Lecture 17
Femtoscopy

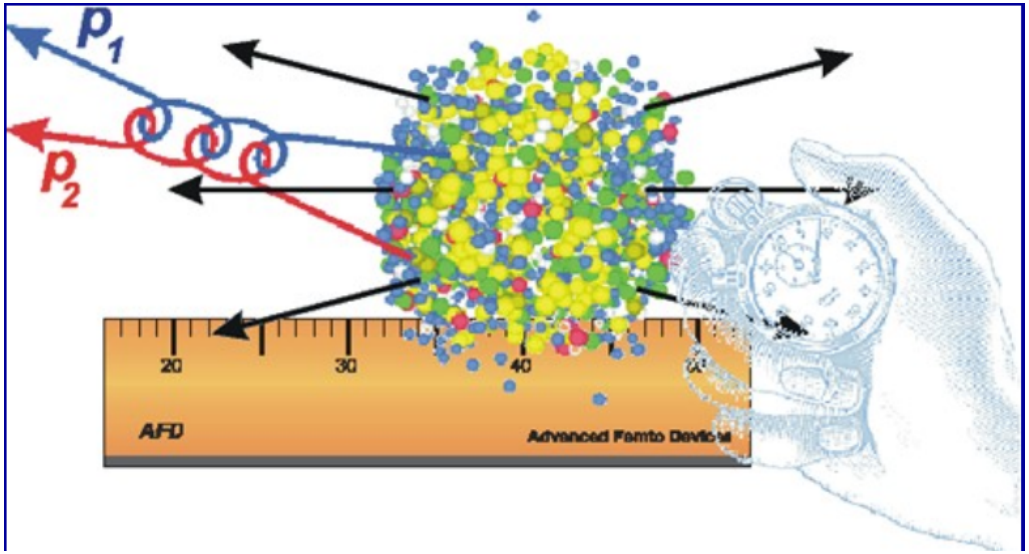
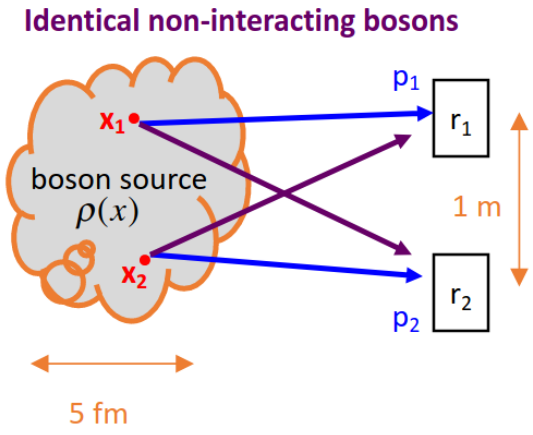


Illustration S.Pratt

Intensity interferometry or HBT interferometry or femtoscopy, is a method to evaluate dimensions and duration of emission regions. It is sensitive to first order phase transitions for example. It can also be used in conjonction with analysis of elliptic flow.

Simple model

Assume two identical bosons (typically pions) are produced respectively with momentum $\vec{p}_1$ and $\vec{p}_2$, at the locations $\vec{x}_1$ and $\vec{x}_2$ and the distribution of the points is given by the density distribution $\rho(\vec{x})$ and later detected.



The two-particle distribution of identical pions detected with momentum $\vec{p}_1$ and $\vec{p}_2$ is

$$\mathcal{P}_2(\vec{p}_1, \vec{p}_2) = \int d^3x_1 d^2x_2 \rho(x_1)\rho(x_2)|\psi_{12}|^2$$

where

$$\psi_{12} = \frac{1}{\sqrt{2}(2\pi)^3}\left(e^{i\vec{p}_1 \cdot (\vec{r}_1 - \vec{x}_1) + i\vec{p}_2 \cdot (\vec{r}_2 - \vec{x}_2)} + e^{i\vec{p}_1 \cdot (\vec{r}_1 - \vec{x}_2) + i\vec{p}_2 \cdot (\vec{r}_2 - \vec{x}_1)}\right)$$

Note that the second term appears to obey Bose-Eistein statistics.

