# Relatividade

15 de dezembro

# Quadrivetores

$$\vec{\mathbf{r}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

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# Quadrivetores

contravariante

$$\vec{\mathbf{r}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

covariante

$$\vec{\mathbf{r}} = \begin{pmatrix} x & y & z \end{pmatrix} \longrightarrow x_\mu = \begin{pmatrix} -ct & x & y & z \end{pmatrix}$$

$$\vec{\mathbf{r}} \cdot \vec{\mathbf{r}} = \begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x^2 + y^2 + z^2 \longrightarrow x_\mu x^\mu = -c^2 t^2 + x^2 + y^2 + z^2$$

# Quadrivetores

$$\begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$\bar{x}^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$$

$$\Lambda \equiv \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Quadrivetores

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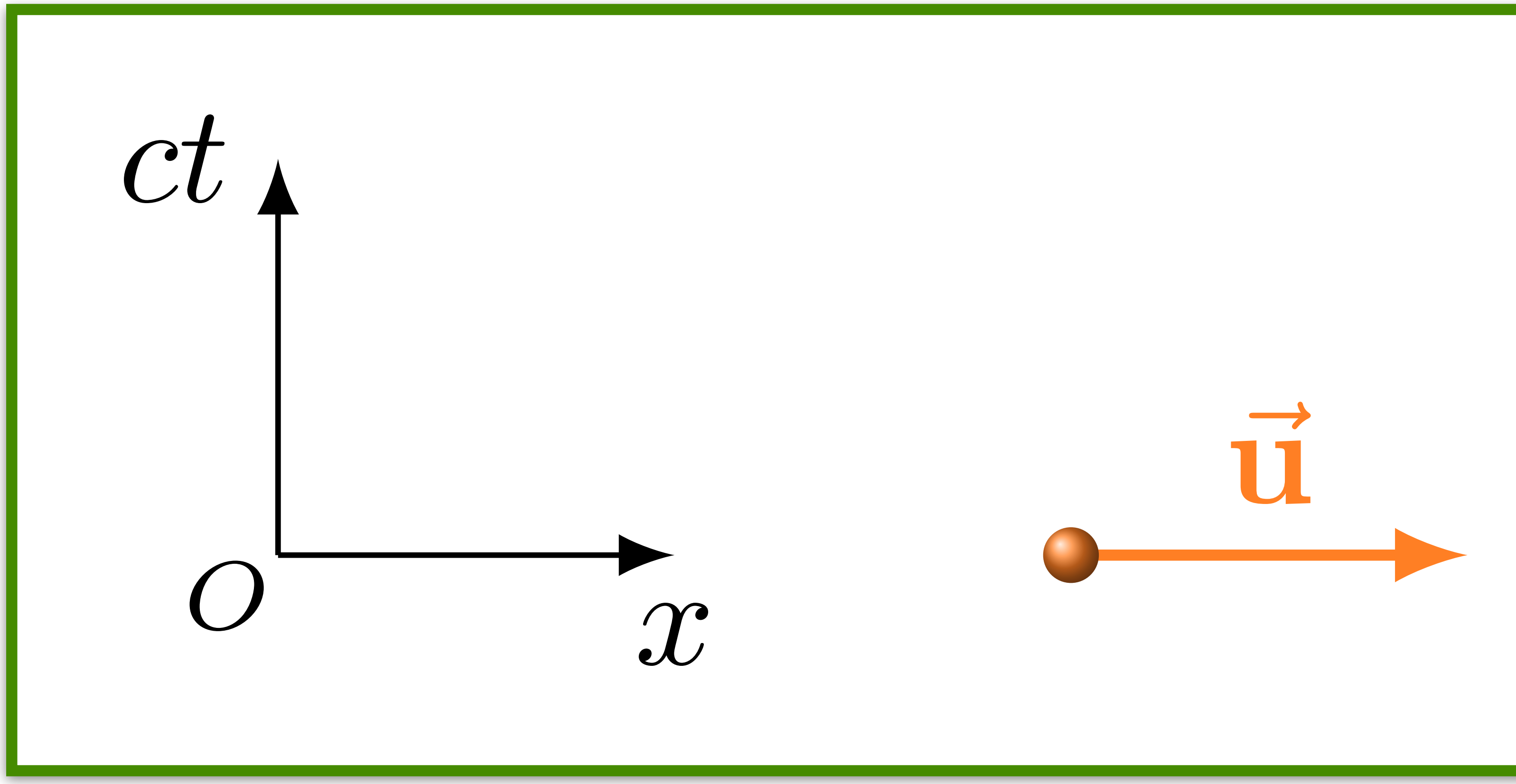
$$\boxed{\bar{x}^\mu = \Lambda^\mu_\nu x^\nu}$$

$$\Lambda \equiv \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

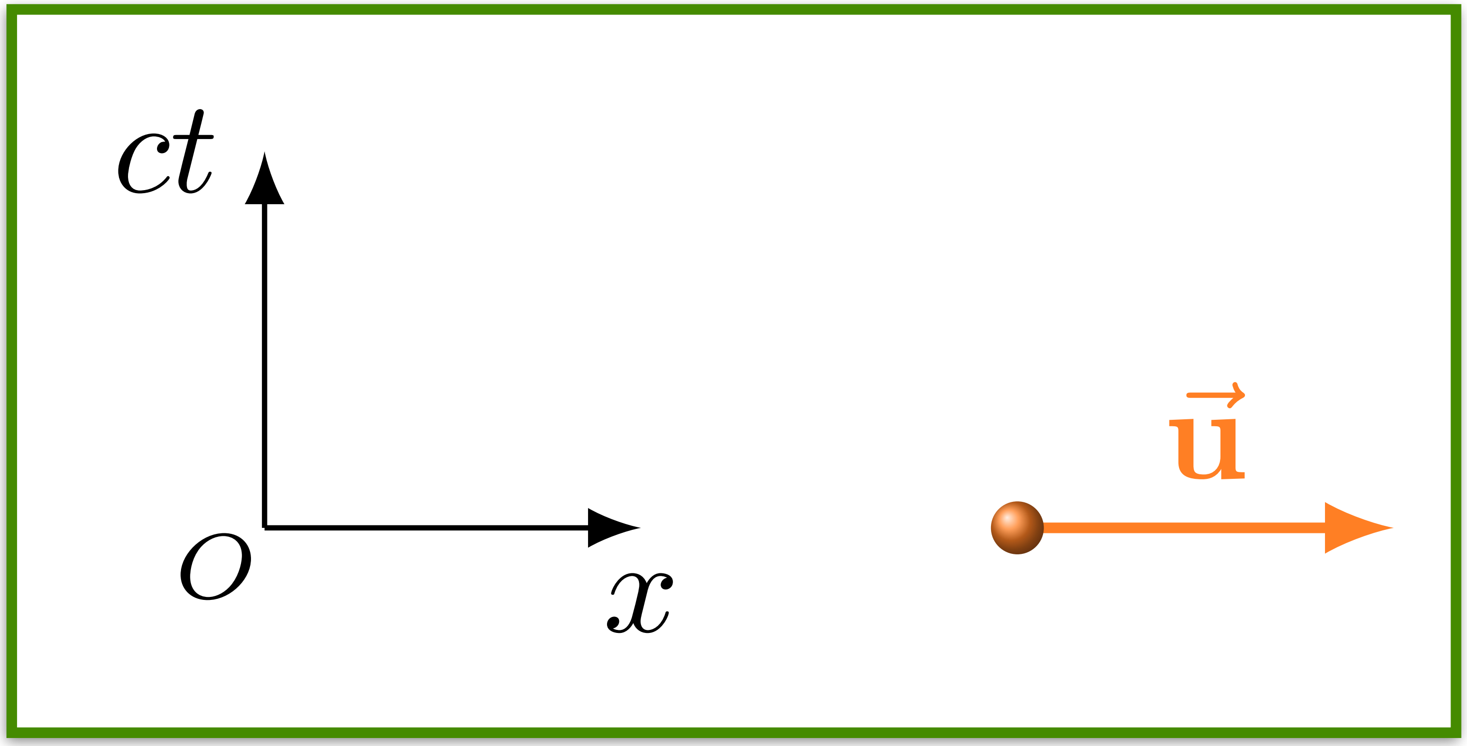
$$a_\mu b^\mu \equiv a_0 b^0 + a_1 b^1 + a_2 b^2 + a_3 b^3 = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3$$

