Eletromagnetismo

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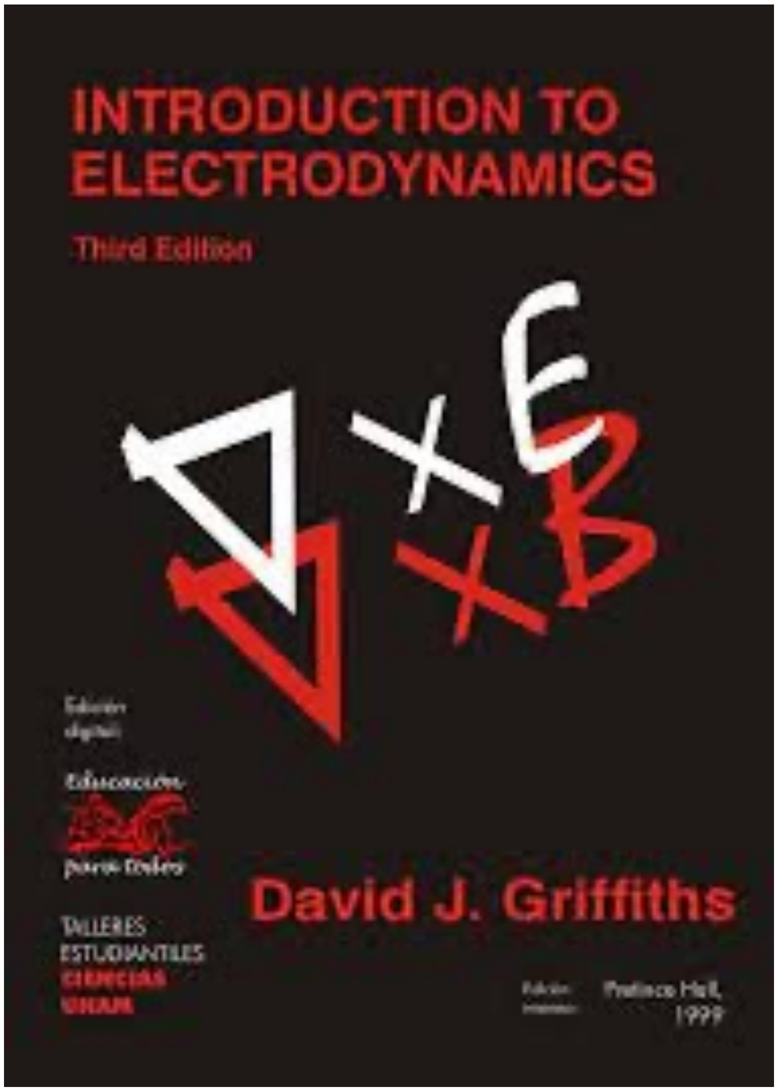
Plano do Curso

16/08	13/09	11/10	08/11
19/08	16/09	14/10	11/11
23/08	20/09 P1	18/10	15/11
26/08	23/09	21/10 P2	18/11
30/08	27/09	25/10	22/11
02/09	30/09	28/10	25/11 P3
06/09	04/10	01/11	29/11 correção
09/09	07/10	04/11	02/12 S1
			06/12 revisão
			09/12 S2

Bibliografia



Capítulo 2 : eletrostática

Capítulo 5 : magnetostática

Capítulo 7 : eletrodinâmica

Capítulo 8 : leis de conservação

Capítulo 9 : ondas eletromagnéticas

Capítulo 10 : campos e potenciais

Capítulo 11 : radiação

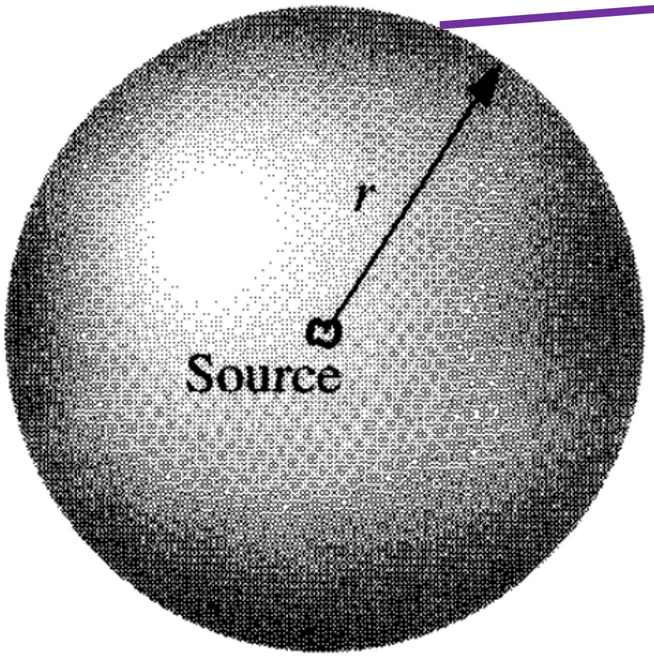
Aula 21

Radiação

Griffiths - Capítulo 11

Radiação: produção de ondas eletromagnéticas

Radiação: emissão de energia eletromagnética, que sai de uma fonte finita e vai até o infinito.



