

Graduate Statistical Mechanics -3st Exam- 24/11/2022

Instruções para a realização da avaliação: A prova deverá ser enviada até 19:00h de 06/01/2023 para os emails fiorecarlos.cf@gmail.com e william2.castilho@usp.br. Ela pode ser realizada com a consulta de livros e/ou materiais, porém as respostas e interpretações são **individuais**.

Todas as respostas deverão ser devidamente justificadas com base em argumentos físicos e matemáticos adequados. A não justificativa das respostas implicará na não integralidade da nota. Utilizem o idioma que for melhor para você.

Para ajudar na correção e garantir que tudo corra bem, procurem seguir as seguintes recomendações:

- procurem mandar a prova em um único arquivo PDF, pode ser foto ou scan. Sugestão de app: Adobe Scan;
- verifiquem a ordem das páginas, a orientação delas e a legibilidade do texto;
- mandem para o email do professor e do monitor e confirmem o envio.

Boa prova!

Monte Carlo simulations and nonequilibrium phase transitions

In the original MV, each site i of the lattice can have spins $+1$ or -1 and with probability $1 - f$ it tends to align itself with its local neighborhood majority and, with complementary probability f , the majority rule is not followed. The transition rate from σ_i to $-\sigma_i$ is given as follows:

$$w_i(\sigma) = \frac{1}{2} \left\{ 1 - (1 - 2f)\sigma_i S\left(\sum_{j=1}^z \sigma_j\right) \right\}, \quad (1)$$

where $\sum_{j=1}^z \sigma_j$ are the nearest neighbor sites of the site i and $S(X) = \pm 1$ and 0 , according to whether $X > 0$, < 0 and $X = 0$, respectively. In the case of a square lattice, $z = 4$.

As discussed in my lectures, by increasing the misalignment parameter f , a continuous order-disorder phase transition takes place at f_c , irrespective the lattice topology. The order parameter for such phase transition is the magnetization per spin $m = \langle \sigma_i \rangle$.

a) For a square lattice and system sizes $L = 10, 20, 30$ and 40 , show curves of the magnetization per spin m versus f , for f ranging from 0.01 to 0.25 in increments of $\Delta f = 0.01$. (2,5)

b) Plot the behavior of the order parameter variance $\chi = L^2[\langle m^2 \rangle - \langle m \rangle^2]$ for the above system sizes and verify that its maximum becomes sharper as L increases. (2,5)

c) In order to locate the critical point f_c , evaluate the reduced fourth-order cumulant $U_4 = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$ and plot its behavior for the same system size as before. Verify that all curves cross at the vicinity of the critical point. (2,5)

d) Once the critical point f_c is found, evaluate the critical exponents β/ν and γ/ν and verify that they are the same to the Ising model in a bidimensional (square) lattice. (2,5)

e) Obtain curves for the entropy production per site for the same system sizes and parameters as those from item a). (2,0)

Hint: Adapt the Ising model code I sent you by including the majority vote transition rate.