# Velocidade própria

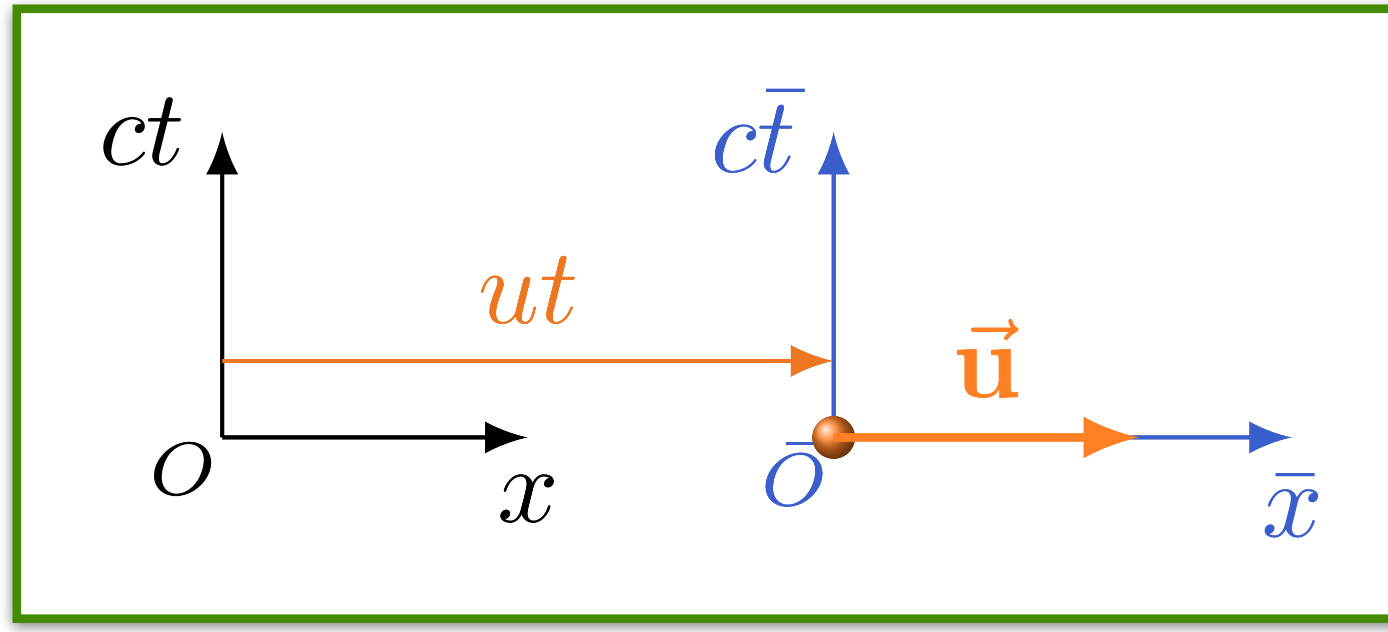


# Velocidade própria

$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$ 🥺



# Velocidade própria

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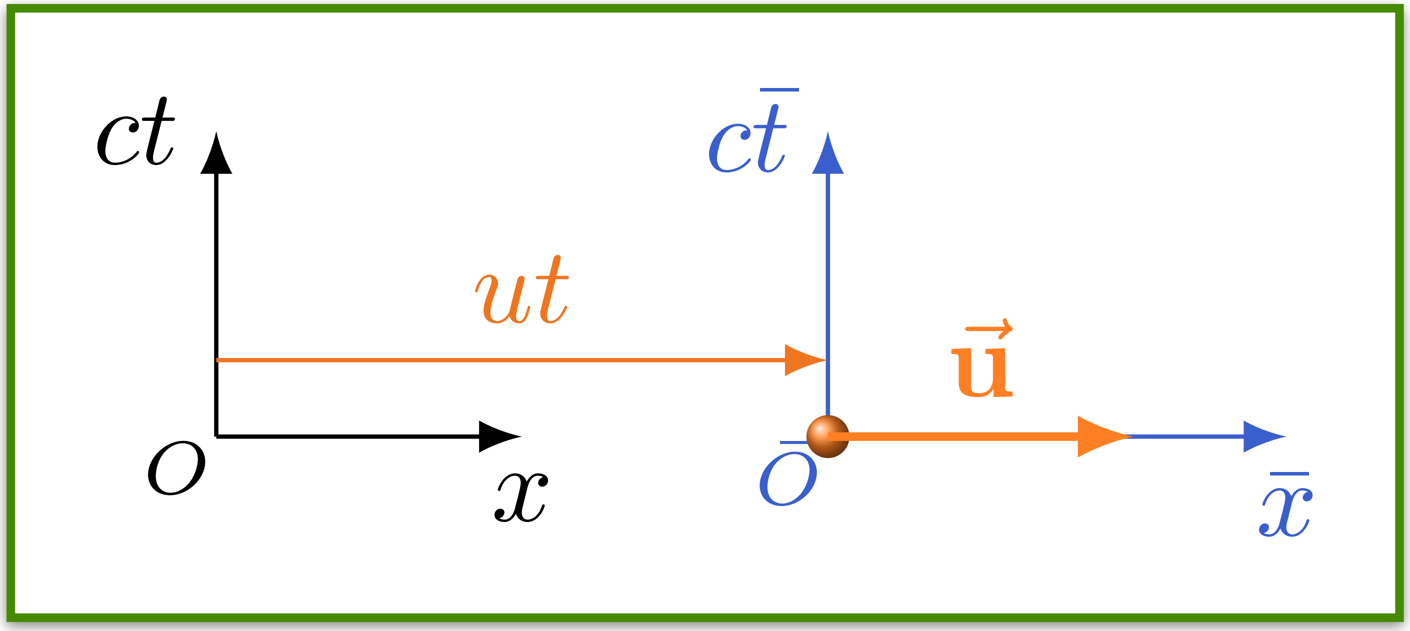
$$\bar{\eta}^{\mu} = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Velocidade própria




$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$\bar{\eta}^\mu = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta^\mu = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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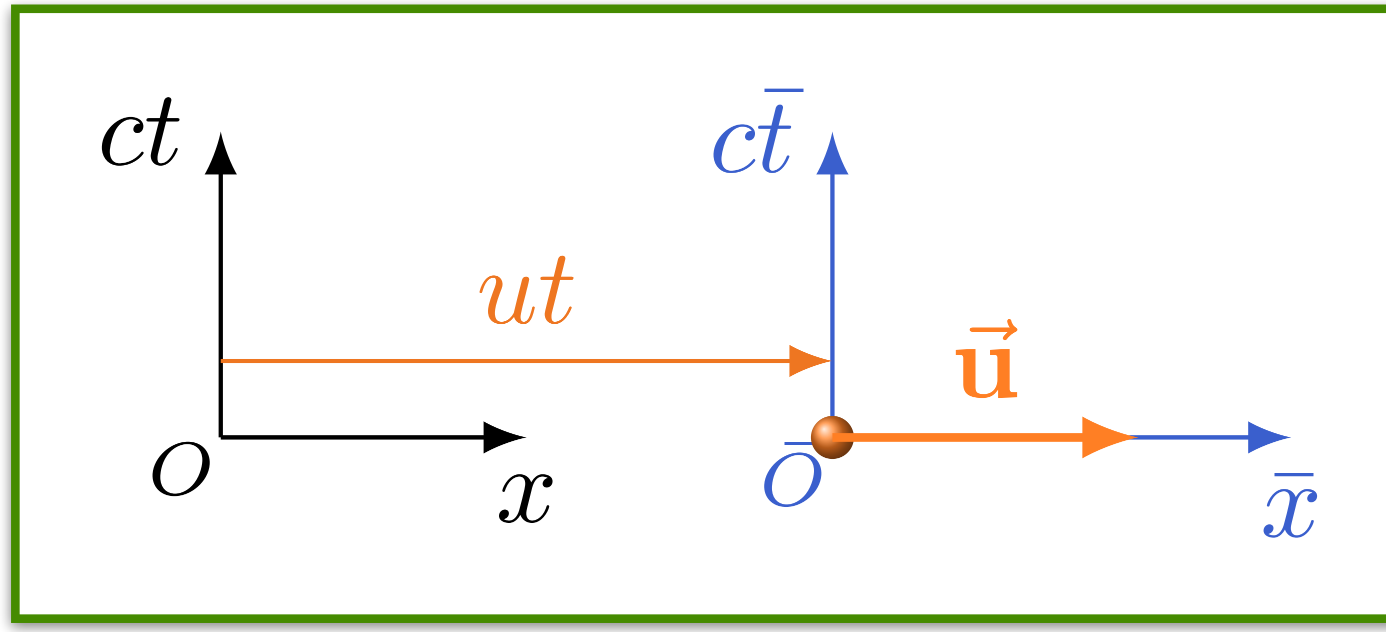
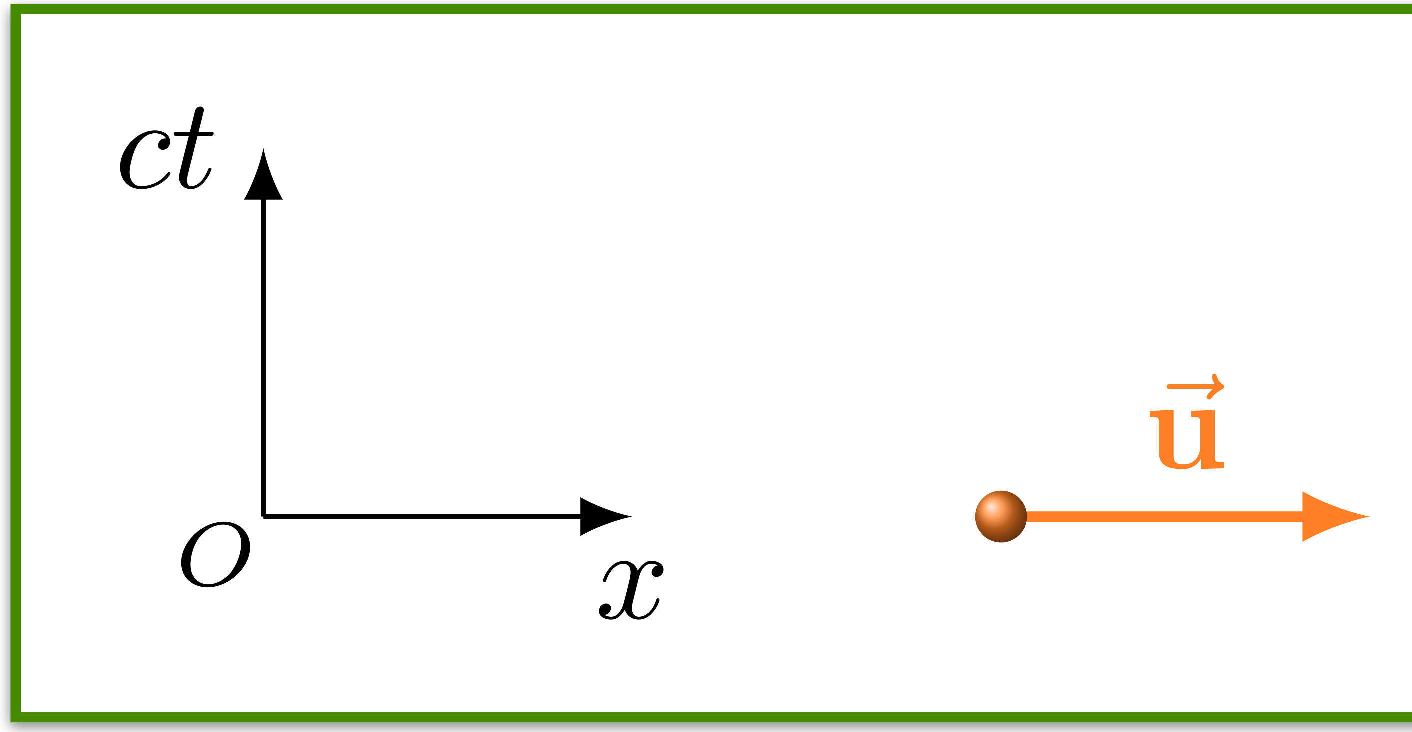



$$\bar{\eta}^{\mu} = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta^{\mu} = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta^{\mu} = \begin{pmatrix} \gamma c \\ \beta\gamma c \\ 0 \\ 0 \end{pmatrix}$$