Potência atravessando esta superfície:

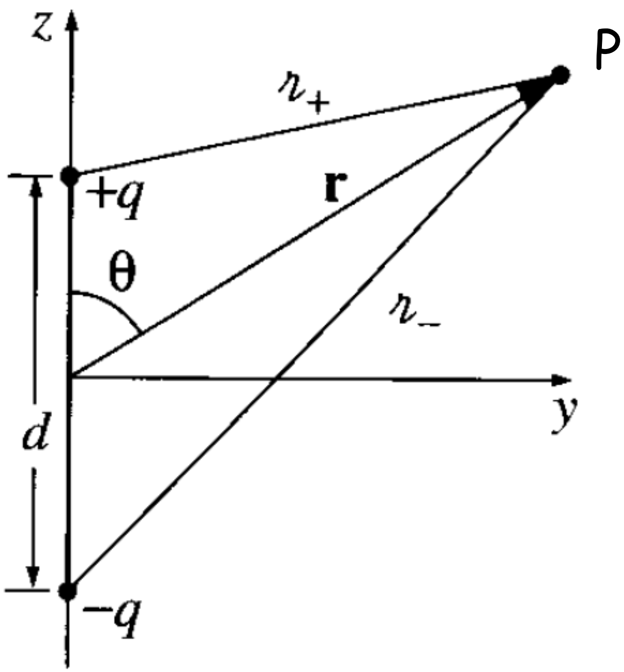
$$P = \oint \vec{S} \, . \, \vec{da} \;\; = \oint \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \, . \, \vec{da}$$

Potência Radiada é o limite de P quando o raio r vai ao infinito

$$P_{rad} = \lim_{r \to \infty} P(r)$$

$$\begin{cases} P_{rad} = 0 & \text{Fonte não irradia} \\ P_{rad} \neq 0 & \text{Fonte irradia} \end{cases}$$

Uma antena simples: o dipolo elétrico de brinquedo



A carga em cada extremidade varia com o tempo :

$$q_+(t) = +\, q_0\, cos(\omega t)$$

$$q_-(t) = -\, q_0\, cos(\omega t)$$

$$r_+ = \sqrt{(\frac{d}{2})^2 + r^2 - d\, r\, cos\theta}$$

$$r_- = \sqrt{(\frac{d}{2})^2 + r^2 + d\, r\, cos\theta}$$

O que é sentido em P foi produzido antes:

$$t_r^+ = t - \frac{r_+}{c}$$

$$t_r^- = t - \frac{r_-}{c}$$

$$V(\vec{r}, t_r) = \frac{q_+(t_r^+)}{4\,\pi\,\epsilon_0}\,\frac{1}{r_+} + \frac{q_-(t_r^-)}{4\,\pi\,\epsilon_0}\,\frac{1}{r_-}$$

$$V(\vec{r}, t_r) = \frac{q_+(t_r^+)}{4\,\pi\,\epsilon_0}\frac{1}{r_+} + \frac{q_-(t_r^-)}{4\,\pi\,\epsilon_0}\frac{1}{r_-}$$

$$\boxed{V(\vec{r}, t_r) = \frac{1}{4\,\pi\,\epsilon_0}\left[\frac{q_0\,cos[\omega(t-\frac{r_+}{c})]}{r_+} - \frac{q_0\,cos[\omega(t-\frac{r_-}{c})]}{r_-}\right]}$$

Aproximação : $r >> \lambda >> d$

$$\omega = 2\,\pi\,\nu = 2\,\pi\,\frac{c}{\lambda} \qquad \frac{\omega}{c} = \frac{2\,\pi}{\lambda} \qquad \frac{\omega\,d}{c} = \frac{2\,\pi\,d}{\lambda} \qquad \frac{\omega\,d}{c} = \frac{2\,\pi\,d}{\lambda} << 1$$

