Chapter 26

OPTIMAL FISCAL AND MONETARY POLICY *

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Abstract

We provide an introduction to optimal fiscal and monetary policy using the primal approach to optimal taxation. We use this approach to address how fiscal and monetary policy should be set over the long run and over the business cycle.

We find four substantive lessons for policymaking: Capital income taxes should be high initially and then roughly zero; tax rates on labor and consumption should be roughly constant; state-contingent taxes on assets should be used to provide insurance against adverse shocks; and monetary policy should be conducted so as to keep nominal interest rates close to zero.

We begin by studying optimal taxation in a static context. We then develop a general framework to analyze optimal fiscal policy. Finally, we analyze optimal monetary policy in three commonly used models of money: a cash–credit economy, a money-in-the-utility-function economy, and a shopping-time economy.

Keywords

primal approach, Ramsey problems, capital income taxation, Friedman rule, tax smoothing

JEL classification: E5, E6, E52, E62, H3, H21

Introduction

A fundamental question in macroeconomics is, How should fiscal and monetary policy be set over the long run and over the business cycle? Answering this question requires integrating tools from public finance into macroeconomics. The purpose of this chapter is to lay out and extend recent developments in the attempts to do that within a framework which combines two distinguished traditions in economics: the public finance tradition and the general equilibrium tradition in macroeconomics. The public finance tradition we follow in this chapter stems from the work of Ramsey (1927), who considers the problem of choosing an optimal tax structure in an economy with a representative agent when only distorting taxes are available. The general equilibrium tradition stems from the work of Cass (1965), Koopmans (1965), Kydland and Prescott (1982), and Lucas and Stokey (1983).

Within the public finance tradition, our framework builds on the *primal approach* to optimal taxation. [See, for example, Atkinson and Stiglitz (1980), Lucas and Stokey (1983), and Chari et al. (1991).] This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a resource constraint and an implementability constraint. The implementability constraint is the consumer budget constraint in which the consumer and the firm first-order conditions are used to substitute out for prices and policies. Thus both constraints depend only on allocations. This characterization implies that optimal allocations are solutions to a simple programming problem. We refer to this optimal tax problem as the *Ramsey problem* and to the solutions and the associated policies as the *Ramsey allocations* and the *Ramsey plan*.

We study optimal fiscal and monetary policy in variants of neoclassical growth models. This analysis leads to four substantive lessons for policymaking:

- Capital income taxes should be high initially and then roughly zero.
- Tax rates on labor and consumption should be roughly constant.
- State-contingent taxes on assets should be used to provide insurance against adverse shocks.
- Monetary policy should be conducted so as to keep nominal interest rates close to zero.

The basic logic behind these policymaking lessons is that Ramsey policies smooth distortions over time and states of nature. Smoothing tax distortions over time implies that capital tax rates should be roughly zero and labor and consumption taxes should be roughly constant [Lucas and Stokey (1983) and Chari et al. (1994)]. Ramsey policies also imply that heavily taxing inelastically supplied inputs is optimal. Thus Ramsey policies involve taxing capital income at initially high rates, but then dropping these rates, to zero in the long run. [See Judd (1985) and Chamley (1986).]

Since keeping capital, labor, and consumption taxes roughly constant is optimal, the government needs some source of revenue to ensure that taxes need not be sharply changed when the economy is hit by shocks. One way to provide such revenue insurance is to have explicitly state-contingent debt, in the sense that the rate of return on the debt varies with the shocks. Another way is to have non-state-contingent debt with taxes on interest income which vary with the shocks. Revenue insurance can also be provided by having taxes on capital income that vary with the shocks while still being roughly zero on average.

In terms of monetary policy, Friedman (1969) advocates a simple rule: set nominal interest rates to zero. In the models we consider, the Friedman rule is optimal if the consumption elasticity of money demand is one. We think that this rule deserves attention because the weight of the empirical evidence is that the consumption elasticity of money demand is indeed one. [See Stock and Watson (1993).]

Throughout the chapter, we emphasize that the primal approach, in essence, involves finding optimal wedges between marginal rates of substitution and marginal rates of transformation. Typically, many tax systems can decentralize the Ramsey allocations. Thus optimal tax theory yields results on optimal wedges, and thus the prescriptions for optimal taxes depend on the details of the particular tax system. For example, in the one-sector growth model, a tax system which includes any two of consumption, labor, and capital income taxes can decentralize the Ramsey allocations. In such a model, it is optimal to set intertemporal marginal rates of substitution equal to intertemporal marginal rates of transformation in the long run. With a tax system that consists of capital and labor taxes, this is accomplished by setting capital income taxes equal to zero. With a tax system that consists of consumption and labor taxes, this is accomplished by making consumption taxes constant. Thus the Ramsey allocations can be implemented either with zero capital income taxes or with constant consumption taxes.

Throughout this chapter, we focus on economies in which the government effectively has access to a commitment technology. As is well known, without such a technology, there are time inconsistency problems, so the equilibrium outcomes with commitment are not necessarily sustainable without commitment. Economies with commitment technologies can be interpreted in two ways. One is that the government can simply commit to its future actions by, say, restrictions in its constitution. The other, and the way we prefer, is that the government has no access to a commitment technology, but the commitment outcomes are sustained by reputational mechanisms. [See, for example, Chari et al. (1989), Chari and Kehoe (1990, 1993), and Stokey (1991) for analyses of optimal policy in environments without commitment.] Throughout this chapter we also restrict attention to proportional tax systems.

The results we develop all come from environments with an infinite number of periods and include some combination of uncertainty, capital, debt, and money. Many of the basic principles, however, can be developed in a simple static context in which the ideas are easiest to digest. In Section 1, in a static context, we develop two of the three main results in public finance which show up repeatedly in macroeconomic models. First, under appropriate separability and homotheticity conditions on preferences, it is optimal to tax goods at a uniform rate. Second, if all consumption goods, types of labor income, and pure profits can be taxed, then it is optimal not to tax intermediate goods. The uniform commodity tax result shows up repeatedly in analyses of fiscal policy, and this result and the intermediate-goods result show up repeatedly in analyses of monetary policy. We defer to the next section the development of the third main result, that it is optimal to set taxes on capital income equal to zero in the long run.

In Section 2, we lay out a stochastic neoclassical growth model to analyze fiscal policy. We begin with a deterministic version of this model to highlight the long-run properties of optimal fiscal policy. In this version, we develop the results of Chamley (1980, 1986) on the optimality of zero capital-income taxation in a steady state, the generalizations by Judd (1985) to environments with heterogeneous agents, and some qualifications by Stiglitz (1987) when there are restrictions on the tax system. Next, we show that for a commonly used class of utility functions, optimal capital taxes are zero not only in a steady state, but also after the first period.

Next, we consider a stochastic model without capital to highlight how optimal fiscal policy should respond to shocks. We illustrate how, by using debt as a shock absorber, taxes on labor income are optimally smoothed in response to shocks to government consumption and technology [as in Lucas and Stokey (1983) and Chari et al. (1991)]. We then contrast these results with the assertions in Barro (1979) about tax-smoothing in a reduced-form model. We argue that the work of Marcet et al. (1996) on taxation with incomplete markets partially affirms Barro's assertions. We also consider the quantitative features of optimal fiscal policy in a standard real business cycle model [as in Chari et al. (1994)].

We go on to discuss how the results developed in a closed economy with infinitely lived agents and only exogenous growth extend to other environments. We first show that in an endogenous growth framework along a balanced growth path, all taxes are zero. [See Bull (1992) and Jones et al. (1997).] Essentially, in this framework, capital income taxes distort physical capital accumulation, and labor income taxes distort human capital accumulation. Hence it is optimal to front-load both taxes. We then consider an open economy and show that under both source-based and residence-based taxation, optimal capital income taxation is identically zero. The intuition for these results is that with capital mobility, each country faces a perfectly elastic supply of capital and therefore optimally chooses to set capital income tax rates to zero. [See Atkeson et al. (1999) and Garriga (1999).] Finally, we consider an overlapping generations model and show that only under special conditions is the tax rate on capital income zero in a steady state. The conditions are that certain homotheticity and separability conditions hold. [See Atkeson et al. (1999) and Garriga (1999).]

In Section 3, we lay out a general framework for the analysis of monetary policy. We consider three commonly used models of money: a cash–credit monetary economy, a money-in-the-utility-function monetary economy, and a shopping-time monetary economy. For each model, we provide sufficient conditions for the optimality of the Friedman rule. These conditions for the cash–credit economy and the money-in-the-utility-function economy are analyzed by Chari et al. (1996), while conditions for the shopping-time economy are analyzed by Kimbrough (1986), Faig (1988), Woodford (1990), Guidotti and Végh (1993), and Correia and Teles (1996), as well as by

Chari et al. (1996). The common features of the requirements for optimality are simple homotheticity and separability conditions similar to those in the public finance literature on optimal uniform commodity taxation.

There have been conjectures in the literature – by Kimbrough (1986) and Woodford (1990), among others – about the connection between the optimality of the Friedman rule and the intermediate-goods results. For all three monetary economies, we show that when the homotheticity and separability conditions hold, the optimality of the Friedman rule follows from the intermediate-goods result.

Finally, we report results for a quantitative monetary business cycle model. We find that if debt has nominal non-state-contingent returns, so that asset markets are incomplete, inflation can be used to make real returns contingent, so that debt can serve as a shock absorber.

1. The primal approach to optimal taxation

The general approach to characterizing competitive equilibria with distorting taxes described in this section is known in the public finance literature as the *primal approach* to taxation. [See Atkinson and Stiglitz (1980).] The basic idea is to recast the problem of choosing optimal taxes as a problem of choosing allocations subject to constraints which capture the restrictions on the type of allocations that can be supported as a competitive equilibrium for some choice of taxes. In this section, we lay out the primal approach and use it to establish some basic principles of optimal taxation, together with the results on uniform commodity taxation and intermediate-goods taxation.

The rest of this chapter applies these basic principles of optimal taxation to a variety of environments of interest to macroeconomists. These environments all have an infinite number of periods and include some combination of uncertainty, capital, debt, and money. As such, the derivations of the results look more complicated than the derivations here, but the basic ideas are quite similar.

1.1. The Ramsey allocation problem

Consider a model economy in which n types of consumption goods are produced with labor. The resource constraint is given by

$$F(c_1+g_1,\ldots,c_n+g_n,l)=0, \tag{1.1}$$

where c_i and g_i denote private and government consumption of good i, l denotes labor, and F denotes a production process that satisfies constant returns to scale. The consumer's problem is to maximize utility:

$$\max U(c_1,\ldots,c_n,l) \tag{1.2}$$

$$\text{subject to } \sum_i p_i(1+\tau_i)\,c_i = l, \tag{1.3}$$

where p_i is the price of good i and τ_i is the ad valorem tax rate on good i. Thus there are n linear commodity taxes. We normalize the wage to 1.

A representative firm operates the constant returns technology F and solves

$$\max_{(x,l)} \sum_i p_i x_i - l \tag{1.4}$$

$$\text{subject to } F(x_1, \ldots, x_n, l) = 0, \tag{1.5}$$

where x_i denotes output of good i. The government budget constraint is

$$\sum_i p_i g_i = \sum_i p_i \tau_i c_i. \tag{1.6}$$

Market clearing requires that

$$c_i + g_i = x_i \quad \text{for} \quad i = 1, \ldots, n. \tag{1.7}$$

Throughout this chapter, we take government expenditures as given. A *competitive equilibrium* is a policy $\pi = (\tau_i)_{i=1}^n$; allocations c, l, and x; and a price system p that satisfy the following: (i) the allocations c and l maximize Equation (1.2) subject to (1.3), (ii) the allocations x and l solve Equation (1.4), (iii) the government budget constraint (1.6) holds, and (iv) the allocations c and x satisfy condition (1.7).

Throughout this chapter, we assume that first-order conditions are necessary and sufficient and that all allocations are interior. The sufficiency of the first-order conditions for firms and consumers holds under appropriate concavity assumptions, and interiority can be assured with appropriate monotonicity and Inada conditions.

Proposition 1. *The allocations in a competitive equilibrium satisfy*

$$F(c_1 + g_1, \ldots, c_n + g_n, l) = 0 \tag{1.8}$$

and the implementability constraint

$$\sum_i U_i c_i + U_l l = 0. \tag{1.9}$$

Furthermore, given allocations which satisfy Equations (1.8) and (1.9), we can construct policies and prices which, together with the given allocations, constitute a competitive equilibrium.

Remark: The literature usually refers to Equation (1.9) as the *implementability constraint* because it is a constraint on the set of allocations that can be implemented as a competitive equilibrium with distorting taxes. This constraint can be thought of as the consumer budget constraint with both the taxes and the prices substituted out by using first-order conditions.