It is usual to introduce: $\vec{q} = \vec{p}_1 - \vec{p}_2$ (relative momentum) and $\vec{k} = (1/2)(\vec{p}_1 + \vec{p}_2)$ (center-of-mass momentum).

Then for the two-particle distribution

$$\mathcal{P}_2(\vec{p}_1,\vec{p}_2) = \int \frac{d^3x_1}{(2\pi)^3}\frac{d^2x_2}{(2\pi)^3}\rho(x_1)\rho(x_2)\left[1+\frac{1}{2}\left(e^{i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}+e^{-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}\right)\right]$$

and the one-particle distribution

$$\mathcal{P}_1(\vec{p}) = \int d^3x\rho(x)\left|\frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\right|^2 = \int \frac{d^3x}{(2\pi)^3}\rho(\vec{x})$$

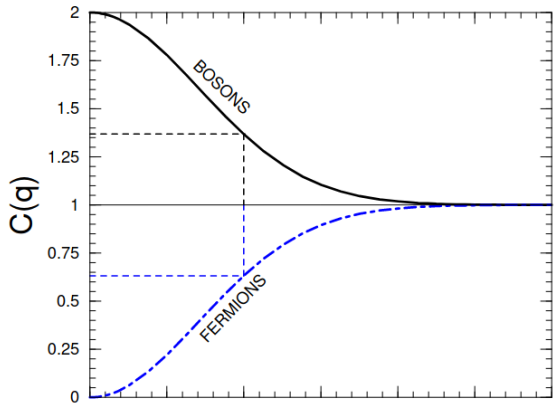
So we obtain the two-particle correlation function

$$C(\vec{p}_1,\vec{p}_2) \equiv \frac{\mathcal{P}_2(\vec{p}_1,\vec{p}_2)}{\mathcal{P}_1(\vec{p}_1)\mathcal{P}_1(\vec{p}_2)} = 1 + \frac{|\int d^3x e^{-i\vec{q}\cdot\vec{x}}\rho(\vec{x})|^2}{|\int d^3x\rho(\vec{x})|^2} = C(\vec{k},\vec{q})$$

For illustration we use a Gaussian profile in 1 dimension:

$$\rho(x) = e^{-x^2/(2R)^2} \longrightarrow C(\vec{k},\vec{q}) = C(q) = 1 + e^{-q^2R^2}$$

This function is plotted below:



There is an enhancement above 1. When $q = 1/R$, $C(q) = 1 + e^{-1} = 1.37$ (dashed lines). The typical size of the emission region is related to $C(\vec{p}_1,\vec{p}_2)$ plotted as function of q.

The emission function

The main problem of the simple model above is that the time dependence was not included. We proceed to do that in the general case. Now the notations x and p will designate 4-vectors and $\vec{p}$ a particle 3-momentum.

We introduce the emission function
$S(x,\vec{p}) = \int d\sigma_\mu(x')p^\mu\delta^{(4)}(x-x')f(x',p)$
We note that
$\int d^4x S(x,\vec{p}) = \int f(x,p)p^\mu d\sigma_\mu = E\frac{dN}{d^3p}$ (= Cooper-Frye formula)
It is useful to introduce as well
$S(\vec{k},\vec{q}) = \int d^4x e^{iq\cdot x}S(x,\vec{k}) = \int d\sigma_\mu k^\mu e^{iq\cdot x}f(x,\vec{k})$
which looks like a Cooper-Frye formula written for the center-of-mass momentum $\vec{k}$ with an $e^{iq\cdot x}$

Then we generalize For a more rigourous derivation, see Florkowski

$$C(\vec{k},\vec{q}) = 1 + \frac{|\int d^3x e^{-i\vec{q}\cdot\vec{x}}\rho(\vec{x})|^2}{|\int d^3x\rho(\vec{x})|^2} \rightarrow \boxed{C(\vec{k},\vec{q}) = 1 + \frac{|\int d^4x e^{iq\cdot x}S(x,\vec{k})|^2}{|\int d^4x S(x,\vec{k})|^2}}$$