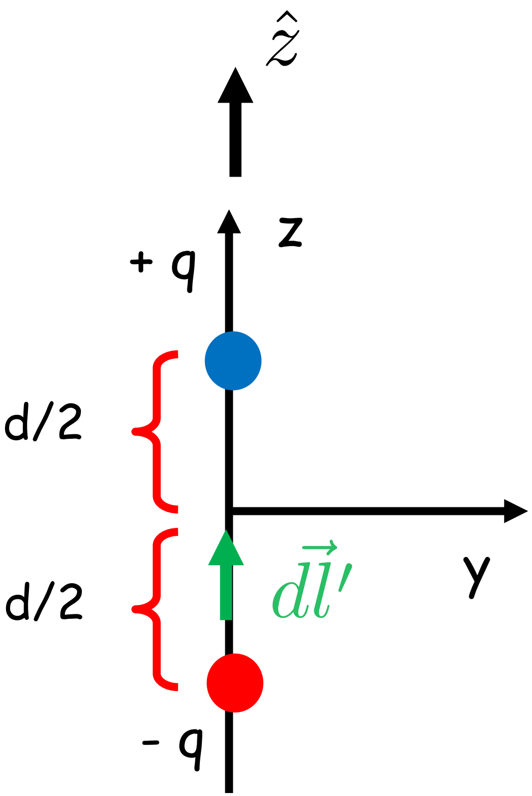
$$\frac{1}{r} << \frac{\omega}{c}$$

$$\boxed{V = -\,\frac{q_0\,d\,\omega}{4\,\pi\,\epsilon_0\,c}\left(\frac{cos\theta}{r}\right)\,sen[\omega(t-\frac{r}{c})]}$$

"onda" de potencial que enfraquece com a distância

Cálculo do potencial A

Corrente ao longo do eixo do dipolo:



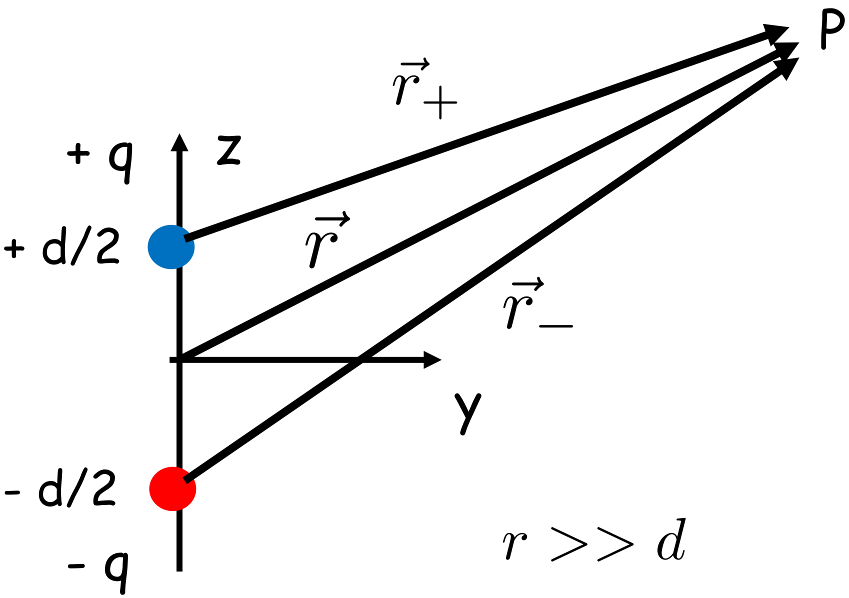
$$\vec{I} = \frac{dq}{dt}\,\hat{z}$$

$$q = q_0 \cos \omega t$$

$$\frac{dq}{dt} = -q_0\,\omega\, sen\,\omega t$$

$$\vec{I} = -q_0\,\omega\, sen\,\omega t\ \hat{z}$$

$$\vec{A}(t) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(t_r)}{r}\,dl'$$



$$r >> d$$

$$r_+ = \sqrt{(\cancelto{0}{\frac{d}{2}})^2 + r^2 - \cancelto{0}{d}\, r \, cos\theta}$$

$$r_- = \sqrt{(\cancelto{0}{\frac{d}{2}})^2 + r^2 + \cancelto{0}{d}\, r \, cos\theta}$$

$$\vec{r}_+ = \vec{r}_- = \vec{r} \quad \Rightarrow \quad t_r = t - \frac{r}{c}$$

$$\vec{I} = -q_0\, \omega\, sen[\omega \left(t - \frac{r}{c}\right)] \; \hat{z}$$

