CHAPTER 16

COMPOSITES

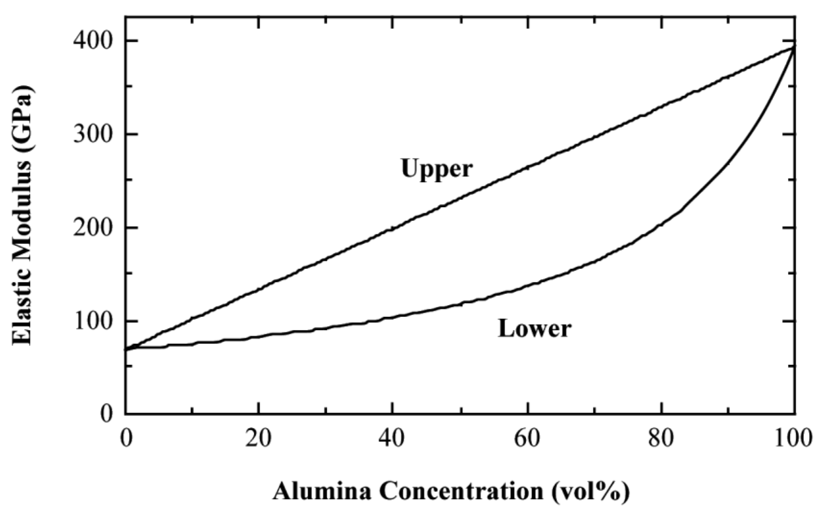
PROBLEM SOLUTIONS

Large-Particle Composites

16.1 *The mechanical properties of aluminum may be improved by incorporating fine particles of aluminum oxide (Al_2O_3). Given that the moduli of elasticity of these materials are, respectively, 69 GPa (10×10^6 psi) and 393 GPa (57×10^6 psi), plot modulus of elasticity versus the volume percent of Al_2O_3 in Al from 0 to 100 vol%, using both upper- and lower-bound expressions.*

Solution

The elastic modulus versus the volume percent of Al_2O_3 is shown below, on which is included both upper and lower bound curves; these curves were generated using Equations 16.1 and 16.2, respectively, and using the moduli of elasticity for aluminum and Al_2O_3 that were given in the problem statement.



16.2 *Estimate the maximum and minimum thermal conductivity values for a cermet that contains 85 vol% titanium carbide (TiC) particles in a cobalt matrix. Assume thermal conductivities of 27 and 69 W/m-K for TiC and Co, respectively.*

Solution

This problem asks for the maximum and minimum thermal conductivity values for a TiC-Co cermet. Using a modified form of Equation 16.1 the maximum thermal conductivity k_{max} is calculated as

$$k_{\text{max}} = k_m V_m + k_p V_p$$

$$= k_{\text{Co}} V_{\text{Co}} + k_{\text{TiC}} V_{\text{TiC}}$$

$$= (69 \text{ W/m-K})(0.15) + (27 \text{ W/m-K})(0.85) = 33.3 \text{ W/m-K}$$

Using a modified form of Equation 16.2, the minimum thermal conductivity k_{min} will be

$$k_{\text{min}} = \frac{k_{\text{Co}} k_{\text{TiC}}}{V_{\text{Co}} k_{\text{TiC}} + V_{\text{TiC}} k_{\text{Co}}}$$

$$= \frac{(69 \text{ W/m-K})(27 \text{ W/m-K})}{(0.15)(27 \text{ W/m-K}) + (0.85)(69 \text{ W/m-K})}$$

$$= 29.7 \text{ W/m-K}$$

16.3 *A large-particle composite consisting of tungsten particles within a copper matrix is to be prepared. If the volume fractions of tungsten and copper are 0.60 and 0.40, respectively, estimate the upper limit for the specific stiffness of this composite given the data that follow.*

	Specific Gravity	***Modulus of Elasticity (GPa)***
Copper	*8.9*	*110*
Tungsten	*19.3*	*407*

Solution

Given the elastic moduli and specific gravities for copper and tungsten we are asked to estimate the upper limit for specific stiffness when the volume fractions of tungsten and copper are 0.60 and 0.40, respectively. There are two approaches that may be applied to solve this problem. The first is to estimate both the upper limits of elastic modulus [$E_c(u)$] and specific gravity (ρ_c) for the composite, using expressions of the form of Equation 16.1, and then take their ratio. Using this approach

$$E_c(u) = E_{\mathrm{Cu}}V_{\mathrm{Cu}} + E_{\mathrm{W}}V_{\mathrm{W}}$$

$$= (110 \text{ GPa})(0.40) + (407 \text{ GPa})(0.60)$$

$$= 288 \text{ GPa}$$

And

$$\rho_c = \rho_{\mathrm{Cu}}V_{\mathrm{Cu}} + \rho_{\mathrm{W}}V_{\mathrm{W}}$$

$$= (8.9)(0.40) + (19.3)(0.60) = 15.14$$

Therefore

$$\text{Specific Stiffness} = \frac{E_c(u)}{\rho_c} = \frac{288 \text{ GPa}}{15.14} = 19.0 \text{ GPa}$$

With the alternate approach, the specific stiffness is calculated, again employing a modification of Equation 16.1, but using the specific stiffness-volume fraction product for both metals, as follows:

$$\text{Specific Stiffness} = \frac{E_{\text{Cu}}}{\rho_{\text{Cu}}} V_{\text{Cu}} + \frac{E_{\text{W}}}{\rho_{\text{W}}} V_{\text{W}}$$

$$= \frac{110 \text{ GPa}}{8.9}(0.40) + \frac{407 \text{ GPa}}{19.3}(0.60) = 17.6 \text{ GPa}$$

16.4 *(a) What is the distinction between cement and concrete?*

(b) Cite three important limitations that restrict the use of concrete as a structural material.

(c) Briefly explain three techniques that are used to strengthen concrete by reinforcement.

Solution

(a) Concrete consists of an aggregate of particles that are bonded together by a cement.

(b) Three limitations of concrete are: (1) it is a relatively weak and brittle material; (2) it experiences relatively large thermal expansions (contractions) with changes in temperature; and (3) it may crack when exposed to freeze-thaw cycles.

(c) Three reinforcement strengthening techniques are: (1) reinforcement with steel wires, rods, etc.; (2) reinforcement with fine fibers of a high modulus material; and (3) introduction of residual compressive stresses by prestressing or posttensioning.

Dispersion-Strengthened Composites

16.5 *Cite one similarity and two differences between precipitation hardening and dispersion strengthening.*

Solution

The similarity between precipitation hardening and dispersion strengthening is the strengthening mechanism--i.e., the precipitates/particles effectively hinder dislocation motion.

The two differences are: (1) the hardening/strengthening effect is not retained at elevated temperatures for precipitation hardening--however, it is retained for dispersion strengthening; and (2) the strength is developed by a heat treatment for precipitation hardening--such is not the case for dispersion strengthening.