# Velocidade própria




$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$\bar{\eta}^{\mu} = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta^{\mu} = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\boxed{\eta^{\mu} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}}$$

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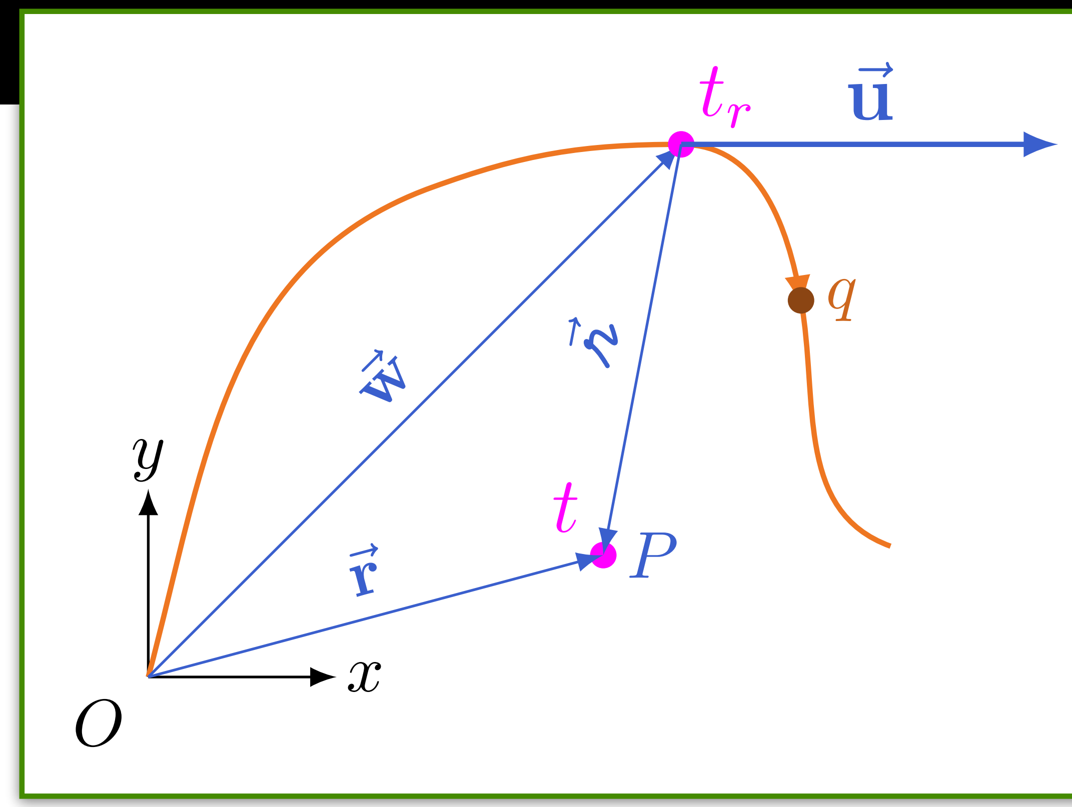
# Pratique o que aprendeu




$$\mathscr{r}_{\mu} \eta^{\mu} = ?$$

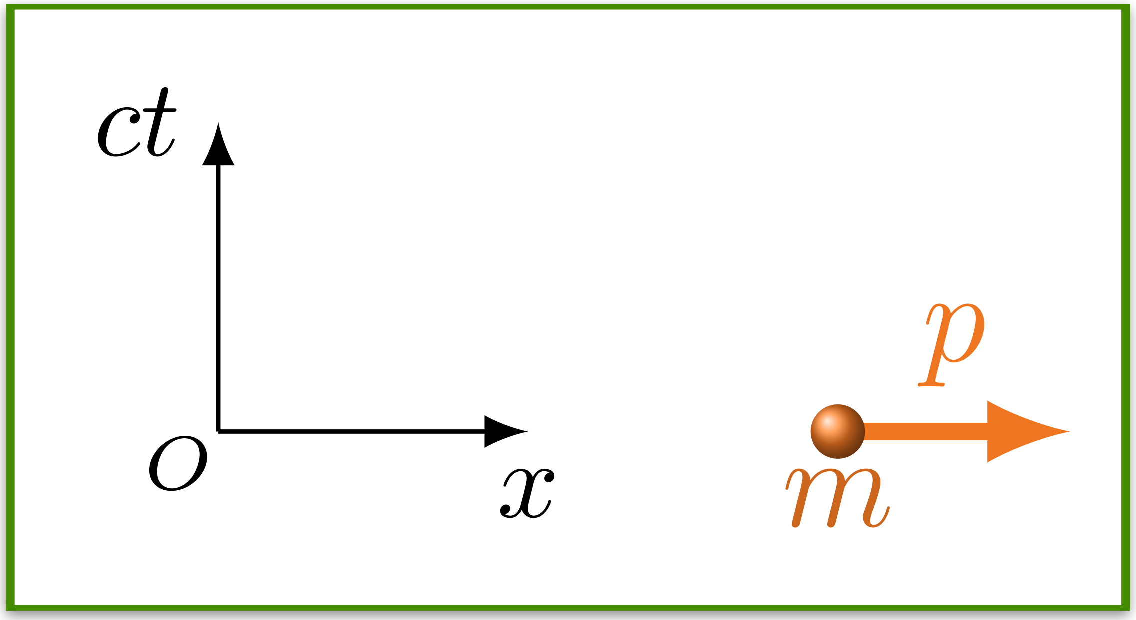
$$\mathscr{r}_{\mu} = \begin{pmatrix} -c(t - t_r) & \mathscr{r}_1 & \mathscr{r}_2 & \mathscr{r}_3 \end{pmatrix}$$

$$c(t - t_r) = \mathscr{r}$$

$$\mathscr{r}_{\mu} \eta^{\nu} = -\gamma (c\mathscr{r} - \vec{\mathbf{u}} \cdot \vec{\mathscr{r}})$$

# Dinâmica relativística

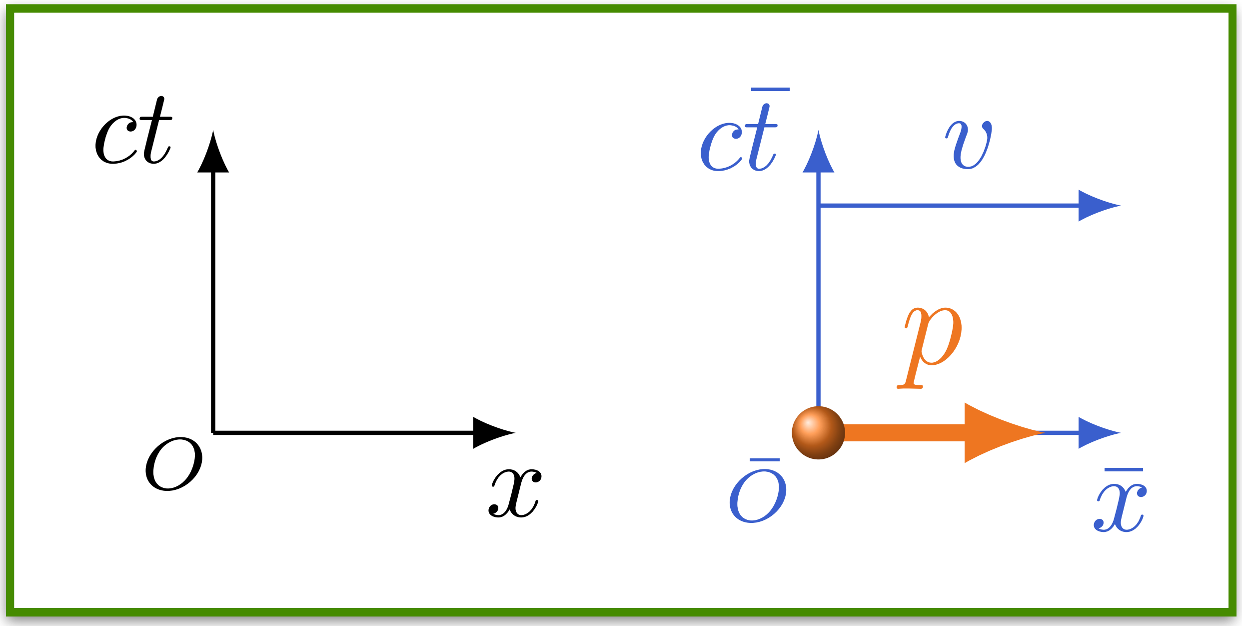
$$p^\mu = \begin{pmatrix} p^0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$



# Dinâmica relativística

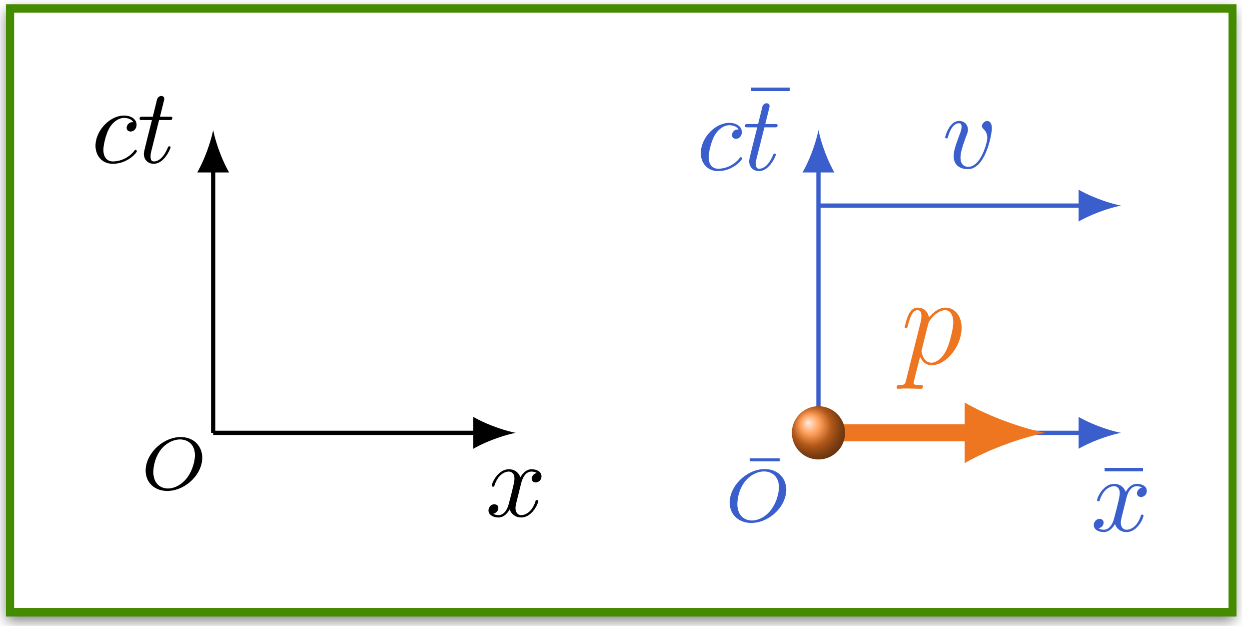
$$p^\mu = \begin{pmatrix} p^0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{p}^\mu = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} p^0 \\ mu \\ 0 \\ 0 \end{pmatrix}$$



