Mathematical Morphology for image processing

scc0251/scc5830

www.icmc.usp.br/∼moacir — moacir@icmc.usp.br

ICMC/USP — São Carlos, SP

2020

Agenda

Agenda

Definition

- A technique for analysis and processing of geometrical structures
- often applied to digital images, but can also be applied to: graphs, surface meshes, solids and other

Definition

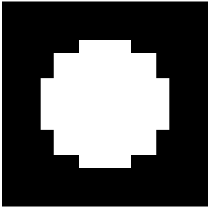
- Mathematical morphology concerns the study of sets
- We define objects of an image as sets

Definition

- Mathematical morphology concerns the study of sets
- We define objects of an image as sets

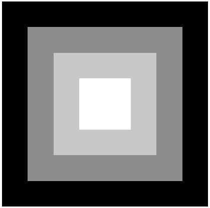
Binary images are defined by a set Z^2 (bi-dimensional integers), usually for the white pixels.

Each element of a set is a vector with the coordinates (x, y) of pixels.



Definition

Grey level images are defined by sets Z^3, two components are coordinates (x, y), the third is the intensity value



Definition

Given two sets $A, B \in Z^2$, we can define union, intersection, complement and difference:

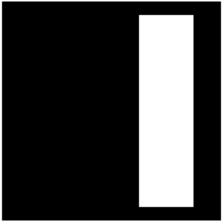
- Complement: $A^c = \{x | x \notin A\}$
- Difference: $A - B = \{x | x \in A \vee x \notin B\} = A \cap B^c$

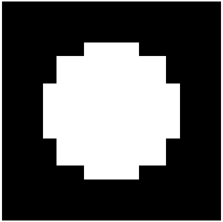
Definition

Given two sets $A, B \in Z^2$, we can define union, intersection, complement and difference:

- Complement: $A^c = \{x | x \notin A\}$
- Difference: $A - B = \{x | x \in A \vee x \notin B\} = A \cap B^c$

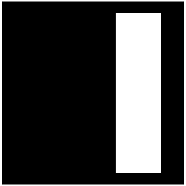
Consider the images as A and B, respectively



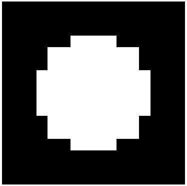


Definition - examples

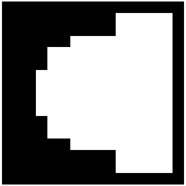
A



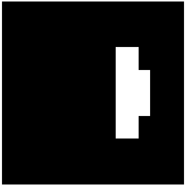
B



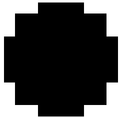
Union



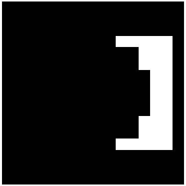
Intersection



Complement B



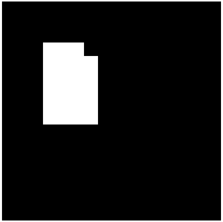
Difference A-B



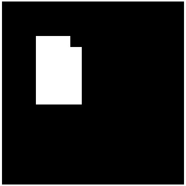
Definition

Translation and reflection are two other important morphological operations

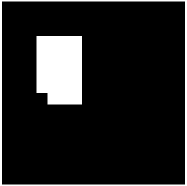
- Translation: $A_z = \{c | c = a + z, a \in A\}$
- Reflection: $\widehat{B} = \{x | x = -b, b \in B\}$



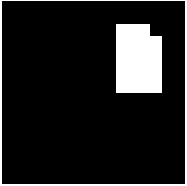
Definition - examples



Reflection



Translation



Definition

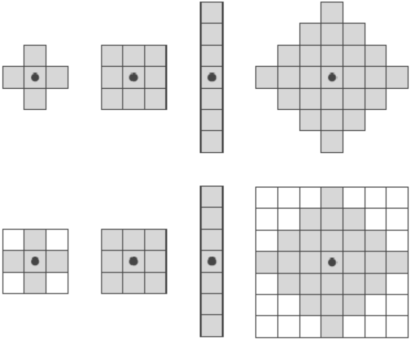
Modeling the image as sets, we can also apply logical operators