Influence of Fiber Length

16.6 *For some glass fiber-epoxy matrix combination, the critical fiber length-fiber diameter ratio is 50. Using the data in Table 16.4, determine the fiber-matrix bond strength.*

Solution

This problem asks that, for a glass fiber-epoxy matrix combination, to determine the fiber-matrix bond strength if the critical fiber length-fiber diameter ratio is 50. Thus, we are to solve for τ_c in Equation 16.3. Since we are given that σ_f^* = 3.45 GPa from Table 16.4, and that $\frac{l_c}{d}$ = 50, then

$$\tau_c = \sigma_f^* \left(\frac{d}{2l_c}\right) = (3.45 \times 10^3 \text{ MPa})\left(\frac{1}{2}\right)\left(\frac{1}{50}\right) = 34.5 \text{ MPa}$$

16.7 *(a) For a fiber-reinforced composite, the efficiency of reinforcement η is dependent on fiber length l according to*

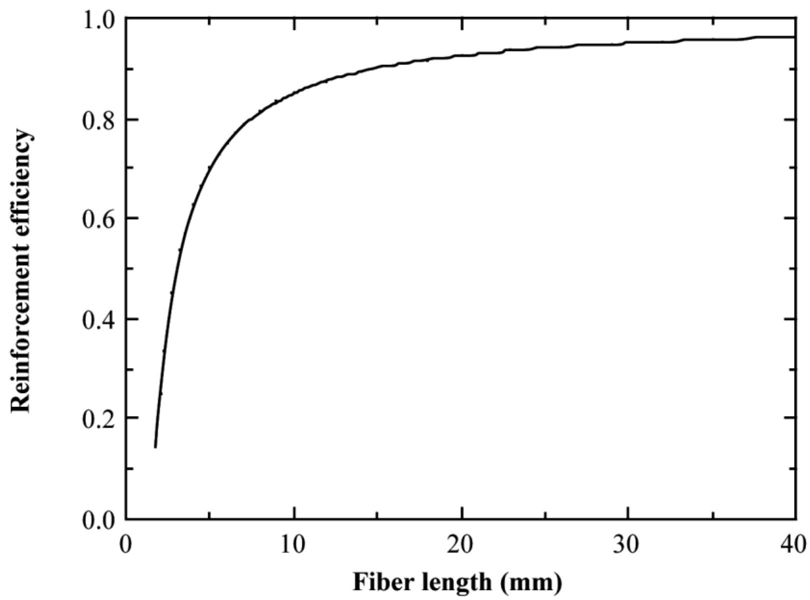
$$\eta = \frac{l - 2x}{l}$$

where x represents the length of the fiber at each end that does not contribute to the load transfer. Make a plot of η versus l to l = 40 mm (1.6 in.) assuming that x = 0.75 mm (0.03 in.).

(b) What length is required for a 0.80 efficiency of reinforcement?

Solution

(a) The plot of reinforcement efficiency versus fiber length is given below.



(b) This portion of the problem asks for the length required for a 0.80 efficiency of reinforcement. Solving for l from the given expression

$$l = \frac{2x}{1 - \eta}$$

Or, when x = 0.75 mm (0.03 in.) and η = 0.80, then

$$l = \frac{(2)(0.75 \text{ mm})}{1 - 0.80} = 7.5 \text{ mm} \ \ (0.30 \text{ in.})$$

Influence of Fiber Orientation and Concentration

16.8 *A continuous and aligned fiber-reinforced composite is to be produced consisting of 30 vol% aramid fibers and 70 vol% of a polycarbonate matrix; mechanical characteristics of these two materials are as follows:*

	***Modulus of Elasticity* [*GPa* (*psi*)]**	***Tensile Strength* [*MPa* (*psi*)]**
Aramid fiber	*131 (19×10^6)*	*3600 (520,000)*
Polycarbonate	*2.4 (3.5×10^5)*	*65 (9425)*

Also, the stress on the polycarbonate matrix when the aramid fibers fail is 45 MPa (6500 psi).

For this composite, compute the following:

(a) the longitudinal tensile strength, and

(b) the longitudinal modulus of elasticity

Solution

This problem calls for us to compute the longitudinal tensile strength and elastic modulus of an aramid fiber-reinforced polycarbonate composite.

(a) The longitudinal tensile strength is determined using Equation 16.17 as

$$\sigma_{cl}^{*} = \sigma_m'\left(1 - V_f\right) + \sigma_f^{*}V_f$$

$$= (65\text{ MPa})(0.70) + (3600)(0.30)$$

$$= 1125\text{ MPa}\ \ (162{,}600\text{ psi})$$

(b) The longitudinal elastic modulus is computed using Equation 16.10a as

$$E_{cl} = E_m V_m + E_f V_f$$

$$= (2.4\text{ GPa})(0.70) + (131\text{ GPa})(0.30)$$

$$= 41\text{ GPa}\ \ (5.95 \times 10^6\text{ psi})$$

16.9 *Is it possible to produce a continuous and oriented aramid fiber-epoxy matrix composite having longitudinal and transverse moduli of elasticity of 57.1 GPa (8.28 $\times$ 10^6 psi) and 4.12 GPa (6 $\times$ 10^5 psi), respectively? Why or why not? Assume that the modulus of elasticity of the epoxy is 2.4 GPa (3.50 $\times$ 10^5 psi).*

Solution

This problem asks for us to determine if it is possible to produce a continuous and oriented aramid fiber-epoxy matrix composite having longitudinal and transverse moduli of elasticity of 57.1 GPa and 4.12 GPa, respectively, given that the modulus of elasticity for the epoxy is 2.4 GPa. Also, from Table 16.4 the value of E for aramid fibers is 131 GPa. The approach to solving this problem is to calculate values of V_f for both longitudinal and transverse cases using the data and Equations 16.10b and 16.16; if the two V_f values are the same then this composite is possible.

For the longitudinal modulus E_{cl} (using Equation 16.10b),

$$E_{cl} = E_m\left(1 - V_{fl}\right) + E_f V_{fl}$$

$$57.1 \text{ GPa} = (2.4 \text{ GPa})\left(1 - V_{fl}\right) + (131 \text{ GPa})V_{fl}$$

Solving this expression for V_{fl} (i.e., the volume fraction of fibers for the longitudinal case) yields V_{fl} = 0.425.

Now, repeating this procedure for the transverse modulus E_{ct} (using Equation 16.16)

$$E_{ct} = \frac{E_m E_f}{\left(1 - V_{ft}\right)E_f + V_{ft}E_m}$$

$$4.12 \text{ GPa} = \frac{(2.4 \text{ GPa})(131 \text{ GPa})}{\left(1 - V_{ft}\right)(131 \text{ GPa}) + V_{ft}(2.4 \text{ GPa})}$$

Solving this expression for V_{ft} (i.e., the volume fraction of fibers for the transverse case), leads to V_{ft} = 0.425. Thus, since V_{fl} and V_{ft} are equal, the proposed composite *is possible*.

16.10 *For a continuous and oriented fiber-reinforced composite, the moduli of elasticity in the longitudinal and transverse directions are 19.7 and 3.66 GPa (2.8 $\times 10^6$ and 5.3 $\times 10^5$ psi), respectively. If the volume fraction of fibers is 0.25, determine the moduli of elasticity of fiber and matrix phases.*

Solution

This problem asks for us to compute the elastic moduli of fiber and matrix phases for a continuous and oriented fiber-reinforced composite. We can write expressions for the longitudinal and transverse elastic moduli using Equations 16.10b and 16.16, as

$$E_{cl} = E_m\left(1 - V_f\right) + E_f V_f$$

$$19.7 \text{ GPa} = E_m(1 - 0.25) + E_f(0.25)$$

And

$$E_{ct} = \frac{E_m E_f}{\left(1 - V_f\right) E_f + V_f E_m}$$

$$3.66 \text{ GPa} = \frac{E_m E_f}{(1 - 0.25)E_f + 0.25 E_m}$$

Solving these two expressions simultaneously for E_m and E_f leads to

$$E_m = 2.79 \text{ GPa} \quad (4.04 \times 10^5 \text{ psi})$$

$$E_f = 70.4 \text{ GPa} \quad (10.2 \times 10^6 \text{ psi})$$

16.11 *(a) Verify that Equation 16.11, the expression for the fiber load–matrix load ratio (F_f/F_m), is valid. (b) What is the F_f/F_c ratio in terms of E_f, E_m, and V_f?*

Solution

(a) In order to show that the relationship in Equation 16.11 is valid, we begin with Equation 16.4—i.e.,

$$F_c = F_m + F_f$$

which may be manipulated to the form

$$\frac{F_c}{F_m} = 1 + \frac{F_f}{F_m}$$

or

$$\frac{F_f}{F_m} = \frac{F_c}{F_m} - 1$$

For elastic deformation, combining Equations 6.1 and 6.5

$$\sigma = \frac{F}{A} = \varepsilon E$$

or

$$F = A\varepsilon E$$

We may write expressions for F_c and F_m of the above form as

$$F_c = A_c \varepsilon E_c$$

$$F_m = A_m \varepsilon E_m$$

which, when substituted into the above expression for F_f/F_m, gives

$$\frac{F_f}{F_m} = \frac{A_c \varepsilon E_c}{A_m \varepsilon E_m} - 1$$

But, $V_m = A_m/A_c$, which, upon rearrangement gives

$$\frac{A_c}{A_m} = \frac{1}{V_m}$$

which, when substituted into the previous expression leads to

$$\frac{F_f}{F_m} = \frac{E_c}{E_m V_m} - 1$$

Also, from Equation 16.10a, $E_c = E_m V_m + E_f V_f$, which, when substituted for E_c into the previous expression, yields

$$\frac{F_f}{F_m} = \frac{E_m V_m + E_f V_f}{E_m V_m} - 1$$

$$= \frac{E_m V_m + E_f V_f - E_m V_m}{E_m V_m} = \frac{E_f V_f}{E_m V_m}$$

the desired result.