Proof: We first prove that the allocations in a competitive equilibrium must satisfy Equations (1.8) and (1.9). Condition (1.8) follows from substituting the market-clearing condition (1.7) into (1.5). To derive Equation (1.9), notice that the consumer's first-order conditions are

$$U_i = \alpha p_i(1+\tau_i) \quad \text{for} \quad i = 1, \ldots, n, \tag{1.10}$$

$$-U_l = \alpha, \tag{1.11}$$

$$\sum_i p_i(1+\tau_i)\, c_i = l, \tag{1.12}$$

where α is the Lagrange multiplier on the budget constraint. Substituting Equations (1.10) and (1.11) into (1.12) gives (1.9). Next, we prove that if c and l satisfy (1.8) and (1.9), then a price system p, a policy π, and an allocation x, together with the given allocations, constitute a competitive equilibrium. We use the first-order conditions for the firm, which are

$$p_i = -F_i/F_l \quad \text{for} \quad i = 1, \ldots, n. \tag{1.13}$$

We construct x, p, and π as follows: $x_i = c_i + g_i$, p_i is from (1.13), and π is from

$$1 + \tau_i = \frac{U_i}{U_l}\,\frac{F_l}{F_i}.$$

Given our assumptions on the utility function, the first-order conditions are necessary and sufficient for consumer and firm maximization. With x, p, and π so defined, (c, l, x, p, π) clearly satisfies firm maximization. When $\alpha = -U_l$, conditions (1.10) and (1.11) clearly are also satisfied. Substituting for U_i and U_l in Equation (1.9), we have

$$\sum_i c_i \alpha p_i(1+\tau_i) - \alpha l = 0.$$

Dividing by α and rearranging gives Equation (1.12). The government budget constraint is satisfied by Walras' law. □

We can now define a type of optimal tax equilibrium in which the government objective is to maximize the utility of consumers. We think of the government as first choosing policies and of private agents as then choosing their actions. Let Π denote the set of policies for which a competitive equilibrium exists. A *Ramsey equilibrium* is a policy $\pi = (\tau_i)_{i=1}^n$ in Π; allocation rules $c(\cdot), l(\cdot)$, and $x(\cdot)$; and a price function $p(\cdot)$ that satisfy the following: (i) the policy π solves

$$\max_{\pi'} U(c(\pi'), l(\pi'))$$

$$\text{subject to} \sum_i p_i(\pi')\, g_i = \sum_i p_i(\pi')\, \tau_i' c_i(\pi') \tag{1.14}$$

and (ii) for every π', the allocations $c(\pi')$, $l(\pi')$, $x(\pi')$, the price system $p(\pi')$, and the policy π' constitute a competitive equilibrium.

Notice that we require optimality by consumers and firms for all policies that the government might choose. This requirement is analogous to the requirement of subgame perfection in a game. To see why this requirement is important, suppose we had not imposed it. That is, suppose we required optimality by consumers and firms only at the equilibrium policies, but allowed allocation and price rules to be arbitrary elsewhere. Then the set of equilibria is much larger. For example, allocation rules that prescribe zero labor supply for all policies other than some particular policy would satisfy all the equilibrium conditions. Since the government's budget constraint is then satisfied only at the particular policy, the government optimally chooses that policy. We think that such equilibria do not make sense. That is, we think the requirement that consumers and firms behave optimally for all policies is the sensible way to solve the government's problem of forecasting private behavior.

If the competitive equilibrium associated with each policy is unique, clearly the Ramsey equilibrium is also unique. If there are multiple competitive equilibria associated with some policies, our definition of a Ramsey equilibrium requires that a selection be made from the set of competitive equilibria. In this case, there may be many Ramsey equilibria, depending on the particular selection made. In this chapter, we focus on the Ramsey equilibrium that yields the highest utility for the government. In such a Ramsey equilibrium, a particular allocation and price system are realized, namely, c, l, and p. We call these the *Ramsey allocations and prices*. We then have the following proposition as an immediate corollary of Proposition 1.

Proposition 2. *The Ramsey allocations solve the Ramsey problem, which is to choose c and l to maximize $U(c,l)$ subject to conditions (1.8) and (1.9).*

We have studied an economy in which the government uses consumption-goods taxes to raise revenues and have shown how the problem of solving for the Ramsey equilibrium reduces to the simpler problem of solving for the Ramsey allocations. Other tax systems lead to the same Ramsey problem. For example, consider a tax system that includes taxes on the n consumption goods as well as taxes on labor income. It can be shown that the Ramsey allocations can be supported by a tax system that uses any n of the $n+1$ instruments. For example, the Ramsey allocations can be supported by taxes on consumption goods 2 through n and labor income or by taxes on consumption goods alone. The fact that the Ramsey allocations can be decentralized in many ways implies that it is more useful to think about optimal taxation in terms of the implied wedges between marginal rates of substitution and marginal rates of transformation rather than in terms of the particular tax system used to decentralize the Ramsey allocations.

The form of the Ramsey allocation problem depends on the assumption that the tax system contains at least n independent instruments. We call such a tax system *complete*. An example of an *incomplete* tax system is one in which taxes on the first consumption good and labor are constrained to be zero. For such an incomplete tax system, the analog of Proposition 1 is that a set of allocations is part of a competitive equilibrium if and only if the set satisfies conditions (1.8) and (1.9), together with

$$\frac{U_1}{U_l} = \frac{F_1}{F_l}.$$

Intuitively, this constraint captures the fact that the government has no tax instruments that drive a wedge between the marginal rate of substitution of the first consumption good and labor and the marginal rate of transformation of the same commodities. The reader will find proving this analog useful, in part, because the proof illustrates that condition (1.9) must hold regardless of the nature of the tax system. That is, when the tax system is incomplete, the implementability constraint is unchanged, and the new constraints that reflect this incompleteness must be added to the Ramsey problem.

1.2. Elasticities and commodity taxation

We can use the Ramsey allocation problem to derive some simple results on optimal commodity taxes. We show that with additively separable preferences, tax rates depend on income elasticities, with necessities being taxed more than luxuries. The discussion here closely follows Atkinson and Stiglitz (1980, chap. 12).

Consider the first-order conditions for the Ramsey problem:

$$(1+\lambda)\,U_i - \lambda U_i H_i = \gamma F_i, \tag{1.15}$$

$$(1+\lambda)\,U_l - \lambda U_l H_l = -\gamma F_l, \tag{1.16}$$

where λ and γ are the Lagrange multipliers on the implementability constraint and the resource constraint, respectively; $H_i \equiv -(\sum_j U_{ji}c_j + U_{li}l)/U_i$; and $H_l \equiv -(\sum_j U_{lj}c_j + U_{ll}l)/U_l$. Using Equations (1.10), (1.11), and (1.13) in (1.15) and (1.16) and simplifying gives

$$\frac{\tau_i}{1+\tau_i} = \frac{\lambda(H_i - H_l)}{1+\lambda-\lambda H_l}.$$

Rearranging shows that the relative tax rates for two goods i and j are determined by

$$\frac{\tau_i/(1+\tau_i)}{\tau_j/(1+\tau_j)} = \frac{H_i - H_l}{H_j - H_l}. \tag{1.17}$$

Now, Equation (1.17) is not an explicit formula for optimal tax rates, since the H_i, H_j, and H_l terms depend on endogenous variables. Nevertheless, (1.17) shows that if $H_i > H_j$, then $\tau_i > \tau_j$. Suppose next that the utility function is additively separable. Then

$$H_i = -\frac{U_{ii}c_i}{U_i}. \tag{1.18}$$

Let $c(p,m)$, $l(p,m)$ denote the solution to the problem of maximizing utility subject to $\sum p_i c_i = l + m$, where m is nonlabor income, so that $c_i(p,m)$ is the demand function for good i. Letting α denote the Lagrange multiplier on the budget constraint, we can differentiate the first-order condition $U_i(c(p,m)) = \alpha(p,m)p_i$ with respect to nonlabor income m to obtain

$$U_{ii}\frac{\partial c_i}{\partial m} = p_i\frac{\partial \alpha}{\partial m} = \frac{U_i}{\alpha}\frac{\partial \alpha}{\partial m}$$

or

$$H_i\frac{1}{c_i}\frac{\partial c_i}{\partial m} = -\frac{1}{\alpha}\frac{\partial \alpha}{\partial m} \tag{1.19}$$

so that

$$\frac{H_i}{H_j} = \frac{\eta_j}{\eta_i}, \tag{1.20}$$

where η_i is the income elasticity of demand for good i. Thus necessities should be taxed more than luxuries.

The standard partial equilibrium result is that goods with low price elasticities of demand should be taxed more heavily than goods with high price elasticities. In general equilibrium, this result does not necessarily hold. It does hold if preferences are additively separable and there are no income effects. That is, utility is quasi-linear and is given by

$$\sum_i V^i(c_i) - l. \tag{1.21}$$

For such a utility function, Equation (1.20) is not helpful because the income elasticities for all the consumption goods are zero. It is easy to show that for a utility function of the form (1.21), $H_i = 1/\varepsilon_i$, where $\varepsilon_i = -(\partial c_i/\partial p_i)p_i/c_i$ is the price elasticity of demand. To see this, differentiate the first-order condition with respect to p_i,

$$U_i(c(p,m), l(p,m)) = \alpha p_i, \tag{1.22}$$

to obtain

$$U_{ii}\frac{\partial c_i}{\partial p_i} = \alpha, \tag{1.23}$$

where α is constant because of quasi-linearity. Substituting Equations (1.22) and (1.23) into (1.18) gives $H_i = 1/\varepsilon_i$. Since $\tau_i > \tau_j$ when $H_i > H_j$, consumption goods which are relatively more price inelastic (have low ε_i) should be taxed relatively heavily.

To summarize, with additive separability, the general result is that tax rates depend on income elasticities, with necessities taxed more than luxuries. Moreover, the familiar intuition from partial equilibrium that goods with low price elasticities should be taxed heavily does not necessarily apply in a general equilibrium setting.

1.3. Uniform commodity taxation

Here we set up and prove the classic result on uniform commodity taxation. This result specifies a set of conditions under which taxing all goods at the same rate is optimal. [See Atkinson and Stiglitz (1972).]

Consider a utility function of the form

$$U(c, l) = W(G(c), l) \tag{1.24}$$

where $c = (c_1, \ldots, c_n)$ and G is homothetic.

Proposition 3. *If utility satisfies condition (1.24) – that is, utility is weakly separable across consumption goods and is homothetic in consumption – then $U_i/U_j = F_i/F_j$ for $i = 1, \ldots, n$. That is, optimal commodity taxation is uniform in the sense that the Ramsey taxes satisfy $\tau_i = \tau_j$ for $i = 1, \ldots, n$.*

Proof: Substituting the firm's first-order conditions (1.13) into the consumer's first-order condition, we have that

$$1 + \tau_i = \frac{U_i}{U_l} \frac{F_l}{F_i}.$$

Thus $\tau_i = \tau_j$ if and only if $U_i/F_i = U_j/F_j$.

Note that a utility function which satisfies condition (1.24) satisfies

$$\sum_j \frac{c_j U_{ij}}{U_i} = \sum_j \frac{c_j U_{kj}}{U_k} \quad \text{for all} \quad i, k. \tag{1.25}$$

To see this, notice that from homotheticity, it follows that

$$\frac{U_i(\alpha c, l)}{U_k(\alpha c, l)} = \frac{U_i(c, l)}{U_k(c, l)}$$

or

$$U_i(\alpha c, l) = \left[\frac{U_i(c, l)}{U_k(c, l)}\right] U_k(\alpha c, l). \tag{1.26}$$

Differentiating Equation (1.26) with respect to α and evaluating it at $\alpha = 1$ gives (1.25).

Consider next the first-order condition for c_i from the Ramsey problem, namely,

$$(1+\lambda)\, U_i + \lambda \left[\sum_j c_j U_{ij} + l U_{il} \right] = \gamma F_i, \tag{1.27}$$

where, again, λ is the multiplier on the implementability constraint and γ is the multiplier on the resource constraint. From Equation (1.25), we have that there is some constant A such that $\sum_j c_j U_{ij} = A U_i$ for all i. Using this fact and the form of utility function, we can rewrite Equation (1.27) as

$$(1+\lambda)\, W_1 G_i + \lambda \left[A W_1 G_i + l W_{12} G_i \right] = \lambda F_i. \tag{1.28}$$

Since Equation (1.28) holds for all i and j, $G_i/F_i = G_j/F_j$ for all i and j and

$$\frac{U_i}{F_i} = \frac{W_1 G_i}{F_i} = \frac{W_1 G_j}{F_j} = \frac{U_j}{F_j}.$$

□

Note that the Ramsey allocations can be decentralized in many ways. For example, taxes on goods can all be set to an arbitrary constant, including zero, and remaining revenues raised by taxing labor income.

Consider some generalizations of this proposition. Suppose that the utility function is homothetic and separable over a subgroup of goods, in the sense that the utility function can be written as

$$U(c_1, \ldots, c_k, \phi(c_{k+1}, \ldots, c_n), l)$$

with ϕ homothetic. Then it is easy to show that the Ramsey taxes $\tau_{k+1} = \ldots = \tau_n$. Next, if there is some untaxed income, then we need to modify Proposition 3. Suppose that we add to the model an endowment of good 1, y_1, which is not taxed. Then the implementability constraint becomes

$$\sum_i U_i c_i + U_l l = U_1 y_1.$$

Then even if U satisfies $U(\phi(c_1, \ldots, c_n), l)$ with ϕ homothetic, it is not true that optimal taxes are uniform (because of the extra terms $U_{1j} y_1$ from the derivatives of $U_1 y_1$). If we add the assumption that U is additively separable across $c_1, \ldots, c_n$, then the Ramsey taxes for goods 2 through n will be uniform, but not equal to the tax on good 1. Next, suppose that the tax system is incomplete in the sense that the government is restricted to setting the tax on good 1 to some fixed number, say, $\tau_1 = 0$. Then the Ramsey problem now must include the constraint

$$\frac{U_i}{U_l} = \frac{F_1}{F_l}$$

in addition to the resource constraint and the implementability constraint. Then even if U satisfies condition (1.24), optimal commodity taxes on goods 2 through n are not necessarily uniform. Finally, in order to connect this result on uniform commodity taxation to some of the later results, suppose that the utility function is defined over an infinite sequence of consumption and labor goods as $U(c_1, c_2, \ldots, l_1, l_2, \ldots)$. The assumption that the utility is of the form $V(\phi(c_1, \ldots, c_t, \ldots), l_1, l_2, \ldots)$ with ϕ homothetic and separable between consumption and all labor goods $l_1, l_2, \ldots$, together with the assumption that the utility function is additively separable across time with constant discount factor β, restricts the utility function to the form

$$\sum_t \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + G(l_t) \right].$$

1.4. Intermediate goods

Here we establish the classic intermediate-goods result for a simple example. (This example turns out to be useful when we study monetary economies.) Recall the standard result in public finance that under a wide variety of circumstances, an optimal tax system maintains aggregate production efficiency. [See Diamond and Mirrlees (1971).] In the context of an economy with multiple production sectors, transactions between firms can be taxed. Taxing such transactions distorts the relations between the marginal rate of transformation in one sector and the marginal rate of transformation in another sector and yields aggregate production inefficiency. In such a setup, the standard result on aggregate production efficiency immediately implies that taxing intermediate goods is not optimal.