- NOT
- AND
- OR
- XOR

... and combination of those operations

Definition

Translation and reflection are used to process images using *structuring elements*



Agenda

Erosion

With A and B as sets Z^2, the **erosion** of A by B is:

$$A \ominus B = \left\{ z | \hat{B}_z \subseteq A \right\},$$

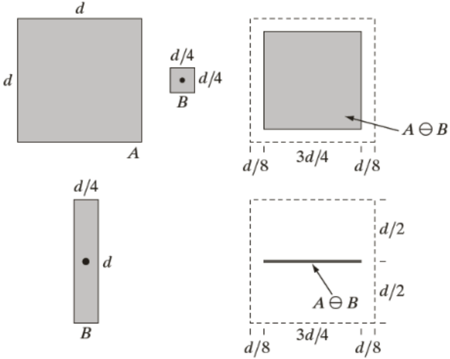
i.e. the set of all points z so that the reflection version of B is contained in A.

Erosion

With A and B as sets Z^2, the **erosion** of A by B is:

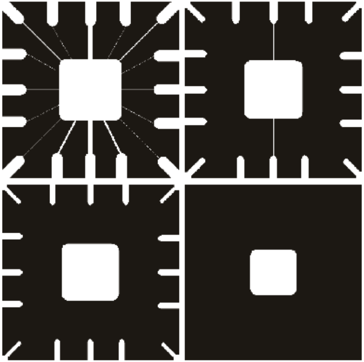
$$A \ominus B = \left\{ z | \hat{B}_z \subseteq A \right\},$$

i.e. the set of all points z so that the reflection version of B is contained in A.



Erosion : example

Erosion by structuring elements (disk) of sizes 11×11, 15×15 and 45×45



Dilation

With A and B with sets of Z^2, the **Dilation** of A by B is:

$$A \oplus B = \left\{ z | \hat{B}_z \cap A = \varnothing \right\},$$

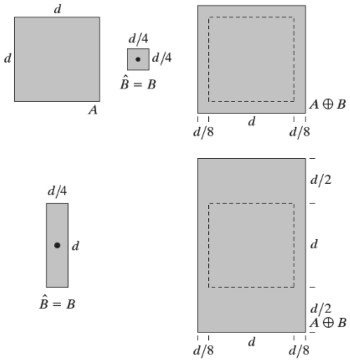
i.e. the set of all shifts z so that $\hat{B}$ and A are overlayed by at least one element.

Dilation

With A and B with sets of Z^2, the **Dilation** of A by B is:

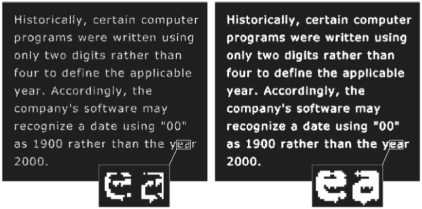
$$A \oplus B = \left\{ z | \hat{B}_z \cap A = \varnothing \right\},$$

i.e. the set of all shifts z so that $\hat{B}$ and A are overlayed by at least one element.



Erosion : example

OCR



0	1	0
1	1	1
0	1	0

Duality

Dilation / Erosion are dual operators with respect to the complement and reflection of sets:

$$(A \ominus B)^C = A^C \oplus \hat{B}$$
$$(A \oplus B)^C = A^C \ominus \hat{B}$$

... when the structuring element is symmetric, $\hat{B} = B$ and:

- *Dilation of the object* can be obtained by the *Erosion of the background*
- *Erosion of the object* can be obtained by the *Dilation of the background*

Agenda

Opening

Smooth out the contours of the object, eliminating small saliences
Opening of A by the structuring element B is:

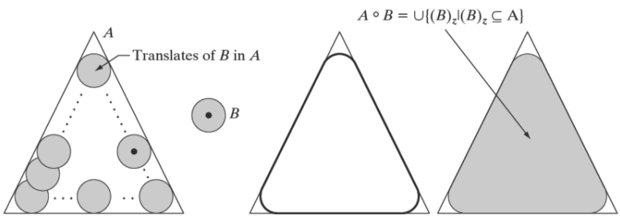
$$A \circ B = (A \ominus B) \oplus B$$

Opening

Smooth out the contours of the object, eliminating small saliences
Opening of A by the structuring element B is:

$$A \circ B = (A \ominus B) \oplus B$$

Geometric interpretation



i.e. the union of all translations of B that fits in A:

$$A \circ B = \cup \{B_z | B_z \subseteq A\},$$

Closing

Smooth out contours, eliminate small discontinuities and small holes

Closing of A by the structuring element B is:

$$A \bullet B = (A \oplus B) \ominus B$$

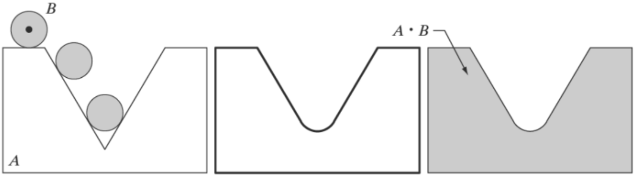
Closing

Smooth out contours, eliminate small discontinuities and small holes

Closing of A by the structuring element B is:

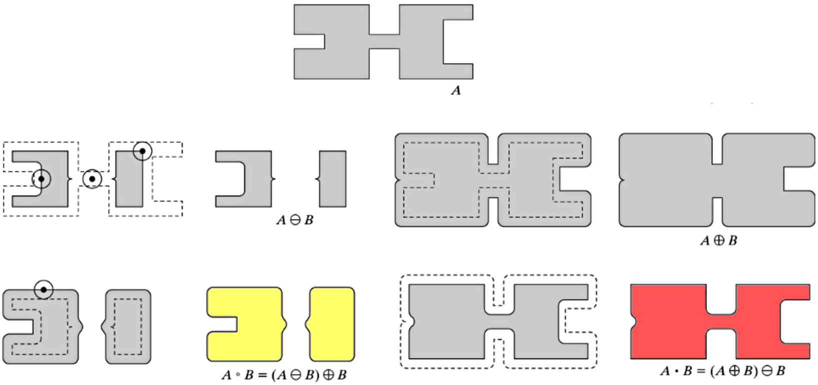
$$A \bullet B = (A \oplus B) \ominus B$$

Geometric interpretation

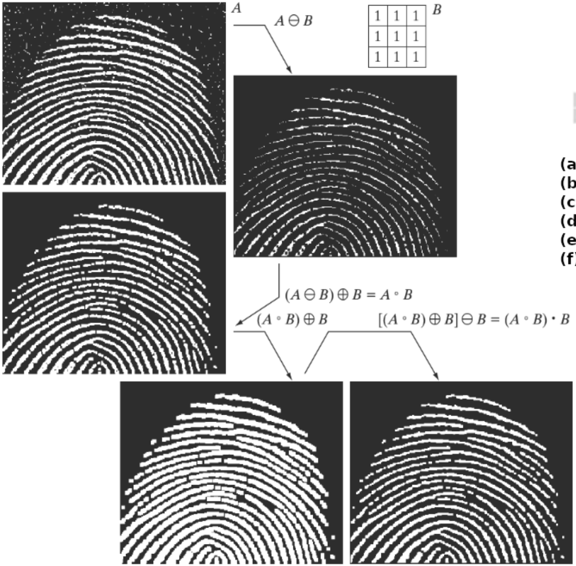


i.e. points in which $B_z \cap A \neq \varnothing$

Opening and Closing: example



Combination of morphological operators



(a) Noisy image
(b) Structuring element
(c) Eroded image
(d) Opening of A
(e) Dilation of opening
(f) Closing of opening

Agenda

Hit-or-Miss

Shape detection using the structuring element D

- Erosion $A \ominus D$ can be seen as the set of (*hits*).