(b) This portion of the problem asks that we establish an expression for F_f/F_c. We determine this ratio in a similar manner. Now $F_c = F_f + F_m$ (Equation 16.4), or division by F_c leads to

$$1 = \frac{F_f}{F_c} + \frac{F_m}{F_c}$$

which, upon rearrangement, gives

$$\frac{F_f}{F_c} = 1 - \frac{F_m}{F_c}$$

Now, substitution of the expressions in part (a) for F_m and F_c that resulted from combining Equations 6.1 and 6.5 results in

$$\frac{F_f}{F_c} = 1 - \frac{A_m \varepsilon E_m}{A_c \varepsilon E_c} = 1 - \frac{A_m E_m}{A_c E_c}$$

Since the volume fraction of fibers is equal to $V_m = A_m/A_c$, then the above equation may be written in the form

$$\frac{F_f}{F_c} = 1 - \frac{V_m E_m}{E_c}$$

And, finally substitution of Equation 16.10(a) for E_c into the above equation leads to the desired result as follows:

$$\frac{F_f}{F_c} = 1 - \frac{V_m E_m}{V_m E_m + V_f E_f}$$

$$= \frac{V_m E_m + V_f E_f - V_m E_m}{V_m E_m + V_f E_f}$$

$$= \frac{V_f E_f}{V_m E_m + V_f E_f}$$

$$= \frac{V_f E_f}{\left(1 - V_f\right) E_m + V_f E_f}$$

16.12 *In an aligned and continuous glass fiber-reinforced nylon 6,6 composite, the fibers are to carry 94% of a load applied in the longitudinal direction.*

(a) Using the data provided, determine the volume fraction of fibers that will be required.

(b) What will be the tensile strength of this composite? Assume that the matrix stress at fiber failure is 30 MPa (4350 psi).

	***Modulus of Elasticity* [*GPa* (*psi*)]**	***Tensile Strength* [*MPa* (*psi*)]**
Glass fiber	*72.5 (10.5 × 10^6)*	*3400 (490,000)*
Nylon 6,6	*3.0 (4.35 × 10^5)*	*76 (11,000)*

Solution

(a) Given some data for an aligned and continuous glass-fiber-reinforced nylon 6,6 composite, we are asked to compute the volume fraction of fibers that are required such that the fibers carry 94% of a load applied in the longitudinal direction. From Equation 16.11

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{E_f V_f}{E_m \left(1 - V_f\right)}$$

Now, using values for F_f and F_m from the problem statement

$$\frac{F_f}{F_m} = \frac{0.94}{0.06} = 15.67$$

And when we substitute the given values for E_f and E_m into the first equation leads to

$$\frac{F_f}{F_m} = 15.67 = \frac{(72.5 \text{ GPa})V_f}{(3.0 \text{ GPa})\left(1 - V_f\right)}$$

And, solving for V_f yields, $V_f = 0.393$.

(b) We are now asked for the tensile strength of this composite. From Equation 16.17,

$$\sigma_{cl}^{*} = \sigma_m' \left(1 - V_f\right) + \sigma_f^{*} V_f$$

$$= (76 \text{ MPa})(1 - 0.393) + (3400 \text{ MPa})(0.393)$$

$$= 1382 \text{ MPa} \ \ (199{,}200 \text{ psi})$$

since values for σ_f^* (3400 MPa) and σ_m' (76 MPa) are given in the problem statement.

16.13 *Assume that the composite described in Problem 16.8 has a cross-sectional area of 320 mm^2 (0.50 in.^{2}) and is subjected to a longitudinal load of 44,500 N (10,000 lb$_f$).*

(a) Calculate the fiber–matrix load ratio.

(b) Calculate the actual loads carried by both fiber and matrix phases.

(c) Compute the magnitude of the stress on each of the fiber and matrix phases.

(d) What strain is experienced by the composite?

Solution

The problem stipulates that the cross-sectional area of a composite, A_c, is 320 mm^2 (0.50 in.^{2}), and the longitudinal load, F_c, is 44,500 N (10,000 lb$_f$) for the composite described in Problem 16.8. Also, for this composite

$V_f = 0.30$

$V_m = 0.70$

$E_f = 131$ GPa

$E_m = 2.4$ GPa

(a) First, we are asked to calculate the F_f/F_m ratio. According to Equation 16.11

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{(131\ \text{GPa})(0.30)}{(2.4\ \text{GPa})(0.70)} = 23.4$$

Or, $F_f = 23.4F_m$

(b) Now, the actual loads carried by both phases are called for. From Equation 16.4

$$F_f + F_m = F_c = 44{,}500\ \text{N}$$

$$23.4F_m + F_m = 44{,}500\ \text{N}$$

which leads to

$$F_m = 1824\ \text{N} \quad (410\ \text{lb}_\text{f})$$

$$F_f = F_c - F_m = 44{,}500\ \text{N} - 1824\ \text{N} = 42{,}676\ \text{N} \quad (9590\ \text{lb}_\text{f})$$

(c) To compute the stress on each of the phases, it is first necessary to know the cross-sectional areas of both fiber and matrix. These are determined as

$$A_f = V_f A_c = (0.30)(320 \text{ mm}^2) = 96 \text{ mm}^2 \ \ (0.15 \text{ in.}^2)$$

$$A_m = V_m A_c = (0.70)(320 \text{ mm}^2) = 224 \text{ mm}^2 \ \ (0.35 \text{ in.}^2)$$

Now, the stresses are determined using Equation 6.1 as

$$\sigma_f = \frac{F_f}{A_f} = \frac{42{,}676 \text{ N}}{(96 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 445 \times 10^6 \text{ N/m}^2 = 445 \text{ MPa} \ \ (63{,}930 \text{ psi})$$

$$\sigma_m = \frac{F_m}{A_m} = \frac{1824 \text{ N}}{(224 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 8.14 \times 10^6 \text{ N/m}^2 = 8.14 \text{ MPa} \ \ (1170 \text{ psi})$$

(d) The strain on the composite is the same as the strain on each of the matrix and fiber phases; applying Equation 6.5 to both matrix and fiber phases leads to

$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{8.14 \text{ MPa}}{2.4 \times 10^3 \text{ MPa}} = 3.39 \times 10^{-3}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{445 \text{ MPa}}{131 \times 10^3 \text{ MPa}} = 3.39 \times 10^{-3}$$

16.14 *A continuous and aligned fiber-reinforced composite having a cross-sectional area of 1130 mm^2 (1.75 in.^{2}) is subjected to an external tensile load. If the stresses sustained by the fiber and matrix phases are 156 MPa (22,600 psi) and 2.75 MPa (400 psi), respectively, the force sustained by the fiber phase is 74,000 N (16,600 lb$_f$) and the total longitudinal strain is 1.25 $\times$ 10^{-3}, determine the following:*

(a) The force sustained by the matrix phase

(b) The modulus of elasticity of the composite material in the longitudinal direction

(c) The moduli of elasticity for fiber and matrix phases

Solution

(a) For this portion of the problem we are asked to calculate the force sustained by the matrix phase. It is first necessary to compute the volume fraction of the matrix phase, V_m. This may be accomplished by first determining V_f and then V_m from $V_m = 1 - V_f$. The value of V_f may be calculated since, from the definition of stress (Equation 6.1), and realizing $V_f = A_f/A_c$ as

$$\sigma_f = \frac{F_f}{A_f} = \frac{F_f}{V_f A_c}$$

Or, solving for V_f

$$V_f = \frac{F_f}{\sigma_f A_c} = \frac{74{,}000 \text{ N}}{(156 \times 10^6 \text{ N/m}^2)(1130 \text{ mm}^2)(1 \text{ m/1000 mm})^2} = 0.420$$

