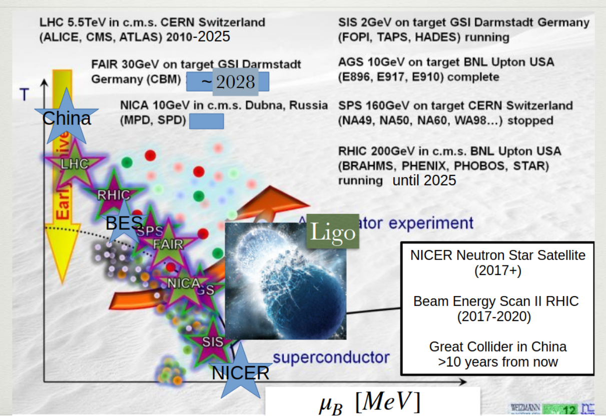
Lecture 25
Exploration of the QCD phase diagram and search for its critical point (fim)

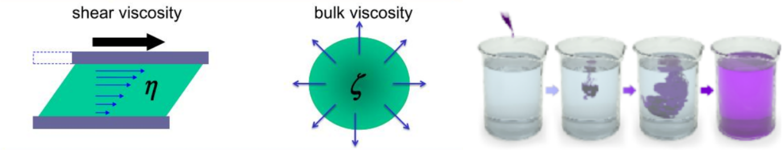


In this lecture, we discuss how to extend the standard model of heavy ion collisions to deal with lower energies, as well as look at more data on the QCD phase diagram.

Hydrodynamics without critical effect

Inclusion of baryon diffusion (green text) in the hydro equations

diffusion



Generally speaking, diffusion refers to the transport of substance against a concentration gradient. The end result is a gradual mixing. It is very important for exemple when computing net-baryon cumulants.

The hydro equations are:

$$\boxed{\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu (n\, u^\mu) = 0}$$

where to incorporate shear and bulk viscosities:

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \pi^{\mu\nu} + (g^{\mu\nu} - u^\mu u^\nu)\Pi$$

$$j^\mu = j^\mu_{ideal} + q^\nu$$

As seen in lecture 10, to ensure causality, in Israel-Stewart theory, DNMR theory, etc, evolution equations must be given (with the Laudau choice):

$$\tau_\Pi D\Pi + \Pi = \Pi_{NS} + etc.$$

$$\tau_q \Delta^{\mu\nu} Dq_\nu + q^\mu = q^\mu_{NS} + etc.$$

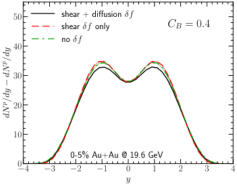
$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = \pi^{\mu\nu}_{NS} + etc$$

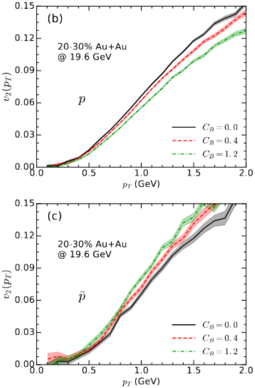
where $\Pi_{NS} = -\zeta(d_\mu u^\mu)$, $q_{NS} = \kappa_n \nabla^\mu(\mu_b/T)$ and $\pi^{\mu\nu}_{NS} = 2\eta\Delta^{\mu\nu}_{\alpha\beta}\nabla^\alpha u^\beta$
and $D = u_\mu d^\mu$, $\nabla^\mu = \Delta^{\mu\nu} d_\nu$,
$\Delta^{\mu\nu}_{\alpha\beta} = 1/2(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta) - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$

Inclusion of baryon diffusion in the Cooper-Frye formula

Similarly to the cases of shear and bulk viscosities, a term must be added to the thermodynamic part of the distribution function. As one would expect, its effect is mostly seen in protons/antiprotons variables:

$$E\frac{d^3N}{d^3p} = \int f(x,p)p^\mu d\sigma_\mu \text{ with } f(x,p) = f_{eq}(x,p) + \delta f_{shear} + \delta f_{bulk} + \delta f_{diff}$$



