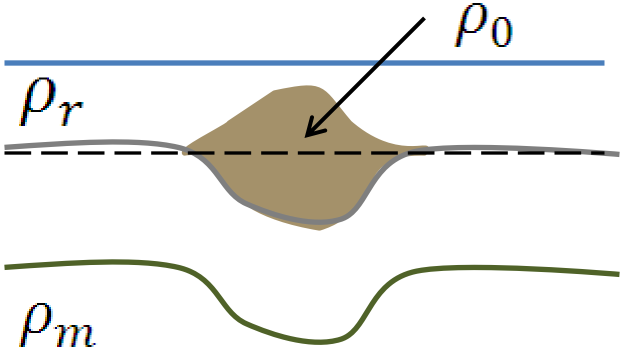
Flexura de placa elástica por diferenças finitas

Victor Sacek

IAG - USP



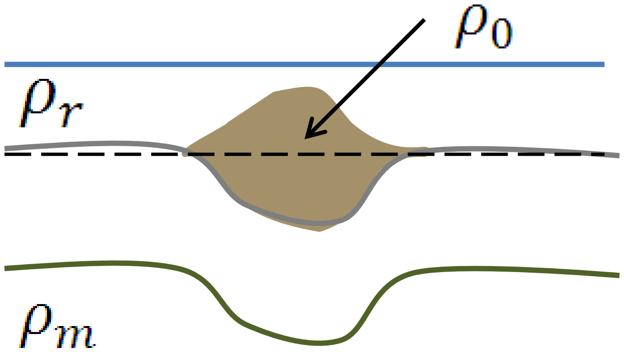
Comparação

h Espessura total do carregamento:

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_r)gw = (\rho_0 - \rho_r)gh$$

h' Espessura do carregamento acima da paleotopografia/paleobatimetria:

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0)gw = (\rho_0 - \rho_r)gh'$$



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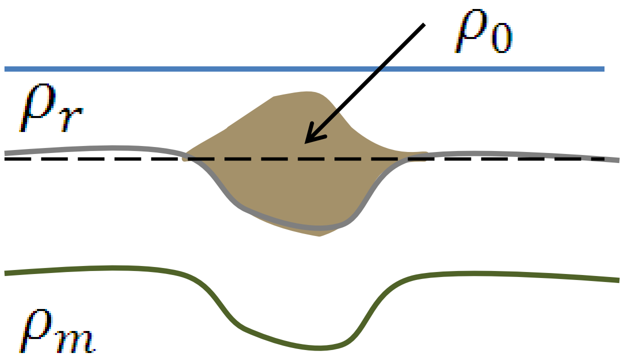

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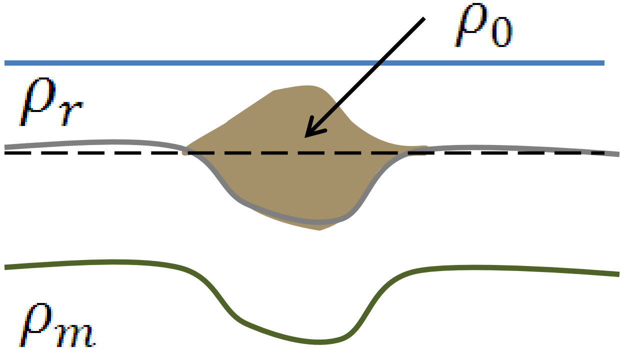

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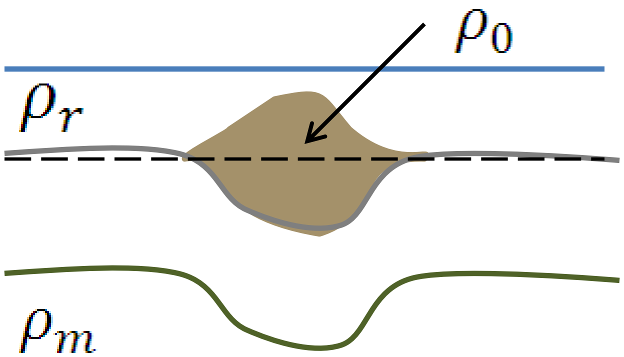
D constante $\quad \Delta\rho \quad p$



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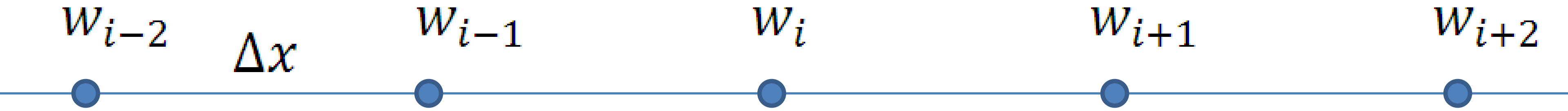
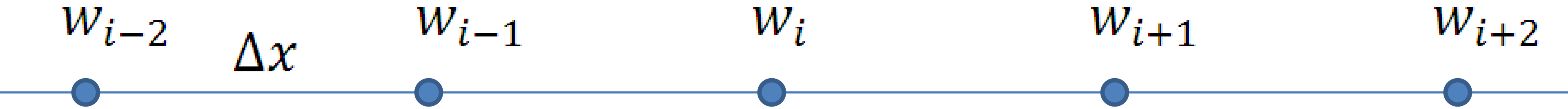
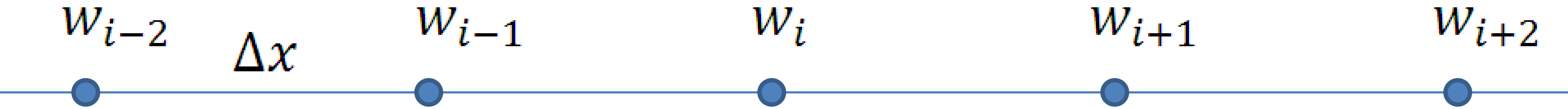
D constante $\quad \Delta\rho \quad p$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$