Consider an economy with three final goods – private consumption x, government consumption g, and labor l – and an intermediate good z. The utility function is $U(x, l)$. The technology set for producing the final consumption good using labor l_1 and the intermediate good is described by

$$f(x, z, l_1) \leqslant 0, \tag{1.29}$$

where f is a constant returns to scale production function. There is a technology set for producing the intermediate good and government consumption using labor l_2 described by

$$h(z, g, l_2) \leqslant 0, \tag{1.30}$$

where h also is a constant returns to scale production function. The consumer's problem is to maximize

$$U(x, l_1 + l_2)$$
$$\text{subject to } p(1+\tau)x \leqslant w(l_1 + l_2),$$

where p and w are the prices of the consumption good and labor and τ is the tax on the consumption good. The firm that produces private consumption goods maximizes profits

$$px - wl_1 - q(1+\eta)z$$

subject to condition (1.29); q is the price of intermediate goods, and η is the tax on intermediate goods. The firm that produces intermediate goods and government consumption goods maximizes profits

$$qz + rg - wl_2,$$

where r is the price of government consumption, subject to condition (1.30).

We can easily show that the Ramsey allocation problem is given by

$$\max U(x, l_1 + l_2)$$

subject to conditions (1.29), (1.30), and

$$xU_x + (l_1 + l_2)\, U_l = 0. \tag{1.31}$$

We then have

Proposition 4. *The solution to the Ramsey allocation problem satisfies production efficiency; namely, the marginal rates of transformation are equated across technologies. Equivalently, setting the tax on intermediate goods $\eta = 0$ is optimal.*

Proof: For this economy, production efficiency is equivalent to

$$\frac{f_z}{f_l} = -\frac{h_z}{h_l}. \tag{1.32}$$

Solving the Ramsey allocation problem, we obtain the following first-order conditions for z, l_1, and l_2, respectively:

$$\upsilon f_z = -\mu h_z, \tag{1.33}$$

$$U_l + \lambda(xU_{lx} + U_l + lU_{ll}) + \upsilon f_l = 0, \tag{1.34}$$

$$U_l + \lambda(xU_{lx} + U_l + lU_{ll}) + \mu h_l = 0, \tag{1.35}$$

where υ, μ, and λ are the multipliers on (1.29), (1.30), and (1.31). Combining Equations (1.34) and (1.35) gives $\upsilon f_l = \mu h_l$, which, combined with (1.33), establishes Equation (1.32).

The first-order conditions for profit maximization for the firms imply that

$$\frac{f_z}{f_l} = \frac{q(1+\eta)}{w} = -\frac{h_z}{h_l}(1+\eta). \tag{1.36}$$

Thus, if condition (1.32) holds, Equation (1.36) implies that $\eta = 0$. □

The intermediate-goods result holds in general settings in which there are (possibly infinitely) many goods and many production technologies. We have assumed that the production technologies satisfy constant returns to scale. If there are increasing returns to scale, then there are standard problems with the existence of a competitive equilibrium. If there are decreasing returns to scale, then the intermediate-goods result continues to hold, provided that pure profits can be fully taxed away.

It turns out that the result for uniform commodity taxation follows from the intermediate-goods result. To see this, consider a utility function of the form

$$U(c,l) = W(G(c),l), \tag{1.37}$$

where $c = (c_1,\ldots,c_n)$ and G is homogeneous of degree 1. We can reinterpret this economy as an economy with a single consumption good x, which is produced using n intermediate-goods inputs $(c_1,\ldots,c_n)$ with the constant returns to scale technology $x = G(c)$. The intermediate-goods result requires that in an optimal tax system, the taxes on the intermediate-goods inputs be zero, so that there are taxes only on final goods x and l. This is clearly equivalent to a uniform tax on $(c_1,\ldots,c_n)$.

2. Fiscal policy

In this section, we begin by setting out a general framework for analyzing optimal fiscal policy in a stochastic one-sector growth model. We use a deterministic version of this model to develop results on the taxation of capital income, in both the short and long run. We first show that the optimal capital income taxes are zero in a steady state, even if there are heterogeneous consumers. We then show that for a class of utility functions, there is only one period with nonzero capital income taxes, following which capital income taxes are zero along a transition to the steady state. We then turn to the cyclical properties of optimal fiscal policy. In a stochastic model without capital, we illustrate how debt can act as a shock absorber. We briefly discuss how incomplete markets can alter these results. We then illustrate the main features of optimal fiscal policy over a business cycle using a calibrated version of the model with capital. Finally, we discuss how these results are altered in three other environments: an endogenous growth model, an open economy model, and an overlapping generations model.

2.1. *General framework*

Consider a production economy populated by a large number of identical, infinitely lived consumers. In each period $t = 0, 1, \ldots$, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0,\ldots,s_t)$ the history of events up to and including period t. The probability, as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period t, the economy has two goods: a consumption-capital good and labor. A constant returns to scale technology which satisfies the standard Inada conditions is available to transform capital $k(s^{t-1})$ and labor $l(s^t)$ into output via $F(k(s^{t-1}), l(s^t), s_t)$. Notice that the production function incorporates a stochastic shock s_t. The output can be used for private consumption $c(s^t)$, government consumption $g(s^t)$, and new capital $k(s^t)$. Throughout, we will take government consumption to be exogenously specified. Feasibility requires that

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)\, k(s^{t-1}), \tag{2.1}$$

where δ is the depreciation rate on capital. The preferences of each consumer are given by

$$\sum_{t,s^t} \beta^t \mu(s^t)\, U(c(s^t), l(s^t)), \tag{2.2}$$

where $0 < \beta < 1$ and U is strictly increasing in consumption, is strictly decreasing in labor, is strictly concave, and satisfies the Inada conditions.

Government consumption is financed by proportional taxes on the income from labor and capital and by debt. Let $\tau(s^t)$ and $\theta(s^t)$ denote the tax rates on the income from labor and capital. Government debt has a one-period maturity and a state-contingent return. Let $b(s^t)$ denote the number of units of debt issued at state s^t and $R_b(s^{t+1})$ denote the return at any state $s^{t+1} = (s^t, s_{t+1})$. The consumer's budget constraint is

$$c(s^t) + k(s^t) + b(s^t) \leqslant \left[1 - \tau(s^t)\right] w(s^t)\, l(s^t) + R_k(s^t)\, k(s^{t-1}) + R_b(s^t)\, b(s^{t-1}), \tag{2.3}$$

where $R_k(s^t) = 1 + [1 - \theta(s^t)][r(s^t) - \delta]$ is the gross return on capital after taxes and depreciation and $r(s^t)$ and $w(s^t)$ are the before-tax returns on capital and labor. Consumers' debt holdings are bounded by $b(s^t) \geqslant -M$ for some large constant M. Competitive pricing ensures that these returns equal their marginal products, namely, that

$$r(s^t) = F_k(k(s^{t-1}), l(s^t), s_t), \tag{2.4}$$

$$w(s^t) = F_l(k(s^{t-1}), l(s^t), s_t). \tag{2.5}$$

Consumers' purchases of capital are constrained to be nonnegative, and the purchases of government debt are bounded above and below by some arbitrarily large constants. We let $x(s^t) = (c(s^t), l(s^t), k(s^t), b(s^t))$ denote an allocation for consumers at s^t and let $x = (x(s^t))$ denote an allocation for all s^t. We let $(w, r, R_b) = (w(s^t), r(s^t), R_b(s^t))$ denote a price system.

The government sets tax rates on labor and capital income and returns for government debt to finance the exogenous sequence of government consumption. The government's budget constraint is

$$b(s^t) = R_b(s^t)\, b(s^{t-1}) + g(s^t) - \tau(s^t)\, w(s^t)\, l(s^t) - \theta(s^t)[r(s^t) - \delta]\, k(s^{t-1}). \tag{2.6}$$

We let $\pi(s^t) = (\tau(s^t), \theta(s^t))$ denote the government policy at s^t and let $\pi = (\pi(s^t))$ denote the infinite sequence of policies. The initial stock of debt, b_{-1}, and the initial stock of capital, k_{-1}, are given.

Notice that for notational simplicity, we have not explicitly included markets in private claims, so all borrowing and lending is between consumers and the government. Since all consumers are identical, such claims will not be traded in equilibrium; hence their absence will not affect the equilibrium. Thus we can always interpret the current model as having complete contingent private claims markets.

A *competitive equilibrium* for this economy is a policy π, an allocation x, and a price system (w, r, R_b) such that given the policy and the price system, the resulting allocation maximizes the representative consumer's utility (2.2) subject to the sequence of budget constraints (2.3), the price system satisfies (2.4) and (2.5), and the government's budget constraint (2.6) is satisfied. Notice that we do not need to impose the feasibility condition (2.1) in our definition of equilibrium. Given our assumptions on the utility function, constraint (2.3) is satisfied with equality in an equilibrium, and this feature, together with (2.6), implies (2.1).

Consider now the policy problem faced by the government. We suppose that there is an institution or a *commitment technology* through which the government, in period 0, can bind itself to a particular sequence of policies once and for all. We model this by having the government choose a policy π at the beginning of time and then having consumers choose their allocations. Formally, allocation rules are sequences of functions $x(\pi) = (x(s^t \mid \pi))$ that map policies π into allocations $x(\pi)$. Price rules are sequences of functions $w(\pi) = (w(s^t \mid \pi))$ and $r(\pi) = (r(s^t \mid \pi))$ that map policies π into price systems. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices must be described by rules that associate government policies with allocations. We will impose a restriction on the set of policies that the government can choose. Since the capital stock in period 0 is inelastically supplied, the government has an incentive to set the initial capital tax rate as high as possible. To make the problem interesting, we will require that the initial capital tax rate, $\theta(s_0)$, be fixed at some rate.

A *Ramsey equilibrium* is a policy π, an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ that satisfy the following: (i) the policy π maximizes

$$\sum_{t,s^t} \beta^t \mu(s^t)\, U(c(s^t \mid \pi), l(s^t \mid \pi))$$

subject to constraint (2.6), with allocations and prices given by $x(\pi)$, $w(\pi)$, and $r(\pi)$; and (ii) for every π', the allocation $x(\pi')$, the price system $w(\pi')$, $r(\pi')$, and $R_b(\pi')$, and the policy π' constitute a competitive equilibrium.

We now turn to characterizing the equilibrium policies and allocations. In terms of notation, it will be convenient here and throughout the chapter to let $U_c(s^t)$ and $U_l(s^t)$ denote the marginal utilities of consumption and leisure at state s^t and let $F_k(s^t)$ and $F_l(s^t)$ denote the marginal products of capital and labor at state s^t. We will show that a competitive equilibrium is characterized by two fairly simple conditions: the resource constraint

$$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), l(s^t), s_t) + (1-\delta)\, k(s^{t-1}) \tag{2.7}$$

and the implementability constraint

$$\sum_{t,s^t} \beta^t \mu(s^t) \left[U_c(s^t)\, c(s^t) + U_l(s^t)\, l(s^t)\right] = U_c(s_0) \left[R_k(s_0)\, k_{-1} + R_b(s_0)\, b_{-1}\right], \tag{2.8}$$

where $R_k(s_0) = 1 + [1 - \theta(s_0)][F_k(s_0) - \delta]$. The implementability constraint should be thought of as an infinite-horizon version of the budget constraint of either the consumer or the government, where the consumer and firm first-order conditions have been used to substitute out the prices and policies. We have

Proposition 5. *The consumption, labor, and capital allocations and the capital tax rate and return on debt in period 0 in a competitive equilibrium satisfy conditions (2.7) and (2.8). Furthermore, given allocations and period 0 policies that satisfy (2.7) and (2.8), we can construct policies, prices, and debt holdings that, together with the given allocations and period-0 policies, constitute a competitive equilibrium.*

Proof: We first show that a competitive equilibrium must satisfy (2.7) and (2.8). To see this, note that we can add (2.3) and (2.6) to get (2.7), and thus feasibility is satisfied in equilibrium. Next, consider the allocation rule $x(\pi)$. The necessary and sufficient conditions for c, l, b, and k to solve the consumer's problem are given as follows. Let $p(s^t)$ denote the Lagrange multiplier on constraint (2.3). Then by Weitzman's (1973) and Ekeland and Scheinkman's (1986) theorems, these conditions are constraint (2.3), together with first-order conditions for consumption and labor:

$$\beta^t \mu(s^t)\, U_c(s^t) \leqslant p(s^t), \quad \text{with equality if} \quad c(s^t) > 0, \tag{2.9}$$

$$\beta^t \mu(s^t)\, U_l(s^t) \leqslant -p(s^t)(1 - \tau(s^t))\, w(s^t), \quad \text{with equality if} \quad l(s^t) > 0; \tag{2.10}$$

first-order conditions for capital and government debt:

$$\left[p(s^t) - \sum_{s^{t+1}} p(s^{t+1})\, R_b(s^{t+1})\right] b(s^t) = 0, \tag{2.11}$$

$$\left[p(s^t) - \sum_{s^{t+1}} p(s^{t+1})\, R_k(s^{t+1})\right] k(s^t) = 0; \tag{2.12}$$

and the two transversality conditions

$$\lim_{t \to \infty} \sum_{s^t} p(s^t)\, b(s^t) = 0, \tag{2.13}$$

$$\lim_{t \to \infty} \sum_{s^t} p(s^t)\, k(s^t) = 0. \tag{2.14}$$

We claim that any allocation which satisfies (2.3) and (2.9)–(2.14) must also satisfy (2.8). To see this, multiply (2.3) by $p(s^t)$, sum over t and s^t, and use (2.11)–(2.14) to obtain

$$\sum_{t,s^t} p(s^t)\{c(s^t) - [1 - \tau(s^t)]\, w(s^t)\, l(s^t)\} = p(s_0)[R_k(s_0)\, k_{-1} + R_b(s_0)\, b_{-1}]. \tag{2.15}$$

Using (2.9) and (2.10) and noting that interiority follows from the Inada conditions, we can rewrite Equation (2.15) as

$$\sum_{t,s^t} \beta^t \mu(s^t) \left[U_c(s^t)\, c(s^t) + U_l(s^t)\, l(s^t)\right] = U_c(s_0)\left[R_k(s_0)\, k_{-1} + R_b(s_0)\, b_{-1}\right]. \tag{2.16}$$

Thus (2.7) and (2.8) are necessary conditions that any competitive equilibrium must satisfy.

