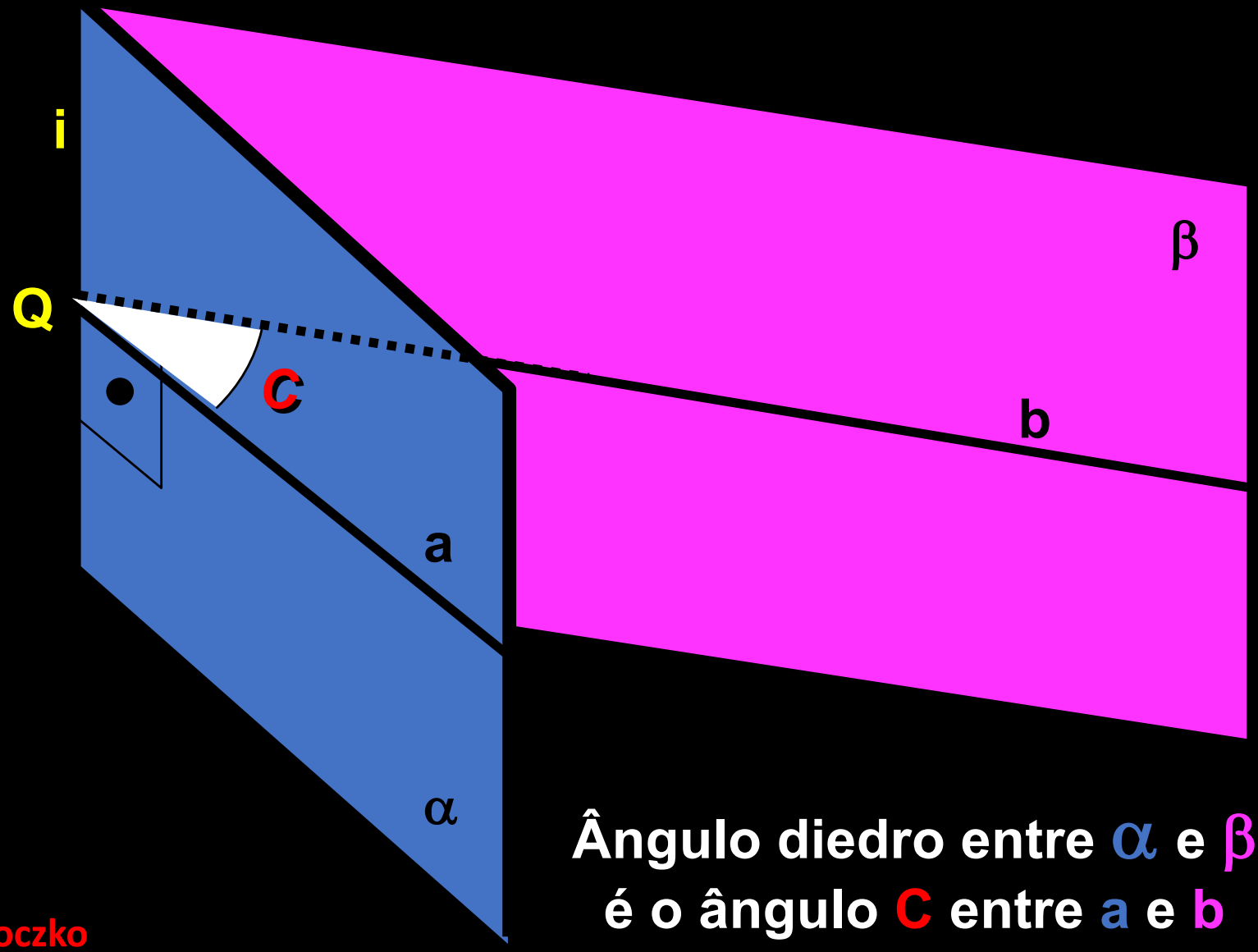
Astronomia de Posição
2º semestre - 2022
Aula_11 – 17/10/2022