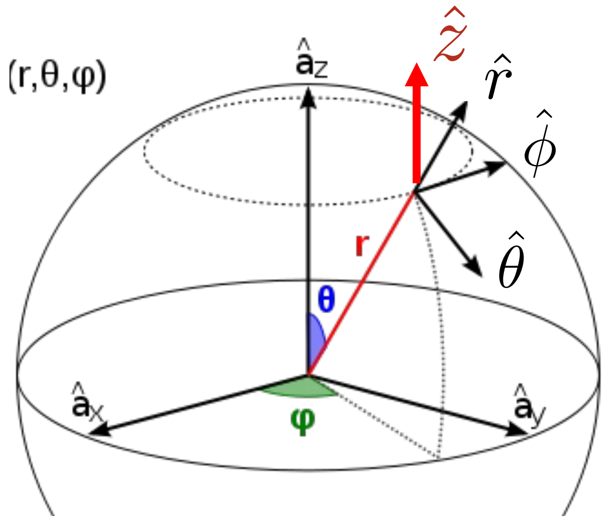
$$\vec{A}(t) = \frac{\mu_0}{4\,\pi} \int_{-d/2}^{+d/2} \frac{(-q_0\, \omega)}{r} sen[\omega \left(t - \frac{r}{c}\right)] \, \hat{z}\, dz$$

$$\boxed{\vec{A}(t) = -\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi} \, \frac{1}{r} sen[\omega \left(t - \frac{r}{c}\right)] \, \hat{z}}$$

Agora no modo guerra !

Cálculo do campo magnético : $\vec{B} = \vec{\nabla} \times \vec{A}$

Coordenadas esféricas:



$$\hat{z} = cos\,\theta\,\hat{r} - sen\,\theta\,\hat{\theta}$$

$$\vec{A}(t) = -\frac{\mu_0\,q_0\,\omega\,d}{4\,\pi}\,\frac{1}{r} sen[\omega\left(t - \frac{r}{c}\right)]\,\hat{z}$$

$$= -\frac{\mu_0\,q_0\,\omega\,d}{4\,\pi}\,\frac{1}{r} sen[\omega\left(t - \frac{r}{c}\right)]\;\left[cos\,\theta\,\hat{r} - sen\,\theta\,\hat{\theta}\right]$$

$$\begin{cases} A_r = -\dfrac{\mu_0\,q_0\,\omega\,d}{4\,\pi}\,\dfrac{cos\,\theta}{r}\,sen[\omega\left(t - \dfrac{r}{c}\right)] \\[2ex] A_\theta = \dfrac{\mu_0\,q_0\,\omega\,d}{4\,\pi}\,\dfrac{sen\,\theta}{r}\,sen[\omega\left(t - \dfrac{r}{c}\right)] \end{cases}$$

Rotacional em esféricas:

$$\vec{\nabla}\times\vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\cancelto{0}{A_\varphi}\sin\theta\right) - \cancelto{0}{\frac{\partial A_\theta}{\partial\varphi}}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\cancelto{0}{\frac{\partial A_r}{\partial\varphi}} - \frac{\partial}{\partial r}\left(r\cancelto{0}{A_\varphi}\right)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}$$

$$A_\theta = \frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{sen\,\theta}{r}\,sen[\omega\left(t-\frac{r}{c}\right)]$$

$$r\,A_\theta = \frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,sen\,\theta\,sen[\omega\left(t-\frac{r}{c}\right)]$$

$$\frac{\partial}{\partial r}(r\,A_\theta) = \frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,sen\,\theta\,(-\frac{\omega}{c})\,cos[\omega\left(t-\frac{r}{c}\right)]$$

$$= -\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{\omega}{c}\,sen\,\theta\,cos[\omega\left(t-\frac{r}{c}\right)]$$

$$\frac{1}{r} << \frac{\omega}{c}$$

$$A_r = -\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{cos\,\theta}{r}\,sen[\omega\left(t-\frac{r}{c}\right)]$$

$$-\frac{\partial}{\partial\theta}A_r = \cancelto{0}{-\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{sen\,\theta}{r}\,sen[\omega\left(t-\frac{r}{c}\right)]}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{1}{r}\,\frac{\partial}{\partial r}(r\,A_\theta)\,\hat{\phi}$$

$$\boxed{\vec{B} = -\frac{\mu_0\,q_0\,d}{4\,\pi}\,\frac{\omega^2}{c}\,\left(\frac{sen\,\theta}{r}\right)\,cos[\omega\left(t - \frac{r}{c}\right)]\hat{\phi}}$$

Cálculo do campo elétrico

$$\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

$$V = -\,\frac{q_0\,d\,\omega}{4\,\pi\,\epsilon_0\,c}\,\left(\frac{cos\theta}{r}\right)\,sen[\omega(t - \frac{r}{c})]$$

Gradiente em esféricas :

$$\vec{\nabla}\,V = \frac{\partial V}{\partial r}\,\hat{r} + \frac{1}{r}\,\frac{\partial V}{\partial\theta}\,\hat{\theta} + \frac{1}{r\,sen\,\theta}\,\cancelto{0}{\frac{\partial V}{\partial\phi}}\,\hat{\phi}$$

$$\vec{\nabla}\,V = \frac{\partial V}{\partial r}\,\hat{r} + \frac{1}{r}\,\frac{\partial V}{\partial\theta}\,\hat{\theta}$$