Also

$$V_m = 1 - V_f = 1 - 0.420 = 0.580$$

And, an expression for σ_m analogous to the one for σ_f above is

$$\sigma_m = \frac{F_m}{A_m} = \frac{F_m}{V_m A_c}$$

From which

$$F_m = V_m \sigma_m A_c = (0.580)(2.75 \times 10^6 \text{ N/m}^2)(1.13 \times 10^{-3} \text{ m}^2) = 1802 \text{ N} \quad (406 \text{ lb}_\text{f})$$

(b) We are now asked to calculate the modulus of elasticity in the longitudinal direction. This is possible realizing that $E_c = \dfrac{\sigma_c}{\varepsilon}$ (from Equation 6.5) and that $\sigma_c = \dfrac{F_m + F_f}{A_c}$ (from Equation 6.1). Thus

$$E_c = \frac{\sigma_c}{\varepsilon} = \frac{\dfrac{F_m + F_f}{A_c}}{\varepsilon} = \frac{F_m + F_f}{\varepsilon A_c}$$

$$= \frac{1802 \text{ N} + 74{,}000 \text{ N}}{\left(1.25 \times 10^{-3}\right)\left(1130 \text{ mm}^2\right)(1 \text{ m}/1000 \text{ mm})^2} = 53.7 \times 10^9 \text{ N/m}^2 = 53.7 \text{ GPa} \quad (7.77 \times 10^6 \text{ psi})$$

(c) Finally, it is necessary to determine the moduli of elasticity for the fiber and matrix phases. This is possible assuming Equation 6.5 for the matrix phase—i.e.,

$$E_m = \frac{\sigma_m}{\varepsilon_m}$$

and, since this is an isostrain state, $\varepsilon_m = \varepsilon_c = 1.25 \times 10^{-3}$. Thus

$$E_m = \frac{\sigma_m}{\varepsilon_c} = \frac{2.75 \times 10^6 \text{ N/m}^2}{1.25 \times 10^{-3}} = 2.2 \times 10^9 \text{ N/m}^2$$

$$= 2.2 \text{ GPa} \quad (3.2 \times 10^5 \text{ psi})$$

The elastic modulus for the fiber phase may be computed in an analogous manner:

$$E_f = \frac{\sigma_f}{\varepsilon_f} = \frac{\sigma_f}{\varepsilon_c} = \frac{156 \times 10^6 \text{ N/m}^2}{1.25 \times 10^{-3}} = 1.248 \times 10^{11} \text{ N/m}^2$$

$$= 124.8 \text{ GPa} \quad (18.1 \times 10^6 \text{ psi})$$

16.15 *Compute the longitudinal strength of an aligned carbon fiber-epoxy matrix composite having a 0.25 volume fraction of fibers, assuming the following: (1) an average fiber diameter of 10* $\times 10^{-3}$ *mm (3.94* $\times 10^{-4}$ *in.), (2) an average fiber length of 5 mm (0.20 in.), (3) a fiber fracture strength of 2.5 GPa (3.625* $\times 10^{5}$ *psi), (4) a fiber-matrix bond strength of 80 MPa (11,600 psi), (5) a matrix stress at fiber failure of 10.0 MPa (1450 psi), and (6) a matrix tensile strength of 75 MPa (11,000 psi).*

Solution

It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine the longitudinal strength using Equation 16.17, otherwise, use of either Equation 16.18 or Equation 16.19 is necessary. Thus, from Equation 16.3

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(2.5 \times 10^3 \text{ MPa})(10 \times 10^{-3} \text{ mm})}{2(80 \text{ MPa})} = 0.16 \text{ mm}$$

Inasmuch as $l >> l_c$ (5.0 mm >> 0.16 mm), then use of Equation 16.17 is appropriate. Therefore,

$$\sigma_{cl}^* = \sigma_m'(1 - V_f) + \sigma_f^* V_f$$

$$= (10 \text{ MPa})(1 - 0.25) + (2.5 \times 10^3 \text{ MPa})(0.25)$$

$$= 633 \text{ MPa} \quad (91{,}700 \text{ psi})$$

16.16 *It is desired to produce an aligned carbon fiber-epoxy matrix composite having a longitudinal tensile strength of 750 MPa (109,000 psi). Calculate the volume fraction of fibers necessary if (1) the average fiber diameter and length are 1.2 × 10^-2 mm (4.7 × 10^-4 in.) and 1 mm (0.04 in.), respectively; (2) the fiber fracture strength is 5000 MPa (725,000 psi); (3) the fiber-matrix bond strength is 25 MPa (3625 psi); and (4) the matrix stress at fiber failure is 10 MPa (1450 psi).*

Solution

It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine V_f using Equation 16.17, otherwise, use of either Equation 16.18 or Equation 16.19 is necessary. Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(5000\ \text{MPa})(1.2 \times 10^{-2}\ \text{mm})}{2(25\ \text{MPa})} = 1.20\ \text{mm}$$

Inasmuch as $l < l_c$ (1.0 mm < 1.20 mm), then use of Equation 16.19 is required. Therefore,

$$\sigma_{cd'}^* = \frac{l\tau_c}{d} V_f + \sigma_m'(1 - V_f)$$

$$750\,\text{MPa} = \frac{(1.0 \times 10^{-3}\,\text{m})(25\ \text{MPa})}{0.012 \times 10^{-3}\,\text{m}}(V_f) + (10\ \text{MPa})(1 - V_f)$$

Solving this expression for V_f leads to $V_f = 0.357$.

16.17 *Compute the longitudinal tensile strength of an aligned glass fiber-epoxy matrix composite in which the average fiber diameter and length are 0.010 mm (4 × 10^{-4} in.) and 2.5 mm (0.10 in.), respectively, and the volume fraction of fibers is 0.40. Assume that (1) the fiber-matrix bond strength is 75 MPa (10,900 psi), (2) the fracture strength of the fibers is 3500 MPa (508,000 psi), and (3) the matrix stress at fiber failure is 8.0 MPa (1160 psi).*

Solution

It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine σ_{cl}^* using Equation 16.17, otherwise, use of either Equations 16.18 or 16.19 is necessary. Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(3500 \text{ MPa})(0.010 \text{ mm})}{2(75 \text{ MPa})} = 0.233 \text{ mm} \quad (0.0093 \text{ in.})$$

Inasmuch as $l > l_c$ (2.5 mm > 0.233 mm), but since l is not much greater than l_c, then use of Equation 16.18 is necessary. Therefore,

$$\sigma_{cd}^* = \sigma_f^* V_f \left(1 - \frac{l_c}{2l}\right) + \sigma_m' (1 - V_f)$$

$$= (3500 \text{ MPa})(0.40)\left[1 - \frac{0.233 \text{ mm}}{(2)(2.5 \text{ mm})}\right] + (8.0 \text{ MPa})(1 - 0.40)$$

$$= 1340 \text{ MPa} \quad (194{,}400 \text{ psi})$$

16.18 *(a) From the moduli of elasticity data in Table 16.2 for glass fiber-reinforced polycarbonate composites, determine the value of the fiber efficiency parameter for each of 20, 30, and 40 vol% fibers.*

(b) Estimate the modulus of elasticity for 50 vol% glass fibers.