Noções de trigonometria esférica

Ramachrisna Teixeira
IAG-USP
rama.teixeira@iag.usp.br

Ângulo Diedro



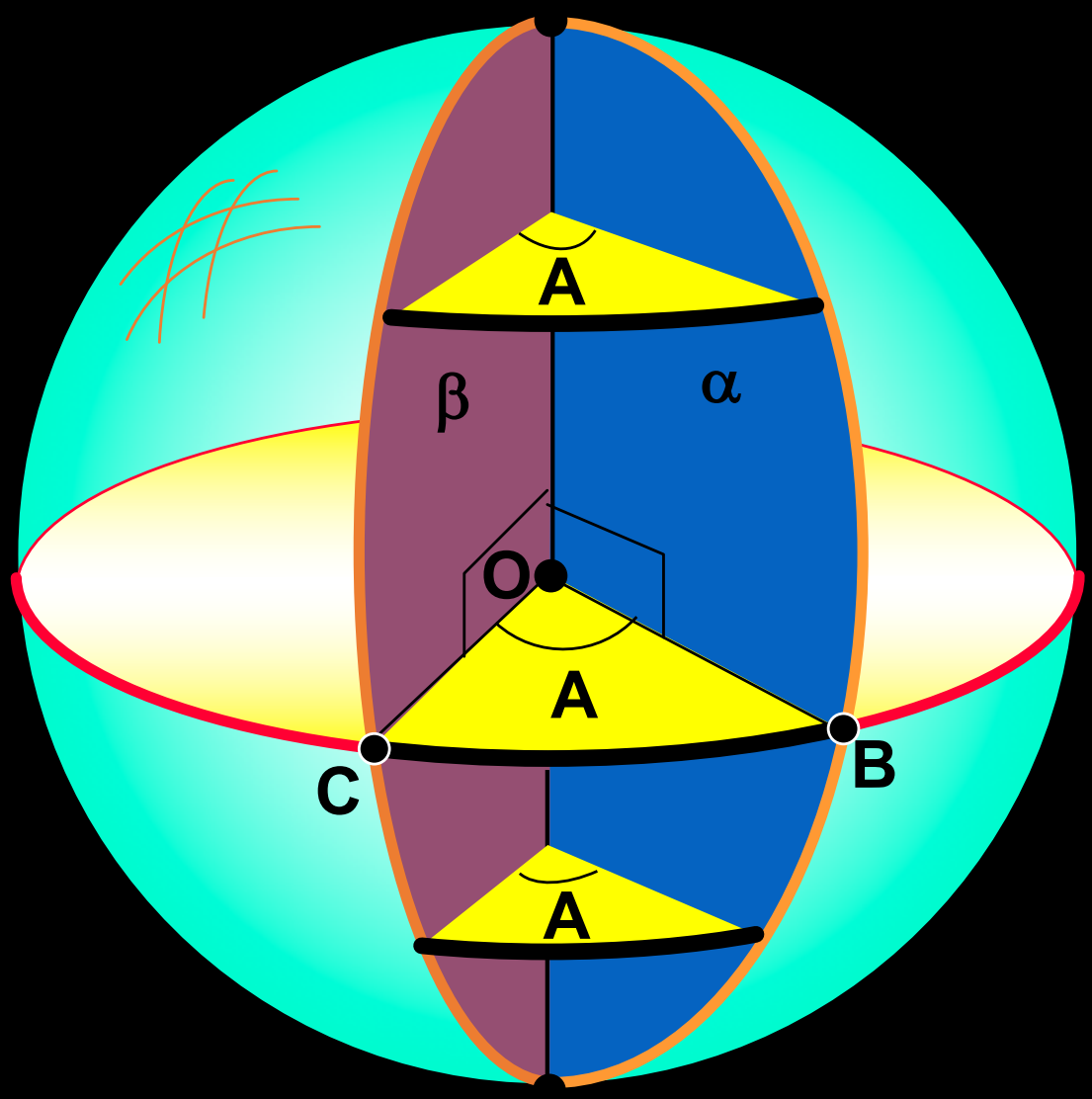
$a \in \alpha$
$b \in \beta$

$a \perp i$
$b \perp i$

Ângulo diedro entre α e β
é o ângulo C entre a e b

Adaptado de R. Boczko

Ângulo Diedro



Adaptado de R. Boczko

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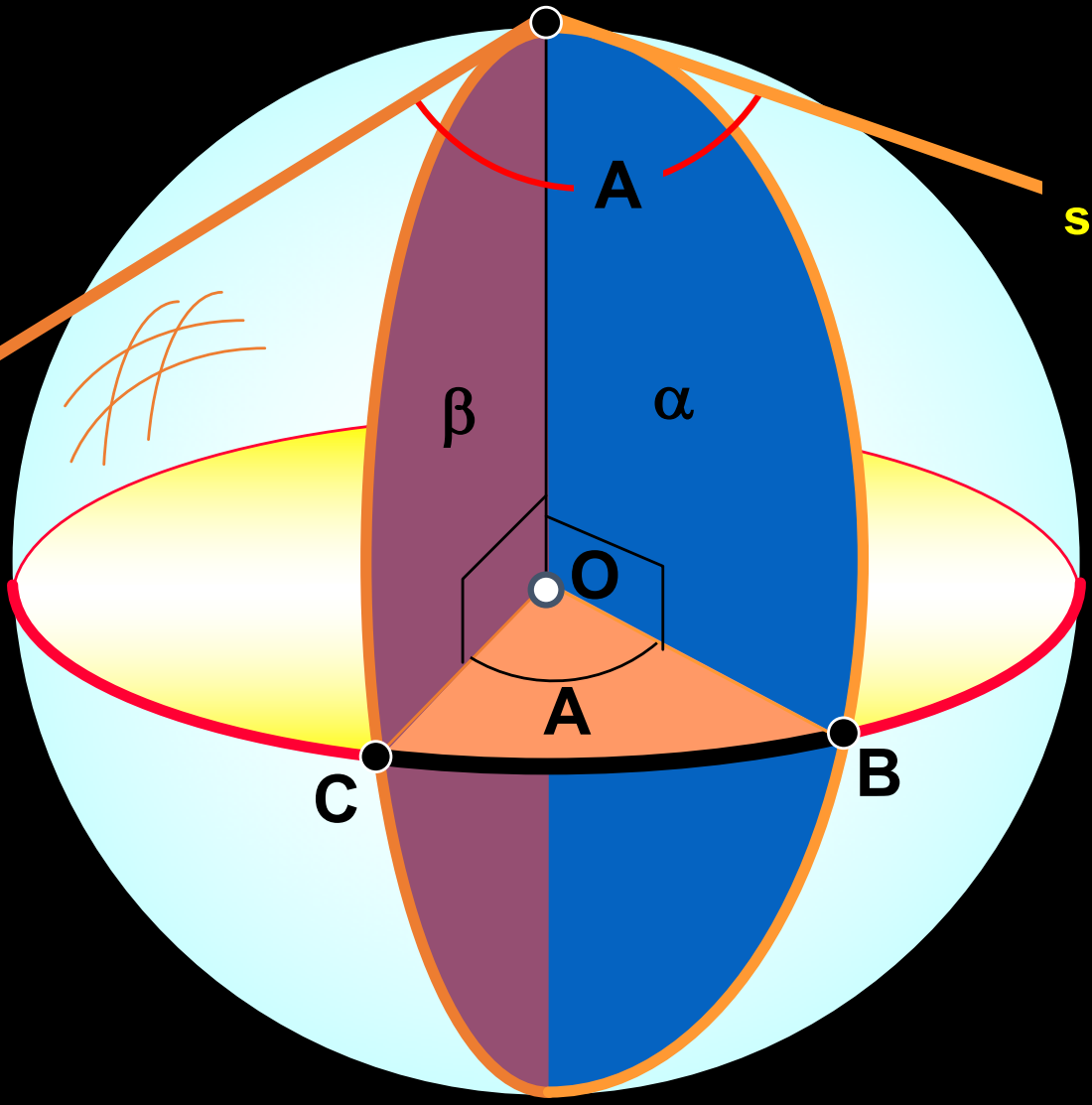


s // AB

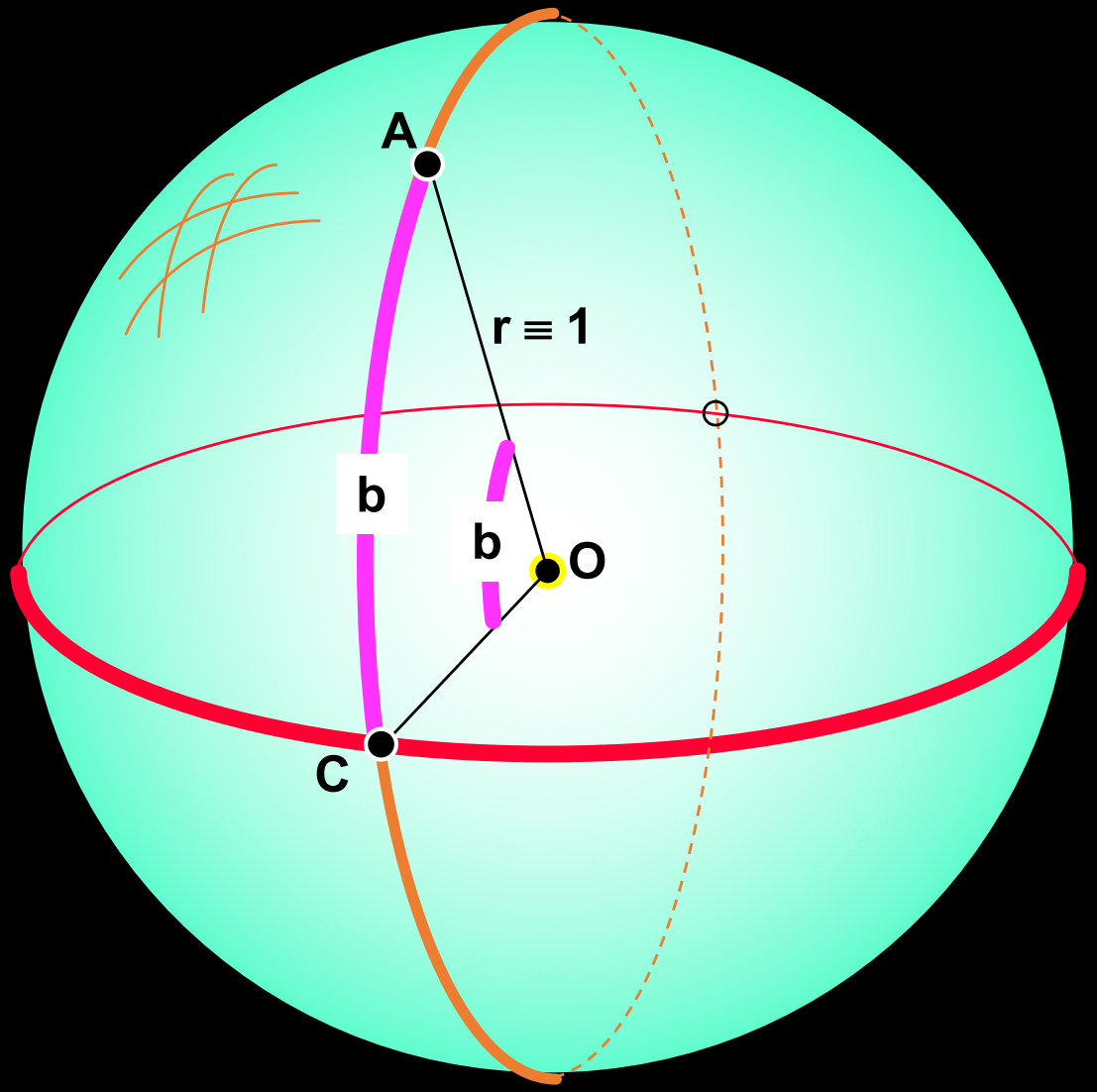
Tangente, por A, ao arco AB. Será paralela ao raio OB.

r // AC

Tangente, por A, ao arco AC. Será paralela ao raio OC.

