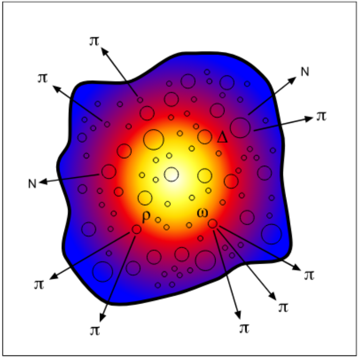
Lecture 14
Freeze out and Particlization (part II)



Abundances

The Cooper-Frye for hadron species i leads to:

$N_i = \int \frac{d^3p}{E} \int p^\mu d\sigma_\mu f(x,p)_i = \int d\sigma_\mu n_i^\mu(x)$

where (using $d^3p/2E = d^4p\delta(p^2 - m^2)\theta(E)$)

$$
\begin{aligned}
n_i^\mu(x) &= 2\int d^4p\delta(p^2 - m^2)\theta(E)p^\mu f_i(p\cdot u) \\
&= 2\int d^4p\delta(p^2 - m^2)\theta(E)p^\mu \frac{g_i}{(2\pi)^3}\frac{1}{exp\left(\frac{p\cdot u - \mu_i}{T}\right) \pm 1},
\end{aligned}
$$

which is written in a Lorentz-covariant way.

In this formula: $\mu_i = m_B B_i + \mu_S S_i + \mu_I I_{3i}$,
B_i, S_i I_{3i} are the baryon number, strangeness and third component of isospin of the species.
μ_B, μ_S and μ_I allows to satisfy the conservation rules for baryon number, zero total strangeness and eletric charge (related to I_{3i} through $Q_i = I_{3i} + (B_i + S_i)/2$).

In thermal equilibrium $n_i^\mu(x) = n_i(x)u^\mu(x)$, where, using the expression in previous slide for $n_i^\mu(x)$ and computing in the local rest frame:

$$n_i(x) = u_\mu(x)\, n_i^\mu(x) = 2\int d^4p\delta(p^2 - m^2)\theta(E)p^\mu u_\mu f_i(p \cdot u)$$
$$= \int d^3p' f_i(p'; T(x), \mu_i(x)) = n_i(T(x), \mu_i(x))$$

If T and μ_i are independent of x :
$N_i = \int d^3\sigma_\mu n_i(T(x), \mu_i(x))u^\mu(x) = \mathrm{n}_i(T, \mu_i) \int_{f.out} d\sigma_\mu(x)u^\mu(x)$
Doing ratios, we can get rid of $\int d\sigma_\mu(x)u^\mu(x)$
For exemple at mid-rapidity:

$$\boxed{\frac{dN_i/dy}{dN_j/dy}_{|y=0} = \frac{N_i}{N_j} = \frac{n_i}{n_j}}$$

Exercise:

An experiment measures for Au+Au at 200 GeV the following ratios: $\bar{p}/p = 0.70$ and $\bar{p}/\pi^- = 0.083$. What are the values of T and μ_B at freeze out? (Ignore isospin).

Using the Boltzmann approximation (limit for large argument in K_2, see lecture 5)

$$\frac{\bar{p}}{p} = \frac{n_p}{n_{\bar{p}}} = \frac{e^{-\mu_B/T}}{e^{\mu_B/T}} = e^{-2\mu_B/T} = 0.70 \Rightarrow \mu_B/T = 0.18$$

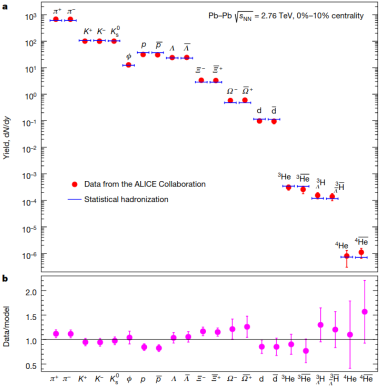
$$\frac{\bar{p}}{\pi^-} = \frac{\frac{2m^2T}{2\pi^2}\sqrt{\frac{\pi T}{2m}}e^{-m/T}e^{-\mu_B/T}}{\frac{m_\pi^2 T}{2\pi^2}\sqrt{\frac{\pi T}{2m_\pi}}e^{-m_\pi/T}} \Rightarrow T \sim 140\,MeV \text{ and } \mu_B \sim 25\,MeV$$

We note 2 important points:

1) μ_B is small so the net baryon density at midrapidity is small

2) $T_{ch\,f.out}$ is close to T_{deconf}.