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\hat{\theta}$$

$$V = -\frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\left(\frac{cos\theta}{r}\right) sen[\omega(t - \frac{r}{c})]$$

0

$$\frac{\partial V}{\partial r} = \frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\left[\frac{1}{r}\left(\frac{cos\theta}{r}\right) sen[\omega(t - \frac{r}{c})] + \frac{\omega}{c}\left(\frac{cos\theta}{r}\right) cos[\omega(t - \frac{r}{c})]\right]$$

$$\frac{\partial V}{\partial \theta} = \frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\left(\frac{sen\theta}{r}\right) sen[\omega(t - \frac{r}{c})]$$

$$\frac{1}{r} << \frac{\omega}{c}$$

0

$$\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\frac{1}{r}\left(\frac{sen\theta}{r}\right) sen[\omega(t - \frac{r}{c})]$$

$$\boxed{\vec{\nabla} V = \frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\left[\frac{\omega}{c}\left(\frac{cos\theta}{r}\right) cos[\omega\left(t - \frac{r}{c}\right)]\right]\hat{r}}$$

$$\vec{A}(t) = -\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{1}{r}\,sen[\omega\left(t-\frac{r}{c}\right)]\,\hat{z} \;=\; -\frac{\mu_0\, q_0\, \omega\, d}{4\,\pi}\,\frac{1}{r}\,sen[\omega\left(t-\frac{r}{c}\right)]\;\left[cos\,\theta\,\hat{r} - sen\,\theta\,\hat{\theta}\right]$$

$$-\frac{\partial\vec{A}}{\partial t} = \cancel{\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi}\left(\frac{cos\,\theta}{r}\right)}\; cos[\omega\left(t-\frac{r}{c}\right)]\;\hat{r}\; - \;\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi}\left(\frac{sen\,\theta}{r}\right)\; cos[\omega\left(t-\frac{r}{c}\right)]\;\hat{\theta}$$

$$\vec{\nabla} V = \frac{q_0\, d\, \omega}{4\,\pi\,\epsilon_0\, c}\left[\frac{\omega}{c}\left(\frac{cos\theta}{r}\right)\; cos[\omega\left(t-\frac{r}{c}\right)]\right]\;\hat{r}$$

$$\frac{1}{c^2} = \mu_0\,\epsilon_0$$

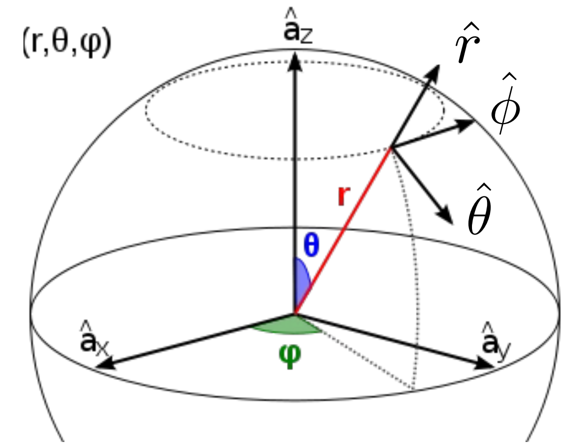
$$-\vec{\nabla} V = \cancel{-\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi}\left(\frac{cos\theta}{r}\right)}\; cos[\omega\left(t-\frac{r}{c}\right)]\;\hat{r}$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial\vec{A}}{\partial t}$$

$$\boxed{\vec{E} = -\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi}\left(\frac{sen\,\theta}{r}\right)\; cos[\omega\left(t-\frac{r}{c}\right)]\;\hat{\theta}}$$

$$\boxed{\vec{E} = -\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi} \left(\frac{sen\,\theta}{r}\right) cos[\omega \left(t - \frac{r}{c}\right)]\ \hat{\theta}}$$

$$\boxed{\vec{B} = -\frac{\mu_0\, q_0\, d}{4\,\pi}\, \frac{\omega^2}{c} \left(\frac{sen\,\theta}{r}\right) cos[\omega \left(t - \frac{r}{c}\right)]\hat{\phi}}$$