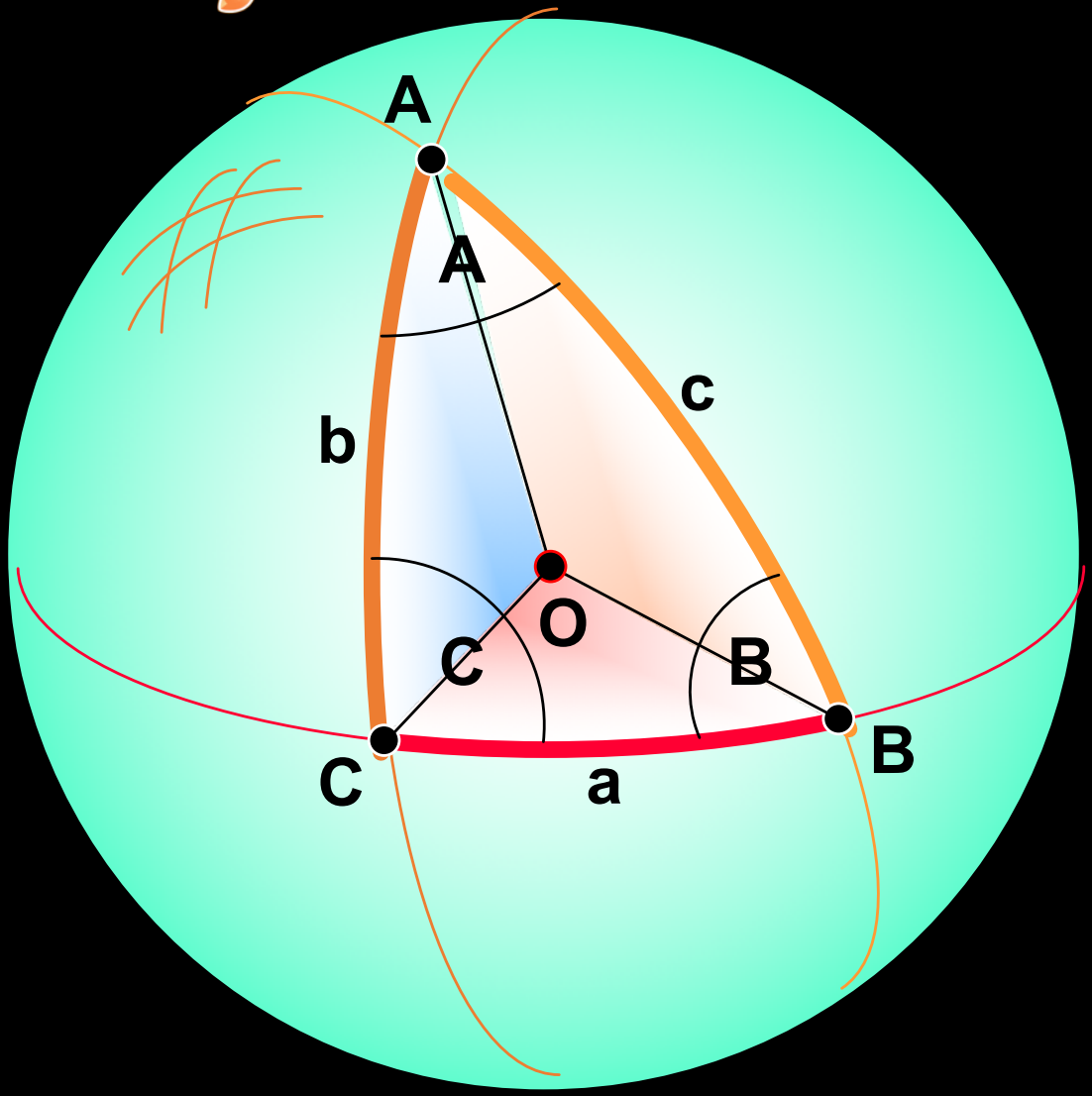
Adaptado de R. Boczko

Ângulo central



Adaptado de R. Boczko

Triângulo esférico



Adaptado de R. Boczko

Formado pela intersecção, dois a dois, de 3 grandes círculos

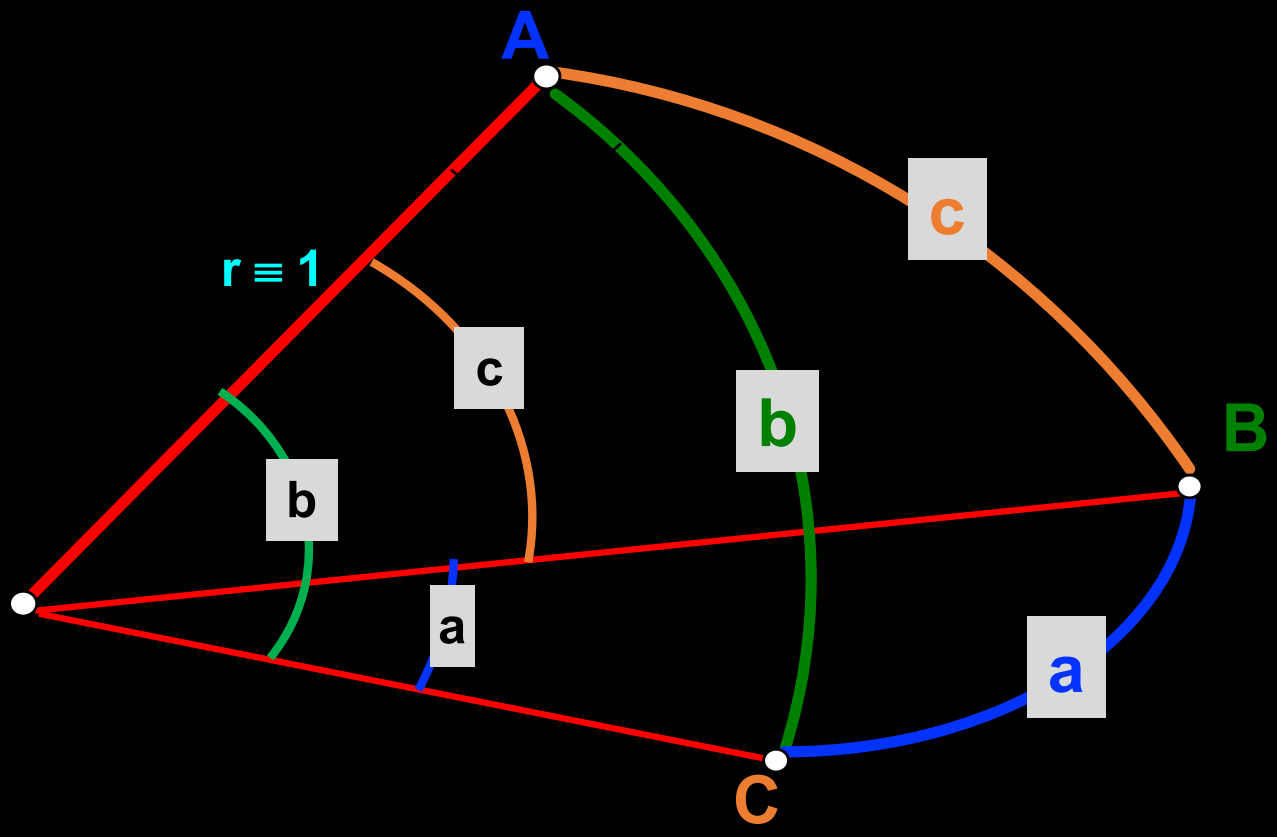
A, B, C = vértices

A, B, C = ângulos diedro

a, b, c = lados

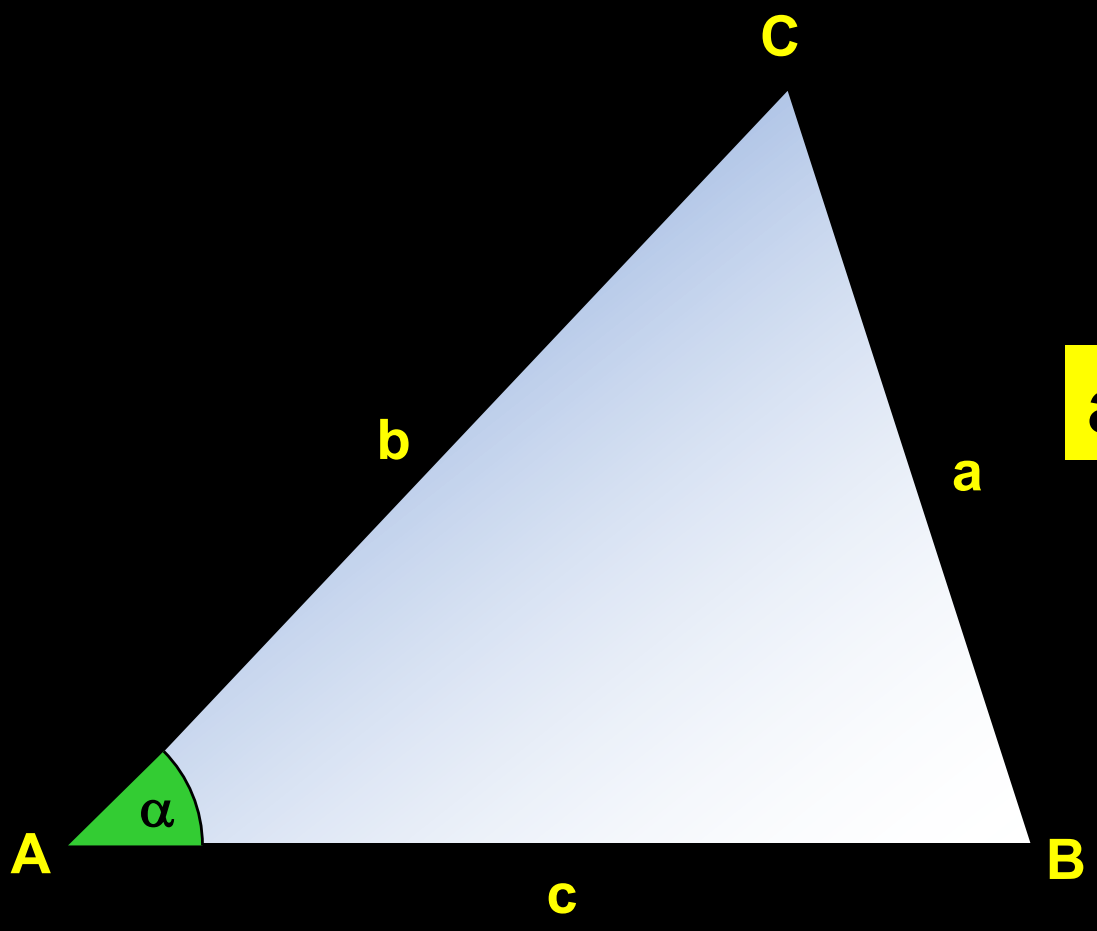
- **soma de dois lados é sempre maior que o terceiro.**
- **soma dos ângulos não é fixa e resulta sempre maior que 180º.**
- **pode ter um, dois ou mesmo, três ângulos retos.**