Next, suppose that we are given allocations and period-0 policies that satisfy (2.7) and (2.8). We construct the competitive equilibrium as follows. First, note that for an allocation to be part of a competitive equilibrium, it must satisfy conditions (2.3) and (2.9)–(2.14). Multiplying (2.3) by $p(s^t)$ and summing over all periods and states following s^r and using (2.9)–(2.14), we get

$$b(s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu(s^t \mid s^r) \frac{U_c(s^t)\, c(s^t) + U_l(s^t)\, l(s^t)}{U_c(s^r)} - k(s^r). \tag{2.17}$$

Thus any competitive equilibrium debt allocation must satisfy (2.17), and hence (2.17) defines the unique debt allocations given consumption, labor, and capital allocations. The wage rate and the rental rate on capital are determined by (2.4) and (2.5) from the capital and labor allocations. The labor tax rate is determined from (2.5), (2.9), and (2.10) and is given by

$$-\frac{U_l(s^t)}{U_c(s^t)} = [1 - \tau(s^t)] F_l(s^t). \tag{2.18}$$

We can use Equations (2.3), (2.9), (2.11), and (2.12) to construct the capital tax rate and the return on debt. From these conditions, it is clear that given the allocations, the tax rate on capital and the return on debt satisfy

$$\mu(s^t)\, U_c(s^t) = \sum_{s^{t+1}|s^t} \beta \mu(s^{t+1})\, U_c(s^{t+1})\, R_k(s^{t+1}), \tag{2.19}$$

$$\mu(s^t)\, U_c(s^t) = \sum_{s^{t+1}|s^t} \beta \mu(s^{t+1})\, U_c(s^{t+1})\, R_b(s^{t+1}), \tag{2.20}$$

$$\begin{aligned} &c(s^{t+1}) + k(s^{t+1}) + b(s^{t+1}) \\ &\quad = [1 - \tau(s^{t+1})]\, w(s^{t+1})\, l(s^{t+1}) + R_k(s^{t+1})\, k(s^t) + R_b(s^{t+1})\, b(s^t), \end{aligned} \tag{2.21}$$

where $R_k(s^{t+1}) = 1 + [1 - \theta(s^{t+1})][r(s^{t+1}) - \delta]$. It turns out that these conditions do not uniquely determine the tax rate on capital and the return on debt. To see this, suppose that s_{t+1} can take on one of N values. Then counting equations and unknowns in Equations (2.19)–(2.21) gives $N + 2$ equations and $2N$ unknowns in each period and state. Actually, however, there is one linear dependency across these equations. To see this, multiply (2.21) by $\beta\mu(s^{t+1})\,U_c(s^{t+1})$ and sum across states in period $t+1$. Use (2.17), (2.19), and (2.20) to obtain an equation that does not depend on R_b and θ. Since we can replace any of the N equations from (2.21) with this equation, there are only $N+1$ equations left to determine R_b and θ. Thus there are $N-1$ degrees of indeterminacy in setting the tax rate on capital and determining the return on debt. One particular set of policies supporting a competitive equilibrium has the capital tax rate not contingent on the current state. That is, suppose for each s^t,

$$\theta(s^t, s_{t+1}) = \bar{\theta}(s^t) \quad \text{for all} \quad s_{t+1}. \tag{2.22}$$

We can then use (2.19) to define $\bar{\theta}(s^t)$ and use the period-$t+1$ version of (2.21) to define $R_b(s^{t+1})$. It is straightforward to check that the constructed return on debt satisfies (2.20). Another set of policies supporting the same competitive equilibrium has the return on debt not contingent on the current state. [For details, see Chari et al. (1994), and for a more general discussion of this kind of indeterminacy, see Bohn (1994).] □

If the competitive equilibrium associated with each policy is unique, clearly the Ramsey equilibrium is also unique. If there are multiple competitive equilibria associated with some policies, our definition of a Ramsey equilibrium requires that a selection be made from the set of competitive equilibria. We focus on the Ramsey equilibrium that yields the highest utility for the government. Given our characterization of a competitive equilibrium, the characterization of this Ramsey equilibrium is immediate. We have

Proposition 6. *The allocations in a Ramsey equilibrium solve the following programming problem:*

$$\max \sum_{s^t} \sum_{t} \beta^t \mu(s^t)\, U(c(s^t), l(s^t)) \tag{2.23}$$

subject to (2.7) and (2.8).

For convenience, write the Ramsey allocation problem in Lagrangian form:

$$\max \sum_{t,s^t} \beta^t \mu(s^t) \left\{ W(c(s^t), l(s^t), \lambda) - \lambda U_c(s_0)\left[R_k(s_0)\,k_{-1} + R_b(s_0)\,b_{-1}\right]\right\} \tag{2.24}$$

subject to (2.7). The function W simply incorporates the implementability constraint into the maximand and is given by

$$W(c(s^t), l(s^t), \lambda) = U(c(s^t), l(s^t)) + \lambda\left[U_c(s^t)\,c(s^t) + U_l(s^t)\,l(s^t)\right], \tag{2.25}$$

where λ is the Lagrange multiplier on the implementability constraint, (2.8). The first-order conditions for this problem imply that, for $t \geqslant 1$,

$$-\frac{W_l(s^t)}{W_c(s^t)} = F_l(s^t) \tag{2.26}$$

and

$$W_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1} \mid s^t)\, W_c(s^{t+1}) \left[1 - \delta + F_k(s^{t+1})\right] \quad \text{for} \quad t = 0, 1, 2, \ldots. \tag{2.27}$$

A property of the Ramsey allocations which is useful in our analysis of the cyclical properties of optimal fiscal policy is the following. If the stochastic process on s follows a Markov process, then from Equations (2.26) and (2.27) it is clear that the allocations from period 1 onward can be described by time-invariant allocation rules $c(k, s; \lambda)$, $l(k, s; \lambda)$, $k'(k, s; \lambda)$, and $b(k, s; \lambda)$. The period-0 first-order conditions include terms related to the initial stocks of capital and debt and are therefore different from the other first-order conditions. The period-0 allocation rules are thus different from the stationary allocation rules, which govern behavior from period 1 onward.

Thus far, we have considered a tax system with capital income taxes and labor income taxes. A wide variety of other tax systems lead to the same Ramsey allocation problem. For example, consider a tax system that includes consumption taxes, denoted $\tau_c(s^t)$, as well as labor and capital income taxes. For such a system, the implementability constraint is given by

$$\sum_{t, s^t} \beta^t \mu(s^t) \left[U_c(s^t)\, c(s^t) + U_l(s^t)\, l(s^t)\right] = \frac{U_c(s_0)}{(1 + \tau_{c0})} \left[R_k(s_0)\, k_{-1} + R_b(s_0)\, b_{-1}\right], \tag{2.28}$$

where $R_k(s_0) = 1 + [1 - \theta(s_0)][F_k(s_0) - \delta]$ and τ_{c0} is the tax rate on consumption in period 0. The first-order conditions of the competitive equilibrium with such a tax system are given by

$$-\frac{U_l(s^t)}{U_c(s^t)} = \frac{1 - \tau(s^t)}{1 + \tau_c(s^t)} F_l(s^t) \tag{2.29}$$

and

$$\frac{U_c(s^t)}{1 + \tau_c(s^t)} = \sum_{s^{t+1} \mid s^t} \beta\mu(s^{t+1} \mid s^t) \frac{U_c(s^{t+1})}{1 + \tau_c(s^{t+1})} R_k(s^{t+1}), \tag{2.30}$$

where $R_k(s^{t+1}) = 1 + [1 - \theta(s^{t+1})][F_k(s^{t+1}) - \delta]$. Inspection of these first-order conditions shows that if an allocation satisfies the implementability constraint (2.28) and the resource constraint (2.1), it can be decentralized as a competitive equilibrium under a variety of tax systems. Examples of such tax systems include those with only consumption taxes and labor income taxes and those with only consumption taxes and capital income taxes. More complicated examples include those in which tax rates on capital and labor income are required to be the same, but are allowed to be different from tax rates on consumption. The message of this analysis is that optimal tax theory implies optimal wedges between marginal rates of substitution and marginal rates of transformation and is typically silent on the detailed taxes used to implement these wedges.

Recall that with a capital and labor income tax system, we ruled out lump-sum taxes by imposing a constraint on period-0 capital income taxes. In a consumption and labor tax system, an analogous constraint is necessary. Notice that if consumption taxes are constant so that $\tau_c(s^t) = \tau_{c0}$ for all s^t and that if labor is subsidized appropriately so that $\tau(s^t) = -\tau_{c0}$, then (2.29) and (2.30) become the undistorted first-order conditions. By setting τ_{c0} arbitrarily high, it is possible to satisfy (2.28) at the lump-sum tax allocations and thus to achieve the undistorted optimum. One way to rule this out is to impose an upper bound on τ_{c0}. (There seems to be some confusion about this point in the literature.)

2.2. Capital income taxation

2.2.1. In a steady state

Here we develop the results on the optimality of zero capital income taxes in a steady state, and we consider various generalizations and qualifications for that result.

For simplicity, we consider a nonstochastic version of the model in which the stochastic shock in the production function is constant and government consumption is also constant, so $g(s^t) = g$. Suppose that under the Ramsey plan, the allocations converge to a steady state. In such a steady state, W_c is constant. Thus, from Equation (2.27),

$$1 = \beta(1 - \delta + F_k). \tag{2.31}$$

The consumer's intertemporal first-order condition (2.19) in a steady state reduces to

$$1 = \beta[1 + (1 - \theta)(F_k - \delta)]. \tag{2.32}$$

Comparing (2.31) and (2.32), we can see that in a steady state, the optimal tax rate on capital income, θ, is zero. This result is due to Chamley (1980, 1986).

A natural conjecture is that with heterogeneous consumers, a nonzero tax on capital income is optimal to redistribute income from one type of consumer to another. We examine this conjecture in an economy with two types of consumers, indexed $i = 1, 2$, whose preferences are given by

$$\sum_{t=0}^{\infty} \beta^t U^i(c_{it}, l_{it}), \tag{2.33}$$

where c_{it} and l_{it} denote the consumption and labor supply of a consumer of type i. Notice that the discount factors are assumed to be the same for both types of consumers. The resource constraint for this economy is given by

$$c_{1t} + c_{2t} + g + k_{t+1} = F(k_t, l_{1t}, l_{2t}) + (1 - \delta)\, k_t, \tag{2.34}$$

where the production function F has constant returns to scale. Notice that the production function allows for imperfect substitutability between the two types of labor and capital. For this economy, the implementability constraints for the two types of consumers $i = 1, 2$ are given by

$$\sum_t \beta^t \left(U^i_{ct} c_{it} + U^i_{lt} l_{it} \right) = U^i_{c0} \left(R_{k0} k^i_0 + R_{b0} b^i_0 \right), \tag{2.35}$$

where k^i_0 and b^i_0 denote the initial ownership of capital and debt by consumers of type i [1]. If the tax system allows tax rates on capital income and labor income to differ across consumer types, then it is straightforward to establish that the resource constraint and the two implementability constraints completely characterize a competitive equilibrium.

For the Ramsey equilibrium, we suppose that the government maximizes a weighted sum of consumers' utilities of the form

$$\omega_1 \sum_{t=0}^{\infty} \beta^t U^1(c_{1t}, l_{1t}) + \omega_2 \sum_{t=0}^{\infty} \beta^t U^2(c_{2t}, l_{2t}), \tag{2.36}$$

where the welfare weights $\omega_i \in [0, 1]$ satisfy $\omega_1 + \omega_2 = 1$. The Ramsey problem is to maximize Equation (2.36) subject to the resource constraint (2.34) and the implementability constraints (2.35). Let us define

$$W(c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2) = \sum_{i=1,2} \left[\omega_i U^i(c_{it}, l_{it}) + \lambda_i (U^i_{ct} c_{it} + U^i_{lt} l_{it}) \right] \tag{2.37}$$

for $t \geqslant 1$; and for $t = 0$, W equals the right-hand side of Equation (2.37) evaluated at $t = 0$ minus $\sum \lambda_i U^i_{c0}(R_{k0} k^i_0 + R_{b0} b^i_0)$. Here λ_i is the Lagrange multiplier on the implementability constraint for the consumer of type i. The Ramsey problem is, then, to maximize

$$\sum_{t=0}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2)$$

subject to the resource constraint (2.34). The first-order conditions for this problem imply that

$$W_{cit} = \beta W_{cit+1} \left(1 - \delta + F_{kt+1}\right) \qquad \text{for} \quad i = 1, 2 \quad \text{and} \quad t = 0, 1, 2, \ldots. \tag{2.38}$$

In a steady state, W_{ct} is a constant, and thus

$$1 = \beta(1 - \delta + F_k), \tag{2.39}$$

which as before implies that the steady-state tax on capital income is zero. This result is due to Judd (1985).