# Dinâmica relativística

$$p^\mu = \begin{pmatrix} p^0 \\ mu \\ 0 \\ 0 \end{pmatrix}$$




$$p^\mu = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \bar{p}^0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p^0 = \gamma\bar{p}^0$$

$$mv = \gamma\beta\bar{p}^0 \qquad \Rightarrow \qquad mc = \gamma\bar{p}^0$$

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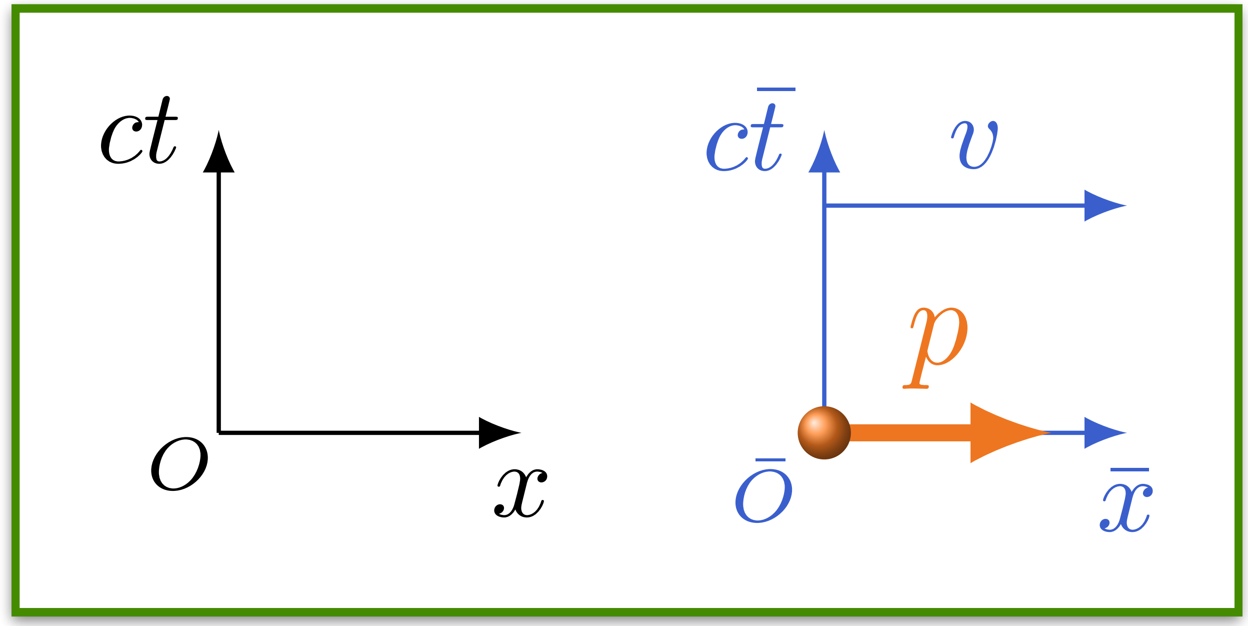
$$p^\mu = \begin{pmatrix} p^0 \\ mu \\ 0 \\ 0 \end{pmatrix}$$




$$p^\mu = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \bar{p}^0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p^0 = \gamma \bar{p}^0$$

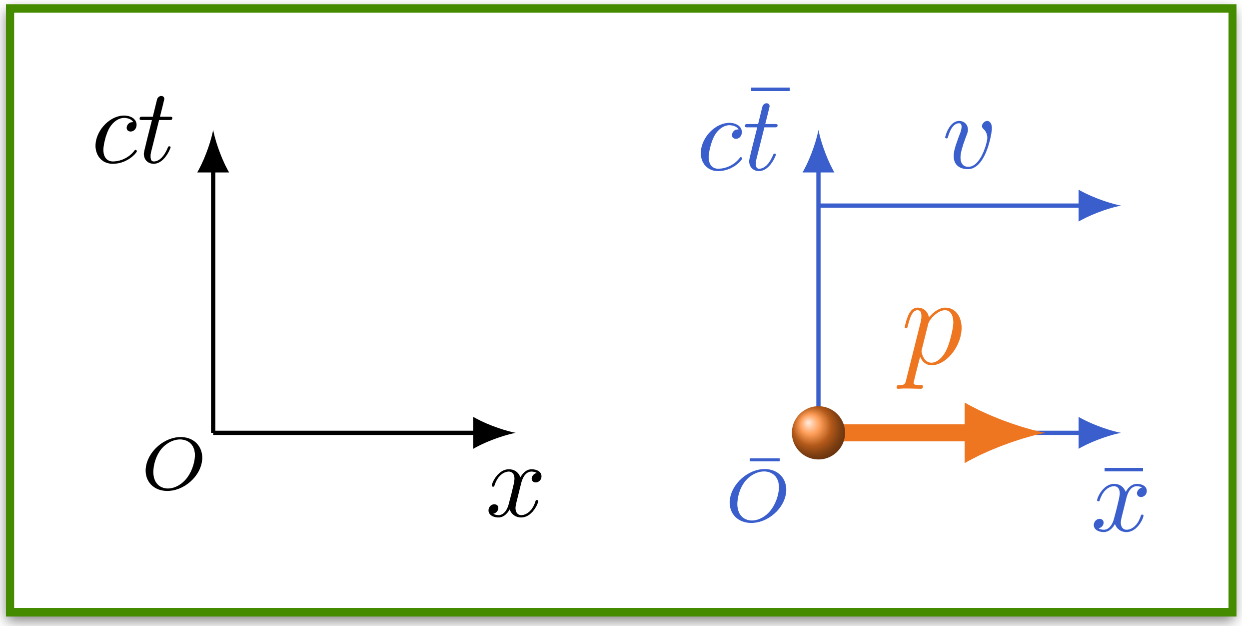
$$mv = \gamma\beta\bar{p}^0 \qquad \Rightarrow \qquad mc = \gamma\bar{p}^0$$

$$\boxed{\bar{p}^0 = m_0 c} \qquad \boxed{m = \gamma m_0}$$

# Dinâmica relativística

$$p^\mu = \begin{pmatrix} mc \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}$$