Hit-or-Miss

Shape detection using the structuring element D

- Erosion $A \ominus D$ can be seen as the set of (*hits*).
- Only erosion cannot guarantee that the object is disjunct

Hit-or-Miss

Shape detection using the structuring element D

- Erosion $A \ominus D$ can be seen as the set of (*hits*).
- Only erosion cannot guarantee that the object is disjunct
- Using local background information $W - D$ (W is a window containing D) makes each object surrounded by background pixels,

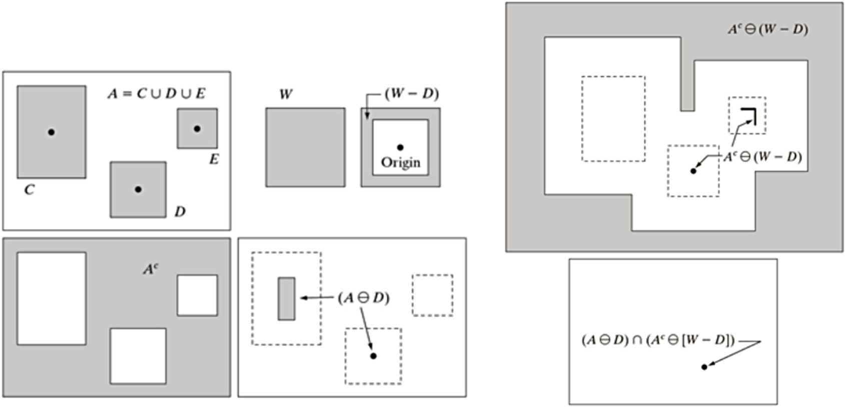
Hit-or-Miss

Shape detection using the structuring element D

- Erosion $A \ominus D$ can be seen as the set of (*hits*).
- Only erosion cannot guarantee that the object is disjunct
- Using local background information $W - D$ (W is a window containing D) makes each object surrounded by background pixels,
- Transform is composed of two operations:

$$A \circledast B = (A \ominus D) \cap \left[A^C \ominus (W - D)\right]$$

Hit-or-Miss


$A = C \cup D \cup E$
C
D
E
W
(W − D)
Origin
A^c
$(A \ominus D)$
$A^c \ominus (W - D)$
$A^c \ominus (W - D)$
$(A \ominus D) \cap (A^c \ominus [W - D])$

Hit-or-Miss

Modify the notation writing $D = (B_1, B_2)$, and:

$$A \circledast B = (A \ominus B_1) \cap \left[A^C \ominus B_2\right]$$

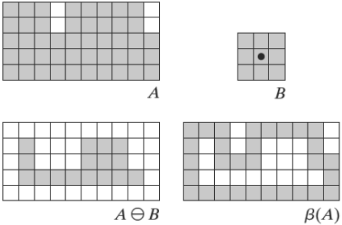
Using difference and dual relationship between Erosion and Dilation:

$$A \circledast B = (A \ominus B_1) - \left(A \oplus \hat{B}_2\right)$$

The three equations represent the hit-or-miss transform.

Edge-detection

$$\beta(A) = A - (A \ominus B)$$

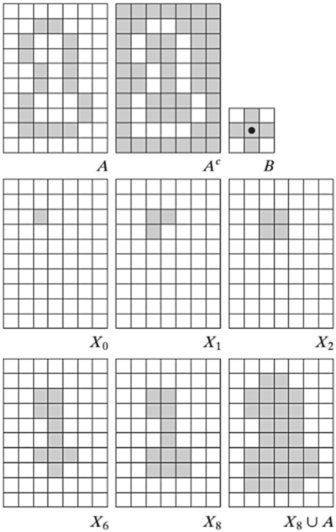


Filling

Consider A as an closed edge 8-conected

1. From a point (seed) X_0, inside the edge
2. For $k = 1, 2, 3, \cdots$:
   - $X_k = (X_{k-1} \oplus B) \cap A^C$
   - if $(X_k = X_{k-1})$, terminate.

This operation is also called conditional Dilation.