Solution

(a) This portion of the problem calls for computation of values of the fiber efficiency parameter. From Equation 16.20

$$E_{cd} = KE_f V_f + E_m V_m$$

Solving this expression for K yields

$$K = \frac{E_{cd} - E_m V_m}{E_f V_f} = \frac{E_{cd} - E_m(1 - V_f)}{E_f V_f}$$

For glass fibers, E_f = 72.5 GPa (Table 16.4); using the data in Table 16.2, and taking an average of the extreme E_m values given, E_m = 2.29 GPa (0.333 × 10^6 psi). And, for V_f = 0.20

$$K = \frac{5.93 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.2)}{(72.5 \text{ GPa})(0.2)} = 0.283$$

For V_f = 0.3

$$K = \frac{8.62 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.3)}{(72.5 \text{ GPa})(0.3)} = 0.323$$

And, for V_f = 0.4

$$K = \frac{11.6 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.4)}{(72.5 \text{ GPa})(0.4)} = 0.353$$

(b) For 50 vol% fibers (V_f = 0.50), we must assume a value for K. Since it is increasing with V_f, let us estimate it to increase by the same amount as going from 0.3 to 0.4—that is, by a value of 0.03. Therefore, let us assume a value for K of 0.383. Now, from Equation 16.20

$$E_{cd} = KE_f V_f + E_m V_m$$

$$= (0.383)(72.5 \text{ GPa})(0.5) + (2.29 \text{ GPa})(0.5)$$

$$= 15.0 \text{ GPa} \quad (2.18 \times 10^6 \text{ psi})$$

The Fiber Phase

The Matrix Phase

16.19 *For a polymer-matrix fiber-reinforced composite,*

(a) List three functions of the matrix phase.

(b) Compare the desired mechanical characteristics of matrix and fiber phases.

(c) Cite two reasons why there must be a strong bond between fiber and matrix at their interface.

Solution

(a) For polymer-matrix fiber-reinforced composites, three functions of the polymer-matrix phase are: (1) to bind the fibers together so that the applied stress is distributed among the fibers; (2) to protect the surface of the fibers from being damaged; and (3) to separate the fibers and inhibit crack propagation.

(b) The matrix phase must be ductile and is usually relatively soft, whereas the fiber phase must be stiff and strong.

(c) There must be a strong interfacial bond between fiber and matrix in order to: (1) maximize the stress transmittance between matrix and fiber phases; and (2) minimize fiber pull-out, and the probability of failure.

16.20 *(a) What is the distinction between matrix and dispersed phases in a composite material?*

(b) Contrast the mechanical characteristics of matrix and dispersed phases for fiber-reinforced composites.

Solution

(a) The matrix phase is a continuous phase that surrounds the noncontinuous dispersed phase.

(b) In general, the matrix phase is relatively weak, has a low elastic modulus, but is quite ductile. On the other hand, the fiber phase is normally quite strong, stiff, and brittle.

Polymer-Matrix Composites

16.21 *(a) Calculate and compare the specific longitudinal strengths of the glass-, carbon-, and aramid-fiber reinforced epoxy composites in Table 16.5 with the following alloys: tempered (315°C) 440A martensitic stainless steel, normalized 1020 plain-carbon steel, 2024-T3 aluminum alloy, cold-worked (HO2 temper) C36000 free-cutting brass, rolled AZ31B magnesium alloy, and annealed Ti-6Al-4V titanium alloy.*

(b) Compare the specific moduli of the same three fiber-reinforced epoxy composites with the same metal alloys. Densities (i.e., specific gravities), tensile strengths, and moduli of elasticity for these metal alloys may be found in Tables B.1, B.4, and B.2, respectively, in Appendix B.

Solution

(a) This portion of the problem calls for us to calculate the specific longitudinal strengths of glass-fiber, carbon-fiber, and aramid-fiber reinforced epoxy composites, and then to compare these values with the specific strengths of several metal alloys.

The longitudinal specific strength of the glass-reinforced epoxy material (V_f = 0.60) in Table 16.5 is just the ratio of the longitudinal tensile strength and specific gravity as

$$\frac{1020 \text{ MPa}}{2.1} = 486 \text{ MPa}$$

For the carbon-fiber reinforced epoxy

$$\frac{1520 \text{ MPa}}{1.6} = 950 \text{ MPa}$$

And, for the aramid-fiber reinforced epoxy

$$\frac{1240 \text{ MPa}}{1.4} = 886 \text{ MPa}$$

Now, for the metal alloys we use data found in Tables B.1 and B.4 in Appendix B (using the density values from Table B.1 for the specific gravities). For the 440A tempered martensitic steel

$$\frac{1790 \text{ MPa}}{7.80} = 229 \text{ MPa}$$

For the normalized 1020 plain carbon steel, the ratio is

$$\frac{440 \text{ MPa}}{7.85} = 56 \text{ MPa}$$

For the 2024-T3 aluminum alloy

$$\frac{485 \text{ MPa}}{2.77} = 175 \text{ MPa}$$

For the C36000 brass (cold worked)

$$\frac{400 \text{ MPa}}{8.50} = 47 \text{ MPa}$$

For the AZ31B (rolled) magnesium alloy

$$\frac{290 \text{ MPa}}{1.77} = 164 \text{ MPa}$$

For the annealed Ti-6Al-4V titanium alloy

$$\frac{900 \text{ MPa}}{4.43} = 203 \text{ MPa}$$

(b) The longitudinal specific modulus is just the longitudinal tensile modulus-specific gravity ratio. For the glass-fiber reinforced epoxy, this ratio is

$$\frac{45 \text{ GPa}}{2.1} = 21.4 \text{ GPa}$$

For the carbon-fiber reinforced epoxy

$$\frac{145 \text{ GPa}}{1.6} = 90.6 \text{ GPa}$$

And, for the aramid-fiber reinforced epoxy

$$\frac{76\ \text{GPa}}{1.4} = 54.3\ \text{GPa}$$

The specific moduli for the metal alloys (Tables B.1 and B.2) are as follows:

For the 440A tempered martensitic steel

$$\frac{200\ \text{GPa}}{7.80} = 25.6\ \text{GPa}$$

For the normalized 1020 plain-carbon steel

$$\frac{207\ \text{GPa}}{7.85} = 26.4\ \text{GPa}$$

For the 2024-T3 aluminum alloy

$$\frac{72.4\ \text{GPa}}{2.77} = 26.1\ \text{GPa}$$

For the cold-worked C36000 brass

$$\frac{97\ \text{GPa}}{8.50} = 11.4\ \text{GPa}$$

For the rolled AZ31B magnesium alloy

$$\frac{45\ \text{GPa}}{1.77} = 25.4\ \text{GPa}$$

For the Ti-6Al-4V titanium alloy

$$\frac{114\ \text{GPa}}{4.43} = 25.7\ \text{GPa}$$

16.22 *(a) List four reasons why glass fibers are most commonly used for reinforcement.*

(b) Why is the surface perfection of glass fibers so important?

(c) What measures are taken to protect the surface of glass fibers?

Solution

(a) The four reasons why glass fibers are most commonly used for reinforcement are listed at the beginning of Section 16.8 under "Glass Fiber-Reinforced Polymer (GFRP) Composites."

(b) The surface perfection of glass fibers is important because surface flaws or cracks act as points of stress concentration, which will dramatically reduce the tensile strength of the material.

(c) Care must be taken not to rub or abrade the surface after the fibers are drawn. As a surface protection, newly drawn fibers are coated with a protective surface film.

16.23 *Cite the distinction between carbon and graphite.*

Solution

"Graphite" is crystalline carbon having the structure shown in Figure 12.17, whereas "carbon" will consist of some noncrystalline material as well as areas of crystal misalignment (Figure 13.7).

16.24 *(a) Cite several reasons why fiberglass-reinforced composites are utilized extensively.*

(b) Cite several limitations of this type of composite.

Solution

(a) Reasons why fiberglass-reinforced composites are utilized extensively are: (1) glass fibers are very inexpensive to produce; (2) these composites have relatively high specific strengths; and (3) they are chemically inert in a wide variety of environments.

(b) Several limitations of these composites are: (1) care must be exercised in handling the fibers inasmuch as they are susceptible to surface damage; (2) they are lacking in stiffness in comparison to other fibrous composites; and (3) they are limited as to maximum temperature use.

Hybrid Composites

16.25 *(a) What is a hybrid composite?*

(b) List two important advantages of hybrid composites over normal fiber composites.

Solution

(a) A hybrid composite is a composite that is reinforced with two or more different fiber materials in a single matrix.

(b) Two advantages of hybrid composites are: (1) better overall property combinations, and (2) failure is not as catastrophic as with single-fiber composites.