w_{i-2}
Δx
w_{i-1}
w_i
w_{i+1}
w_{i+2}



$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$



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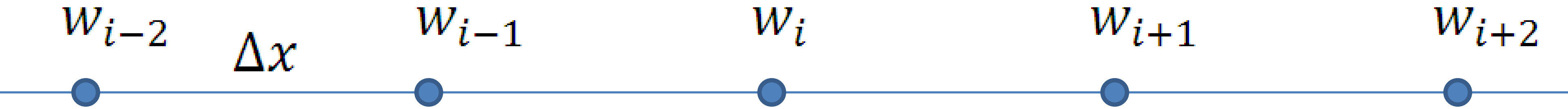
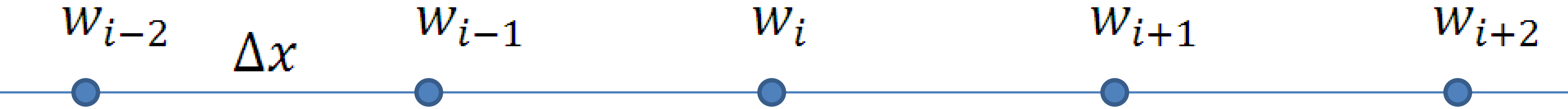


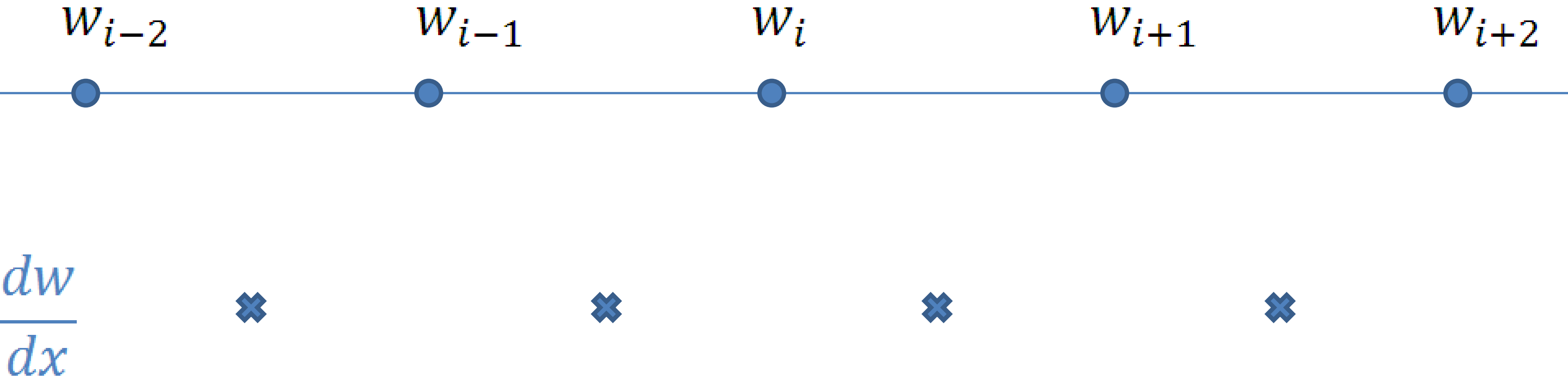
$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$+\Delta\rho g w_i = p_i$$



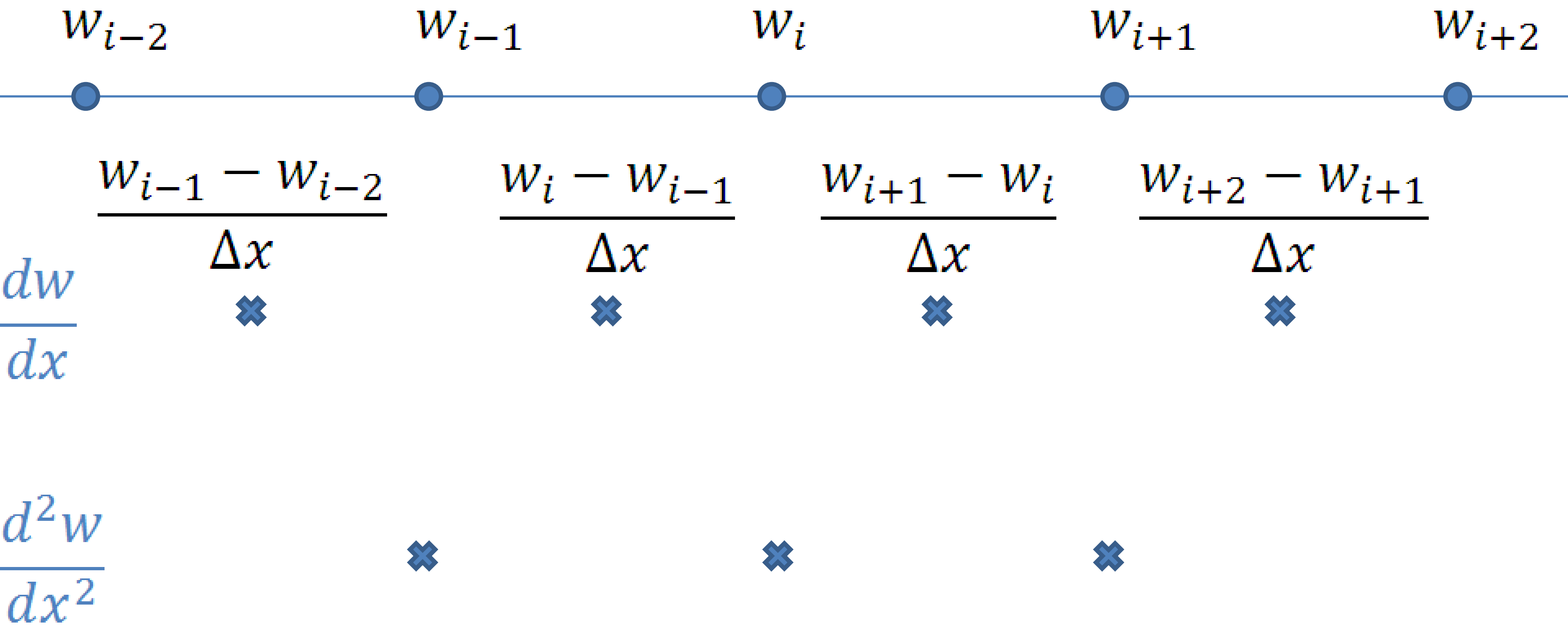
$$D\frac{d^4w}{dx^4}+\Delta\rho g w = p$$