$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)$$

$$= \frac{1}{\mu_0}\left[\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi}\right]^2 \frac{1}{c}\left(\frac{sen\,\theta}{r}\right)^2 \left(cos[\omega \left(t - \frac{r}{c}\right)]\right)^2\ \overbrace{\hat{\theta}\times\hat{\phi}}^{\hat{r}}$$

$$<\vec{S}>= \frac{\mu_0\, q_0^2\, d^2\omega^4}{16\,\pi^2\, c}\, \frac{sen^2\theta}{r^2}\, \underbrace{\frac{1}{T}\int_0^T cos^2\left[\omega\left(t - \frac{r}{c}\right)\right]\, dt}_{1/2}\ \hat{r}$$

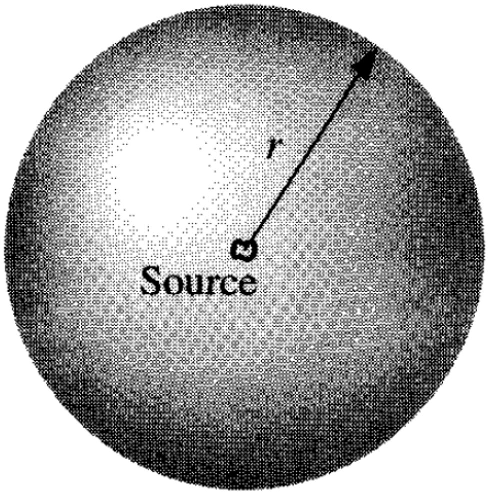
$$<\vec{S}>= \frac{\mu_0\, q_0^2\, d^2\omega^4}{32\,\pi^2\, c}\, \frac{sen^2\theta}{r^2}\, \hat{r}$$

Cálculo da potência média radiada

$$< P >= \oint < \vec{S} > \, . \, \vec{da}$$

$$\vec{da} = da \, \hat{r}$$

$$da = r^2 \, sen\, \theta \, d\theta \, d\phi$$



$$< \vec{S} >= \frac{\mu_0 \, q_0^2 \, d^2 \omega^4}{32 \, \pi^2 \, c} \, \frac{sen^2 \theta}{r^2} \, \hat{r}$$

$$< P >= \frac{\mu_0 \, q_0^2 \, d^2 \omega^4}{32 \, \pi^2 \, c} \underbrace{\int_0^{2\pi} d\phi}_{2\,\pi} \int_0^{\pi} d\theta \, \frac{sen^2\theta}{\cancel{r^2}} \, \cancel{r^2} \, sen\, \theta$$

$$\int_0^{\pi} d\theta \, sen^3 \theta = \frac{4}{3}$$

$$\boxed{\langle P \rangle = \frac{\mu_0 \, q_0 \, d^2 \, \omega^4}{12 \, \pi \, c}}$$

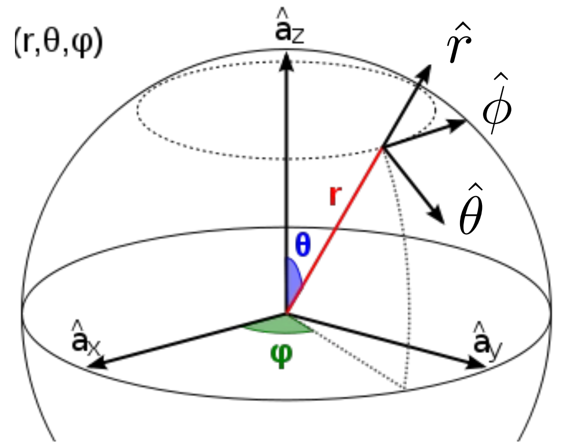
$$< P_{rad} >= \lim_{r \to \infty} < P > \neq 0$$

Sim, este sistema irradia !

Radiação uma antena de brinquedo:

Resumindo...

$$\boxed{\vec{E} = -\frac{\mu_0\, q_0\, \omega^2\, d}{4\,\pi} \left(\frac{sen\,\theta}{r}\right)\, cos[\omega\left(t - \frac{r}{c}\right)]\ \hat{\theta}}$$



$$\boxed{\vec{B} = -\frac{\mu_0\, q_0\, d}{4\,\pi}\, \frac{\omega^2}{c} \left(\frac{sen\,\theta}{r}\right)\, cos[\omega\left(t - \frac{r}{c}\right)]\hat{\phi}}$$

$$\boxed{< \vec{S} >= \frac{\mu_0\, q_0^2\, d^2 \omega^4}{32\,\pi^2\, c}\, \frac{sen^2\theta}{r^2}\, \hat{r}}$$