16.26 *(a) Write an expression for the modulus of elasticity for a hybrid composite in which all fibers of both types are oriented in the same direction.*

(b) Using this expression, compute the longitudinal modulus of elasticity of a hybrid composite consisting of aramid and glass fibers in volume fractions of 0.30 and 0.40, respectively, within a polyester resin matrix [E_m = 2.5 GPa (3.6×10^5 psi)].

Solution

(a) For a hybrid composite having all fibers aligned in the same direction

$$E_{cl} = E_m V_m + E_{f1} V_{f1} + E_{f2} V_{f2}$$

in which the subscripts *f*1 and *f*2 refer to the two types of fibers.

(b) Now we are asked to compute the longitudinal elastic modulus for a glass- and aramid-fiber hybrid composite. From Table 16.4, the elastic moduli of aramid and glass fibers are, respectively, 131 GPa (19×10^6 psi) and 72.5 GPa (10.5×10^6 psi). Thus, from the previous expression

$$E_{cl} = (2.5\ \text{GPa})(1.0 - 0.30 - 0.40) + (131\ \text{GPa})(0.30) + (72.5\ \text{GPa})(0.40)$$

$$= 69.1\ \text{GPa} \quad (10.0 \times 10^6\ \text{psi})$$

16.27 *Derive a generalized expression analogous to Equation 16.16 for the transverse modulus of elasticity of an aligned hybrid composite consisting of two types of continuous fibers.*

Solution

This problem asks that we derive a generalized expression analogous to Equation 16.16 for the transverse modulus of elasticity of an aligned hybrid composite consisting of two types of continuous fibers. Let us denote the subscripts *f*1 and *f*2 for the two fiber types, and *m*, *c*, and *t* subscripts for the matrix, composite, and transverse direction, respectively. For the isostress state, the expressions analogous to Equations 16.12 and 16.13 are

$$\sigma_c = \sigma_m = \sigma_{f1} = \sigma_{f2}$$

And

$$\varepsilon_c = \varepsilon_m V_m + \varepsilon_{f1} V_{f1} + \varepsilon_{f2} V_{f2}$$

Since $\varepsilon = \sigma/E$ (Equation 6.5), making substitutions of the form of this equation into the previous expression yields

$$\frac{\sigma}{E_{ct}} = \frac{\sigma}{E_m} V_m + \frac{\sigma}{E_{f1}} V_{f1} + \frac{\sigma}{E_{f2}} V_{f2}$$

Thus

$$\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_{f1}}{E_{f1}} + \frac{V_{f2}}{E_{f2}}$$

$$= \frac{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}{E_m E_{f1} E_{f2}}$$

And, finally, taking the reciprocal of this equation leads to

$$E_{ct} = \frac{E_m E_{f1} E_{f2}}{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}$$

Processing of Fiber-Reinforced Composites

16.28 *Briefly describe pultrusion, filament winding, and prepreg production fabrication processes; cite the advantages and disadvantages of each.*

Solution

Pultrusion, filament winding, and prepreg fabrication processes are described in Section 16.13.

For pultrusion, the advantages are: the process may be automated, production rates are relatively high, a wide variety of shapes having constant cross-sections are possible, and very long pieces may be produced. The chief disadvantage is that shapes are limited to those having a constant cross-section.

For filament winding, the advantages are: the process may be automated, a variety of winding patterns are possible, and a high degree of control over winding uniformity and orientation is afforded. The chief disadvantage is that the variety of shapes is somewhat limited.

For prepreg production, the advantages are: resin does not need to be added to the prepreg, the lay-up arrangement relative to the orientation of individual plies is variable, and the lay-up process may be automated. The chief disadvantages of this technique are that final curing is necessary after fabrication, and thermoset prepregs must be stored at subambient temperatures to prevent complete curing.

Laminar Composites

Sandwich Panels

16.29 *Briefly describe laminar composites. What is the prime reason for fabricating these materials?*

Solution

Laminar composites are a series of sheets or panels, each of which has a preferred high-strength direction. These sheets are stacked and then cemented together such that the orientation of the high-strength direction varies from layer to layer.

These composites are constructed in order to have a relatively high strength in virtually all directions within the plane of the laminate.

16.30 *(a) Briefly describe sandwich panels.*

(b) What is the prime reason for fabricating these structural composites?

(c) What are the functions of the faces and the core?

Solution

(a) Sandwich panels consist of two outer face sheets of a high-strength material that are separated by a layer of a less-dense and lower-strength core material.

(b) The prime reason for fabricating these composites is to produce structures having high in-plane strengths, high shear rigidities, and low densities.

(c) The faces function so as to bear the majority of in-plane tensile and compressive stresses. On the other hand, the core separates and provides continuous support for the faces, and also resists shear deformations perpendicular to the faces.

DESIGN PROBLEMS

16.D1 *Composite materials are now being utilized extensively in sports equipment.*

(a) List at least four different sports implements that are made of, or contain composites.

(b) For one of these implements, write an essay in which you do the following: (1) Cite the materials that are used for matrix and dispersed phases, and, if possible, the proportions of each phase; (2) note the nature of the dispersed phase (i.e., continuous fibers); and (3) describe the process by which the implement is fabricated.

Solution

Inasmuch as there are a number of different sports implements that employ composite materials, no attempt will be made to provide a complete answer for this question. However, a list of this type of sporting equipment would include skis and ski poles, fishing rods, vaulting poles, golf clubs, hockey sticks, baseball and softball bats, surfboards and boats, oars and paddles, bicycle components (frames, wheels, handlebars), canoes, and tennis and racquetball rackets.

Influence of Fiber Orientation and Concentration

16.D2 *It is desired to produce an aligned and continuous fiber-reinforced epoxy composite having a maximum of 50 vol% fibers. In addition, a minimum longitudinal modulus of elasticity of 50 GPa (7.3 $\times 10^6$ psi) is required, as well as a minimum tensile strength of 1300 MPa (189,000 psi). Of E-glass, carbon (PAN standard modulus), and aramid fiber materials, which are possible candidates and why? The epoxy has a modulus of elasticity of 3.1 GPa (4.5 $\times 10^5$ psi) and a tensile strength of 75 MPa (11,000 psi). In addition, assume the following stress levels on the epoxy matrix at fiber failure: E-glass—70 MPa (10,000 psi); carbon (PAN standard modulus)—30 MPa (4350 psi); and aramid—50 MPa (7250 psi). Other fiber data are contained in Tables B.2 and B.4 in Appendix B. For aramid fibers, use the minimum of the range of strength values.*

Solution

In order to solve this problem, we want to make longitudinal elastic modulus and tensile strength computations assuming 50 vol% fibers for all three fiber materials, in order to see which of these materials meet the stipulated criteria [i.e., a minimum elastic modulus of 50 GPa (7.3 × 10^6 psi), and a minimum tensile strength of 1300 MPa (189,000 psi)]. Thus, it becomes necessary to use Equations 16.10b and 16.17 with V_m = 0.5 and V_f = 0.5, E_m = 3.1 GPa, and σ_m^* = 75 MPa.

For glass, E_f = 72.5 GPa and σ_f^* = 3450 MPa. Therefore,

$$E_{cl} = E_m\left(1 - V_f\right) + E_f V_f$$

$$= (3.1 \text{ GPa})(1 - 0.5) + (72.5 \text{ GPa})(0.5) = 37.8 \text{ GPa} \quad (5.48 \times 10^6 \text{ psi})$$

Since this is less than the specified minimum (i.e., 50 GPa), glass is not an acceptable candidate.

For carbon (PAN standard-modulus), E_f = 230 GPa and σ_f^* = 3800 MPa (the minimum of the range of values in Table B.4), thus, from Equation 16.10b

$$E_{cl} = (3.1 \text{ GPa})(0.5) + (230 \text{ GPa})(0.5) = 116.6 \text{ GPa} \quad (16.9 \times 10^6 \text{ psi})$$

which is greater than the specified minimum. In addition, from Equation 16.17

$$\sigma_{cl}^* = \sigma_m'\left(1 - V_f\right) + \sigma_f^* V_f$$

$$= (30 \text{ MPa})(0.5) + (3800 \text{ MPa})(0.5) = 1915 \text{ MPa} \quad (277{,}200 \text{ psi})$$

which is also greater than the minimum (1300 MPa). Thus, carbon (PAN standard-modulus) is a candidate.