$$? \quad +\Delta\rho g w_i = p_i$$


w_{i-2}
w_{i-1}
w_i
w_{i+1}
w_{i+2}
$\frac{dw}{dx}$

w_{i-2} $\quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

$$\frac{dw}{dx} \qquad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$



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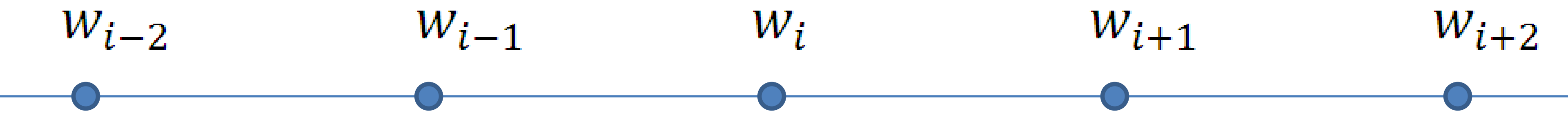

$$\frac{dw}{dx} \qquad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2} \qquad \frac{\dfrac{w_i - w_{i-1}}{\Delta x} - \dfrac{w_{i-1} - w_{i-2}}{\Delta x}}{\Delta x}$$

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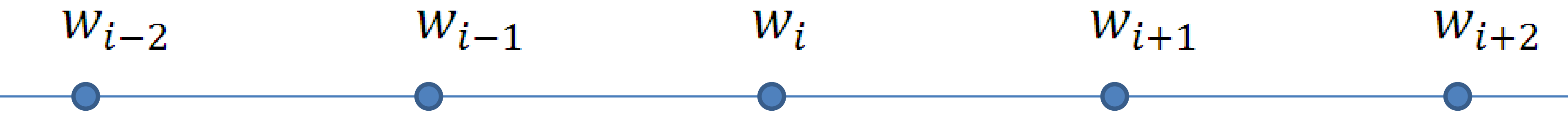
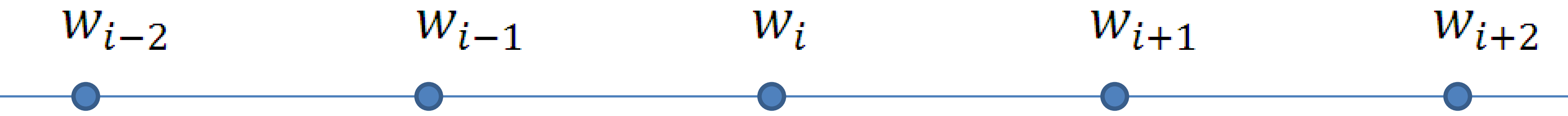


$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2}$$



$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$$

w_{i-2} w_{i-1} w_i w_{i+1} w_{i+2}

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

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$$\frac{d^3w}{dx^3}$$

$$w_{i-2} \qquad w_{i-1} \qquad w_i \qquad w_{i+1} \qquad w_{i+2}$$

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

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$$\frac{d^3w}{dx^3}: \quad \frac{w_{i+1} - 3w_i + 3w_{i-1} - w_{i-2}}{\Delta x^3} \qquad \frac{w_{i+2} - 3w_{i+1} + 3w_i - w_{i-1}}{\Delta x^3}$$

w_{i-2} w_{i-1} w_i w_{i+1} w_{i+2}

$$\frac{dw}{dx}: \quad \frac{w_{i-1}-w_{i-2}}{\Delta x} \qquad \frac{w_i-w_{i-1}}{\Delta x} \qquad \frac{w_{i+1}-w_i}{\Delta x} \qquad \frac{w_{i+2}-w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i-2w_{i-1}+w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1}-2w_i+w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2}-2w_{i+1}+w_i}{\Delta x^2}$$

$$\frac{d^3w}{dx^3}: \quad \frac{w_{i+1}-3w_i+3w_{i-1}-w_{i-2}}{\Delta x^3} \qquad \frac{w_{i+2}-3w_{i+1}+3w_i-w_{i-1}}{\Delta x^3}$$

$$\frac{d^4w}{dx^4}$$

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w_{i-2} $\quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

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$$\frac{d^2w}{dx^2}: \quad \frac{w_i-2w_{i-1}+w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1}-2w_i+w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2}-2w_{i+1}+w_i}{\Delta x^2}$$

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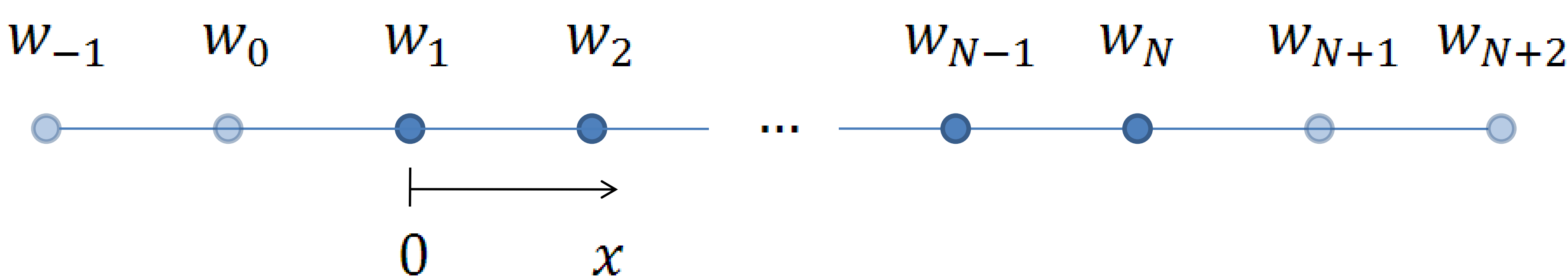
$$\frac{d^4w}{dx^4}: \quad \frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}$$