More complex models lead to precise results for many abundances:

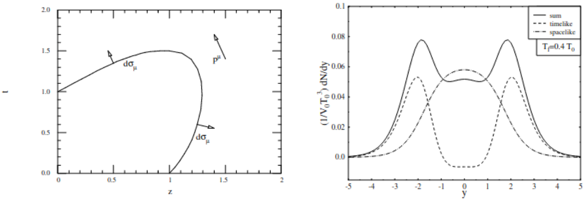


A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel Nature 561 (2018) 321: $T_{ch.f.out} = 156.5 \pm 1.5$ MeV, $\mu_B = 0.7 \pm 3.8$ MeV.

Problems with freeze out

1) Negative contributions in the Cooper-Frye formula

In the Cooper-Frye, $p^\mu d\sigma_\mu$ can be negative. This corresponds to particles that re-enter the fluid and are not counted leading to violation of conservation laws (we remove baryon number, energy and momentum by ignoring these particles)



Figures FG.

How large is the effect is a matter of debate. In SPheRIO, it could be a 20% effect. It depends on the shape of the freeze out surface. Other authors D. Teaney, J. Lauret, E. V. Shuryak nucl-th/0110037 find a smaller effect.

2) Dissipative contributions to f in the Cooper-Frye formula

We write: $f(x,p) = f_{eq}(x,p) + \delta f_{shear} + \delta f_{bulk} + \delta f_{diff}$

There exist various ways to estimate these δf's and they have an influence on observables.

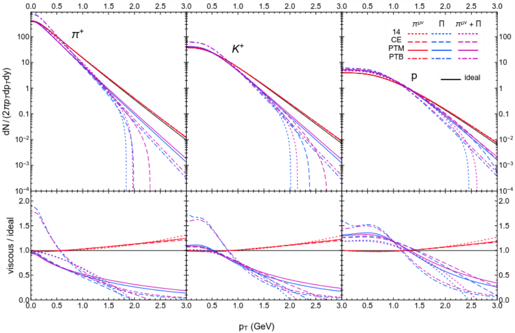
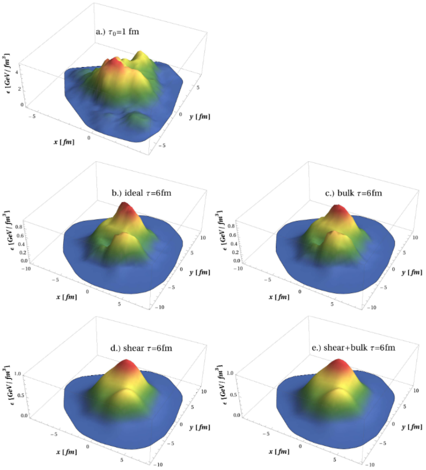


FIG. 6. Same as Fig. 5 but with a $\zeta/\mathcal{S}$ peak temperature of $T_\zeta = 160$ MeV.

M. McNelis and U. Heinz arXiv:2103.03401. For a peak of ζ/s at higher temperature, the effects are smaller

Viscosity is usually small and does not influence much the fluid evolution



J. Noronha-Hostler, J. Noronha, F. Grassi Phys. Rev. C 90, 034907 (2014) /arXiv:1406.3333

Particlization

In the so-called hybrid models, at a certain switching temperature, hadrons are generated (= “Particlization”) using Cooper-Frye formula and used as input in a transport code (UrQMD, RQMD, JAM, SMASH) that describe their collisions, decays, etc.

The 2 problems already mentioned are still present but the description of data is improved (more on this later)

Challenge

Redo the calculation p.3 using as well isospin and $\pi^-/\pi^+ = 1.02$.

Homework

Estimate the temperature and baryonic potential for chemical freeze out for Au+Au collisions at 130 GeV A where at midrapidity $\bar{p}/p$=0.65 and $\bar{p}/\pi^- = 0.08$

Other references on this topic

- W. Florkowski, Phenomenology of Ultra-Relativistic Heavy-Ion Collisions, World Scientific, 2010 (Ch.25)
- U.Heinz, K.S.Lee, E.Schedermann in "Quark Gluon Plasma" (vol. 1) World Scientific, 1990, Ed. Hwa