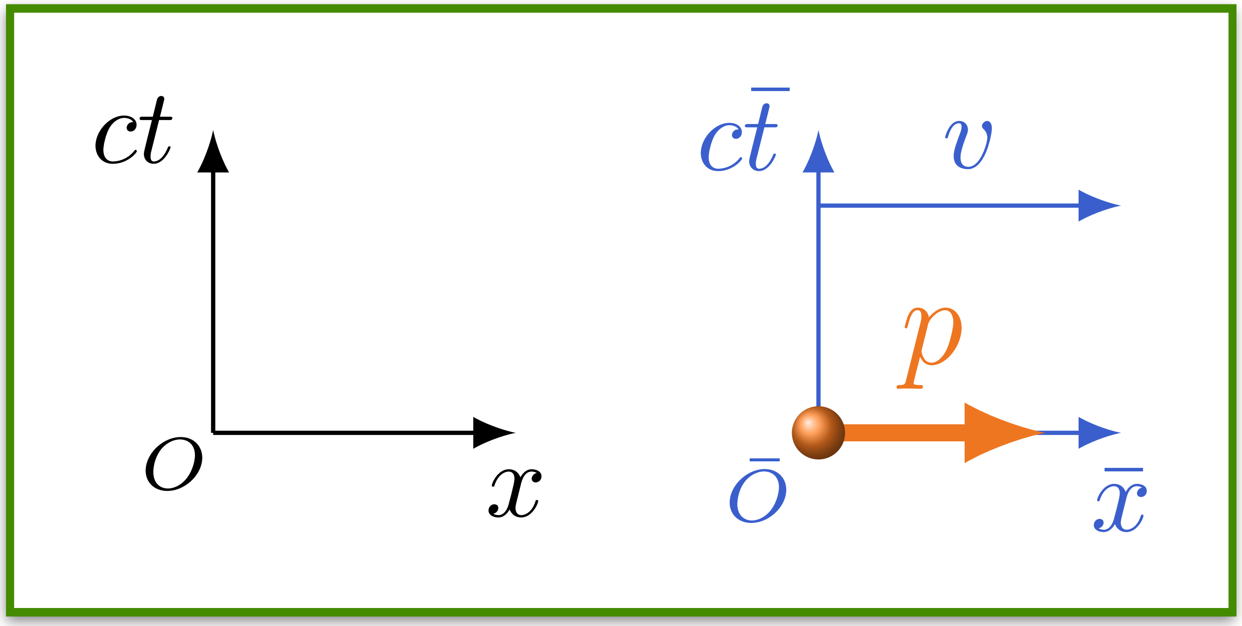
$$\boxed{m = \gamma m_0}$$



# Dinâmica relativística

$$m = m_0\gamma$$

$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$



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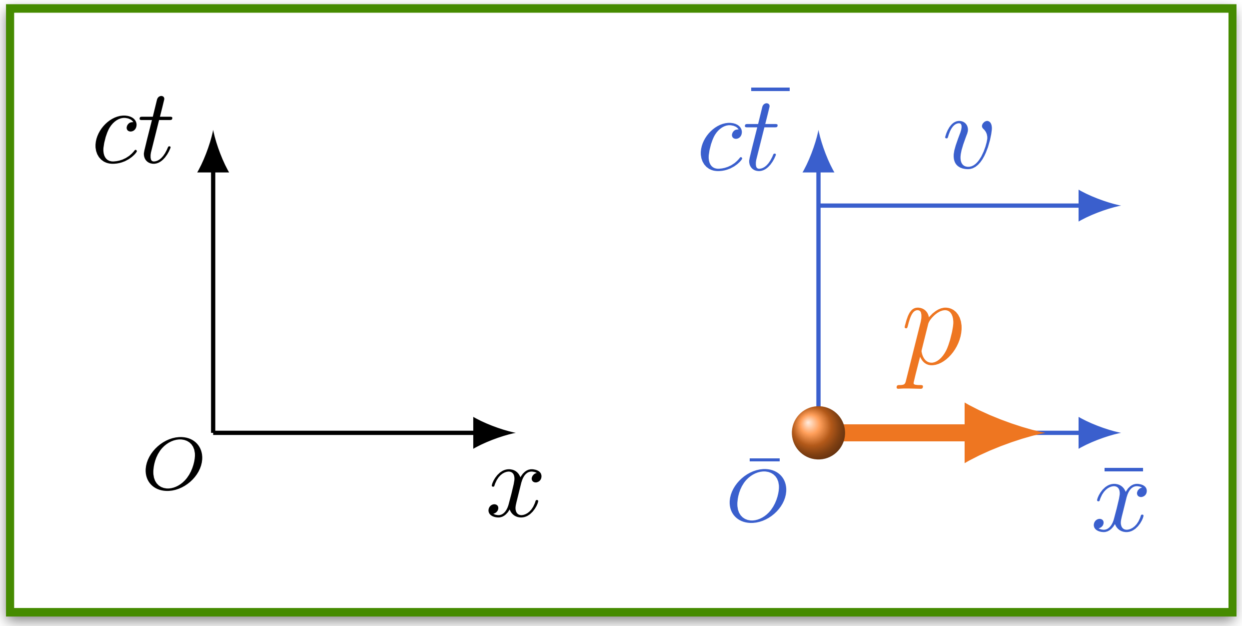


$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$

$$m = m_0(1+\frac{v^2}{2c^2}) + \mathcal{O}\big(v^4/c^4\big)$$

# Dinâmica relativística

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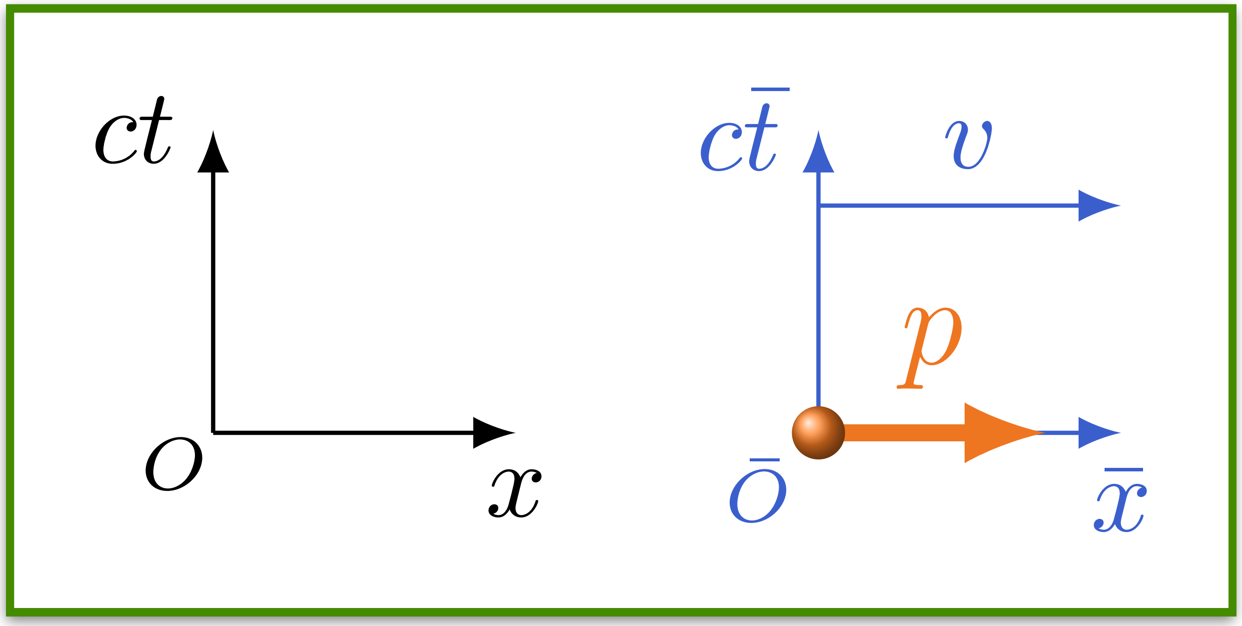
$$mc^2 = m_0c^2 + m_0\frac{v^2}{2}$$



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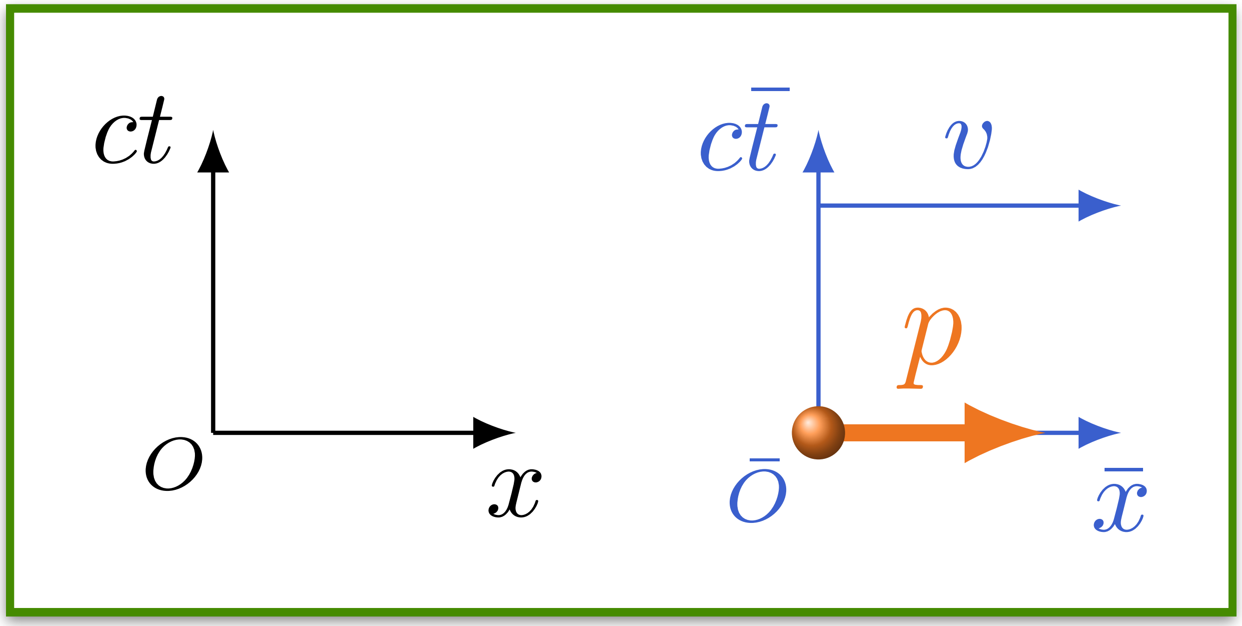


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$$p^\mu = \begin{pmatrix} E/c \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}$$

# Dinâmica relativística

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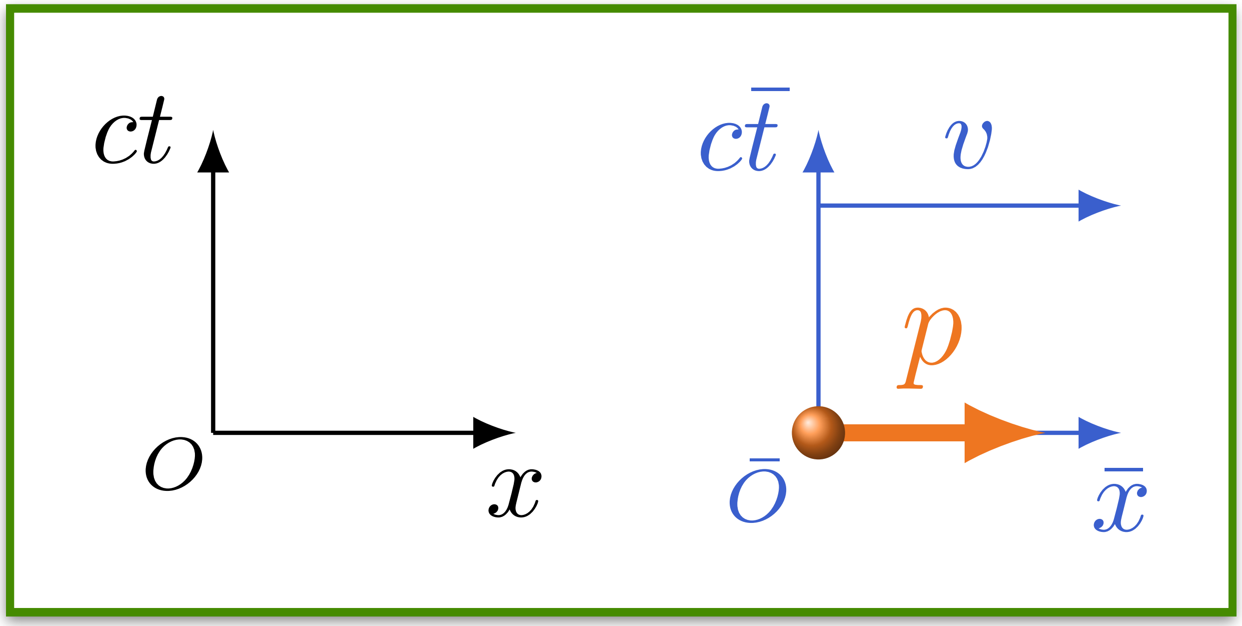