Condições de contorno

- Placa contínua:

$$w \to 0 \text{ para } x \to 0$$
$$w \to 0 \text{ para } x \to x_n$$

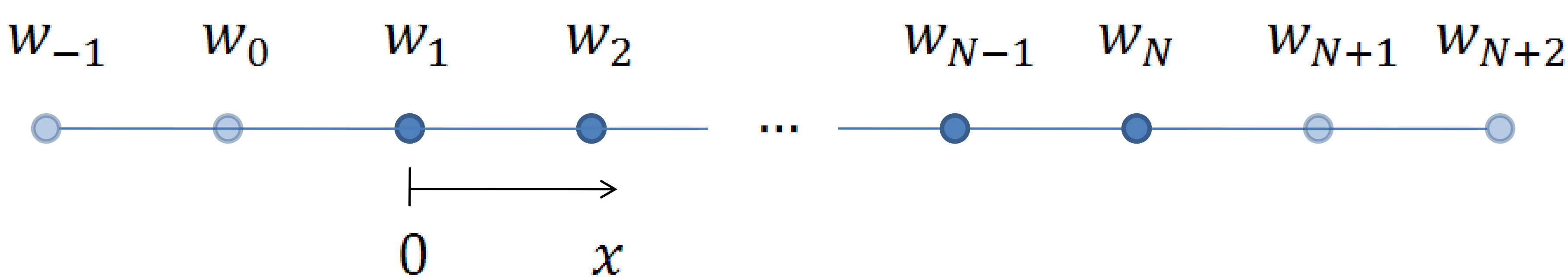


Condições de contorno

- Placa contínua:

$$w \to 0 \ \text{para} \ x \to 0$$
$$w \to 0 \ \text{para} \ x \to x_n$$

$$w_{-1}, w_0, w_{N+1}, w_{N+2} = 0$$



$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D\frac{d^4w}{dx^4}+\Delta\rho g w = p$$

$$D\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}+\Delta\rho g w_i = p_i$$

$$D[w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}]+\Delta x^4\Delta\rho g w_i = \Delta x^4 p_i$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

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$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} =$$

$$= \Delta x^4 p_i$$

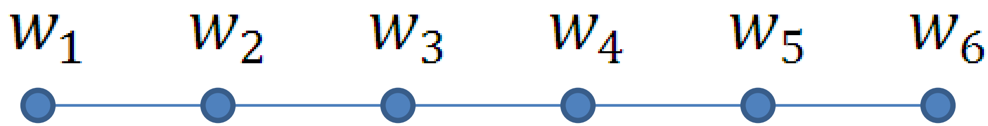
$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i \end{aligned}}$$

Exemplo



$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} =$$
$$= \Delta x^4 p_i$$

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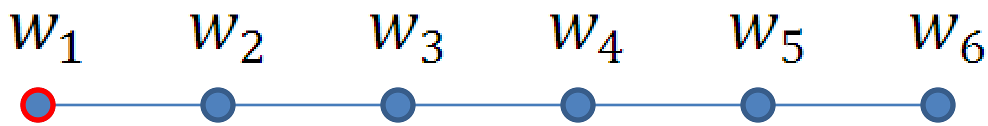
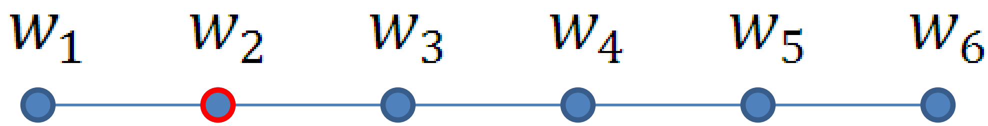


w_1 :

$$[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} &= \\ &= \Delta x^4 p_i \end{aligned}}$$

Exemplo



w_1 :

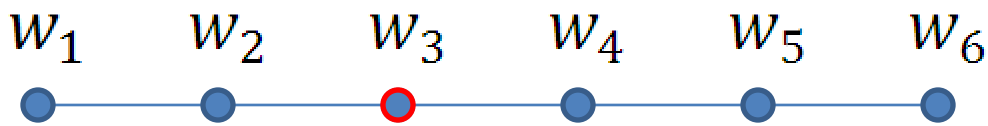
$$[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

w_2 :

$$-4Dw_1 + [6D + \Delta x^4 \Delta\rho g]w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} &= \\ &= \Delta x^4 p_i \end{aligned}}$$

Exemplo



w_1 :

$$[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

w_2 :

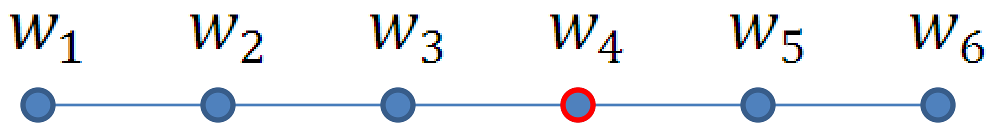
$$-4Dw_1 + [6D + \Delta x^4 \Delta\rho g]w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

w_3 :

$$Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta\rho g]w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$$

$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i}$$

Exemplo



w_1 : $$[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

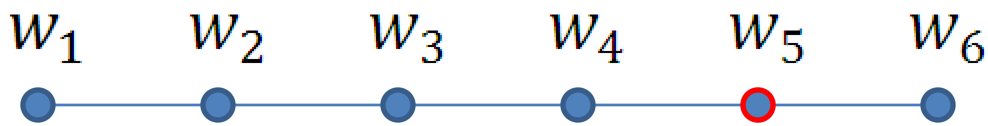
w_2 : $$-4Dw_1 + [6D + \Delta x^4 \Delta\rho g]w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

w_3 : $$Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta\rho g]w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$$

w_4 : $$Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta\rho g]w_4 - 4Dw_5 + Dw_6 = \Delta x^4 p_4$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} &= \\ &= \Delta x^4 p_i \end{aligned}}$$