The aim now is to extract as much information as possible on $S(x,\vec{k})$

Gaussian approximation for $S(x, \vec{k})$

More details in Florkowski's book and B. Tomasik & U.A. Wiedermann

There are various parametrizations that have been used for S, here we use the Gaussian approximation commonly used to interprate data $S(x, \vec{k}) = \mathcal{N}(\vec{k}) exp\left[-\frac{1}{2}\tilde{x}^{\mu}(\vec{k}) B_{\mu\nu} \tilde{x}^{\nu}(\vec{k})\right]$

where

- $\tilde{x}^{\mu}(\vec{k}) = x^{\mu} - \bar{x}^{\mu}(\vec{k})$ and $\bar{x}^{\mu}(\vec{k}) \equiv \left[\int d^4x x^{\mu} S(x, \vec{k})\right] / \int d^4x S(x, \vec{k})$ (= acts as coordinates of an effective center of emission for bosons emitted with $\vec{k}$)
- the choice $(B^{-1}_{\mu\nu}(\vec{k}) =< \tilde{x}^{\mu}\tilde{x}^{\nu} > (\vec{k})$ regulates the width.

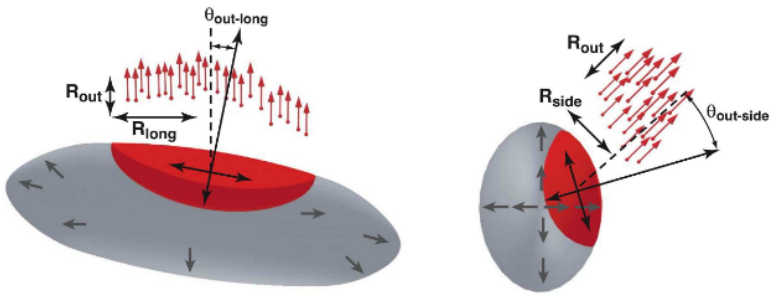
So $C(\vec{k}, \vec{q}) = 1 + exp\left[-q_{\mu} q_{\nu} < \tilde{x}^{\mu}\tilde{x}^{\nu} > (\vec{k})\right]$

Using $k \cdot q = 0 \Rightarrow q_0 = \vec{k} \cdot \vec{q}/k_0 \equiv \vec{\beta} \cdot \vec{q}$

$C(\vec{k}, \vec{q}) = 1 + exp\left[-\sum_{i,j=1}^{3} q^i q^j R^2_{ij}(\vec{k})\right]$

where $R^2_{ij}(\vec{k}) =< (\tilde{x}^i - \beta^i \tilde{t})(\tilde{x}^j - \beta^j \tilde{t}) > (\vec{k})$

Out-side-long coordinate system



It is common to choose the axis as follows:

- The *long* direction is the collision axis
- The *out* direction is chosen along $\vec{k}_\perp$
- The *side* complements the orthogonal right-handed frame.

With this choice:

$$
\begin{aligned}
R^2_{11}(\vec{k}) &= R^2_{out}(\vec{k}) &&=< (\tilde{x} - \beta_\perp \tilde{t})^2 > (\vec{k}) \\
R^2_{22}(\vec{k}) &= R^2_{side}(\vec{k}) &&=< \tilde{y}^2 > (\vec{k}) \\
R^2_{33}(\vec{k}) &= R^2_{long}(\vec{k}) &&=< (\tilde{z} - \beta_l \tilde{t})^2 > (\vec{k}) \\
R^2_{12}(\vec{k}) &= R^2_{out-side}(\vec{k}) &&=< (\tilde{x} - \beta_\perp \tilde{t})\tilde{y} > (\vec{k}) \\
R^2_{23}(\vec{k}) &= R^2_{side-long}(\vec{k}) &&=< \tilde{y}(\tilde{z} - \beta_l \tilde{t}) > (\vec{k}) \\
R^2_{23}(\vec{k}) &= R^2_{long-out}(\vec{k}) &&=< (\tilde{z} - \beta_l \tilde{t})(\tilde{x} - \beta_\perp \tilde{t}) > (\vec{k})
\end{aligned}
$$

