of nerve impulses, and other effects. The model is described by the following first-order differential equations:

$$C\frac{dV}{dt} = -g_K n^4(V - V_K) - g_{Na}m^3h(V - V_{Na}) - g_L(V - V_L) + I_{\text{ext}}(t) \tag{4.30a}$$

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n \tag{4.30b}$$

$$\frac{dm}{dt} = \alpha_m(1 - m) - \beta_m m \tag{4.30c}$$

$$\frac{dh}{dt} = \alpha_h(1 - h) - \beta_h h, \tag{4.30d}$$