[1] Notice in (2.35) the initial assets are denoted k^i_0 and b^i_0, while in (2.8) they are denoted k_{-1} and b_{-1}. Throughout the chapter in deterministic environments initial assets have a subscript 0, while in stochastic environments initial assets have a subscript −1. This unfortunate inconsistency stems from the tradition of using k_{t+1} to denote the capital choise in period t.

This result also holds when type-1 consumers are workers who supply labor, cannot save or borrow, and hold no initial capital, while type-2 consumers are capitalists who own all the capital but supply no labor. Then we replace Equation (2.35) for type-1 consumers with

$$U^1_{ct}c_{1t} + U^1_{lt}l_{1t} = 0 \quad \text{for all } t. \tag{2.40}$$

Notice that in the solution to the Ramsey problem, Equation (2.38) continues to hold for the capitalists, and thus the steady-state tax on capital income is zero. Notice also that this result shows that even if the Ramsey planner puts zero weight on the capitalists, taxing capital in the long run is still not optimal. The reason is that the cumulative distortion of the capital taxes on intertemporal margins makes even the workers prefer the static distortion of marginal rates that comes from labor income taxes.

Now suppose that the tax system does not allow tax rates on either capital income or labor income to differ across consumer types. These restrictions on the tax system imply extra constraints on the allocations that can be achieved in a competitive equilibrium. Consider first the restriction that tax rates on capital income do not differ across consumers. To derive the restrictions that this adds to the Ramsey problem, consider the consumers' intertemporal first-order conditions, which can be written as

$$\frac{U^i_{ct}}{U^i_{ct+1}} = \beta\,[1 + (1 - \theta_{t+1})(F_{kt+1} - \delta)]. \tag{2.41}$$

Since the right-hand side of Equation (2.41) does not vary with i, the restriction

$$\frac{U^1_{ct}}{U^1_{ct+1}} = \frac{U^2_{ct}}{U^2_{ct+1}} \tag{2.42}$$

holds in any competitive equilibrium. Thus Equation (2.42) is an extra restriction that must be added to the Ramsey problem. Let μ_t denote the Lagrange multiplier on (2.42). Defining

$$V(x_t, c_{1t+1}, c_{2t+1}, l_{1t+1}, l_{2t+1}, \mu_t) = W(x_t) + \mu_t\left(\frac{U^1_{ct}}{U^1_{ct+1}} - \frac{U^2_{ct}}{U^2_{ct+1}}\right),$$

where $x_t = (c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2)$, we can use the same argument as before, with V replacing W, to conclude that the steady-state tax on capital income is zero.

Consider next the restriction that tax rates on labor income do not vary across consumers. Consider the consumers' first-order conditions for labor supply, which can be written as

$$-\frac{U^i_{lt}}{U^i_{ct}}\frac{1}{F_{lit}} = 1 - \tau_t. \tag{2.43}$$

Since the right-hand side of Equation (2.43) does not vary with i, the restriction

$$\frac{U_{lt}^1}{U_{ct}^1}\frac{U_{ct}^2}{U_{lt}^2} = \frac{F_{l1t}}{F_{l2t}} \tag{2.44}$$

holds in any competitive equilibrium and thus must be added to the Ramsey problem. We proceed as before and, with no confusion, define

$$V(x_t, k_t, \mu_t) = W(x_t) + \nu_t \left(\frac{U_{lt}^1}{U_{ct}^1}\frac{U_{ct}^2}{U_{lt}^2} - \frac{F_{l1t}}{F_{l2t}} \right), \tag{2.45}$$

where ν_t is the Lagrange multiplier on (2.44). A first-order condition for the Ramsey problem is

$$-\beta V_{kt+1} + V_{c1t} = \beta V_{c1t+1} \left[F_{kt+1} + (1-\delta) \right].$$

In a steady state, this reduces to

$$-\beta \frac{V_k}{V_{c1}} + 1 = \beta \left[F_k + (1-\delta) \right].$$

Clearly, unless $V_k = 0$, the steady-state tax on capital income is not zero. Inspection of Equation (2.45) shows that $V_k = 0$ if and only if F_{l1t}/F_{l2t} does not depend on k. Recall that the production function is separable between k and (l_1, l_2) if F_{l1t}/F_{l2t} does not depend on k. Such separable production functions can be written in the form $F(k, l_1, l_2) = F(k, H(l_1, l_2))$ for some function H. [For some related discussion, see Stiglitz (1987).]

This analysis of fiscal policy with restrictions suggests that other restrictions on tax rates may lead to nonzero taxation of capital income in a steady state even in a representative agent model. Consider an economy with identical consumers, and consider another restriction on the tax system, namely, that tax rates are equal for all periods. Suppose, for example, that taxes on capital income are restricted to being equal for all periods from period 1 onward, while labor tax rates are unrestricted. Using the consumer's first-order conditions, we see that

$$\frac{U_{ct}}{U_{ct+1}} = \beta \left[1 + (1 - \theta_{t+1})(F_{kt+1} - \delta) \right] \tag{2.46}$$

together with the restriction that $\theta_{t+1} = \theta_1$ for all $t > 1$, implies the following restriction across allocations:

$$\left[\frac{U_{ct}}{\beta U_{ct+1}} - 1 \right] \frac{1}{F_{kt+1} - \delta} = \left[\frac{U_{c0}}{\beta U_{c1}} - 1 \right] \frac{1}{F_{k1} - \delta} \quad \text{for all} \quad t > 1. \tag{2.47}$$

The appropriate Ramsey problem, then, has constraints of the form (2.47), as well as the implementability constraint and the resource constraint. We leave it to the reader (as a difficult exercise) to show that, under suitable conditions, the optimal tax on capital income is positive, even in the steady state. The intuition is that with no such restrictions, it is optimal to front-load the capital income taxes by initially making them large and positive and eventually setting them to zero. When taxes are constant, it is optimal to try to balance these two opposing forces and make them somewhat positive throughout.

The discussion of the extra constraints on the Ramsey problem implied by restrictions on the tax system suggests the following observation. Zero capital income taxation in the steady state is optimal if the extra constraints do not depend on the capital stock and is not optimal if these constraints depend on the capital stock (and, of course, are binding).

Another possible restriction is that there is some upper bound on tax rates. Suppose, for example, that capital tax rates are at most 100 percent. Then in addition to satisfying the analogs of conditions (2.7) and (2.8), an allocation must satisfy an extra condition to be part of a competitive equilibrium. Rewrite the analog of Equation (2.19) as

$$U_{ct} = \beta U_{ct+1}(1 + (1 - \theta_{t+1})(F_{kt+1} - \delta)). \tag{2.48}$$

Then if an allocation satisfies

$$F_{kt+1} \geqslant \delta \tag{2.49}$$

and $\theta_{t+1} \leqslant 1$, Equation (2.48) implies that

$$U_{ct} \geqslant \beta U_{ct+1}. \tag{2.50}$$

Thus we can simply impose (2.50) as an extra constraint. With this constraint, for suitable restrictions on the utility function, the optimal policy is to set the tax rate to its upper bound for a finite number of periods. After that, the tax takes on an intermediate value for one period and is zero thereafter.

2.2.2. *In a non-steady state*

In the preceding subsection, we showed that in a variety of circumstances, in a steady state, the optimal tax on capital income is zero. Sometimes one can establish a much stronger result, namely, that optimal capital income taxes are close to zero after only a few periods. [See Chamley (1986), for example.] In this subsection, we show that for a commonly used class of utility functions, it is not optimal to distort the capital accumulation decision in period 1 or thereafter.

The class of utility functions we consider are of the form

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + V(l). \tag{2.51}$$

One might conjecture that if utility functions of this form have the property that optimal capital income taxes are exactly zero after period 1, then for utility functions that are in some sense close to these, keeping capital income tax rates close to zero after period 1 is also optimal.

To motivate our result, we write the consumer's first-order condition for capital as

$$q_{t+1}(1-\delta+F_{kt+1})-1=q_{t+1}\theta_{t+1}(F_{kt+1}-\delta), \tag{2.52}$$

where $q_{t+1}=\beta U_{ct+1}/U_{ct}$ is the Arrow–Debreu price of a unit of consumption in period $t+1$ in units of consumption in period t. Now, in an undistorted equilibrium, the consumer's first-order condition has the same left-hand side as (2.52), but the right-hand side equals zero. Thus the right-hand side of (2.52) measures the size of the wedge between the distorted and undistorted first-order conditions for capital accumulation in period t. We then have

Proposition 7. *For utility functions of the form (2.51), it is not optimal to distort the capital accumulation decision at period 1 or thereafter. Namely, the optimal tax rate on capital income received in period t is zero for $t \geqslant 2$. Equivalently,*

$$q_{t+1}\theta_{t+1}(F_{kt+1}-\delta)=0 \quad \textit{for all} \quad t\geqslant 1. \tag{2.53}$$

Proof: For $t \geqslant 1$, the first-order conditions for the Ramsey problem imply that

$$1=\beta\frac{W_{ct+1}}{W_{ct}}(1-\delta+F_{kt+1}), \tag{2.54}$$

where W is given in Equation (2.25). For $t \geqslant 1$, the consumer's first-order conditions for capital imply that

$$1=\beta\frac{U_{ct+1}}{U_{ct}}\left[1+(1-\theta_{t+1})(F_{kt+1}-\delta)\right]. \tag{2.55}$$

Now, for any utility function of the form (2.51), we can easily show that

$$\frac{W_{ct+1}}{W_{ct}}=\frac{U_{ct+1}}{U_{ct}}. \tag{2.56}$$

Substituting Equation (2.56) into (2.54) and subtracting the resulting equation from (2.55) gives the result. □

Proposition 7 implies that the tax rate on capital income received in period t is zero for $t \geqslant 2$ and is typically different from zero in period 1. In period 0, of course, the tax rate is fixed by assumption.

This result is much stronger than the standard Chamley result, which refers to steady states, and the logic behind this result is actually more connected to the uniform tax results than to the rest of the Chamley-type results. To see this, suppose that the tax system allows the government to levy only proportional taxes on consumption and labor income. For this tax system, the analog of the restriction of the initial tax on capital income is that the initial consumption tax is given. Then with a utility function of the form (2.51), consumption taxes are constant in all periods except period 0.

In a continuous-time version of the deterministic model with instantaneous preferences given by Equation (2.51), Chamley (1986) shows that the tax rate on capital income is constant for a finite length of time and is zero thereafter. The reason for Chamley's different result is that he imposes an exogenous upper bound on the tax rate on capital income. If we impose such an upper bound, the Ramsey problem must be amended to include an extra constraint to capture the restrictions imposed by this upper bound. (See the example in Subsection 2.2.1.) In the deterministic version of the model, with preferences given by Equation (2.51), the tax rate is constant at this upper bound for a finite number of periods, there is one period of transition, and the tax rate is zero thereafter.

In the stochastic version of the model, constraints of this kind can also be imposed. One can derive an upper bound endogenously. Consider the following scenario. At the end of each period t, consumers can rent capital to firms for use in period $t+1$ and pay taxes on the rental income from capital in period $t+1$. Or consumers can choose to hide the capital, say, in their basements. If they hide it, the capital depreciates and is not available for use in $t+1$. Thus, if they hide it, there is no capital income, and consumers pay zero capital taxes.

2.3. Cyclical properties

2.3.1. Debt taxation as a shock absorber

In this subsection, we illustrate how state-contingent returns on debt can be used as a shock absorber in implementing optimal fiscal policy. One interpretation of state-contingent returns on debt is that the government issues debt with a non-state-contingent return and uses taxes or partial defaults to make the return state-contingent. We show that under reasonable assumptions, during periods of high government expenditures such as wartime, the government partially defaults on debt, and during periods of low government expenditures such as peacetime, it does not. Many of the insights here are developed in Lucas and Stokey (1983) and Chari et al. (1991).

We illustrate this shock-absorber role in a version of our model of fiscal policy with no capital. Specifically, we assume that $F(k,l,z)=zl$, where z is a technology shock. The resource constraint is

$$c(s^t)+g(s^t)=z(s^t)\,l(s^t)$$

and the consumer's first-order condition for labor supply is

$$-\frac{U_l(s^t)}{U_c(s^t)}=\left[1-\tau(s^t)\right]z(s^t). \tag{2.57}$$

The first-order condition for debt is

$$U_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1})\, U_c(s^{t+1})\, R_b(s^{t+1})/\mu(s^t). \tag{2.58}$$

For convenience, let $H(s^t) = U_c(s^t)\, c(s^t) + U_l(s^t)\, l(s^t)$. Notice that the resource constraint and the consumer's first-order condition imply $H(s^t) = U_c(s^t)[\tau(s^t)\, z(s^t)\, l(s^t) - g(s^t)]$. Thus $H(s^t)$ is the value of the (primary) government surplus at s^t in units of current marginal utility. The implementability constraint reduces to

$$\sum_{t,s^t} \beta^t \mu(s^t)\, H(s^t) = U_c(s_0)\, R_0\, b_{-1}. \tag{2.59}$$

Expression (2.17) reduces to

$$b(s^r) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^t \mu(s^t)\, H(s^t)/\beta^r \mu(s^r)\, U_c(s^r). \tag{2.60}$$

Now imagine that the government promises a non-state-contingent (gross) rate of return on government debt $\bar{R}(s^{t-1})$ and then levies a state-contingent tax $\nu(s^t)$ on the gross return on government debt. That is, $\bar{R}$ and ν satisfy

$$R_b(s^t) = \left[1 - \nu(s^t)\right] \bar{R}(s^{t-1}).$$

Consider next determining the tax rate and the return on debt. The after-tax return on debt $[1 - \nu(s^r)]\, \bar{R}(s^{r-1})$ in some period r and state s^r is obtained as follows. Multiplying the consumer budget constraint by $\beta^t \mu(s^t)\, U_c(s^t)$ and summing from period r and over all periods and states from period $r+1$ onward, we obtain the familiar requirement that the value of the government's after-tax debt obligation must equal the expected present value of government surpluses:

$$\beta^r \mu(s^r)\, U_c(s^r)\, [1 - \nu(s^r)]\, \bar{R}(s^{r-1})\, b(s^{r-1}) = \beta^r \mu(s^r)\, H(s^r) + \sum_{t=r+1} \sum_{s^t} \beta^t \mu(s^t)\, H(s^t). \tag{2.61}$$

While the after-tax returns are determined by Equation (2.61), the gross returns and the tax rates on debt are not separately determined. The reason is that both consumers and the government care only about the after-tax return on debt. Obviously, there are many ways of combining (before-tax) gross returns and tax rates to give the same after-tax returns. More formally, if v and $\bar{R}$ support a particular set of Ramsey allocations, so do any v and R' that satisfy

$$[1 - v(s^r)]\, R'(s^{r-1}) = [1 - v(s^r)]\, \bar{R}(s^{r-1}) \quad \text{for all} \quad r \text{ and } s^r. \tag{2.62}$$

We resolve this indeterminacy by normalizing gross returns $\bar{R}$ to satisfy

$$\bar{R}(s^{t-1}) = \frac{\mu(s^{t+1})\, U_c(s^{t-1})}{\sum \beta\mu(s^t)\, U_c(s^t)}, \tag{2.63}$$

where $s^t = (s^{t-1}, s_t)$. Notice that the normalization in Equation (2.63), together with (2.62), implies that tax rates on debt satisfy

$$\sum_{s^t} \mu(s^t \mid s^{t-1})\, U_c(s^t) v(s^t) = 0 \quad \text{for all} \quad t \text{ and } s^{t-1}, \tag{2.64}$$

where $\mu(s^t \mid s^{t-1}) = \mu(s^{t+1})/\mu(s^t)$.