Boost-invariant cylindrically symmetric sources

For cylindrically symmetric sources, there is reflection symmetry with respect to the side direction, $\tilde{y} \to -\tilde{y}$, so $R_{out-side} = R_{side-long} = 0$ (since they are linear in $\tilde{y}$. For a boost-invariant system, a similar argument leads to $R_{long-out} = 0$. In the longitudinally moving system (LCMS), where $\beta_l = 0$ ($\vec{\beta} = \vec{k}/k_0$):

$$R^2_{out}(\vec{k}) =< (\tilde{x} - \beta_\perp \tilde{t})^2 > (\vec{k})$$
$$R^2_{side}(\vec{k}) =< \tilde{y}^2 > (\vec{k})$$
$$R^2_{long}(\vec{k}) =< (\tilde{z}^2 > (\vec{k})$$

Other R's are zero.
So

$$\boxed{C(\vec{k}, \vec{q}) = 1 + exp\left[-R^2_{out}(k_\perp)q^2_{out} - R^2_{side}(k_\perp)q^2_{side} - R^2_{long}(k_\perp)q^2_{long}\right]}$$

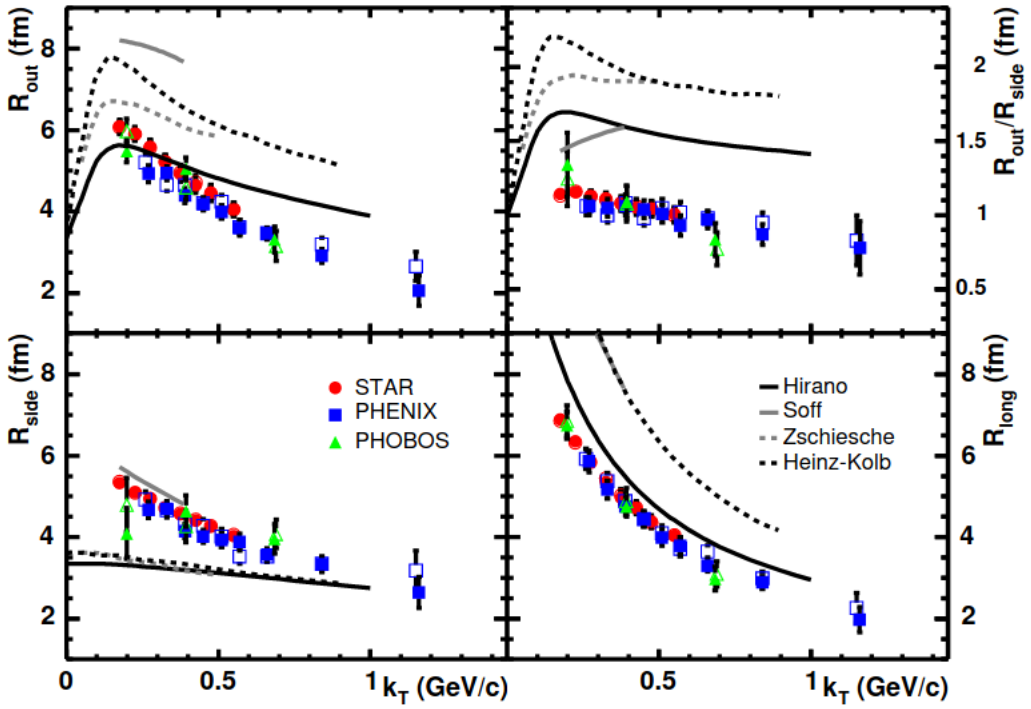
where $q^1 = q_{out}, q_2 = q_{side}, q^3 = q_{long}$.
This generalizes the simple formula we had previously:
$C(\vec{k}, \vec{q}) = 1 + e^{-q^2 R^2}$

The R's give information about the space-time sizes of the regions where pions are correlated (homogeneity lengths) rather than about the actual sizes of the whole system.