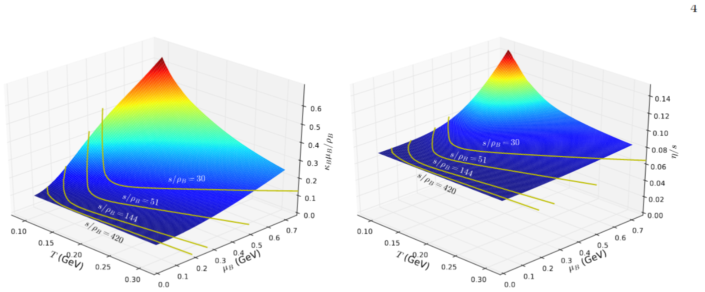


Denicol et al. 1804.10557

Transport coefficients

In addition to η/s and ζ/s, a new transport coefficient is added: κ_n

All coefficients (lecture 12) must depend on both T and μ_b.



Denicol et al. 1804.10557: $\kappa_n\mu_b/n_b$ and η/s

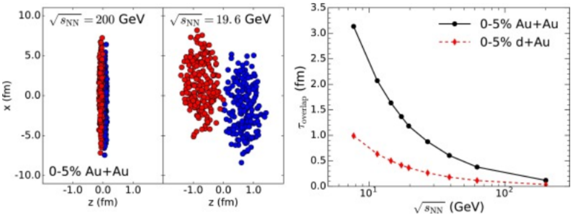
Collision time

- For high $\sqrt{s_{NN}}$, Lorentz contraction is large and the incoming nuclei have negligible width: the collision time is short (= time for the two sheets to pass through each other).

For lower $\sqrt{s_{NN}}$,the the finite thickness of the colliding nuclei along the longitudinal direction cannot be neglected.

$\tau_{overlap} = \frac{2R}{\gamma_z v_z} = \frac{2R}{\sqrt{\gamma_z^2 - 1}}$ with $\gamma_z = \sqrt{s_{NN}}/2/m_N$

Ex.: at $\sqrt{s_{NN}} = 7.7$ GeV, for Au+Au, $\tau_{overlap} \sim 3$ fm.



BEST collaboration: NPA 1017 (2022) 122343

- One needs to re-think how to initialize hydrodynamics.

See e.g. Chen and Schencke 1710.00881; Du, Heinz and Vujanovic 1807.04721

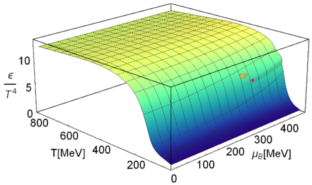
Hydrodynamics with critical effects

Equation of state with critical point

• One possibility to incorporate the critical point is to use the lattice QCD equation of state extrapolated to higher μ_b (see lectures 6 and 24) and add a critical contribution to it.

Nonaka and Asakawa, PRC 71 (2005) 044904; Parotto et al. PRC 101 (2020) 034901/1805.05249 and Stafford et al. EPJP 136 (2021621/2103.08146; Kapusta, Welle and Plumberg 2112.07563

Here is an exemple we met already



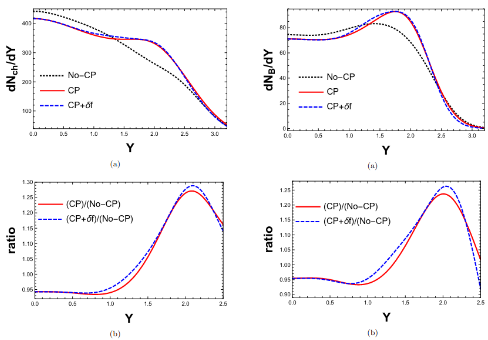
• Some works use an equation of state with crossover such as NEOS and incorporate critical effects only via transport coefficients (see below)

Du, An, Heinz 2107.02302

Transport coefficients with critical effects

• When the system approaches a critical point, the equilibrium correlation length, becomes large and eventually diverges.
In the H-class model: $\eta \sim \xi^{(1/19)(4-d)}$, $\zeta \sim \xi^3$, $\kappa_n \sim \xi$, $\tau_\Pi \sim \xi^3$, $\tau_q \sim \xi^2$, etc.

• While it might be that critical effects on η have negligible consequences, it is not the same for ζ and κ_n in principle.
In fact critical effects on ζ may not matter for observables (2107.02302) but this is not the case for κ_n:



Monnai, Mukherjee and Yin 1606.00771

Fluctuation dynamics near the critical point

• Near the critical popint, it has been argued that since the relaxation times increase (as powers of ξ), quantities do not manage to return to tehir equilibrium value and traditional hydrodynamics would fail.