Next, we derive the first-order conditions for the Ramsey problem. Let λ denote the Lagrange multiplier on the implementability constraint. The first-order conditions for $t \geqslant 1$ imply that

$$z(s^t)\, U_c(s^t) + U_l(s^t) + \lambda \left[z(s^t) H_c(s^t) + H_l(s^t)\right] = 0, \tag{2.65}$$

where $H_c(s^t)$ and $H_l(s^t)$ denote the derivatives of $H(s^t)$. For $t = 0$, the first-order condition is the same as (2.65), except that the right-hand side is replaced by

$$\lambda\, [z(s_0)\, U_{cc}(s_0) + U_{cl}(s_0)]\, [1 - v(s_0)]\, R_{-1} b_{-1}.$$

These first-order conditions can be used to prove the following proposition:

Proposition 8. *For $t \geqslant 1$, there exist functions $\bar{c}$, $\bar{l}$, and $\bar{\tau}$ such that the Ramsey consumption allocations, labor allocations, and labor tax rates can be written as*

$$c(s^t) = \bar{c}(g_t, z_t), \qquad l(s^t) = \bar{l}(g_t, z_t), \qquad \tau(s^t) = \bar{\tau}(g_t, z_t).$$

Moreover, if $b_{-1} = 0$, then $c(s_0)$, $l(s_0)$, and $\tau(s_0)$ are given by these same functions.

Proof: For $t \geqslant 1$, substituting from the resource constraint for $l(s^t)$ into (2.65) gives one equation of the form $F(c, g, z; \lambda) = 0$. Solving this gives the Ramsey consumption allocation as a function of the current levels of government consumption, the technology shock, and the multiplier. From the resource constraint and from Equation (2.57), we know that the labor allocation and the labor tax rate are a function of these same variables. For $t = 0$, the same procedure gives allocations and the labor tax rate in period 0 as a function of g_0, z_0, and λ. We can solve for λ by substituting the allocations into the implementability constraint (2.59). Clearly, for $b_{-1} = 0$, the first-order conditions for $t = 0$ are the same as the first-order conditions for $t \geqslant 1$. □

Proposition 8 says that the allocations and the labor tax rate depend only on the current realizations of the shocks and not separately on the entire history of realizations. This proposition implies that labor tax rates inherit the stochastic properties of the underlying shocks. For example, if government consumption is i.i.d. and the technology shock is constant, then tax rates are i.i.d. (This result does not hold in general with capital.)

If government consumption is persistent, then so are the tax rates. This result of standard neoclassical theory sharply contrasts with claims in the literature that optimal taxation requires labor tax rates to follow a random walk. [See Barro (1979), Mankiw (1987), and our discussion in the following subsection, 2.3.2.]

To understand the nature of the Ramsey outcomes, we consider several examples. In all of them, we let technology shocks be constant, so $z(s_t) = 1$ for all s^t. We begin with a deterministic example that illustrates how Ramsey policies smooth distortions over time.

Example 1. Consider an economy that alternates between wartime and peacetime. Specifically, let $g_t = G$ for t even and $g_t = 0$ for t odd. Let the initial indebtedness $R_{-1}b_{-1} = 0$. We will show that the government runs a deficit in wartime and then pays off the debt in the ensuing peacetime. Consider the first-order condition for the Ramsey problem in peacetime. Using the resource constraint, we have that

$$(1+\lambda)[U_c(0)+U_l(0)]+\lambda c[U_{cc}(0)+2U_{cl}(0)+U_{ll}(0)] = 0,$$

where the partial derivatives are evaluated at $g_t = 0$. By strict concavity, the second bracketed term is negative. Since the multiplier λ is positive, the first term is positive. From Equation (2.57), we have that $U_c + U_l = \tau U_c$. Thus $\tau(0) > 0$. When we use Proposition 8, Equation (2.59) implies that $H(G)+\beta H(0) = 0$, which can be rewritten as

$$U_c(G)[\tau(G)\,l(g) - G]+\beta U_c(0)[\tau(0)\,l(0)] = 0.$$

That is, the discounted value of the government surplus is zero over the two-period cycle of government consumption. Since the government runs a surplus in peacetime, it must run a deficit in wartime. Here the government sells debt $b(G) = G - \tau(G)\,l(G)$ in wartime and retires debt in the next peacetime. The gross return on the debt from wartime to peacetime is $R(G) = U_c(G)/\beta U_c(0)$, and with our normalization, the tax rate on debt is always zero.

Example 2. Consider an economy that has recurrent wars with long periods of peace in between. Specifically let $g_t = G$ for $t = 0, T, 2T, \ldots$, and let $g_t = 0$ otherwise. Let the initial indebtedness $R_{-1}b_{-1} = 0$. Again, by Proposition 8, the budget is balanced over each T-period cycle, that is,

$$U_c(G)[\tau(G)\,l(G) - G] + \beta U_c(0)[\tau(0)\,l(0)] + \cdots + \beta^{T-1} U_c(0)[\tau(0)\,l(0)] = 0.$$

Here, as in Example 1, the government runs a deficit in wartime and a constant surplus in peacetime. The war debt is slowly retired during the following $T-1$ periods of peace. The government enters the next wartime with zero debt and restarts the cycle. Specifically, the government issues debt of level $G - \tau(G)\,l(G)$ in wartime. In the first period of peacetime, the government sells

$$\frac{U_c(G)}{\beta U_c(0)}[G - \tau(G)\,l(G)] - \tau(0)\,l(0)$$

units of debt. In the second period, it sells

$$\frac{U_c(G)}{\beta^2 U_c(0)}[G - \tau(G)\,l(G)] - \frac{\tau(0)\,l(0)}{\beta} - \tau(0), l(0),$$

and so on. Clearly, the debt is decreasing during peacetime.

Example 3. Here we will illustrate the shock-absorbing nature of optimal debt taxes. Let government spending follow a two-state Markov process with a symmetric transition matrix with positive persistence. The two states are $g_t = G$ and $g_t = 0$. Let

$$\pi = \mathrm{Prob}\{g_{t+1} = G \mid g_t = G\} = \mathrm{Prob}\{g_{t+1} = 0 \mid g_t = 0\} > \tfrac{1}{2}.$$

Therefore, the probability of staying at the same state is greater than the probability of switching states. Let $g_0 = G$, and let the initial indebtedness $R_{-1}b_{-1}$ be positive.

The government's period t budget constraint is

$$b(s^t) = \left[1 - \nu(s^t)\right] \bar{R}(s^{t-1})\, b(s^{t-1}) + g(s^t) - \tau(s^t)\, l(s^t). \tag{2.66}$$

From Proposition 8, the allocations and the labor tax rates depend only on the current realization g_t for $t \geqslant 1$. Under the Markov assumption, Equations (2.60) and (2.63) imply that the end-of-period debt $b(s^t)$ and the interest rate $R(s^t)$ depend only on the current realization g_t. From Equation (2.66), we know that the tax rate on debt depends on the current and the previous realizations. Let $b(g_t)$, $R(g_t)$, and $\nu(g_{t-1}, g_t)$ denote the end-of-period debt, the gross interest rate, and the tax rate on debt. For a large class of economies, we can prove the following proposition:

Proposition 9. *Suppose that in the solution to the Ramsey problem, $H(0) > H(G) > 0$; that is, the value of government surpluses is larger in peacetime than in wartime, the* *government's debt is always positive, the marginal utility of consumption is greater in wartime $U_c(G)$ than in peacetime $U_c(0)$, and both $b(G)$ and $b(0)$ are positive. Then*

$$\nu(0,G) > \nu(G,G) > 0 > \nu(0,0) > \nu(G,0). \tag{2.67}$$

That is, the debt tax rates are most extreme in periods of transition: they are highest in transitions from peacetime to wartime and lowest in transitions from wartime to peacetime. Furthermore, debt is taxed in wartime and subsidized in peacetime.

Remark: It is possible to show that the assumptions in this proposition are satisfied for a large class of economies if the initial debt is sufficiently large.

Proof: Let $V(G)$ and $V(0)$ denote the expected present value of government surpluses when the economy is in state G and state 0, respectively. These surpluses are given by the left-hand side of Equation (2.61) multiplied by the marginal utility of consumption in that state, which can be written recursively as

$$V(G) = H(G) + \beta[\pi V(G) + (1-\pi)\,V(0)], \tag{2.68}$$
$$V(0) = H(0) + \beta[\pi V(0) + (1-\pi)\,V(G)]. \tag{2.69}$$

Solving these, we obtain

$$V(G) = \frac{\beta(1-\pi)\,H(0) + (1-\beta\pi)\,H(G)}{D}, \tag{2.70}$$
$$V(0) = \frac{\beta(1-\pi)\,H(G) + (1-\beta\pi)\,H(0)}{D}, \tag{2.71}$$

where $D = (1-\beta\pi)^2 - \beta^2(1-\pi)^2 > 0$. From Equation (2.60), we obtain

$$b(G) = \frac{\beta[\pi V(G) + (1-\pi)\,V(0)]}{U_c(G)}, \tag{2.72}$$
$$b(0) = \frac{\beta[\pi V(0) + (1-\pi)\,V(G)]}{U_c(0)}, \tag{2.73}$$

and from Equation (2.63), we obtain

$$R(G) = \frac{U_c(G)}{\beta[\pi U_c(G) + (1-\pi)\,U_c(0)]}, \tag{2.74}$$
$$R(0) = \frac{U_c(0)}{\beta[\pi U_c(0) + (1-\pi)\,U_c(G)]}. \tag{2.75}$$

Combining these, we obtain expressions for the before-tax obligations of the government:

$$R(G)\,b(G) = \frac{\pi V(G) + (1-\pi)\,V(0)}{\pi U_c(G) + (1-\pi)\,U_c(0)}, \tag{2.76}$$
$$R(0)\,b(0) = \frac{\pi V(0) + (1-\pi)\,V(G)}{\pi U_c(0) + (1-\pi)\,U_c(G)}. \tag{2.77}$$

Since $H(G) < H(0)$, Equations (2.70) and (2.71) imply that $V(G) < V(0)$. Using this result, $\pi > \frac{1}{2}$, and $U_c(0) < U_c(G)$, we can see that Equations (2.76) and (2.77) imply that

$$R(G)\, b(G) < R(0)\, b(0). \tag{2.78}$$

We can rewrite Equation (2.61) as

$$[1 - v(g_{t-1}, g_t)]\, R(g_{t-1})\, b(g_{t-1}) = \frac{V(g_t)}{U_c(g_t)}. \tag{2.79}$$

The right-hand side of Equation (2.79) depends only on the current state; thus (2.78) implies that $v(0, G) > v(G, G)$ and $v(0, 0) > v(G, 0)$. To establish Equation (2.67), we need only show that $v(G, G) > 0 > v(0, 0)$. But this follows from (2.64) and (2.79), using $V(G) < V(0)$ and $U_c(0) < U_c(G)$. □

The intuition for these results is as follows. The Ramsey policy smooths labor tax rates across states. This smoothing implies that the government runs a smaller surplus in wartime than in peacetime. With persistence in the shocks, the expected present value of surpluses starting from the next period is smaller if the economy is currently in wartime than if it is in peacetime. The end-of-period debt is, of course, just the expected present value of these surpluses. [See Equation (2.60).] Thus the end-of-period debt is smaller if the economy is in wartime than if it is in peacetime, so $b(G) < b(0)$.

As was shown in (2.78), $R(G)\, b(G) < R(0)\, b(0)$. That is, the obligations of the government if there was war in the preceding period are smaller than if there was peace. Suppose the economy is currently in wartime, so $g_t = G$. The current deficit and end-of-period debt are the same regardless of the history. Thus, if the inherited debt obligations are larger, the only way to meet the government budget constraint is to tax debt at a higher rate. So a transition from peacetime to wartime results in higher debt taxes than does a continuation of wartime. Similar intuition applies for the comparisons of transitions from wartime to peacetime with continuations of peacetime.