Lados do triângulo esférico



Adaptado de R. Boczko

Relações lados e ângulos de um triângulo plano



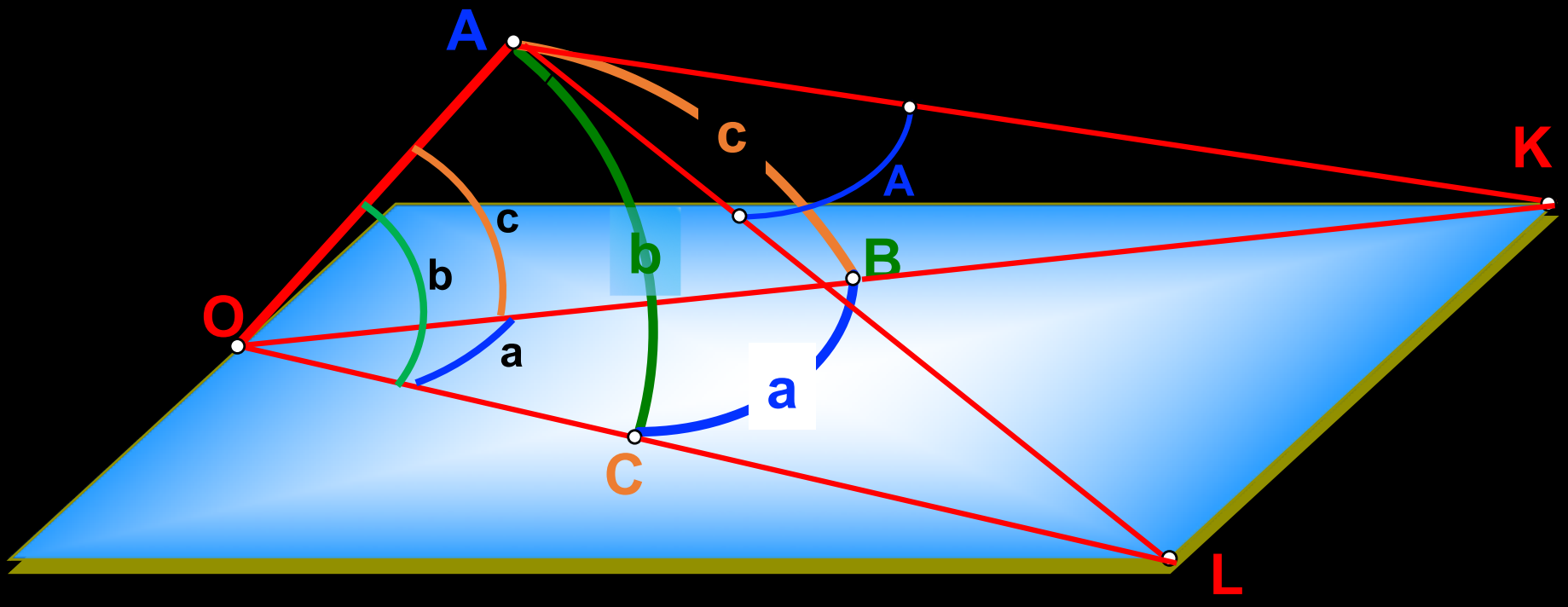
Fórmula do cosseno em um triângulo plano

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

R. Boczko

Fórmula do cosseno em um triângulo esférico

Relações lados e ângulos de um triângulo plano



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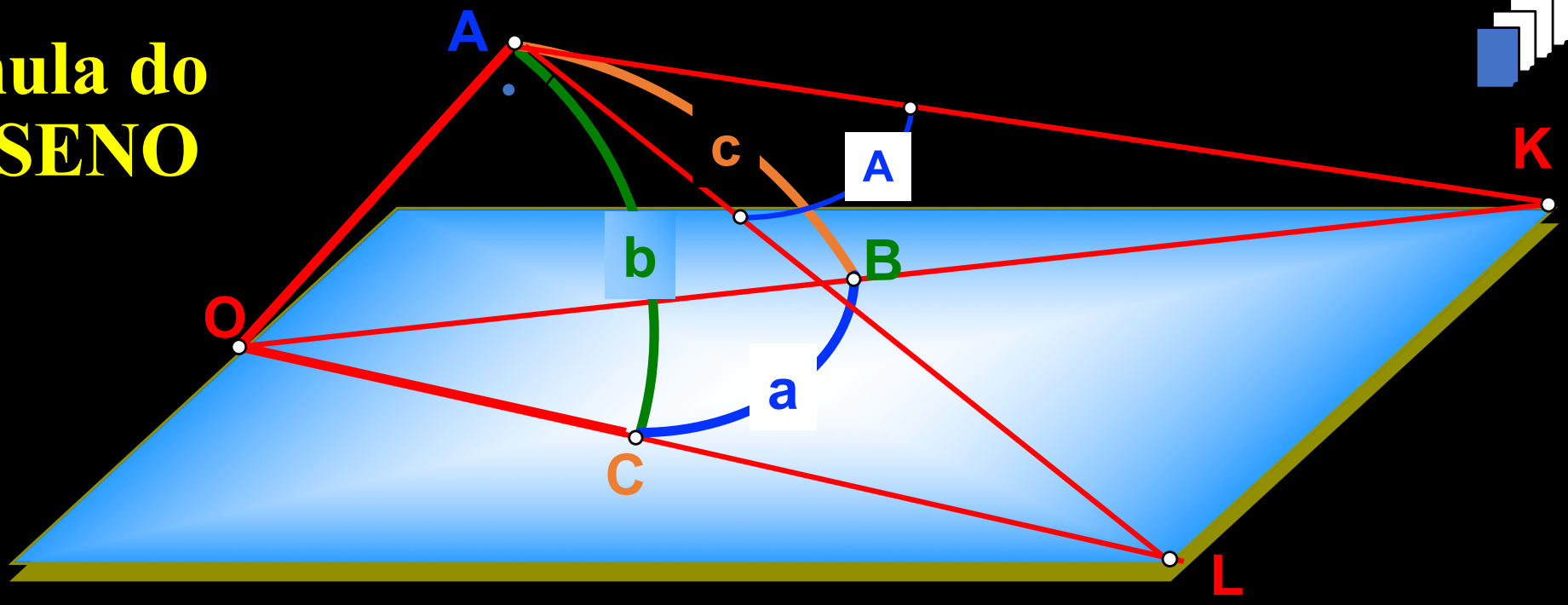
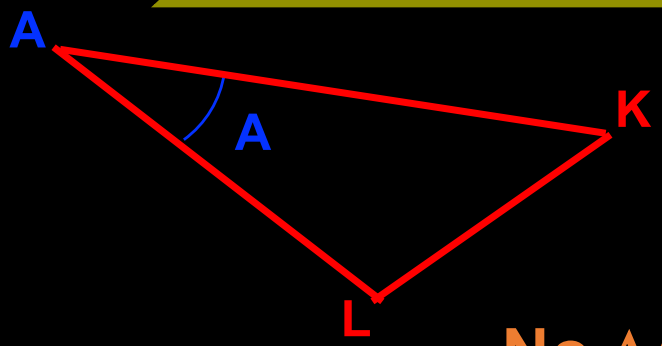
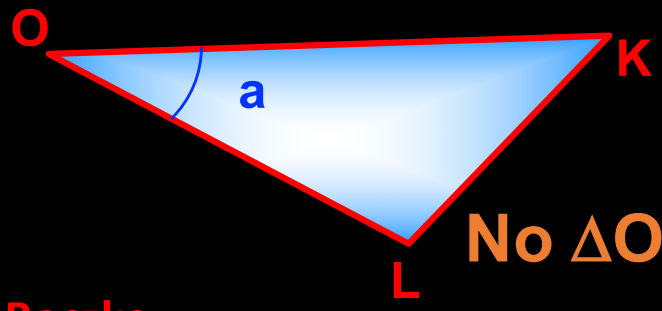
Fórmula do COSSENO





$\Delta_{Plano} \Rightarrow a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$

No ΔAKL: $KL^2 = KA^2 + LA^2 - 2.KA \cdot LA \cdot \cos A$



No ΔOKL: $KL^2 = KO^2 + LO^2 - 2.KO \cdot LO \cdot \cos a$

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Fórmula do COSSENO

No ΔAKL: $KL^2 = KA^2 + LA^2 - 2.KA \,.\, LA \,.\, \cos A$ ①

No ΔOKL: $KL^2 = KO^2 + LO^2 - 2.KO \,.\, LO \,.\, \cos a$ ②

① = ②

$$KO^2 + LO^2 - 2.KO \,.\, LO \,.\, \cos a = KA^2 + LA^2 - 2.KA \,.\, LA \,.\, \cos A$$

$$- 2.KO \,.\, LO \,.\, \cos a = KA^2 - KO^2 + LA^2 - LO^2 - 2.KA \,.\, LA \,.\, \cos A$$

$$2.KO \,.\, LO \,.\, \cos a = - KA^2 + KO^2 - LA^2 + LO^2 + 2.KA \,.\, LA \,.\, \cos A$$