Exemplo



w_1 : $$[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

w_2 : $$-4Dw_1 + [6D + \Delta x^4 \Delta\rho g]w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

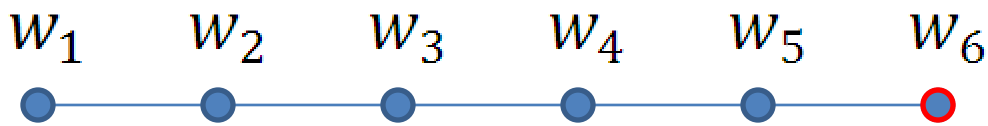
w_3 : $$Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta\rho g]w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$$

w_4 : $$Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta\rho g]w_4 - 4Dw_5 + Dw_6 = \Delta x^4 p_4$$

w_5 : $$Dw_3 - 4Dw_4 + [6D + \Delta x^4 \Delta\rho g]w_5 - 4Dw_6 = \Delta x^4 p_5$$

$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \Delta x^4 p_i}$$

Exemplo



w_1 : $[6D + \Delta x^4 \Delta\rho g]w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$

w_2 : $-4Dw_1 + [6D + \Delta x^4 \Delta\rho g]w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$

w_3 : $Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta\rho g]w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$

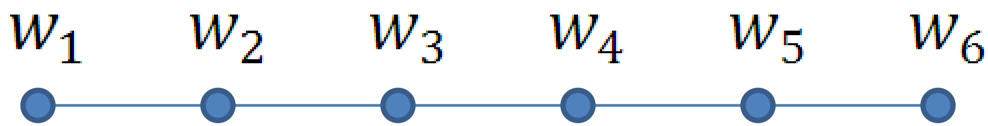
w_4 : $Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta\rho g]w_4 - 4Dw_5 + Dw_6 = \Delta x^4 p_4$

w_5 : $Dw_3 - 4Dw_4 + [6D + \Delta x^4 \Delta\rho g]w_5 - 4Dw_6 = \Delta x^4 p_5$

w_6 : $Dw_4 - 4Dw_5 + [6D + \Delta x^4 \Delta\rho g]w_6 = \Delta x^4 p_6$

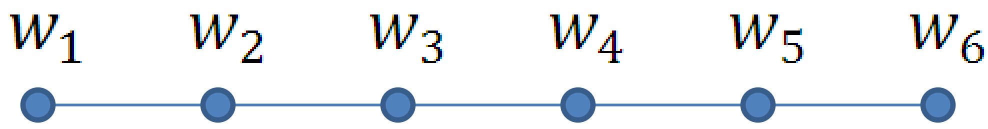
$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i}$$

Exemplo



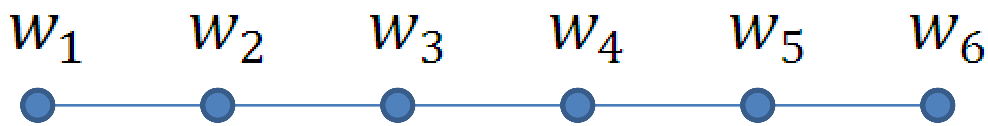
$$\begin{bmatrix} 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 & 0 \\ -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \Delta x^4 \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}$$

Exemplo



$$
\begin{bmatrix}
6D + \Delta x^4 \Delta\rho g & -4D & D & 0 & 0 & 0 \\
-4D & 6D + \Delta x^4 \Delta\rho g & -4D & D & 0 & 0 \\
D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D & D & 0 \\
0 & D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D & D \\
0 & 0 & D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D \\
0 & 0 & 0 & D & -4D & 6D + \Delta x^4 \Delta\rho g
\end{bmatrix}
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}
= \Delta x^4
\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}
$$

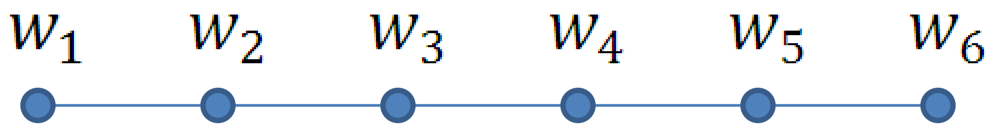
Exemplo



$$\begin{bmatrix} 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 & 0 \\ -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \Delta x^4 \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}$$

$$\mathbf{Aw} = \mathbf{p}$$

Exemplo



$$\begin{bmatrix} 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 & 0 \\ -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \Delta x^4 \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix}$$

$$\mathbf{Aw} = \mathbf{p}$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} - 2) = D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} - 1) = -4D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i}) = 6D + \Delta x^4 \Delta\rho g$$

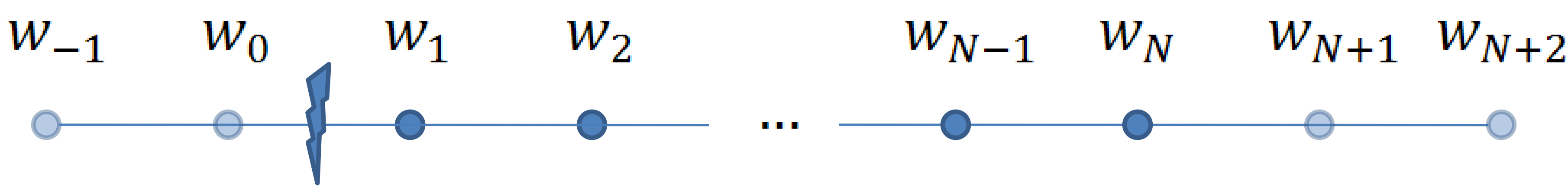
$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} + 1) = -4D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} + 2) = D$$