$$\boxed{\langle P \rangle = \frac{\mu_0\, q_0\, d^2\, \omega^4}{12\,\pi\, c}}$$

$$< P_{rad} >= \lim_{r \to \infty} < P > \neq 0$$

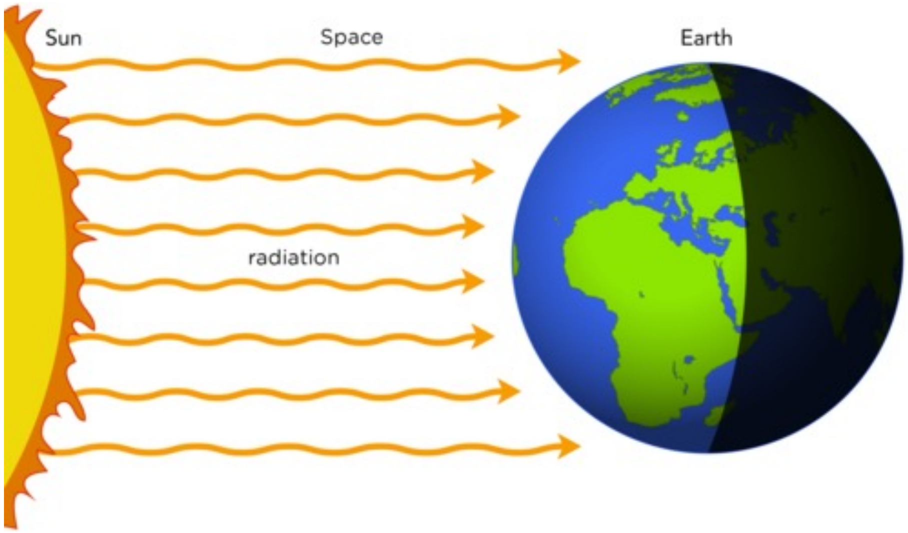
Sim, este sistema irradia !

$$\omega = 2\,\pi\,\nu = 2\,\pi\,\frac{c}{\lambda}$$

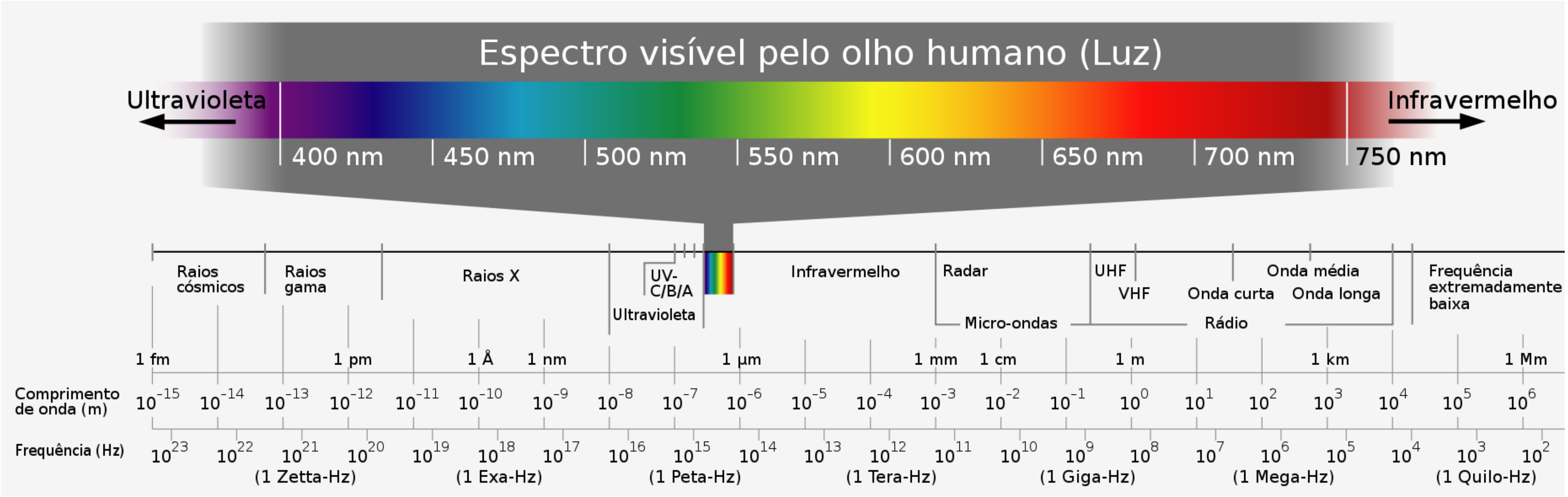
Se a antena emite: mais energia é emitida nas frequências maiores !!!

Se a antena absorve: mais energia absorvida nas frequências maiores !!!