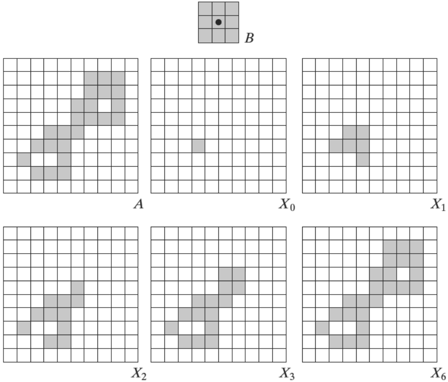


Extracting conected components

- Starting with X_0, a pixel 1 in some place of the image, ad for $k = 1, 2, 3, \cdots$:

$$X_k = (X_{k-1} \oplus B) \cap A$$

- if ($X_k = X_{k-1}$), terminate
- At the end, X_k contains the pixels of a conected components
- continue looking for not visited pixels with value 1, and finding the remaining elements.



Agenda

Erosion

With f and b as sets of Z^3, **Erosion** of f by b:

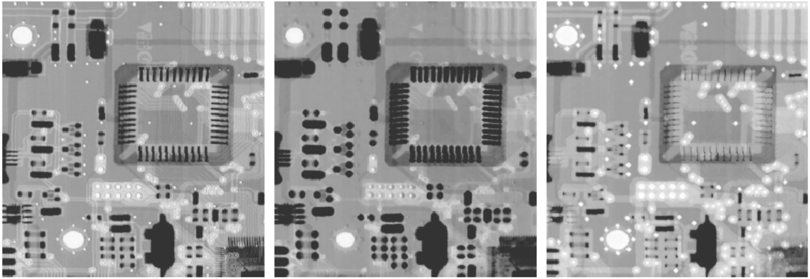
$$f \ominus b = \min_{(s,t) \in b} \{f(x+s, y+t)\},$$

Dilation

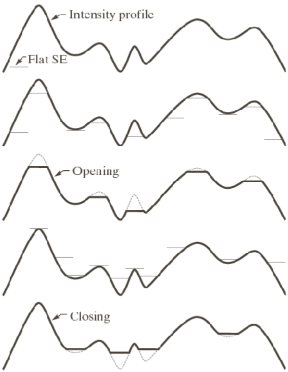
With f and b as sets of Z^3, **Dilation** of f by b:

$$f \oplus b = \max_{(s,t) \in b} \{f(x+s, y+t)\},$$

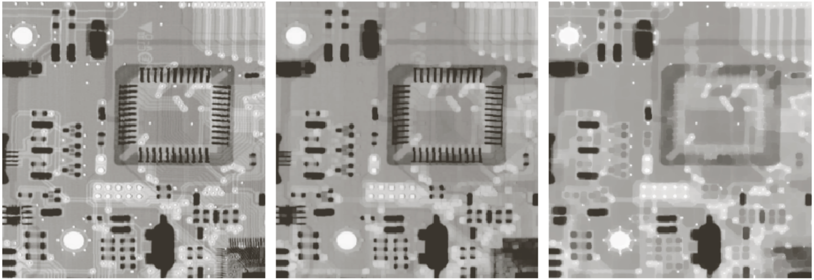
Erosion and Dilation



Opening and Closing effects



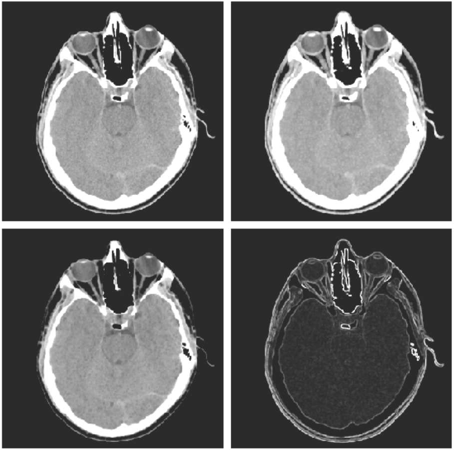
Opening and Closing effects



Morphological gradient

The morphological gradient g of an image f is:

$$g = (f \oplus b) - (f \ominus b)$$



References

- Gonzalez, R.C., Woods, R.E. Processamento Digital de Imagens, 3.ed, 2010, Capítulo 9.