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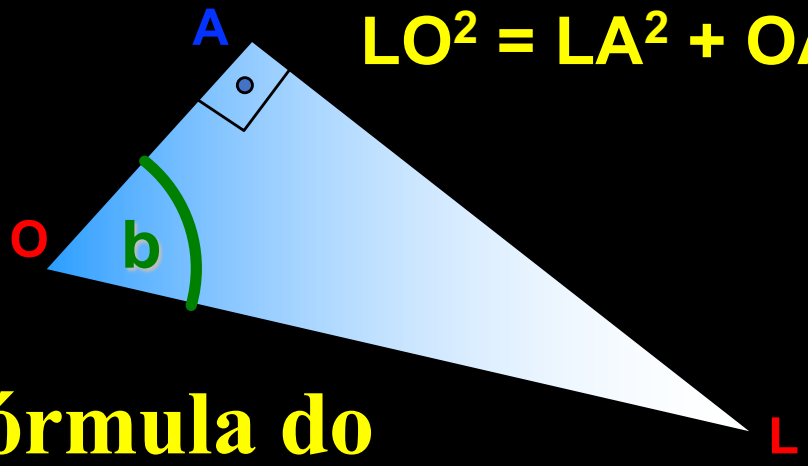
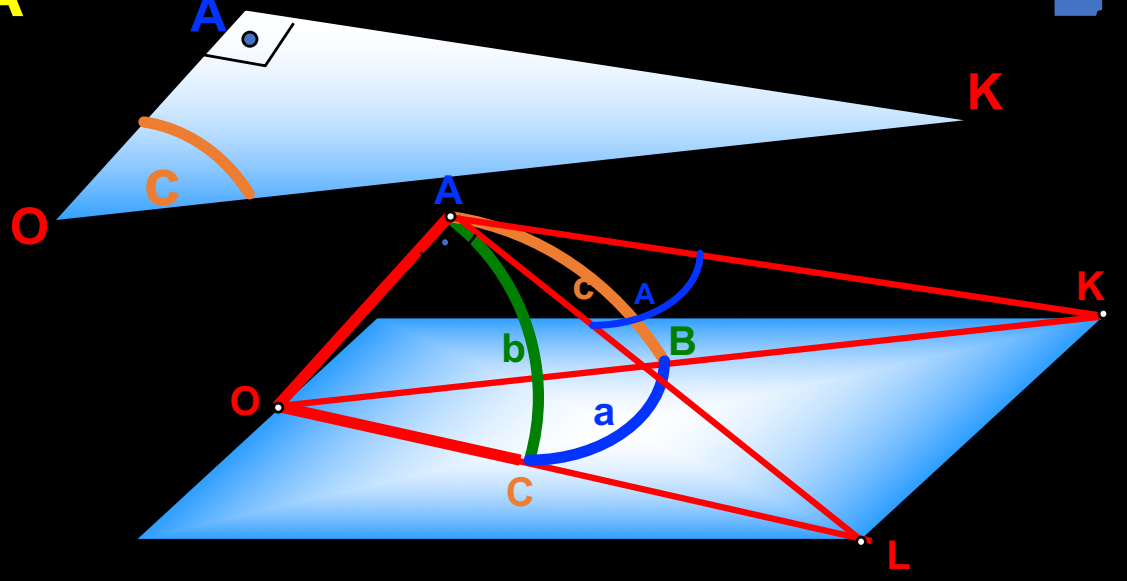
Fórmula do COSSENO



$LO^2 = LA^2 + OA^2$

$KO^2 = KA^2 + OA^2$



$$2.KO \,.\, LO \,.\, \cos a = - KA^2 + KO^2 - LA^2 + LO^2 + 2.KA \,.\, LA \,.\, \cos A$$

$$2.KO \,.\, LO \,.\, \cos a = OA^2 + OA^2 + 2.KA \,.\, LA \,.\, \cos A$$

$$2.KO \,.\, LO \,.\, \cos a = 2.OA^2 + 2.KA \,.\, LA \,.\, \cos A$$

$$KO \,.\, LO \,.\, \cos a = OA^2 + KA \,.\, LA \,.\, \cos A$$

$$KO \,.\, LO \,.\, \cos a = OA \,.\, OA + KA \,.\, LA \,.\, \cos A$$

$$\cos a = \frac{OA \,.\, OA}{LO \,.\, KO} + \frac{KA \,.\, LA}{LO \,.\, KO} \,.\, \cos A$$

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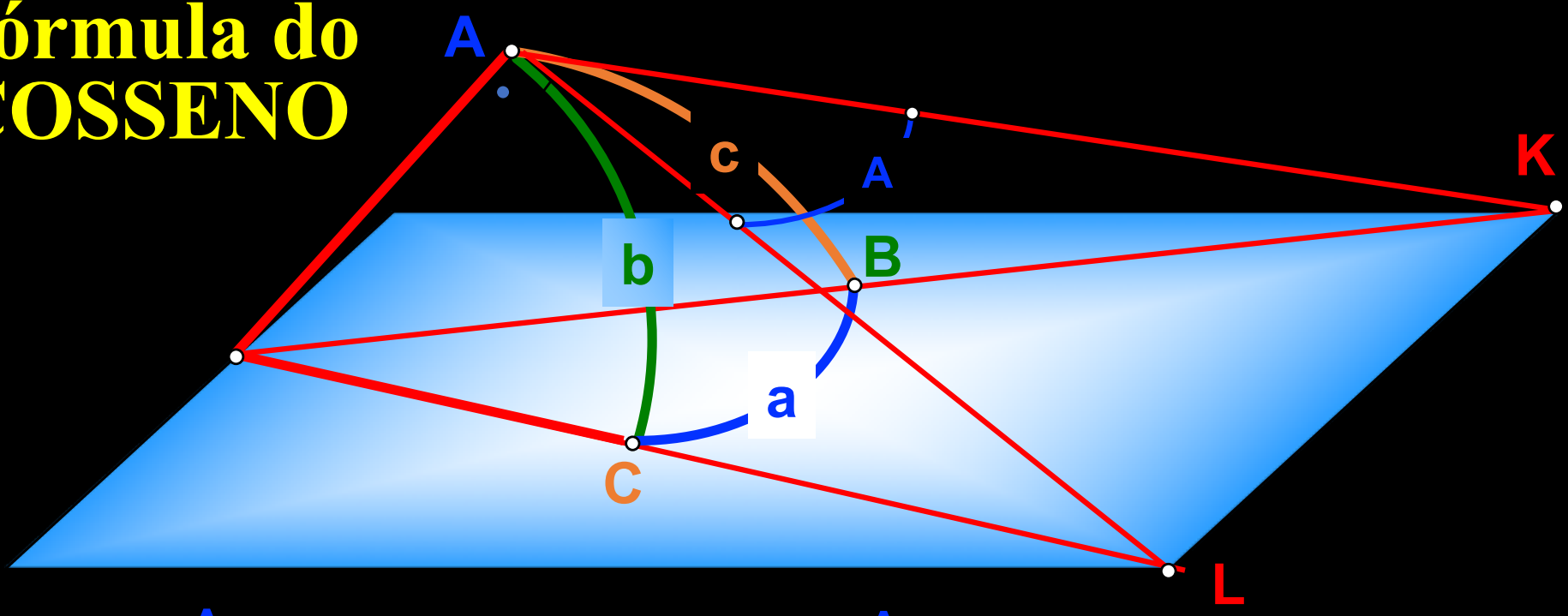
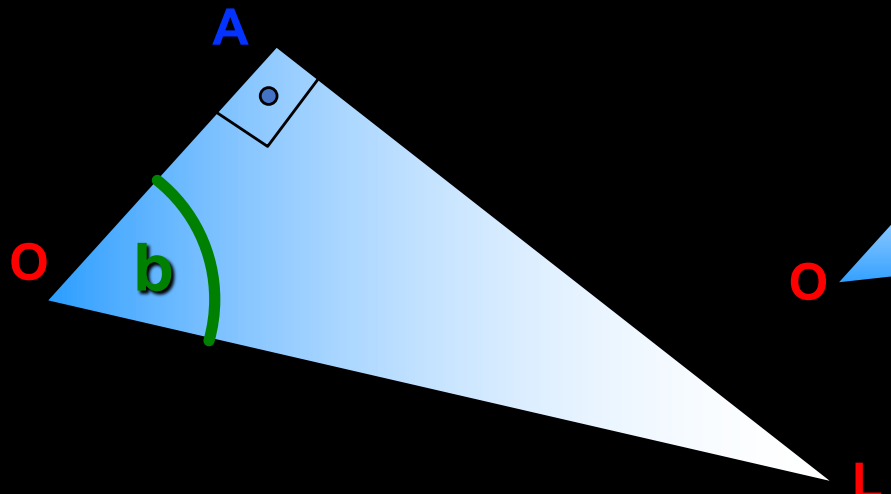
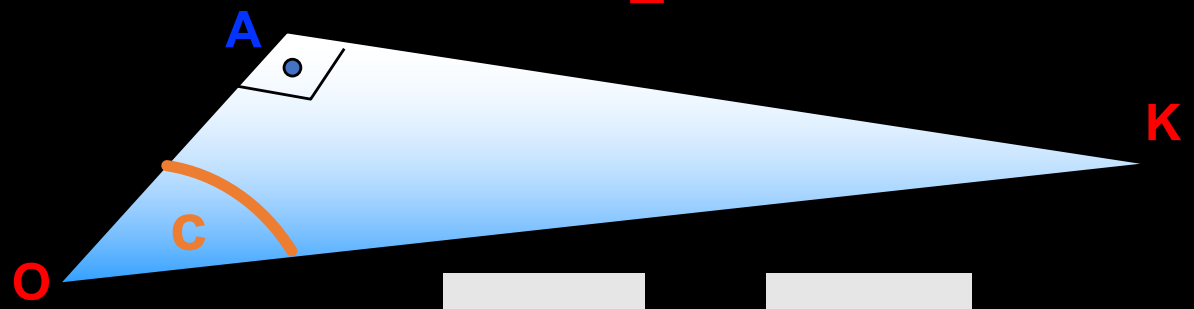
Fórmula do COSSENO