2.3.2. *Tax-smoothing and incomplete markets*

Here we develop Barro's (1979) result on tax-smoothing and compare it to the work of Marcet et al. (1996) on optimal taxation with incomplete markets. In a well-known paper, Barro (1979) analyzes a reduced-form model of optimal taxation. In his theoretical development, there is no uncertainty and the government chooses a sequence of tax rates τ_t on income to maximize

$$\sum_{t=0}^{\infty} \frac{U(\tau_t)\, y_t}{(1+r)^t},$$

where y_t is income in period t and r is an exogenously given interest rate, subject to budget constraints of the form

$$b_t = (1+r)\,b_{t-1} + g_t - \tau_t y_t,$$

where g_t is government spending, b_{-1} is given, and an appropriate boundedness condition on debt is imposed. These constraints are equivalent to the present value budget constraint

$$\sum_{t=0}^{\infty} \frac{\tau_t y_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{g_t}{(1+r)^t} + b_0. \tag{2.80}$$

Barro shows that in this deterministic setup, optimal tax rates are constant.

Barro goes on to assert that the analog of this result with uncertainty is that optimal taxes are a random walk. In an environment with uncertainty, the properties of optimal policy depend on the structure of asset markets. If asset markets are complete, the analogous present value budget constraint is

$$\sum_{t,s^t} \frac{\tau(s^t)\,y(s^t)}{1+r(s^t)} = \sum_{t,s^t} \frac{g(s^t)}{1+r(s^t)} + b_0. \tag{2.81}$$

With this asset structure, optimal tax rates are clearly constant across both time and states of nature. If asset markets are incomplete, then the analysis is much more complicated and depends on precise details of the incompleteness. Suppose, for example, that the only asset available to the government is non-state-contingent debt. The sequence of budget constraints for the government can be written as

$$b(s^t) = (1+r)\,b(s^{t-1}) + g(s^t) - \tau(s^t)\,y(s^t)$$

together with appropriate boundedness conditions on debt. Substituting the first-order conditions to the government's problem into the budget constraints and doing some manipulations yields

$$\sum_{t=r}\sum_{s^t} \beta^{t-r}\mu(s^t \mid s^r)\frac{U_\tau(s^t)\,y(s^t)\left[g(s^t)-\tau(s^t)\,y(s^t)\right]}{U_\tau(s^r)\,y(s^r)} = (1+r)\,b(s^{r-1}). \tag{2.82}$$

The restriction that debt is not state-contingent is equivalent to the requirement that the left-hand side of Equation (2.82) is the same for any two states in period r in the sense that for all s^{r-1},

$$\begin{aligned}&\sum_{t=r}\sum_{s^t} \beta^{t-r}\mu(s^t \mid s^r)\frac{U_\tau(s^t)\,y(s^t)\left[g(s^t)-\tau(s^t)\,y(s^t)\right]}{U_\tau(s^r)\,y(s^r)}\\ &= \sum_{t=r}\sum_{s^t} \beta^{t-r}\mu(s^t \mid s^{r'})\frac{U_\tau(s^t)\,y(s^t)\left[g(s^t)-\tau(s^t)\,y(s^t)\right]}{U_\tau(s^{r'})\,y(s^{r'})},\end{aligned} \tag{2.83}$$

where $s^r = (s^{r-1}, s_r)$ and $s^{r'} = (s^{r-1}, s_{r'})$ for all $s_r, s_{r'}$. Analyzing an economy with incomplete markets requires imposing, in addition to (2.81), an infinite number of constraints of the form (2.83). This problem has not yet been solved. An open question is whether optimal tax rates in such an environment follow a random walk.

In our general equilibrium setup, restrictions on government policy also impose extra constraints. Suppose that neither capital tax rates nor the return on debt can be made state-contingent. Then the additional restrictions that the allocation must satisfy so that we can construct a competitive equilibrium are given as follows. Substituting Equations (2.17) and (2.18) into the consumer's budget constraint yields, after some simplification,

$$\sum_{t=r}\sum_{s^t}\beta^{t-r}\mu(s^t \mid s^r)\frac{U_c(s^t)\,c(s^t)+U_l(s^t)\,l(s^t)}{U_c(s^r)} - \left\{1+[1-\bar{\theta}(s^{r-1})][F_k(s^r)-\delta]\right\}k(s^{r-1}) = \bar{R}_b(s^{r-1})\,b(s^{r-1}), \tag{2.84}$$

where $\bar{\theta}(s^{r-1})$ satisfies

$$U_c(s^{r-1}) = \sum_{s^r}\beta\mu(s^r \mid s^{r-1})U_c(s^r)\left\{1+[1-\bar{\theta}(s^{r-1})][F_k(s^r)-\delta]\right\}. \tag{2.85}$$

The requirement that the debt be non-state-contingent is, then, simply the requirement that the left-hand side of Equation (2.84) with $\bar{\theta}(s^{r-1})$ substituted from (2.85) be the same for all s_r. Furthermore, we need to impose bounds on the absolute value of the debt to ensure that the problem is well posed. We then have that if an allocation satisfies these requirements, together with the resource constraint (2.7) and the implementability constraint (2.8), a competitive equilibrium can be constructed which satisfies the restriction that neither the capital tax rate nor the return on debt be state-contingent. Clearly, computing equilibria with non-state-contingent capital taxes and return on debt is a difficult exercise.

Marcet et al. (1996) analyze an economy with incomplete markets but without capital. When government consumption is serially uncorrelated, they find that the persistence properties of tax rates are a weighted average of a random walk and a serially uncorrelated process. They also find that the allocations are close to the complete markets allocations. They argue that their results partially affirm Barro's (1979) assertion.

In Section 3, we consider a model in which debt is nominal and non-state-contingent. There we show that inflation can be used to make the real returns state-contingent and that the Ramsey allocations are identical to those in an economy with real state-contingent debt. This result is reminiscent of our result that even if debt returns are not state-contingent, as long as capital tax rates are state-contingent, the Ramsey allocations are identical to those in an economy in which all instruments are state-contingent. This feature suggests that for actual economies, judging the extent of market incompleteness can be tricky.

2.3.3. A quantitative illustration

Here we consider a standard real business cycle model and use it to develop the quantitative features of optimal fiscal policy. We follow the development in Chari et al. (1994). In quantitative stochastic growth models, preferences are usually specified to be of the form

$$U(c,l) = \frac{[c^{1-\gamma}(L-l)^{\gamma}]^{\psi}}{\psi},$$

where L is the endowment of labor. This class of preferences has been widely used in the literature [Kydland and Prescott (1982), Christiano and Eichenbaum (1992), Backus et al. (1992)]. The production technology is usually given by

$$F(k,l,z,t) = k^{\alpha}(e^{\rho t+z}l)^{1-\alpha}.$$

Notice that the production technology has two kinds of labor-augmenting technological change. The variable ρ captures deterministic growth in this change. The variable z is a technology shock that follows a symmetric two-state Markov chain with states z_l and z_h and transition probabilities $\text{Prob}(z_{t+1} = z_i \mid z_t = z_i) = \pi$ for $i = l, h$. Government consumption is given by $g_t = ge^{\rho t}$, where again ρ is the deterministic growth rate and g follows a symmetric two-state Markov chain with states g_l and g_h and transition probabilities $\text{Prob}(g_{t+1} = g_i \mid g_t = g_i) = \phi$ for $i = l, h$. Notice that without shocks to technology or government consumption, the economy has a balanced growth path along which private consumption, capital, and government consumption grow at rate ρ and labor is constant. Zhu (1992) shows that in economies of this form, setting capital income tax rates to be identically zero is not optimal. We ask whether capital tax rates are quantitatively quite different from zero.

Recall from the proof of Proposition 5 that certain policies are uniquely determined by the theory, while others are not. Specifically, the labor tax rate is determined, while the state-by-state capital tax rate and return on debt are not. From Equation (2.19), however, we know that the value of revenues from capital income taxation in period $t+1$ in terms of the period-t good is uniquely determined. To turn this variable into a tax rate, consider the ratio of the value of these revenues to the value of capital income, namely,

$$\theta^e(s^t) = \frac{\sum q(s^{t+1})\,\theta(s^{t+1})[F_k(s^{t+1}) - \delta]}{\sum q(s^{t+1})[F_k(s^{t+1}) - \delta]}, \tag{2.86}$$

where $q(s^{t+1}) = \beta\mu(s^{t+1} \mid s^t)\,U_c(s^{t+1})/U_c(s^t)$ is the price of a unit of consumption at state s^{t+1} in units of consumption at s^t. We refer to $\theta^e(s^t)$ as the *ex ante tax rate* on capital income.

Table 1
Parameter values for two models[a]

Model	Parameters and values			
Baseline model				
Preferences	$\gamma = 0.80$	$\psi = 0$	$\beta = 0.97$	$L = 5475$
Technology	$\alpha = 0.34$	$\delta = 0.08$	$\rho = 0.016$	
Markov chains for				
Government consumption	$g_l = 350$	$g_h = 402$	$\phi = 0.95$	
Technology shock	$z_l = 0.04$	$z_h = 0.04$	$\pi = 0.91$	
High risk aversion model				
Preferences	$\psi = -8$			

[a] Source: Chari et al. (1994).

Next, in defining the last variable that is uniquely determined by the theory, it is useful to proceed as follows. Imagine that the government promises a non-state-contingent rate of return on government debt $\bar{r}(s^{t-1})$ and levies a state-contingent tax $v(s^t)$ on interest payments from government debt. That is, $\bar{r}$ and v satisfy

$$R_b(s^t) = 1 + \bar{r}(s^{t-1})[1 - v(s^t)], \tag{2.87}$$

and $\sum q(s^t)v(s^t) = 0$, where $q(s^t)$ is the price of a unit of consumption at state s^t in units of consumption at state s^{t-1}. Thus $\bar{r}(s^{t-1})$ is the equilibrium rate of return on a unit purchased in period $t-1$ at s^{t-1}, which yields a non-state-contingent return $\bar{r}(s^{t-1})$ at all states s^t. It is clear from (2.21) that the theory pins down $R_k(s^t)\,k(s^{t-1}) + R_b(s^t)\,b(s^{t-1})$. Given our definition of v, it is also clear that the theory pins down the sum of the tax revenues from capital income and the interest on debt, which is given by

$$\theta(s^t)\left[F_t(s^t) - \delta\right] k(s^{t-1}) + v(s^t)\,\bar{r}(s^{t-1})\,b(s^{t-1}). \tag{2.88}$$

We transform these revenues into a rate by dividing by the income from capital and debt to obtain the *tax rate on private assets*, given by

$$\eta(s^t) = \frac{\theta(s^t)[F_k(s^t) - \delta]\,k(s^{t-1}) + v(s^t)\bar{r}(s^{t-1})\,b(s^{t-1})}{[F_k(s^t) - \delta]\,k(s^{t-1}) + \bar{r}(s^{t-1})\,b(s^{t-1})}. \tag{2.89}$$

We consider two parametrizations of this model. (See Table 1.) Our *baseline* model has $\psi = 0$ and thus has logarithmic preferences. Our *high risk aversion* model has $\psi = -8$. The remaining parameters of preferences and the parameters for technology are those used by Chari et al. (1994). We choose the three parameters of the Markov chain for government consumption to match three statistics of the postwar US data:

Table 2
Properties of the fiscal policy models[a]

Income tax rates	Percentage in models	
	Baseline	High risk aversion
Labor		
Mean	23.87	20.69
Standard deviation	0.10	0.04
Autocorrelation	0.80	0.85
Capital		
Mean	0.00	–0.06
Standard deviation	0.00	4.06
Autocorrelation	–	0.83
Private assets		
Mean	1.10	–0.88
Standard deviation	53.86	78.56
Autocorrelation	–0.01	0.02

[a] All statistics are based on 400 simulated observations. The means and standard deviations are in percentage terms. For the US economy, the tax rates are constructed as described by Chari et al. (1994). For the baseline model, the capital tax rate is zero; thus, its autocorrelation is not defined.

the average value of the ratio of government consumption to output, the variance of the detrended log of government consumption, and the serial autocorrelation of the detrended log of government consumption. We construct the Markov chain for the technology parameters by setting the mean of the technology shock equal to zero, and we use Prescott's (1986) statistics on the variance and serial correlation of the technology shock to determine the other two parameters.

For each setting of the parameter values, we simulate the Ramsey equilibrium for our economy, starting from the steady state of the deterministic versions of our models. In Table 2, we report some of the resulting properties of the fiscal variables in our models.

In the baseline model, the tax rate on labor income fluctuates very little. For example, if the labor tax rate were approximately normally distributed, then 95 percent of the time, the tax rate would fluctuate between 23.67 percent and 24.07 percent. The tax on capital income is zero. This is to be expected because with $\psi = 0$, the utility function is separable between consumption and leisure and is homothetic in consumption, and the utility function thus satisfies the conditions discussed in Subsection 2.2.2. In the baseline model, the tax on private assets has a large standard deviation. Intuitively, we know that the tax on private asset income acts as a shock absorber. The optimal tax rate on labor does not respond much to shocks to the economy. The government smooths labor tax rates by appropriately adjusting the tax on private assets in response to shocks. This variability of the tax on private assets does not distort capital accumulation, since what matters for the capital accumulation decision is the ex ante tax rate on capital income. This can be seen by manipulating the first-order condition for capital accumulation.

In Table 2, we also report some properties of the fiscal policy variables for the high risk aversion model. Here, too, the tax rate on labor income fluctuates very little. The tax rate on capital income has a mean of −0.06 percent and a standard deviation of 4.06 percent so that the tax rate is close to zero. We find this feature interesting because it suggests that, for the class of utility functions commonly used in the literature, not taxing capital income is optimal. Here, as in the baseline model, we find that the standard deviation of the tax rate on the income from private assets is large.