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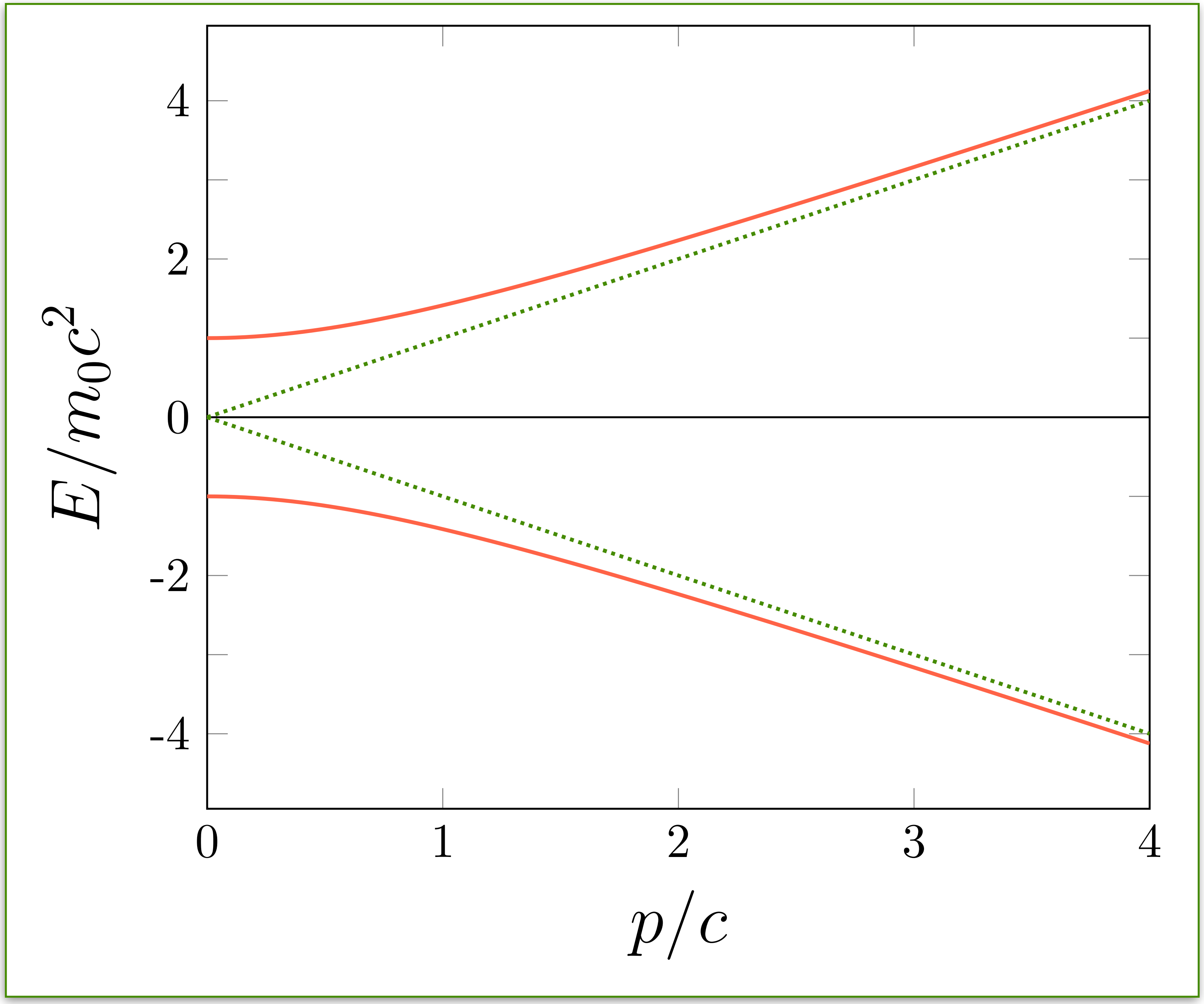
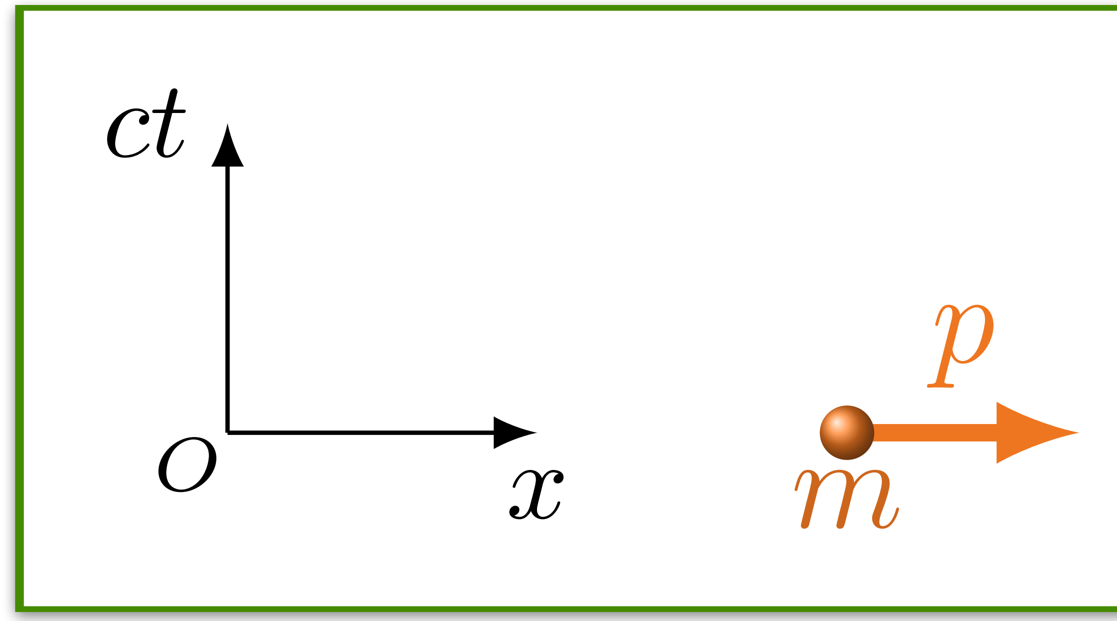
$$p^\mu = \begin{pmatrix} E/c \\ p^1 \\ p^2 \\ p^3 \end{pmatrix} \Rightarrow E^2 = (m_0c^2)^2 + p^2c^2$$

# Dinâmica relativística

$$E = \pm\sqrt{(m_0c^2)^2 + p^2c^2}$$



# Eletromagnetismo




$$p^\mu = \begin{pmatrix} mc \\ p^1 \\ p^1 \\ p^3 \end{pmatrix}$$

$$\bar{p}^\mu = \begin{pmatrix} m_0 c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta^\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

$$p^\mu = m_0 \eta^\mu$$

$$J^\mu = \rho_0 \eta^\mu$$

# Eletromagnetismo

$$J^\mu = \rho_0 \eta^\mu$$

$$\eta^\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

$$J^\mu = \begin{pmatrix} \rho \\ J^1 \\ J^2 \\ J3 \end{pmatrix}$$

# Eletromagnetismo

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$$\eta^\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

$$J^\mu = \begin{pmatrix} c\rho \\ J^1 \\ J^2 \\ J3 \end{pmatrix}$$

$$\frac{\partial J^\mu}{\partial x^\mu} = \frac{\partial \rho}{\partial t} + \frac{\partial J^1}{\partial x^1} + \frac{\partial J^2}{\partial x^2} + \frac{\partial J^3}{\partial x^3}$$

# Eletromagnetismo

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$$\boxed{\frac{\partial J^\mu}{\partial x^\mu} = 0}$$

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$$\boxed{\frac{\partial J^\mu}{\partial x^\mu} = 0}$$

$$\boxed{\partial_\mu J^\mu = 0}$$

# Eletromagnetismo
# Potenciais

$$J^\mu = \rho_0 \eta^\mu$$

$$A^\mu = \begin{pmatrix} V/c \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

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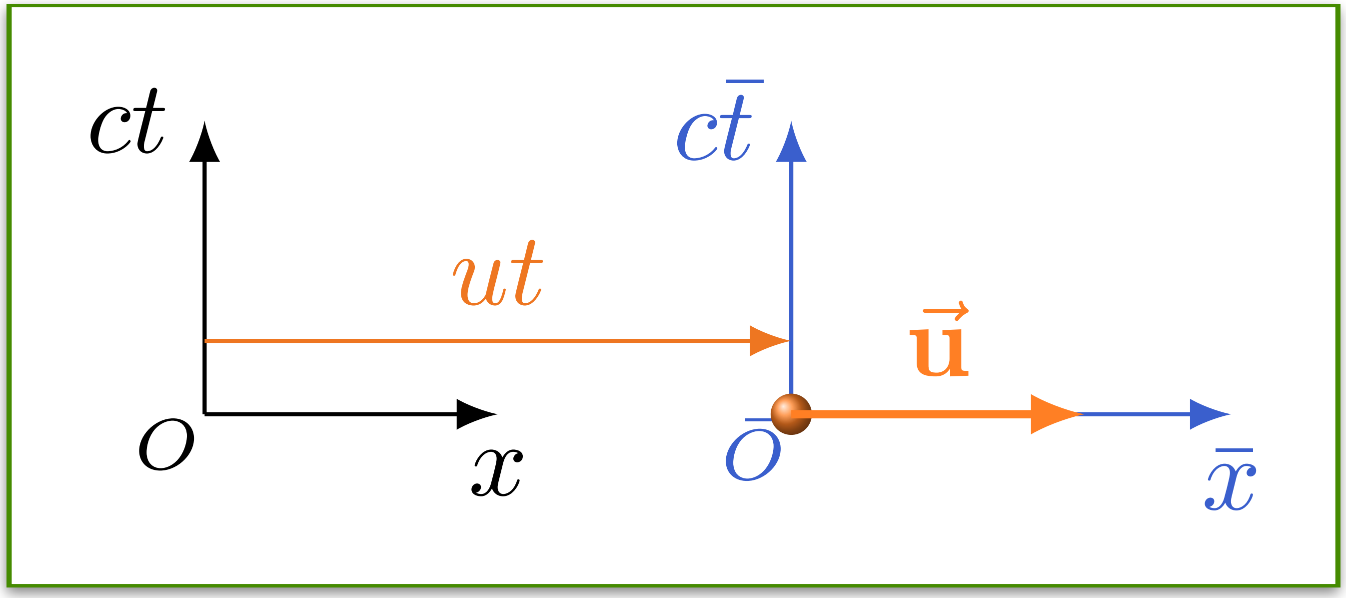