For aramid, E_f = 131 GPa and σ_f^* = 3600 MPa (the minimum of the range of values in Table B.4), thus (Equation 16.10b)

$$E_{cl} = (3.1 \text{ GPa})(0.5) + (131 \text{ GPa})(0.5) = 67.1 \text{ GPa} \quad (9.73 \times 10^6 \text{ psi})$$

which value is greater than the minimum. In addition, from Equation 16.17

$$\sigma_{cl}^* = \sigma_m'(1 - V_f) + \sigma_f^* V_f$$

$$= (50 \text{ MPa})(0.5) + (3600 \text{ MPa})(0.5) = 1825 \text{ MPa} \quad (266{,}100 \text{ psi})$$

which is also greater than the minimum strength value. Therefore, of the three fiber materials, both the carbon (PAN standard-modulus) and the aramid meet both minimum criteria.

16.D3 *It is desired to produce a continuous and oriented carbon fiber-reinforced epoxy having a modulus of elasticity of at least 83 GPa (12 $\times 10^6$ psi) in the direction of fiber alignment. The maximum permissible specific gravity is 1.40. Given the following data, is such a composite possible? Why or why not? Assume that composite specific gravity may be determined using a relationship similar to Equation 16.10a.*

	SpecificGravity	***Modulus of Elasticity [GPa (psi)]***
Carbon fiber	*1.80*	*260 (37 × 10^6)*
Epoxy	*1.25*	*2.4 (3.5 × 10^5)*

Solution

This problem asks us to determine whether or not it is possible to produce a continuous and oriented carbon fiber-reinforced epoxy having a modulus of elasticity of at least 83 GPa in the direction of fiber alignment, and a maximum specific gravity of 1.40. We will first calculate the minimum volume fraction of fibers to give the stipulated elastic modulus, and then the maximum volume fraction of fibers possible to yield the maximum permissible specific gravity; if there is an overlap of these two fiber volume fractions then such a composite is possible.

With regard to the elastic modulus, from Equation 16.10b

$$E_{cl} = E_m\left(1 - V_f\right) + E_f V_f$$

$$83 \text{ GPa} = (2.4 \text{ GPa})\left(1 - V_f\right) + (260 \text{ GPa})(V_f)$$

Solving for V_f yields V_f = 0.31. Therefore, V_f > 0.31 to give the minimum desired elastic modulus.

Now, upon consideration of the specific gravity (or density), ρ, we employ the following modified form of Equation 16.10b

$$\rho_c = \rho_m\left(1 - V_f\right) + \rho_f V_f$$

$$1.40 = 1.25\left(1 - V_f\right) + 1.80\left(V_f\right)$$

And, solving for V_f from this expression gives $V_f = 0.27$. Therefore, it is necessary for $V_f < 0.27$ in order to have a composite specific gravity less than 1.40.

Hence, such a composite is *not possible* since there is no overlap of the fiber volume fractions as computed using the two stipulated criteria.

16.D4 *It is desired to fabricate a continuous and aligned glass fiber-reinforced polyester having a tensile strength of at least 1400 MPa (200,000 psi) in the longitudinal direction. The maximum possible specific gravity is 1.65. Using the following data, determine if such a composite is possible. Justify your decision. Assume a value of 15 MPa for the stress on the matrix at fiber failure.*

	Specific Gravity	***Tensile Strength [MPa (psi)]***
Glass fiber	2.50	3500 (5×10^5)
Polyester	1.35	50 (7.25×10^3)

<u>Solution</u>

This problem asks us to determine whether or not it is possible to produce a continuous and oriented glass fiber-reinforced polyester having a tensile strength of at least 1400 MPa in the longitudinal direction, and a maximum specific gravity of 1.65. We will first calculate the minimum volume fraction of fibers to give the stipulated tensile strength, and then the maximum volume fraction of fibers possible to yield the maximum permissible specific gravity; if there is an overlap of these two fiber volume fractions then such a composite is possible.

With regard to tensile strength, from Equation 16.17

$$\sigma_{cl}^* = \sigma_m'\left(1 - V_f\right) + \sigma_f^* V_f$$

$$1400 \text{ MPa} = (15 \text{ MPa})\left(1 - V_f\right) + (3500 \text{ MPa})(V_f)$$

Solving for V_f yields $V_f = 0.397$. Therefore, $V_f > 0.397$ to give the minimum desired tensile strength.

Now, upon consideration of the specific gravity (or density), ρ, we employ the following modified form of Equation 16.10b:

$$\rho_c = \rho_m\left(1 - V_f\right) + \rho_f V_f$$

$$1.65 = 1.35\left(1 - V_f\right) + 2.50(V_f)$$

And, solving for V_f from this expression gives $V_f = 0.261$. Therefore, it is necessary for $V_f < 0.261$

in order to have a composite specific gravity less than 1.65.

Hence, such a composite is *not possible* since there is no overlap of the fiber volume fractions as computed using the two stipulated criteria.

16.D5 *It is necessary to fabricate an aligned and discontinuous carbon fiber-epoxy matrix composite having a longitudinal tensile strength of 1900 MPa (275,000 psi) using 0.45 volume fraction of fibers. Compute the required fiber fracture strength assuming that the average fiber diameter and length are 8 $\times 10^{-3}$ mm (3.1 $\times 10^{-4}$ in.) and 3.5 mm (0.14 in.), respectively. The fiber-matrix bond strength is 40 MPa (5800 psi), and the matrix stress at fiber failure is 12 MPa (1740 psi).*

<u>Solution</u>

To begin, since the value of σ_f^* is unknown, calculation of the value of l_c in Equation 16.3 is not possible, and, therefore, we are not able to decide which of Equations 16.18 and 16.19 to use. Thus, it is necessary to substitute for l_c in Equation 16.3 into Equation 16.18, solve for the value of σ_f^*, then, using this value, solve for l_c from Equation 16.3. If $l > l_c$, we use Equation 16.18, otherwise Equation 16.19 must be used. *Note:* the σ_f^* parameters in Equations 16.18 and 16.3 are the same. Realizing this, and substituting for l_c in Equation 16.3 into Equation 16.18 leads to

$$\sigma_{cd}^* = \sigma_f^* V_f \left[1 - \frac{l_c}{2l}\right] + \sigma'_m (1 - V_f) = \sigma_f^* V_f \left[1 - \frac{\sigma_f^* d}{4\tau_c l}\right] + \sigma'_m (1 - V_f)$$

$$= \sigma_f^* V_f - \frac{\sigma_f^{*2} V_f d}{4\tau_c l} + \sigma'_m - \sigma'_m V_f$$

This expression is a quadratic equation in which σ_f^* is the unknown. Rearrangement into a more convenient form leads to

$$\sigma_f^{*2}\left[\frac{V_f d}{4\tau_c l}\right] - \sigma_f^*(V_f) + \left[\sigma_{cd}^* - \sigma'_m (1 - V_f)\right] = 0$$

Or

$$a\sigma_f^{*2} + b\sigma_f^* + c = 0$$

where

$$a = \frac{V_f d}{4\tau_c l}$$

$$= \frac{(0.45)\left(8 \times 10^{-6}\text{ m}\right)}{(4)(40\text{ MPa})\left(3.5 \times 10^{-3}\text{ m}\right)} = 6.43 \times 10^{-6}\ (\text{MPa})^{-1} \quad \left[4.29 \times 10^{-8}\ (\text{psi})^{-1}\right]$$

Furthermore,

$$b = -V_f = -0.45$$

And

$$c = \sigma_{cd}^{*} - \sigma_m'\left(1 - V_f\right)$$

$$= 1900\text{ MPa} - (12\text{ MPa})(1 - 0.45) = 1893.4\text{ MPa} \quad (274{,}043\text{ psi})$$