$\cos c$ $\quad$ $\sin c$

$$\cos a = \frac{OA \,.\, OA}{LO \,.\, KO} + \frac{LA \,.\, KA}{LO \,.\, KO} \,.\, \cos A$$

$\cos b$ $\quad$ $\sin b$

$$\cos a = \cos b \,.\, \cos c + \sin b \,.\, \sin c \,.\, \cos A$$

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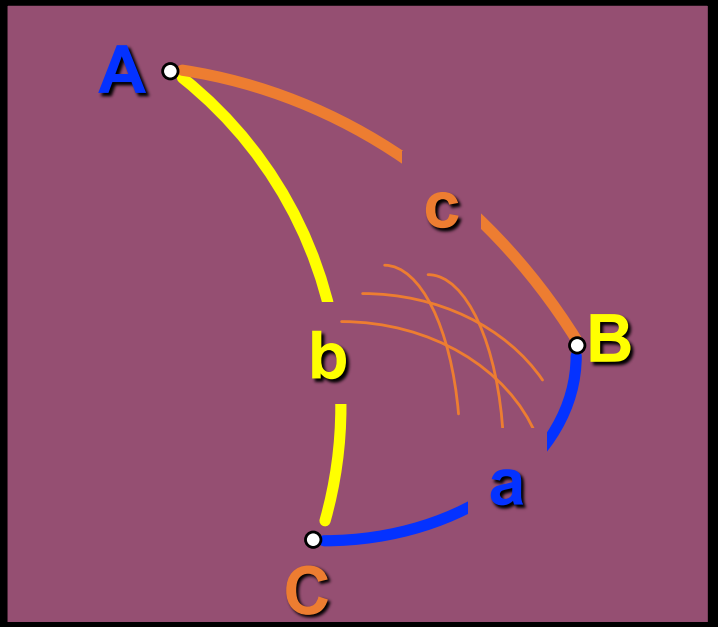
Fórmula do COSSENO



$$\cos a = \cos b \cdot \cos c + \operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A$$

$$\cos b = \cos a \cdot \cos c + \operatorname{sen} a \cdot \operatorname{sen} c \cdot \cos B$$

$$\cos c = \cos b \cdot \cos a + \operatorname{sen} b \cdot \operatorname{sen} a \cdot \cos C$$

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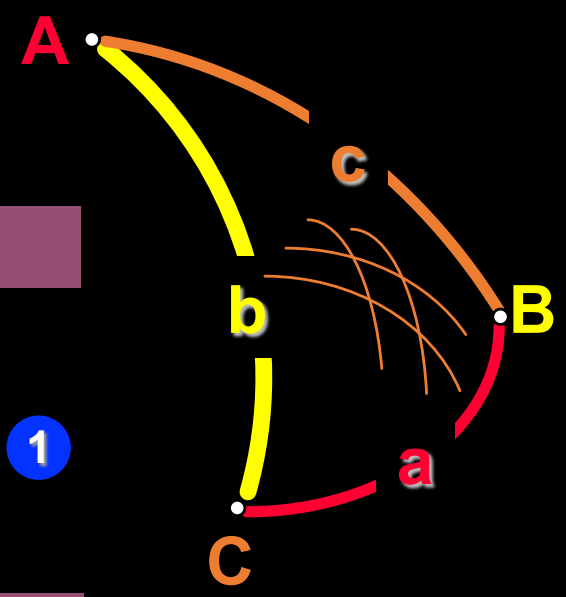
Fórmula do seno em um triângulo esférico

Fórmula do SENO



$$\cos a = \cos b \cdot \cos c + \operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A$$

$$-\operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A = \cos b \cdot \cos c - \cos a$$

$$(-\operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A)^2 = (\cos b \cdot \cos c - \cos a)^2 \quad (1)$$

$$\cos b = \cos a \cdot \cos c + \operatorname{sen} a \cdot \operatorname{sen} c \cdot \cos B$$

$$-\operatorname{sen} a \cdot \operatorname{sen} c \cdot \cos B = \cos a \cdot \cos c - \cos b$$

$$(-\operatorname{sen} a \cdot \operatorname{sen} c \cdot \cos B)^2 = (\cos a \cdot \cos c - \cos b)^2 \quad (2)$$

(1) – (2)

Substituir: $\cos^2 x = 1 - \operatorname{sen}^2 x$

$$\operatorname{sen} a \,/\, \operatorname{sen} A = \operatorname{sen} b \,/\, \operatorname{sen} B = \operatorname{sen} c \,/\, \operatorname{sen} C$$

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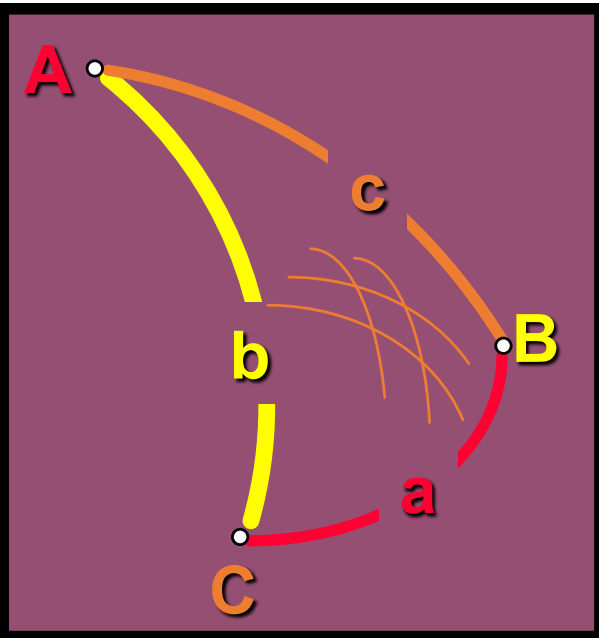
Fórmula do seno e cosseno em um triângulo esférico

Fórmula do SENO e COSSENO



$$\cos a = (\cos b \cdot \cos c + \operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A) \quad ①$$

$$\cos b = (\cos a) \cdot \cos c + \operatorname{sen} a \cdot \operatorname{sen} c \cdot \cos B \quad ②$$

① → ②

Substituir: $\cos^2 c = 1 - \operatorname{sen}^2 c$

$$\operatorname{sen} a \cdot \cos B = \cos b \cdot \operatorname{sen} c - \operatorname{sen} b \cdot \cos c \cdot \cos A$$

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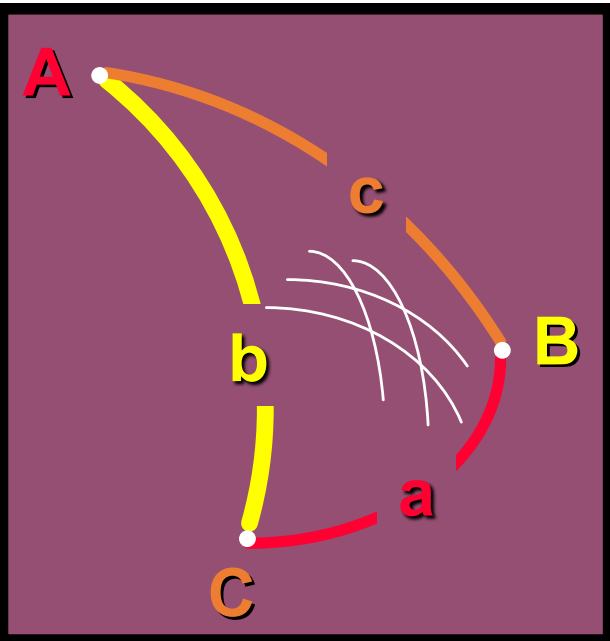
Resumo das Fórmulas de Trigonometria Esférica



Cosseno

$$\cos a = \cos b \cdot \cos c + \operatorname{sen} b \cdot \operatorname{sen} c \cdot \cos A$$

Seno

$$\operatorname{sen} a / \operatorname{sen} A = \operatorname{sen} b / \operatorname{sen} B = \operatorname{sen} c / \operatorname{sen} C$$

Seno e Cosseno

$$\operatorname{sen} a \cdot \cos B = \cos b \cdot \operatorname{sen} c - \operatorname{sen} b \cdot \cos c \cdot \cos A$$