Espalhamento Rayleigh na atmosfera:

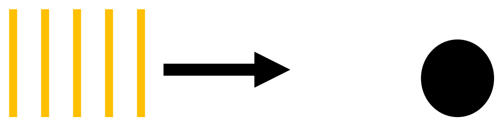


Luz do sol incide sobre a Terra com todas as frequências (cores)

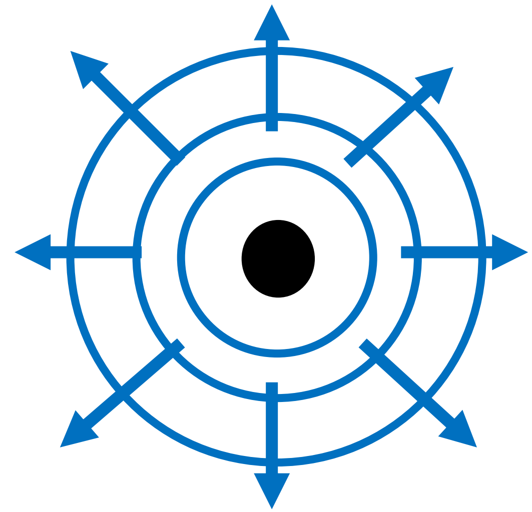


Espalhamento Rayleigh na atmosfera:

Molécula na atmosfera

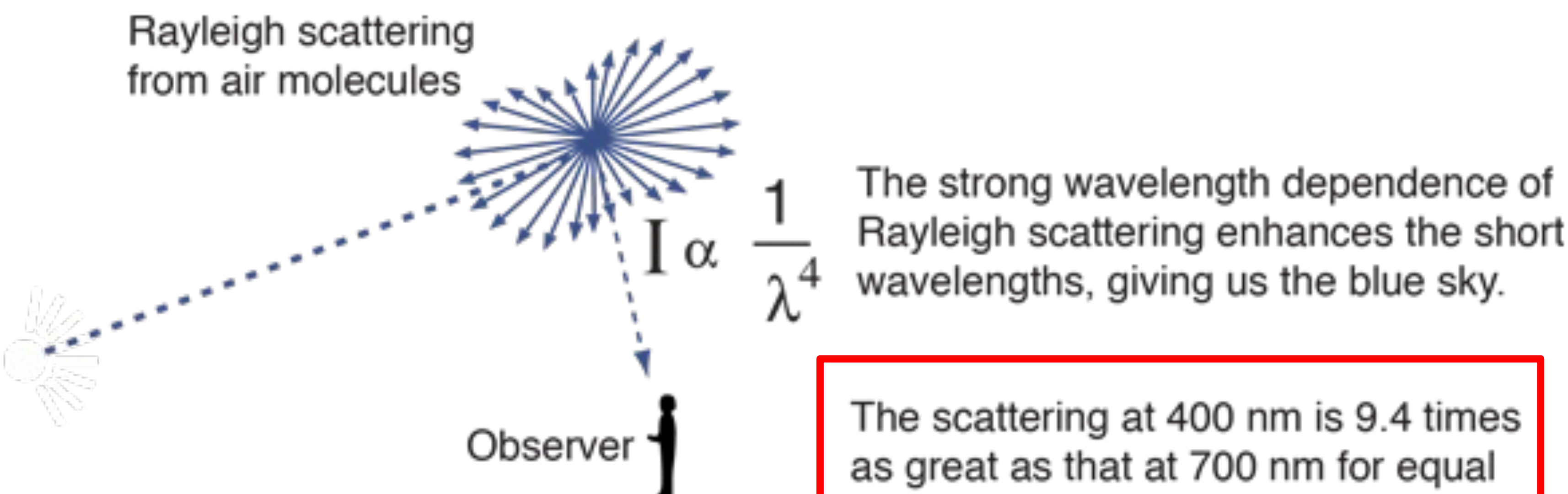


Chega luz com todas as frequências (de todas a cores)



As frequências mais altas são mais espalhadas

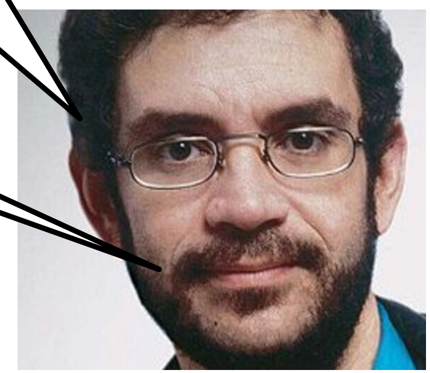
O azul é mais espalhado !!!



The scattering at 400 nm is 9.4 times as great as that at 700 nm for equal incident intensity.

É preciso amaa-aa-aar as pessoas
como se não houvesse amanhã...

Me diz porque que o céu é azul



Renato Russo

$$\langle P \rangle = \frac{\mu_0 \, q_0 \, d^2 \, \omega^4}{12 \, \pi \, c}$$