Condições de contorno

- Placa rompida:

$$\frac{d^2w}{dx^2} = 0 \ \text{ para } \ x = x_1$$



Condições de contorno

- Placa rompida:

$$\frac{d^2w}{dx^2} = 0 \ \text{ para } \ x = x_1$$

$$\frac{w_2 - 2w_1 + w_0}{\Delta x^2} = 0$$

Condições de contorno

- Placa rompida:

$$\frac{d^2w}{dx^2} = 0 \;\text{ para }\; x = x_1$$

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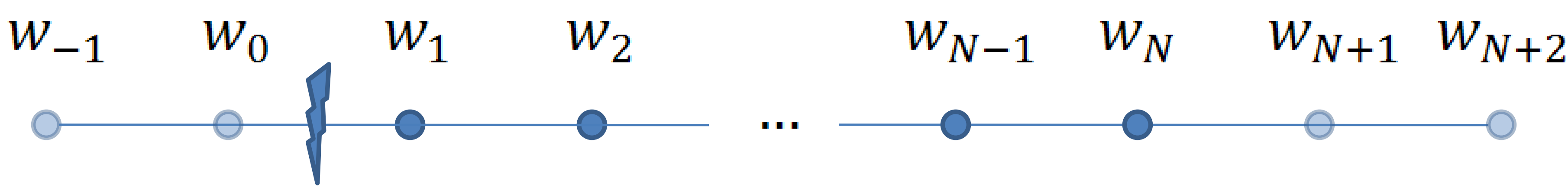
Condições de contorno

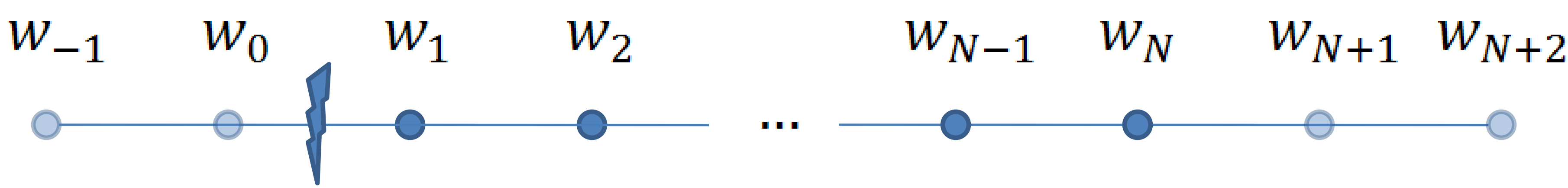
- Placa rompida: $$\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$$



Condições de contorno

- Placa rompida: $$\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$$

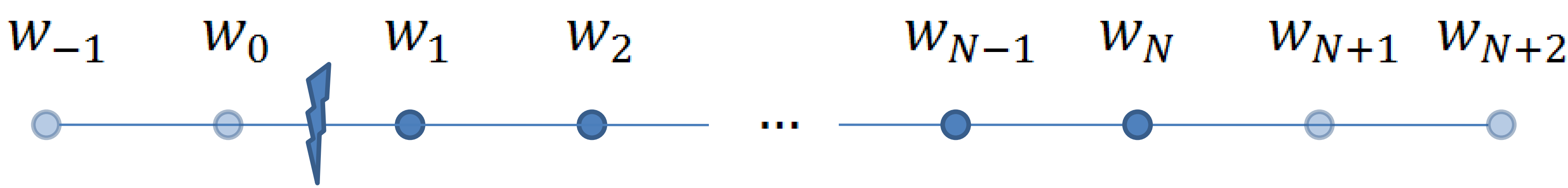
$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$



Condições de contorno

- Placa rompida: $$\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p \rightarrow \int_0^\infty D\frac{d^4w}{dx^4}\, dx + \int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$$



Condições de contorno

- Placa rompida: $\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$

$$D \frac{d^4 w}{dx^4} + \Delta\rho g w = p \rightarrow \int_0^\infty D \frac{d^4 w}{dx^4} \, dx + \cancel{\int_0^\infty \Delta\rho g w \, dx} = \cancel{\int_0^\infty p \, dx}$$

Condições de contorno

- Placa rompida: $\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$

$$D \frac{d^4 w}{dx^4} + \Delta\rho g w = p \rightarrow \int_0^\infty D \frac{d^4 w}{dx^4} \, dx + \cancel{\int_0^\infty \Delta\rho g w \, dx} = \cancel{\int_0^\infty p \, dx}$$

$$\int_0^\infty D \frac{d^4 w}{dx^4} \, dx = 0$$

Condições de contorno

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$$\int_0^\infty D\frac{d^4w}{dx^4}\,dx = 0 \rightarrow D\frac{d^3w}{dx^3}\bigg]_0^\infty = 0$$

Condições de contorno

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Condições de contorno

- Placa rompida:

$$\int_0^\infty D\frac{d^4w}{dx^4}\,dx = 0 \rightarrow D\frac{d^3w}{dx^3}\bigg]_0^\infty = 0$$

Placa semi-infinita

$x \rightarrow \infty$ temos $w \rightarrow 0 \therefore \dfrac{d^3w}{dx^3} \rightarrow 0$

Condições de contorno

- Placa rompida:

Placa semi-infinita

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Condições de contorno

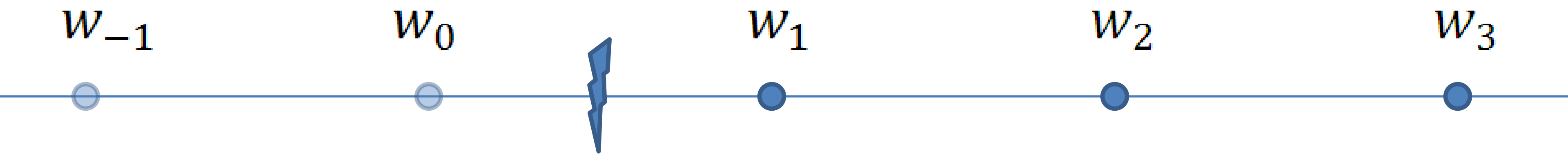
- Placa rompida:

Placa semi-infinita

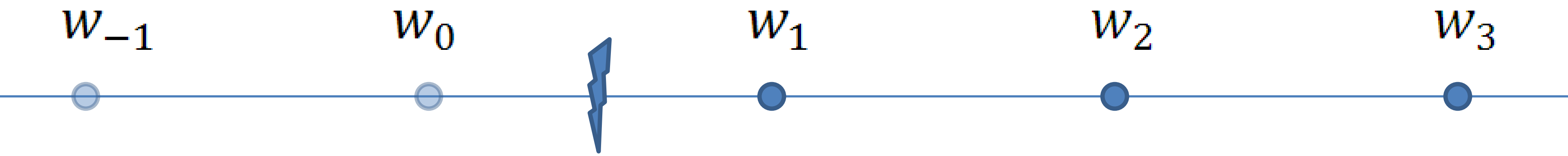
$$x \to \infty \text{ temos } w \to 0 \therefore \frac{d^3w}{dx^3} \to 0$$

$$\int_0^{\infty} D\frac{d^4w}{dx^4}\,dx = 0 \to D\frac{d^3w}{dx^3}\Big]_0^{\infty} = 0 \to D\frac{d^3w}{dx^3}\Big|_0 = 0$$

$$\frac{d^3w}{dx^3}\Big|_0 = 0$$



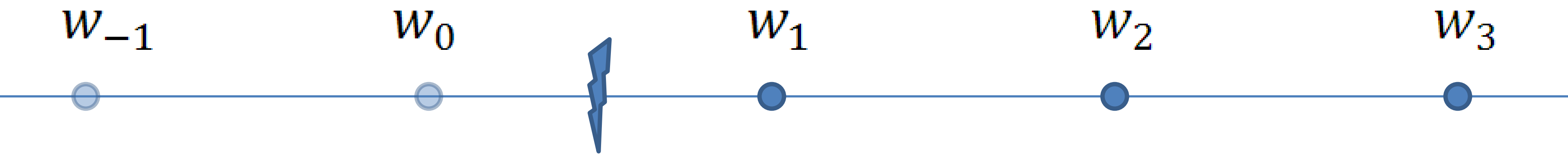
$$\frac{d^3w}{dx^3} \qquad \frac{w_2 - 3w_1 + 3w_0 - w_{-1}}{\Delta x^3} \qquad \frac{w_3 - 3w_2 + 3w_1 - w_0}{\Delta x^3}$$



$$\frac{d^3w}{dx^3}$$

$$\frac{w_2 - 3w_1 + 3w_0 - w_{-1}}{\Delta x^3}$$

$$\frac{w_3 - 3w_2 + 3w_1 - w_0}{\Delta x^3}$$



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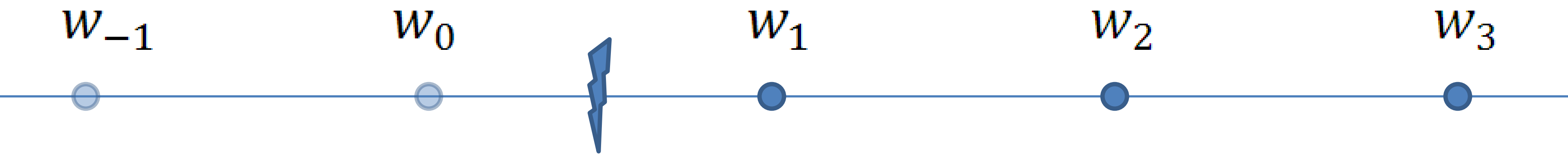
$$\frac{w_3 - 2w_2 + 2w_0 - w_{-1}}{2\Delta x^3}$$



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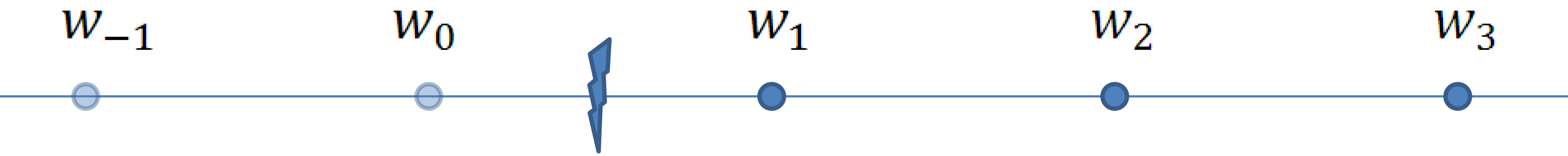

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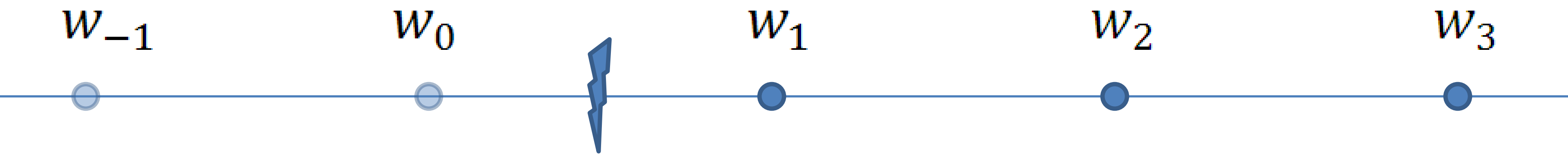

$$\frac{d^3w}{dx^3} \qquad \frac{w_2 - 3w_1 + 3w_0 - w_{-1}}{\Delta x^3} \qquad \frac{w_3 - 3w_2 + 3w_1 - w_0}{\Delta x^3}$$