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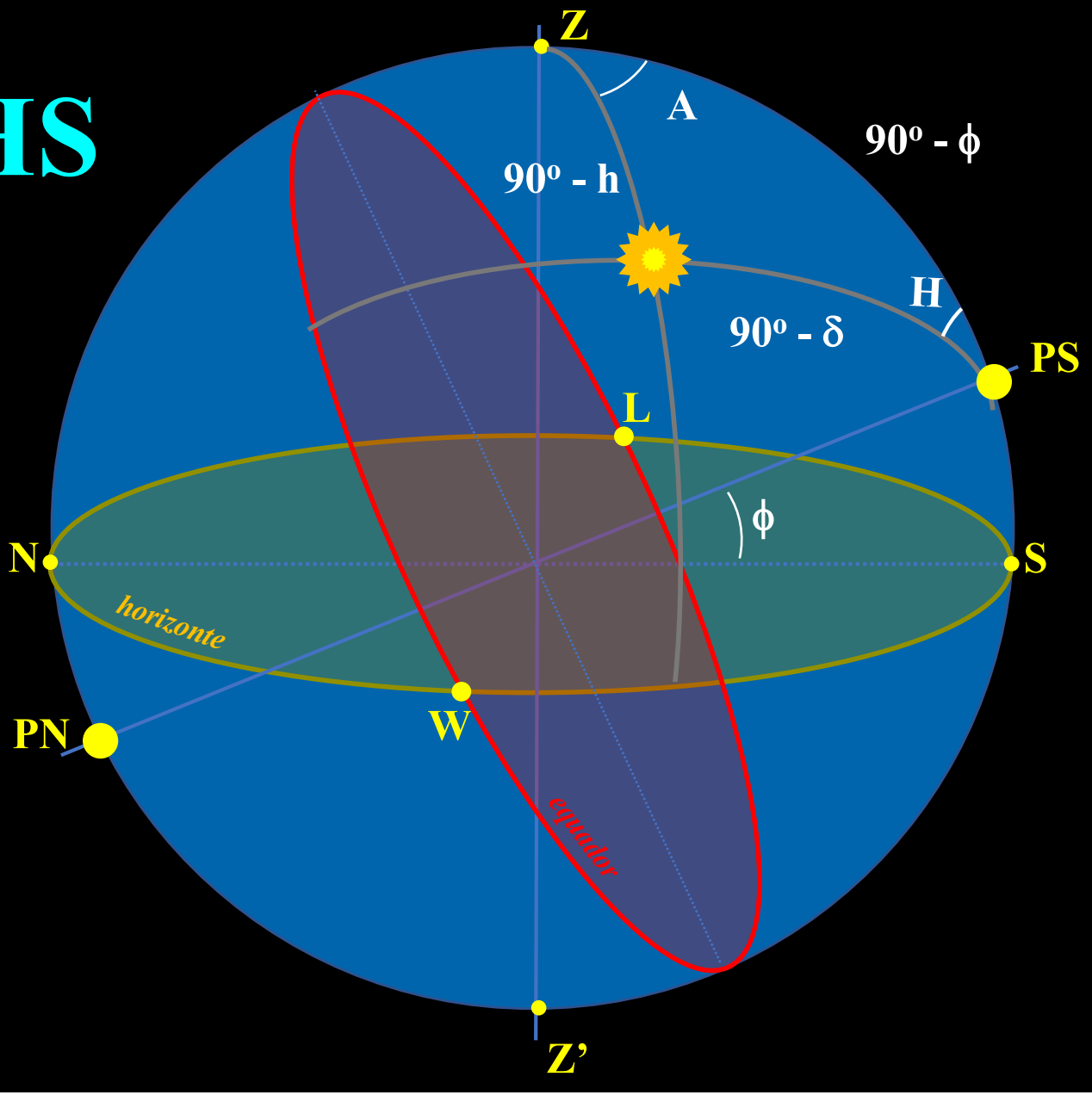
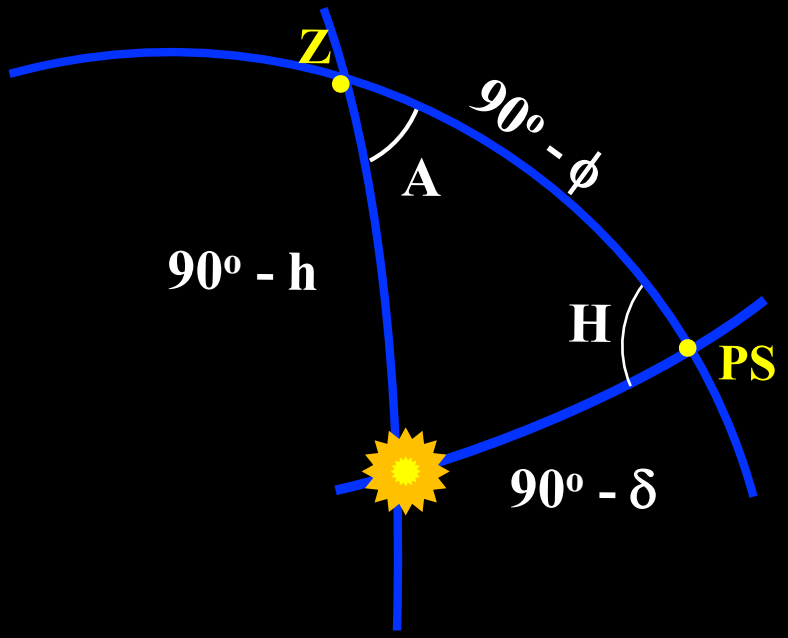
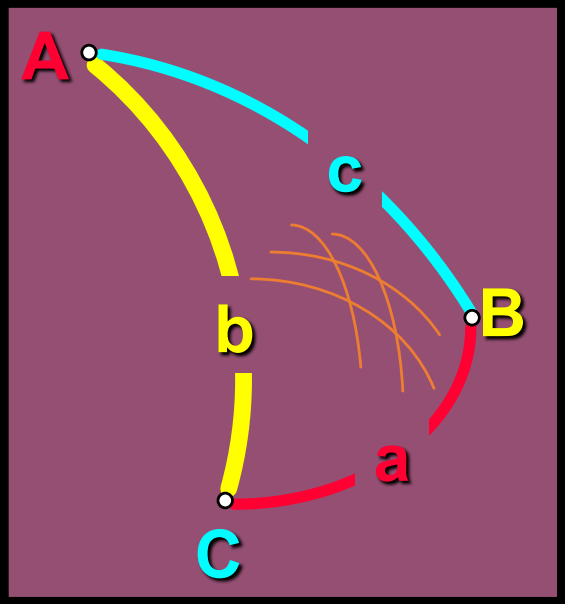
Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.

VERÃO - HS



Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.



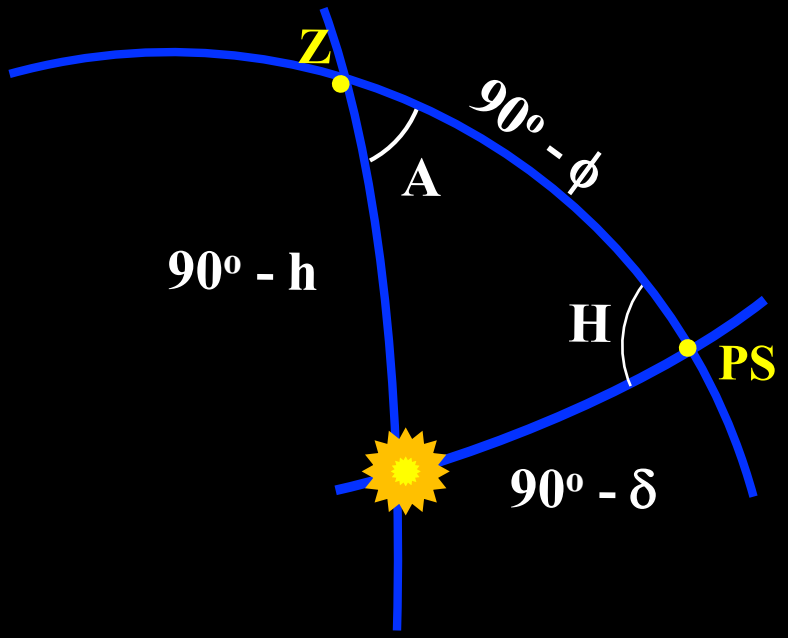


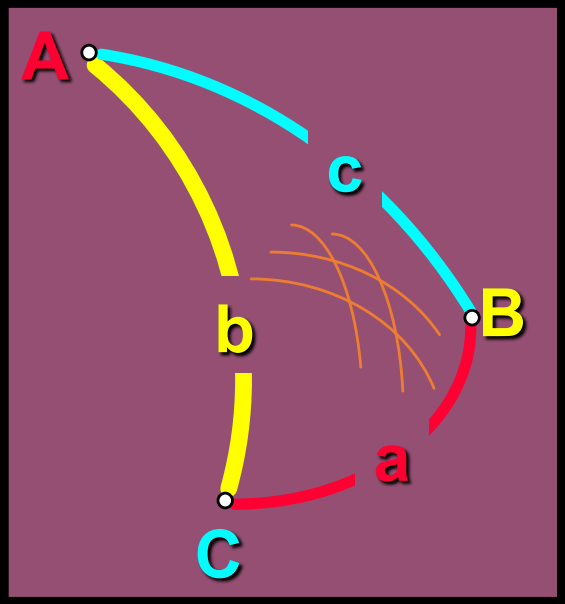
$$\cos a = \cos b \,.\, \cos c + \operatorname{sen} b \,.\, \operatorname{sen} c \,.\, \cos A$$

$$\cos(90^o-\delta) = \cos(90^o-h).\cos(90^o-\phi) + \operatorname{sen}(90^o-h).\operatorname{sen}(90^o-\phi).\cos(A)$$

$$\operatorname{sen}(\delta) = \operatorname{sen}(h).\operatorname{sen}(\phi) + \cos(h).\cos(\phi).\cos(A)$$

Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.





$$sen(\delta) = sen(h).sen(\phi) + cos(h).cos(\phi).cos(A)$$

nascer/ocaso ⟹ $h = 0^o$

$$sen(\delta) = cos(\phi).cos(A)$$

Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.

$$sen(\delta) = cos(\phi).cos(A)$$

$$cos(A) = \frac{sen(\delta)}{cos(\phi)}$$

equinócios ➡ $\delta = 0^o$

$$cos(A) = 0 \begin{cases} \rightarrow A_o = 90^o \\ \rightarrow A_n = 270^o \end{cases}$$

Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.

$$sen(\delta) = cos(\phi).cos(A)$$

$$cos(A) = \frac{sen(\delta)}{cos(\phi)}$$

solstício verão ➡ $\delta = 23{,}5^o$ *figura já organizada com o Sol no hemisfério sul*

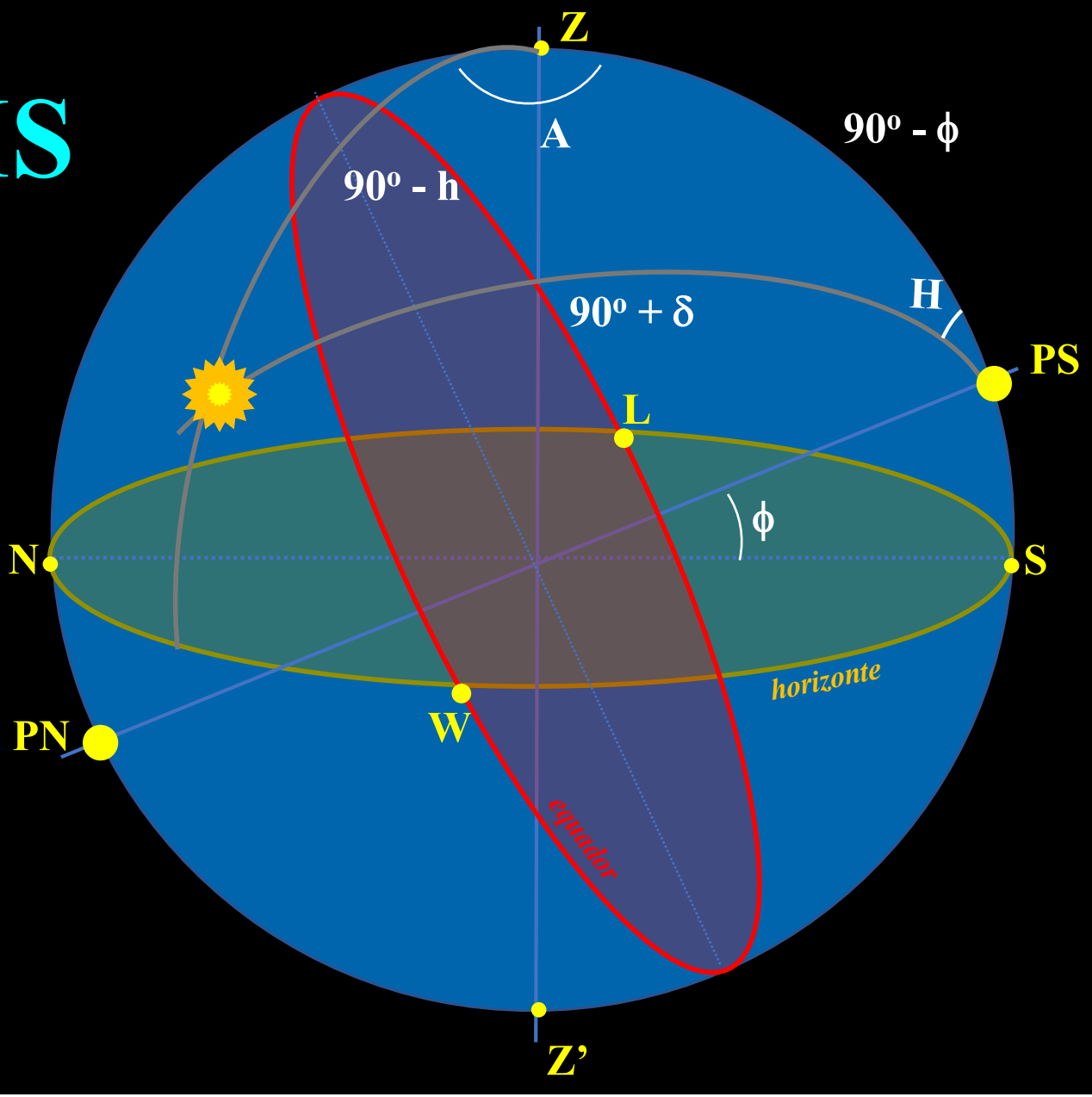
$$cos(A) = +tg(23{,}5^o)$$

➡ $A_n = 295{,}8^o$

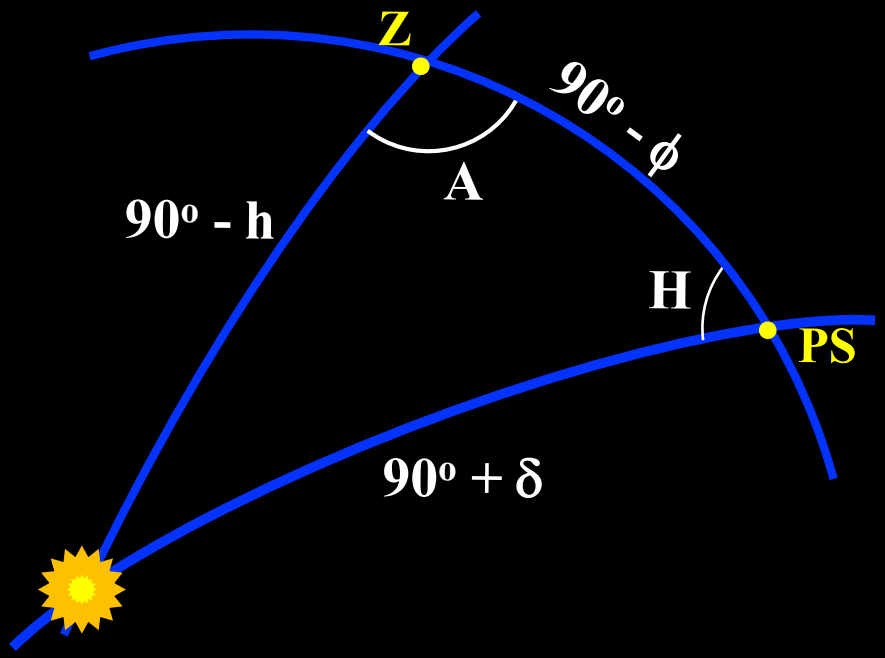
➡ $A_o = 64{,}2^o$

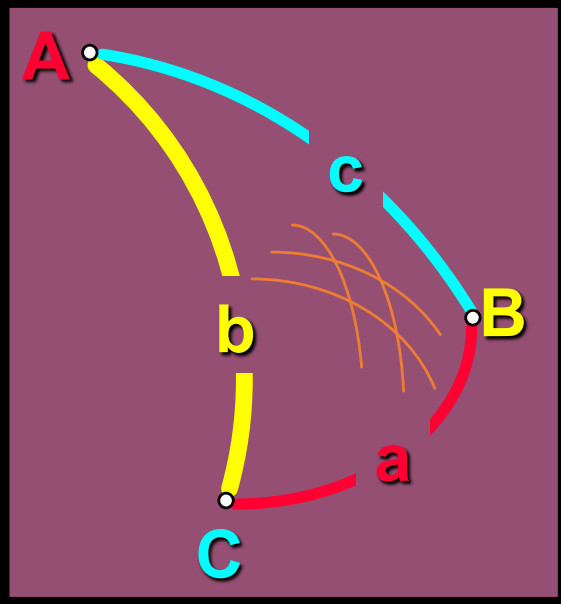
Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.

Inverno - HS



Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.





$$\cos a = \cos b \,.\, \cos c + \operatorname{sen} b \,.\, \operatorname{sen} c \,.\, \cos A$$

$$\cos(90^o+\delta) = \cos(90^o-h).\cos(90^o-\phi) + \operatorname{sen}(90^o-h).\operatorname{sen}(90^o-\phi).\cos(A)$$

$$-\operatorname{sen}(\delta) = \operatorname{sen}(h).\operatorname{sen}(\phi) + \cos(h).\cos(\phi).\cos(A)$$

Exemplo 1 - obter azimute do nascer e de ocaso do Sol nos equinócios e solstícios de um observador em São Paulo.

$$sen(\delta) = -\cos(\phi).\cos(A)$$

$$\cos(A) = \frac{-sen(\delta)}{\cos(\phi)}$$

solstício inverno ➡ $\delta = 23{,}5^o$ *figura já organizada com o Sol no hemisfério norte*

$$\cos(A) = -tg(23{,}5^o)$$

➡ $A_n = 244{,}2^o$

➡ $A_o = 115{,}8^o$

FIM