Expected behavior for the $R(k_\perp)$'s

- Both longitudinal and radial collective expansion reduce the R's because two fluid elements moving rapidly relative to each other are unlikely to contribute particles with small relative momenta, i.e. to the correlation function.
- In the Borken model, the dimension along the beam axis for the source emitting zero-rapidity particles is determined by the distance one can move before the collective velocity z/t overwhelms the thermal velocity to force the emission function back to zero: $R_l \sim v_{therm}/(dv_{flow}/dz = v_{therm}t \sim \sqrt{T/m_\perp}t$ (for large $m_\perp$) where t is the mean time at which emission occurs. More precise calculations can modify the exact $m_\perp$ dependence and blur the t dependance.
- R_{out} and R_{side} are also expected to fall as $1/\sqrt{m_\perp}$ but this relation can also be modified in a more precisa treatment.
- The difference $R^2_{out}(k_\perp) - R^2_{side}(k_\perp) \sim \beta^2_\perp < \tilde{t}^2 >$ if $< \tilde{x}^2 > \sim < \tilde{y}^2 >$ and $\beta_\perp < \tilde{t} >$ is smaller than the other terms. Alternatively one can look for $R_{out}/R_{side} > 1$. $< \tilde{t}^2 >$ measures the spread of emission times. If it is small, emission has to be sudden. If it is sizable, this could indicate a first order phase transition (slower cooling cf. lecture 8).

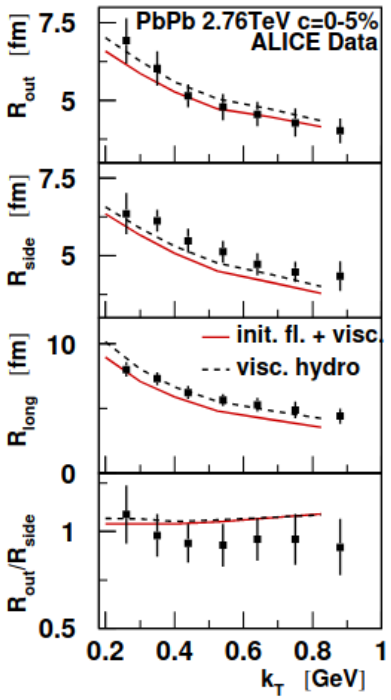
Data at RHIC top energy



- Note the decrease with $k_\perp$ as expected and $R_{out}/R_{side} \sim 1$
- It came as quite a surprise that hydro models (see figure above) that describe very well transverse momentum spectra and anisotropic flow failed to describe HBT data at RHIC
- After various improvements, the HBT puzzle seems to be resolved

S.Pratt Phys.Rev.Lett.102 (2009) 232301 arXiv:0811.3363, W. Broniowski, M.Chojnacki, W. Florkowski, A. Kisiel Phys.Rev.Lett.101 (2008) 022301 arXiv:0801.4361, P.Bozek Phys. Rev C85 (2011) 034901

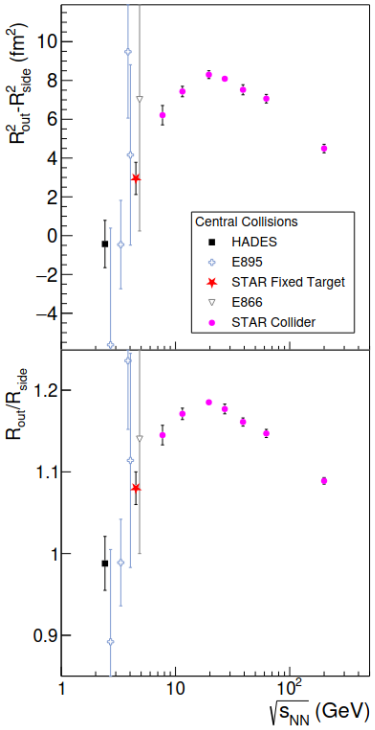
Data at the LHC



Data can be reproduced by hydro

P.Bozek J. Phys. G 38 (2011) 124043 arXiv:1106.5953

Data at RHIC Beam Energy Scan



STAR claims possible evidence of a first order phase transition around $\sqrt{s} = 20$ GeV (cf. peak structure)