$$\bar{V} = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

$$\bar{\mathbf{A}} = 0$$

# Eletromagnetismo
# Potenciais

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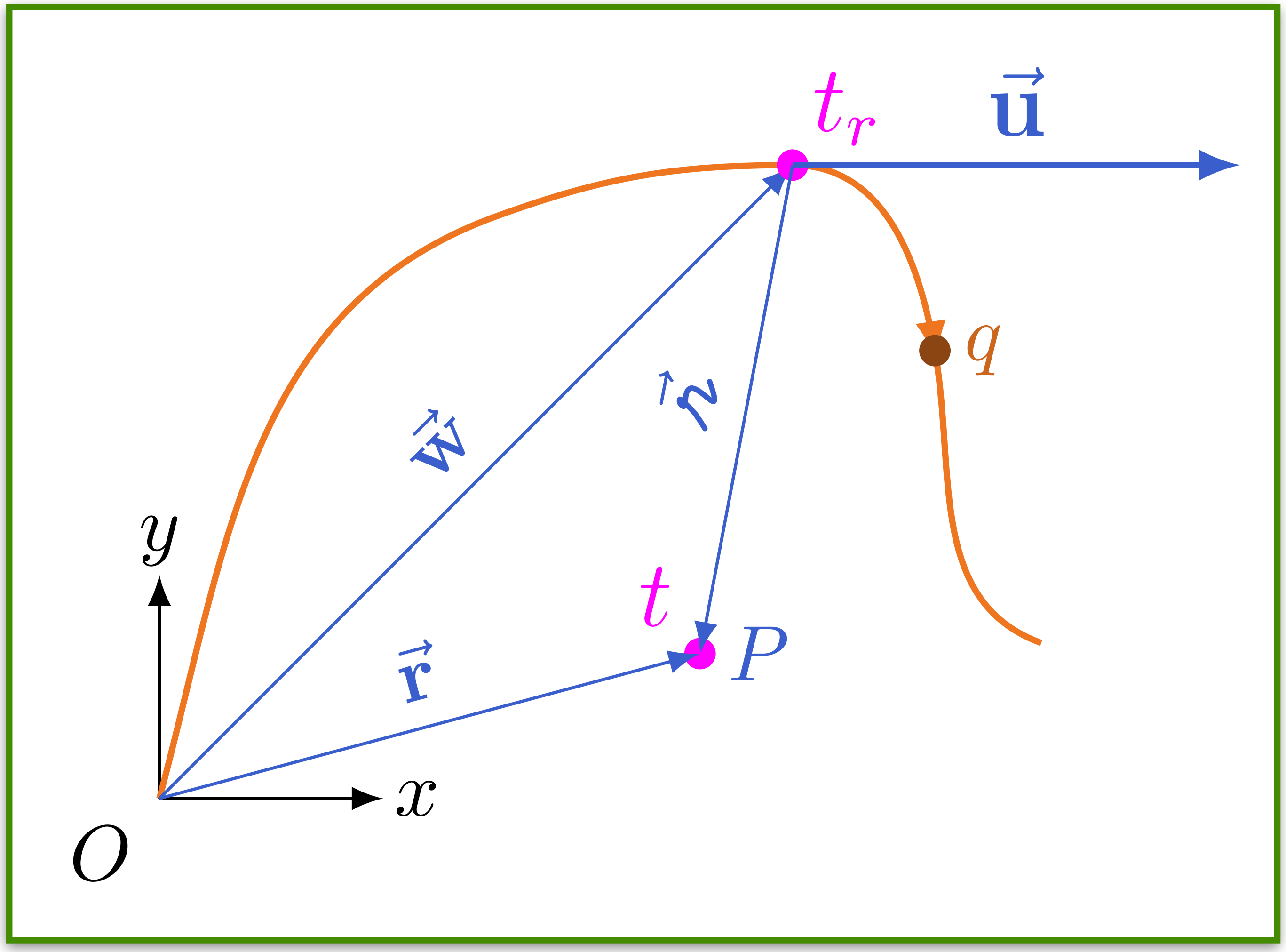

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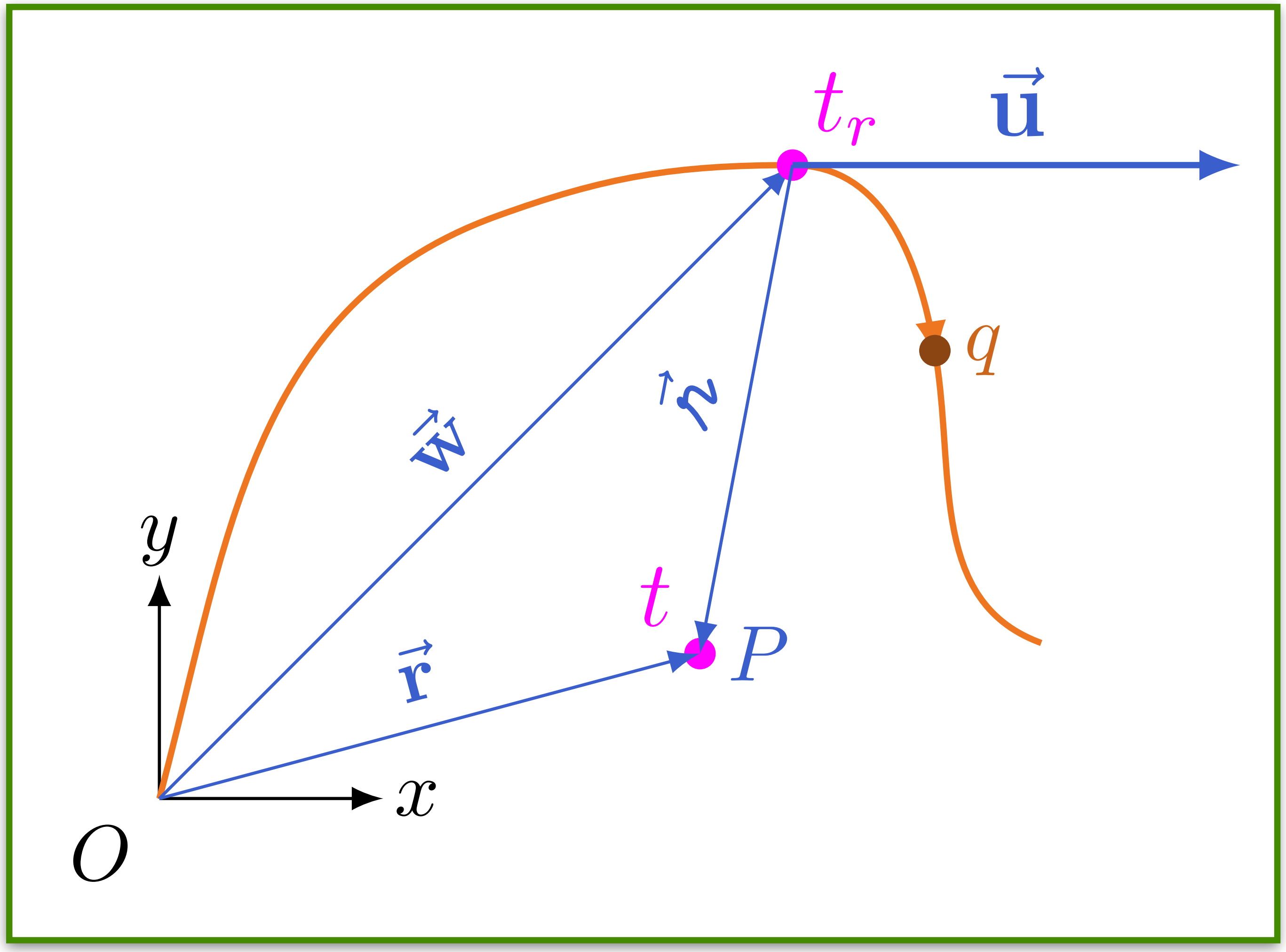
$$\mathscr{r} = ?$$



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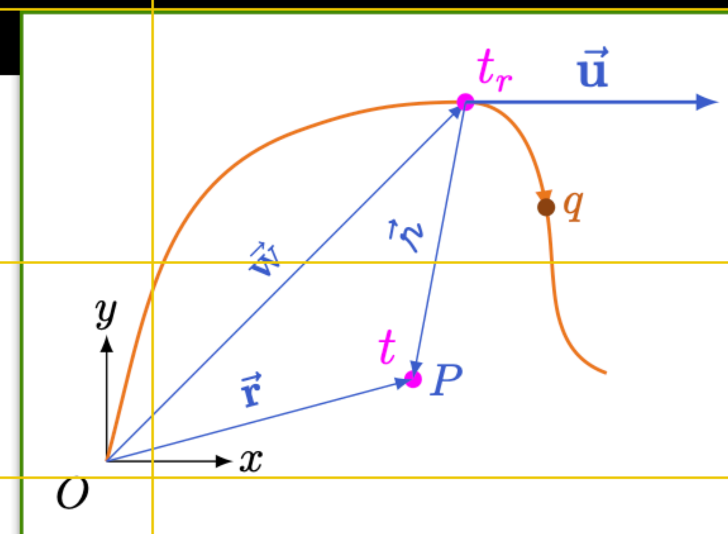
$$\eta^{\mu} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

Pratique o que aprendeu




$$\mathscr{r}_{\mu}\eta^{\mu} = ?$$

$$\mathscr{r}_{\mu} = \begin{pmatrix} -c(t-t_r) & \mathscr{r}_1 & \mathscr{r}_2 & \mathscr{r}_3 \end{pmatrix}$$

$$c(t-t_r) = \mathscr{r}$$

$$\mathscr{r}_{\mu}\eta^{\nu} = -\gamma(c\mathscr{r} - \vec{\mathbf{u}}\cdot\vec{\mathscr{r}})$$