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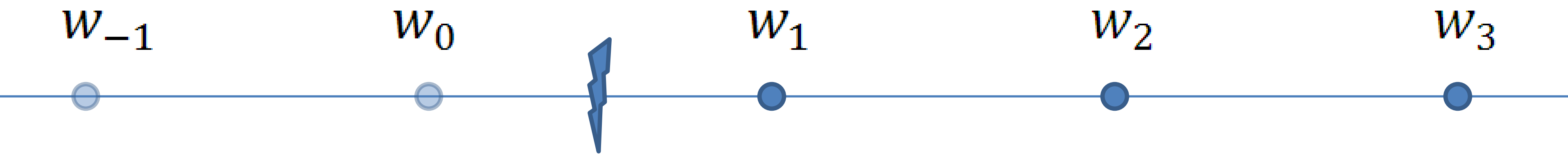
$$\rightarrow w_{-1} = w_3 - 2w_2 + 2w_0$$



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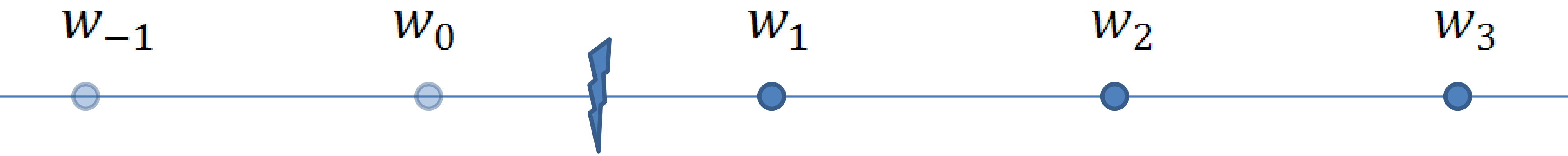

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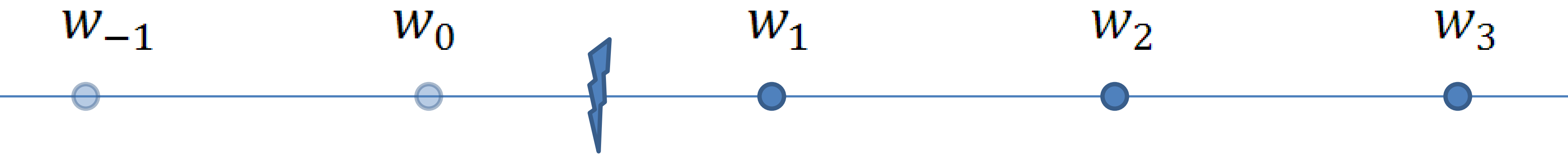

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$$\rightarrow w_{-1} = w_3 - 4w_2 + 4w_1$$

Modificação da matriz $\mathbf{A}$ para placa rompida

$$A(1,1) = 2D + \Delta x^4 \Delta\rho g$$
$$A(1,2) = -4D$$
$$A(1,3) = 2D$$
$$A(2,1) = -2D$$
$$A(2,2) = 5D + \Delta x^4 \Delta\rho g$$

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$$A(2,2) = 5D + \Delta x^4 \Delta\rho g$$

$$\begin{bmatrix} 2D + \Delta x^4\Delta\rho g & -4D & 2D & 0 & 0 & 0 \\ -2D & 5D + \Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D + \Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D + \Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D + \Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D + \Delta x^4\Delta\rho g \end{bmatrix}$$

Condições de contorno

- Placa contínua II:

$$\left.\frac{dw}{dx}\right|_0 = 0$$

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- Placa contínua II: $\int_0^\infty \Delta\rho g w \, dx = \int_0^\infty p \, dx$

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Condições de contorno

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$$\int_0^\infty D \frac{d^4 w}{dx^4} \, dx = 0 \rightarrow \left. \frac{d^3 w}{dx^3} \right|_0 = 0 \rightarrow \frac{w_3 - 2w_2 + 2w_0 - w_{-1}}{2\Delta x^3} = 0$$

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$w_0 = w_2$

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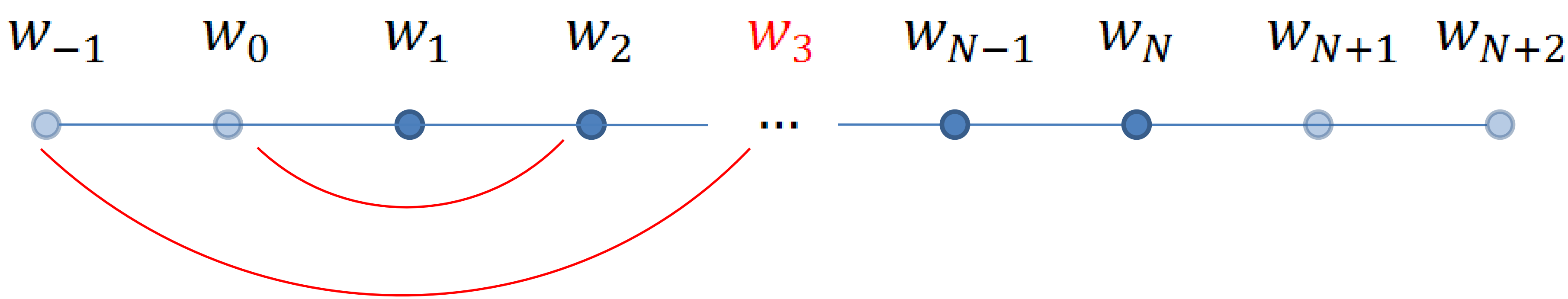

Modificação da matriz $\mathbf{A}$ para placa contínua II

$$A(1,2) = -8D$$
$$A(1,3) = 2D$$
$$A(2,2) = 7D + \Delta x^4 \Delta\rho g$$

Modificação da matriz $\mathbf{A}$ para placa contínua II

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$$\begin{bmatrix} 6D + \Delta x^4\Delta\rho g & -8D & 2D & 0 & 0 & 0 \\ -4D & 7D + \Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D + \Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D + \Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D + \Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D + \Delta x^4\Delta\rho g \end{bmatrix}$$