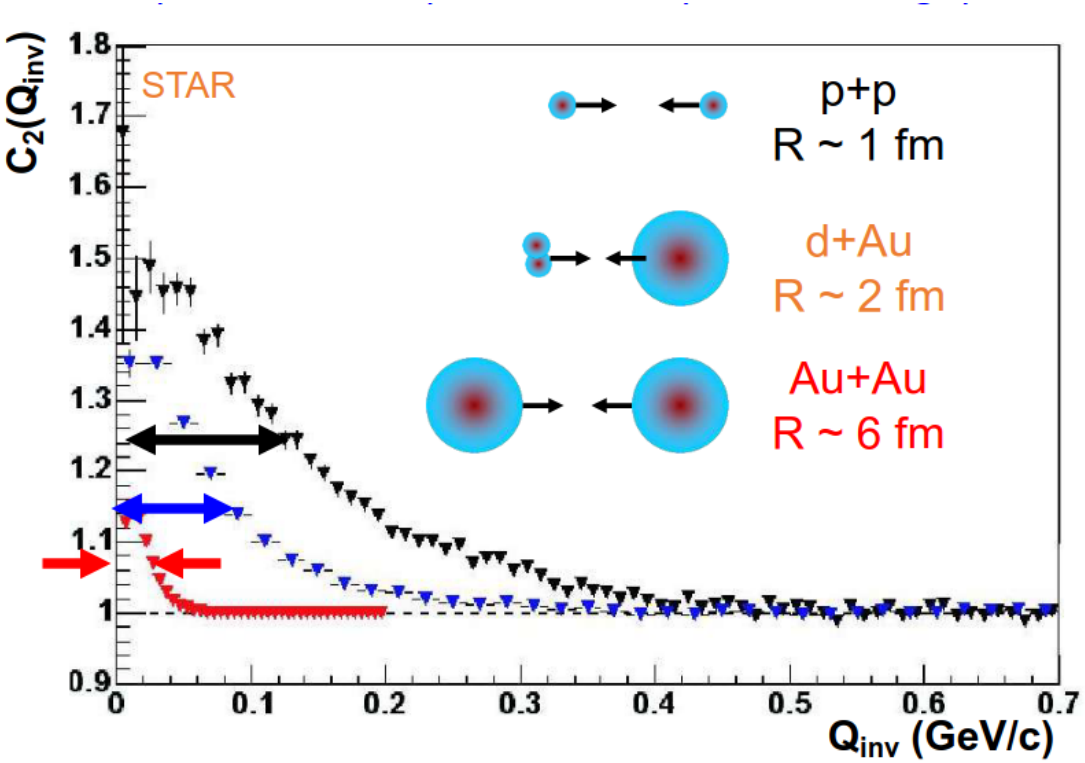
STAR Phys. Rev. C 103 (2021) 034908 arXiv:2007.14005

Challenge

Suppose that $\rho(x) = exp\left(-\frac{|\vec{x}|^2}{2r_0^2} - \frac{t^2}{2\tau_0}\right)$. Compute $C_2(\vec{k}, \vec{q})$

Homework

You are given the results below. Use a fit $C_2(Q_{inv}) = 1 + \lambda\ exp(-R^2 Q_{inv}^2)$ and extract the value of R in fm.



Other references on this topic

- W. Florkowsi “Phenomelogy of Ultra-Relativistic Heavy-Ion Collisions” World Scientific 2010
- B. Tomasik and U.A. Wiedermann “Quark Gluon Plasma 3” World Scientific 2004
- P.F.Kolb and U.Heinz “Quark Gluon Plasma 3” World Scientific 2004
- P.Huovinen and P.V.Ruuskanen “Hydrodynamics Models for Heavy Ion Collisions” Ann.Rev.Nucl. Particle Science 56 (2006) 1056
- M.Lisa, S. Pratt, R.Soltz, U.A. Wiedemann “Femtoscopy in Relativistic Heavy Ion Collisions: Two Decades of Progress” Annu. Rev. Nucl. Part. Sci. 55 (2005) 357 arXiv:nucl-ex/0505014
- U.A. Wiedermann and U.Heinz “Particle Interferometry for Relativistic Heavy-Ion Collisions” Phys.Rept.319 (1999) 145 nucl-th/9901094
- S.S.Padula “HBT Interferometry: Historical Perspective” Braz.J.Phys.35 (2005) 70

- W. Florkowski “The early thermalization and HBT puzzles at RHIC” Acta Phys. Polon. B41 (2010) 2747 arXiv:nucl-th/1009.5459
- S.Pratt `https://www.hken.phys.nagoya-u.ac.jp/hip/Pages/pdf_files/hip5_pratt.pdf` (HBT puzzle)