# Eletromagnetismo
# Potenciais

$$J^\mu = \rho_0 \eta^\mu$$

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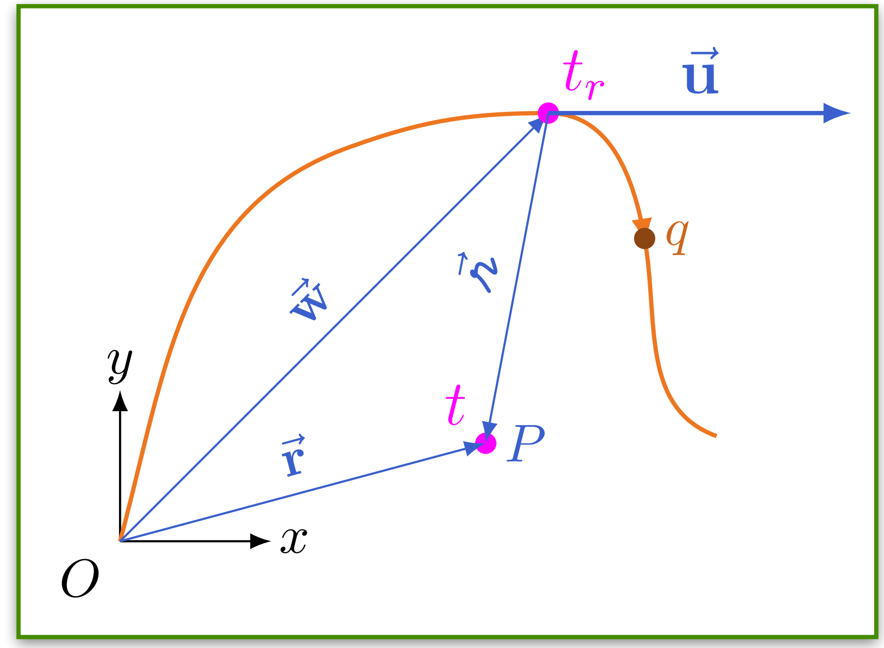



$$\bar{V} = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

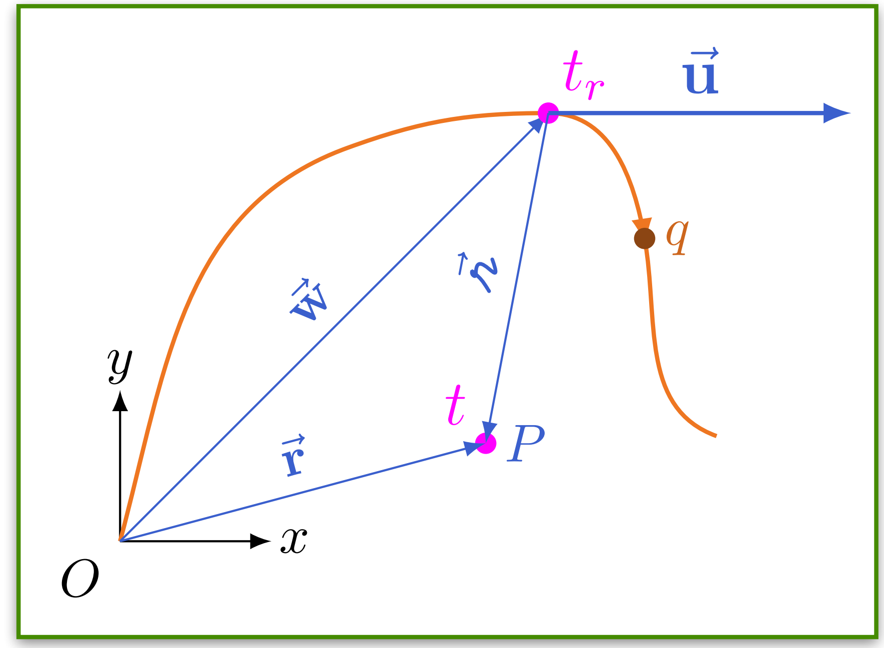
$$\bar{\mathbf{A}} = 0$$

$$\mathscr{r} = ?$$

$$\boxed{-\frac{1}{c}x_\mu \eta^\mu = \gamma(\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c})}$$

$$\bar{V} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{-\frac{1}{c}x_\mu \eta^\mu}\right)$$

# Eletromagnetismo
# Potenciais




$$A^\mu = \begin{pmatrix} V/c \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\bar{V} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{-\frac{1}{c}x_\mu\eta^\mu}\right)$$

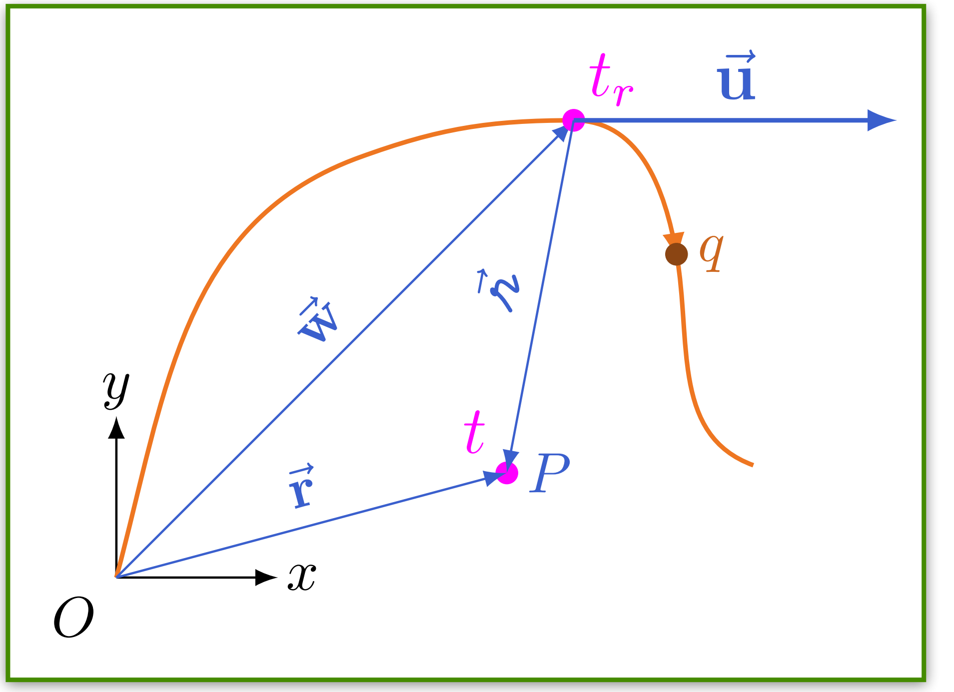
$$\bar{\mathbf{A}} = 0$$

$$\mathscr{r} = ?$$

$$\boxed{-\frac{1}{c}x_\mu\eta^\mu = \gamma\left(\mathscr{r} - \vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}\right)}$$

$$-\frac{1}{c}x_\mu\eta^\nu = \begin{cases} \mathscr{r} & \text{(no referencial da partícula, onde } v = 0) \\ \text{invariante} & \text{(independe do referencial)} \end{cases}$$

# Eletromagnetismo
# Potenciais




$$A^\mu = \begin{pmatrix} V/c \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\bar{V} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{-\frac{1}{c}x_\mu\eta^\mu}\right)$$

$$A^\mu = \begin{pmatrix} \gamma\bar{V}/c \\ \gamma\beta\bar{V} \\ 0 \\ 0 \end{pmatrix}$$

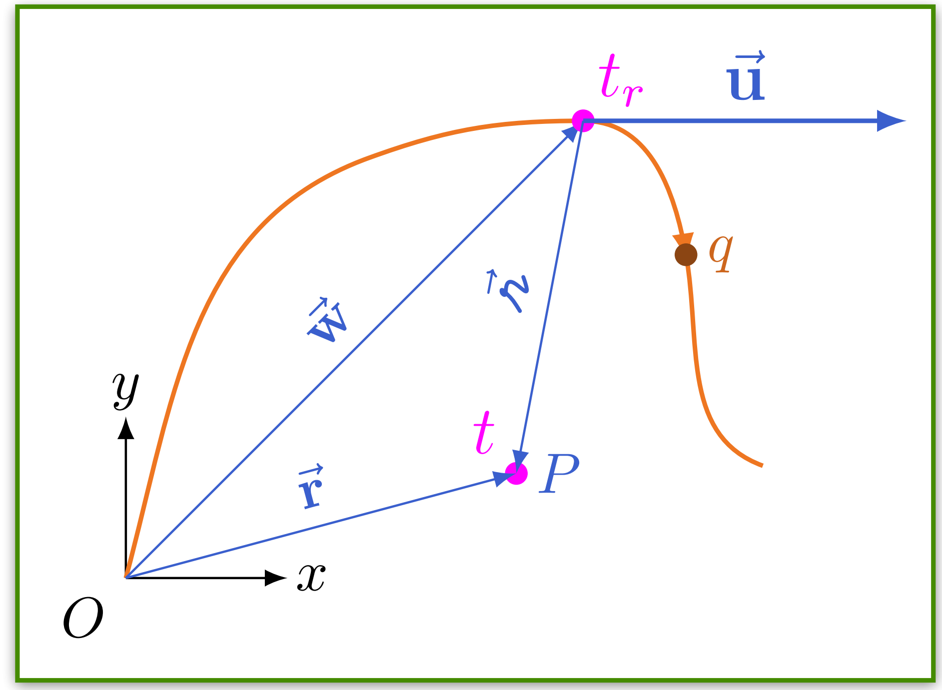
$$\bar{\mathbf{A}} = 0$$

$$\mathscr{r} = ?$$

$$\boxed{-\frac{1}{c}x_\mu\eta^\mu = \gamma(\mathscr{r} - \vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c})}$$

# Eletromagnetismo
# Potenciais

$$A^\mu = \begin{pmatrix} V/c \\ A_x \\ A_y \\ A_z \end{pmatrix}$$




$$\bar{V} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{-\frac{1}{c}x_\mu \eta^\mu}\right)$$

$$\bar{\mathbf{A}} = 0$$

$$A^\mu = \begin{pmatrix} \gamma\bar{V}/c \\ \gamma\beta\bar{V} \\ 0 \\ 0 \end{pmatrix}$$

$$\mathscr{r} = ?$$

$$\boxed{-\frac{1}{c}x_\mu \eta^\mu = \gamma\left(\mathscr{r} - \vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}\right)}$$

$$A^\mu = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r} - \vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}}\begin{pmatrix} 1/c \\ v/c^2 \\ 0 \\ 0 \end{pmatrix}$$

# Eletromagnetismo
# Potenciais

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c}}$$

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}}}{\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c}}$$