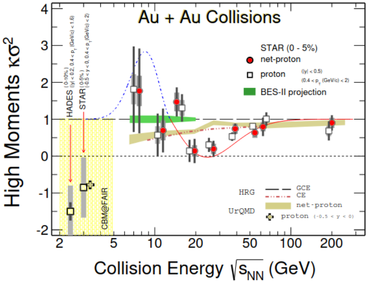
Some models try to circuvent this, in particular Hydro+. For more details, see the review by the BEST collaboration: NPA 1017 (2022) 122343.

• We see that the extension of hydrodynamics to lower energy is still under progress.

Some (tentative) conclusions from data

Net-proton fluctuations

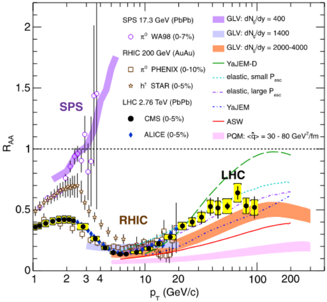
• In lecture 24, we saw that the most promising data on the possible existnce of a critical point is the non-monoticity of kurtosis.

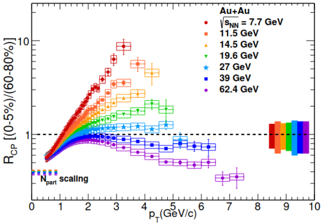


D. Almaalol et al. 2209.05009

• We also noticed that the fact that UrQMD agrees with the data point at $\sqrt{s_{NN}}$= 3 GeV, indicates that the system must be dominated by hadronic interactions at this energy.

High $p_\perp$ suppression





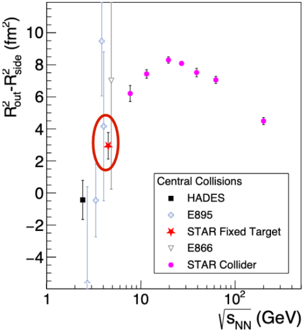
R_{AA} (comparison A+A/p+p) CMS: Eur. Phys. J. C 72 (2012) 1945 and R_{CP} (comparison central/peripheral) STAR:1707.01988

- R_{AA} and R_{CP} probe partonic energy loss in QGP.
- Standard QGP signal only dominates above $\sqrt{s_{NN}} \sim 20$ GeV

Interferometry

• From lecture 17: the long direction is the collision axis, outis along $\vec{k}_\perp = (\vec{p}_{1\perp} + \vec{p}_{2\perp})/2$ and side complements the orthogonal right-handed frame.
In the longitudinally moving system $(\vec{k}_l/k_0 = 0)$: $R^2_{out} - R^2_{side} \propto$ duration of emission (longer for a first order transition)

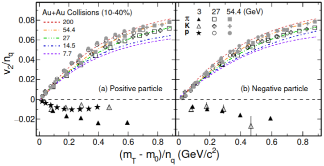
• We already saw that STAR claims possible evidence of a first order phase transition around $\sqrt{s_{NN}}$ = 20 GeV (cf. peak structure i.e. longer duration of emission)



STAR Phys. Rev. C 103 (2021) 034908 arXiv:2007.14005: new fixed target point at $\sqrt{s_{NN}}$ = 4.5 GeV

Anisotropic flow

- In lectures 15 and 16, we saw that elliptic flow (positive) was important to establish the existence of a QGP
- At $\sqrt{s_{NN}} = 3.$ GeV, elliptical flow becomes negative for all particles and is explained by transport models such as JAM and UrQMD c qualitatively reproduce the data



STAR arXiv:2108.00908

Conclusion from BES data

There is some evidence for

- a critical point from non-monoticity of kurtosis
- QGP above $\sqrt{s_{NN}} \sim 20$ GeV (not ruled out below) from high $p_\perp$ suppression
- a hadron medium (ex.UrQMD) at $\sqrt{s_{NN}} \sim 3$ GeV from kurtosis value and elliptic flow
- a first order transition around $\sqrt{s_{NN}} \sim 4.5$ GeV from interferometry radii

We should learn more in the next months