Now solving the above quadratic equation for σ_f^* yields

$$\sigma_f^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.45) \pm \sqrt{(-0.45)^2 - (4)\left[6.43 \times 10^{-6}\ (\text{MPa})^{-1}\right](1893.4\text{ MPa})}}{(2)\left[6.43 \times 10^{-6}\ (\text{MPa})^{-1}\right]}$$

$$= \frac{0.4500 \pm 0.3922}{1.286 \times 10^{-5}}\text{ MPa} \quad \left[\frac{0.4500 \pm 0.3943}{8.58 \times 10^{-8}}\text{ psi}\right]$$

This yields the two possible roots as

$$\sigma_f^*(+) = \frac{0.4500 + 0.3922}{1.286 \times 10^{-5}}\text{ MPa} = 65{,}500\text{ MPa} \quad (9.84 \times 10^6\text{ psi})$$

$$\sigma_f^*(-) = \frac{0.4500 - 0.3922}{1.286 \times 10^{-5}}\text{ MPa} = 4495\text{ MPa} \quad (650{,}000\text{ psi})$$

Upon consultation of the magnitudes of σ_f^* for various fibers and whiskers in Table 16.4, only $\sigma_f^*(-)$ is reasonable. Now, using this value, let us calculate the value of l_c using Equation 16.3 in order to ascertain if use of Equation 16.18 in the previous treatment was appropriate. Thus

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(4495\ \text{MPa})(0.008\ \text{mm})}{(2)(40\ \text{MPa})} = 0.45\ \text{mm}\ \ (0.0173\ \text{in.})$$

Since $l > l_c$ (3.5 mm > 0.45 mm), our choice of Equation 16.18 was indeed appropriate, and σ_f^* = 4495 MPa (650,000 psi).

16.D6 *A tubular shaft similar to that shown in Figure 16.11 is to be designed that has an outside diameter of 80 mm (3.15 in.) and a length of 0.75 m (2.46 ft). The mechanical characteristic of prime importance is bending stiffness in terms of the longitudinal modulus of elasticity. Stiffness is to be specified as maximum allowable deflection in bending; when subjected to three-point bending as in Figure 12.30, a load of 1000 N (225 lb_f) is to produce an elastic deflection of no more than 0.40 mm (0.016 in.) at the midpoint position.*

Continuous fibers that are oriented parallel to the tube axis will be used; possible fiber materials are glass, and carbon in standard-, intermediate-, and high-modulus grades. The matrix material is to be an epoxy resin, and fiber volume fraction is 0.35.

(a) Decide which of the four fiber materials are possible candidates for this application, and for each candidate determine the required inside diameter consistent with the above criteria.

(b) For each candidate, determine the required cost, and on this basis, specify the fiber that would be the least expensive to use.

Elastic modulus, density, and cost data for the fiber and matrix materials are contained in Table 16.6.

Solution

(a) This portion of the problem calls for a determination of which of the four fiber types is suitable for a tubular shaft, given that the fibers are to be continuous and oriented with a volume fraction of 0.35. Using Equation 16.10 it is possible to solve for the elastic modulus of the shaft for each of the fiber types. For example, for glass (using moduli data in Table 16.6)

$$E_{cs} = E_m(1 - V_f) + E_f V_f$$

$$= (2.4 \text{ GPa})(1.00 - 0.35) + (72.5 \text{ GPa})(0.35) = 26.9 \text{ GPa}$$

This value for E_{cs} as well as those computed in a like manner for the three carbon fibers are listed in Table 16.D1.

Table 16.D1 Composite Elastic Modulus for Each of Glass and Three Carbon Fiber Types for $V_f = 0.35$

Fiber Type	E_{cs} (GPa)
Glass	26.9
Carbon—standard modulus	82.1
Carbon—intermediate modulus	101.3
Carbon—high modulus	141.6

It now becomes necessary to determine, for each fiber type, the inside diameter d_i. Rearrangement of Equation 16.23 such that d_i is the dependent variable leads to

$$d_i = \left[d_0^4 - \frac{4FL^3}{3\pi E \Delta y} \right]^{1/4}$$

The d_i values may be computed by substitution into this expression for E the E_{cs} data in Table 16.D1 and the following

F = 1000 N

L = 0.75 m

Δy = 0.40 mm

d_0 = 80 mm

These d_i data are tabulated in the first column of Table 16.D2. Thus, all four materials are candidates for this application, and the inside diameter for each material is given in the first column of this table.

Table 16.D2 Inside Tube Diameter, Total Volume, and Fiber, Matrix, and Total Costs for Three Carbon-Fiber Epoxy-Matrix Composites

Fiber Type	Inside Diameter (mm)	Total Volume (cm^3)	Fiber Cost ($)	Matrix Cost ($)	Total Cost ($)
Glass	70.2	867	1.30	2.25	3.55
Carbon--standard modulus	77.2	259	7.35	0.65	8.00
Carbon--intermediate modulus	77.7	214	12.15	0.55	12.70
Carbon--high modulus	78.4	149	14.10	0.40	14.50

(b) Also included in Table 16.D2 is the total volume of material required for the tubular shaft for each fiber type; Equation 16.24 was utilized for these computations. Since $V_f = 0.35$, 35% this volume is fiber and the other 65% is epoxy matrix. In the manner of Design Example 16.1, the masses and costs of fiber and matrix materials were determined, as well as the total composite cost. These data are also included in Table 16.D2. Here it may be noted that the glass fiber yields the least expensive composite, followed by the standard-, intermediate-, and high-modulus carbon fiber materials.

FUNDAMENTALS OF ENGINEERING QUESTIONS AND PROBLEMS

16.1FE *The mechanical properties of some metals may be improved by incorporating fine particles of their oxides. If the moduli of elasticity of the metal and oxide are, respectively, 55 GPa and 430 GPa, what is the upper-bound modulus of elasticity value for a composite that has a composition of 31 vol% of oxide particles?*

(A) 48.8 GPa	*(C) 138 GPa*
(B) 75.4 GPa	*(D) 171 GPa*

Solution

The upper-bound expression for the elastic modulus of a composite may be computed using Equation 16.1

$$E_c(u) = E_m V_m + E_p V_p$$

where the *m* and *p* subscripts denote matrix and particle phases, respectively. Using data provided in the problem statement the upper-bound elastic modulus for this material is

$$E_c(u) = (55 \text{ GPa})(1 - 0.31) + (430 \text{ GPa})(0.31) = 171 \text{ GPa}$$

which is answer D.

16.2FE *How are continuous fibers typically oriented in fibrous composites?*

(A) Aligned

(B) Partially oriented

(C) Randomly oriented

(D) All of the above

Answer

The correct answer is A. Continuous fibers are typically *aligned* in fibrous composites.

16.3FE *Compared to other ceramic materials, ceramic-matrix composites have better/higher:*

(A) oxidation resistance

(B) stability at elevated temperatures

(C) fracture toughnesses

(D) all of the above

<u>Answer</u>

The correct answer is C. Ceramic-matrix composites have higher *fracture toughnesses* than other ceramic materials.

16.4FE *A continuous and aligned hybrid composite consists of aramid and glass fibers embedded within a polymer resin matrix. Compute the longitudinal modulus of elasticity of this material if the respective volume fractions are 0.24 and 0.28, given the following data:*

Material	***Modulus of Elasticity (GPa)***
Polyester	*2.5*
Aramid fibers	*131*
Glass fibers	*72.5*

(A) 5.06 GPa *(C) 52.9 GPa*

(B) 32.6 GPa *(D) 131 GPa*

Solution

The longitudinal modulus of elasticity for a continuous and aligned hybrid composite consisting of fibers 1 and 2 may be determined using Equation 16.10a with a term added for the second fiber material:

$$E_{cl} = E_m V_m + E_{f1} V_{f1} + E_{f2} V_{f2}$$

Therefore, for the composite described in the problem statement

$$E_{cl} = (2.5 \text{ GPa})(1 - 0.24 - 0.28) + (131 \text{ GPa})(0.24) + (72.5 \text{ GPa})(0.28)$$

$$= 52.9 \text{ GPa}$$

which is answer C.