2.4. Other environments

2.4.1. Endogenous growth models

Thus far, we have considered fiscal policy in models in which the growth rate of the economy is exogenously given. We turn now to models in which this growth rate is determined by the decisions of agents. Our discussion is restricted to a version of the model described in Lucas (1990). Analysis of optimal policy in this model leads to a remarkable result: Along a balanced growth path, all taxes are zero. Bull (1992) and Jones et al. (1997) discuss extensions to a larger class of models.

Consider a deterministic, infinite-horizon model in which the technology for producing goods is given by a constant returns to scale production function $F(k_t, h_t l_{1t})$, where k_t denotes the physical capital stock in period t, h_t denotes the human capital stock in period t, and l_{1t} denotes labor input to goods production in period t. Human capital investment in period t is given by $h_t G(l_{2t})$, where l_{2t} denotes labor input into human capital accumulation and G is an increasing concave function. The resource constraints for this economy are

$$c_t + g + k_{t+1} = F(k_t, h_t l_{1t}) + (1 - \delta_k)\, k_t \tag{2.90}$$

and

$$h_{t+1} = h_t G(l_{2t}) + (1 - \delta_h)\, h_t, \tag{2.91}$$

where c_t is private consumption, g is exogenously given government consumption, and δ_k and δ_h are depreciation rates on physical and human capital, respectively.

The consumer's preferences are given by

$$\sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma} v(l_{1t} + l_{2t})/(1 - \sigma),$$

where v is a decreasing convex function. Government consumption is financed by proportional taxes on the income from labor and capital in the goods production sector and by debt. Let τ_t and θ_t denote the tax rates on the income from labor and capital. Government debt has a one-period maturity. Let b_{t+1} denote the number of units of debt issued in period t and $R_{bt}b_t$ denote the payoff in period t. The consumer's budget constraint is

$$c_t + k_{t+1} + b_{t+1} \leqslant (1-\tau_t)\, w_t h_t l_{1t} + R_{kt}k_t + R_{bt}b_t, \tag{2.92}$$

where $R_{kt} = 1 + (1-\theta_t)(r_t - \delta)$ is the gross return on capital after taxes and depreciation and r_t and w_t are the before-tax returns on capital and labor. Note that human capital accumulation is a nonmarket activity. The consumer's problem is to choose sequences of consumption, labor, physical and human capital, and debt holdings to maximize utility subject to (2.91) and (2.92). We assume that consumer debt holdings are bounded above and below by some arbitrarily large constants. Competitive pricing ensures that the returns to factor inputs equal their marginal products, namely, that

$$r_t = F_k(k_t, h_t l_{1t}), \tag{2.93}$$

$$w_t = F_l(k_t, h_t l_{1t}). \tag{2.94}$$

We let $x_t = (c_t, l_{1t}, l_{2t}, k_t, h_t, b_t)$ denote an allocation for consumers in period t and let $x = (x_t)$ denote an allocation for all t. The government's budget constraint is

$$b_{t+1} = R_{bt}b_t + g - \tau_t w_t h_t l_{1t} - \theta_t(r_t - \delta)\, k_t. \tag{2.95}$$

We let $\pi_t = (\tau_t, \theta_t)$ denote the government policy at period t and let $\pi = (\pi_t)$ denote the infinite sequence of policies. The initial stock of debt, b_{-1}, and the initial stock of capital, k_{-1}, are given. A competitive equilibrium is defined in the usual way. We have the following proposition.

Proposition 10. *The consumption allocation, the labor allocation, the physical and human capital allocations, the capital tax rate, and the return on debt in period 0 in a competitive equilibrium satisfy (2.90), (2.91), and*

$$\sum_{t=0}^{\infty} \beta^t c_t U_{ct} = A_0, \tag{2.96}$$

where

$$A_0 = U_{c0}\{[1 + (1-\theta_0)(F_{k0} - \delta)]\, k_0 + R_{b0}b_0\} - U_{l0}\left[l_{10} + \frac{1 - \delta_h + G(l_{20})}{G'(l_{20})}\right].$$

Furthermore, given any allocations and period-0 policies that satisfy (2.90), (2.91), (2.96), and

$$\frac{U_{lt}}{h_t G'(l_{2t})} = \frac{\beta U_{lt+1}}{h_{t+1} G'(l_{2t+1})}[1 - \delta_h + G(l_{2t+1})] + \frac{\beta U_{lt+1} l_{1t+1}}{h_{t+1}}, \tag{2.97}$$

we can construct policies, prices, and debt holdings which, together with the given allocations and period-0 policies, constitute a competitive equilibrium.

Proof: The procedure we use to derive the implementability constraint is to express the consumer budget constraint in period-0 form with the prices substituted out. Recall that in the model with exogenous growth, this procedure implied that the capital stock from period 1 onward did not appear in the implementability constraint. It turns out that when human capital is accumulable, human capital does not appear in the implementability constraint from period 1 onward either.

The consumer's first-order conditions imply that

$$\beta^t U_{ct} = \lambda_t, \tag{2.98}$$

$$-\beta^t U_{lt} = \lambda_t(1-\tau_t)\, w_t h_t, \tag{2.99}$$

$$-\beta^t U_{lt} = \mu_t h_t G'(l_{2t}), \tag{2.100}$$

$$-\mu_t + \mu_{t+1}\,[1-\delta_h + G(l_{2t+1})] + \lambda_{t+1}(1-\tau_{t+1})\, w_{t+1} l_{1t+1} = 0. \tag{2.101}$$

Multiplying Equation (2.101) by h_{t+1}, substituting for μ_t and μ_{t+1} from (2.99) and (2.100), and using Equation (2.91), we obtain

$$-\frac{\lambda_t(1-\tau_t)\, w_t h_{t+1}}{G'(l_{2t})} + \frac{\lambda_{t+1}(1-\tau_{t+1})\, w_{t+1} h_{t+2}}{G'(l_{2t+1})} + \lambda_{t+1}(1-\tau_{t+1})\, w_{t+1} l_{1t+1} h_{t+1} = 0. \tag{2.102}$$

From Equation (2.102) and a standard transversality condition, we know that

$$\sum_{t=0}^{\infty} \lambda_{t+1}(1-\tau_{t+1})\, w_{t+1} l_{1t+1} h_{t+1} = \frac{\lambda_0(1-\tau_0)\, w_0 h_1}{G'(l_{20})}. \tag{2.103}$$

Similarly, we can show that

$$\sum_{t=0}^{\infty} \lambda_{t+1} R_{kt+1} k_{t+1} = \lambda_0 k_1 + \sum_{t=1}^{\infty} \lambda_t k_t. \tag{2.104}$$

Next, we multiply the consumer budget constraint (2.92) by λ_t and sum from period 0 onward. When we use (2.103) and (2.104), (2.96) follows. To derive (2.97), we substitute (2.99) into (2.102). We leave it to the reader to prove the converse. □

The Ramsey problem is to maximize consumer utility subject to conditions (2.90), (2.91), (2.96), and (2.97). Recall that human capital accumulation occurs outside the market and cannot be taxed. In any competitive equilibrium, the Euler equation for human capital accumulation is undistorted. Therefore, there is no tax instrument that can be used to make the Euler equation for human capital accumulation hold for arbitrary allocations. In contrast, for arbitrary allocations, the Euler equation for physical capital can be made to hold by choosing the tax on capital income appropriately. This incompleteness of the tax system implies that the undistorted Euler equation for human capital accumulation is a constraint on the set of competitive allocations. We have the following proposition.

Proposition 11. *Suppose that the Ramsey allocations converge to a balanced growth path. In such a balanced growth path, all taxes are zero.*

Proof: We prove that along a balanced growth path, the first-order conditions for the Ramsey problem are the same as those for a planner who has access to lump-sum taxes. (This, of course, does not mean that the government can achieve the lump-sum tax allocations, because there are distortions along the transition path.)

Let $W(c_t, l_{1t} + l_{2t}; \lambda) = U(c_t, l_{1t} + l_{2t}) + \lambda c_t U_{ct}$, where λ is the Lagrange multiplier on (2.96). For our specified utility function,

$$W(c_t, l_{1t} + l_{2t}; \lambda) = [1 + \lambda(1 - \sigma)]\, U(c_t, l_{1t} + l_{2t}).$$

The Ramsey problem is to maximize

$$\sum \beta^t W(c_t, l_{1t} + l_{2t}; \lambda) - \lambda A_0$$

subject to (2.90), (2.91), and (2.97). Consider a relaxed problem in which we drop condition (2.97). Since the objective function in this rewritten problem from period 1 onward is proportional to that of a social planner who has access to lump-sum taxes, the solutions to the two problems are the same along a balanced growth path. This solution also satisfies condition (2.97). Thus, along a balanced growth path, the Ramsey problem has the same solution as the lump-sum tax problem. The solutions to these last two problems differ along the transition paths only because the two problems imply different allocations for period 0 and therefore for the capital stocks for the beginning of period 1. □

The reader may be concerned that this result depends on the ratio of government consumption to output going to zero. To see that this concern is not warranted, consider an extension of the model described above. Consider an environment in which the government chooses the path of government consumption optimally. To see this, suppose that the period utility function is given by $U(c_1, l_1 + l_2) + V(g)$, where V is some increasing function of government consumption. The government problem in this setup is to choose both tax rates and government consumption to maximize the consumer utility. We can solve this problem in two parts. In the first part, government consumption is taken as exogenous and tax rates are chosen optimally. In the second part, government consumption is chosen optimally. The proof described above obviously goes through for extensions of this kind. For $V(g) = ag^{1-\sigma}/(1 - \sigma)$, it is easy to show that along a balanced growth path, government consumption is a constant fraction of output.

2.4.2. *Open economy models*

So far, we have considered models of a closed economy. We turn now to considering issues that arise in an open economy. The elasticity of capital supply is likely to be much greater in an open economy than in a closed economy because in the open economy capital is mobile and can flow to the country with the highest rate of return. We consider a small open economy that takes the rates of return on saving in the rest of the world as given. In so doing, we abstract from the interesting strategic issues that arise when more than one authority sets taxes, and we abstract from general equilibrium linkages between an economy's fiscal policy and world prices.

In an open economy, in addition to the standard taxes a government can levy on its citizens, a government can tax foreign owners of factors that are located in its country. To allow this possibility, we allow there to be source-based taxes as well as residence-based taxes. *Source-based taxes* are taxes that governments levy on income generated in their country at the income's source, regardless of ownership. *Residence-based taxes* are taxes that governments levy on the income of their residents regardless of the income's source. We show that source-based taxes on capital income are zero in all periods and that, with a restriction that ensures that the economy has a steady state, residence-based taxes on capital income are zero in all periods as well. This result is much stronger than the corresponding result for closed economies. [See Razin and Sadka (1995) for some closely related work.]

Consider a model with both source-based and residence-based taxation. We model source-based taxes as those levied on a firm and residence-based taxes as those levied on consumers. Let r_t^* denote the world rental rate on capital absent any domestically levied taxes. The firm's problem is to solve

$$\max F(k_t, l_t) - (1 + \theta_{ft})\, r_t^* k_t - (1 + \tau_{ft})\, w_t l_t,$$

where θ_{ft} and τ_{ft} are the source-based tax rates on capital and labor. The first-order conditions are

$$\theta_{ft} r_t^* = F_{kt} - r_t^*, \tag{2.105}$$

$$\tau_{ft} w_t = F_{lt} - w_t. \tag{2.106}$$

Consumers solve

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{2.107}$$

subject to

$$\sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t w_t (1 - \tau_{ct})\, l_t, \tag{2.108}$$

where $p_t = \prod_{s=1}^{t}(1/R_s)$, $R_s = 1 + (1 - \theta_{cs})(r_s - \delta)$, $p_0 = 1$, θ_s and τ_s are the residence-based tax rates on capital and labor, and initial assets are set to zero for convenience. The consumer first-order conditions are summarized by

$$-\frac{U_{lt}}{U_{ct}} = w_t(1 - \tau_{ct}), \tag{2.109}$$

$$\frac{\beta U_{ct+1}}{U_{ct}} = \frac{1}{R_{t+1}}. \tag{2.110}$$

The economy-wide budget constraint (which is simply the sum of the consumer and government budget constraints) is given by

$$\sum_{t=0}^{\infty} q_t[c_t + g + k_{t+1} - (1-\delta)\,k_t] = \sum_{t=0}^{\infty} q_t F(k_t, l_t), \tag{2.111}$$

where $q_t = \prod_{s=1}^{t}(1/R_s^*)$ and $R_s^* = r_s^* + 1 - \delta$. Notice that the economy as a whole borrows and lends at the before-tax rate R_s^*, while consumers borrow and lend at the after-tax rate R_s. Intuitively, we know that any taxes on borrowing or lending levied on consumers are receipts of the government and cancel out in their combined budget constraint.

Notice also that in the closed economy models studied in earlier sections, the competitive equilibrium has consumer budget constraints, a government budget constraint, and a resource constraint. In this small open economy, there is no resource constraint, and it is convenient to replace the government budget constraint by the economy-wide budget constraint.

To derive the constraints for the Ramsey problem, substitute the consumer first-order conditions into Equation (2.108) to get the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t [U_{ct}c_t + U_{lt}l_t] = 0, \tag{2.112}$$

where we have used the fact that Equation (2.110) implies that $p_t = \beta^t U_{ct}/U_{c0}$. Next, notice that the first-order conditions of the firm and the consumer can be summarized by Equations (2.105), (2.110), and

$$-\frac{U_{lt}}{U_{ct}} = F_{lt}\frac{(1-\tau_{ct})}{(1+\tau_{ft})}. \tag{2.113}$$

Thus, for each marginal condition, there is at least one tax rate so that the tax system is complete and there are no additional constraints on the Ramsey problem. Thus, with both source- and residence-based taxes available, the Ramsey problem is to maximize Equation (2.107) subject to (2.111) and (2.112).

With purely source-based taxation, $\tau_{ct} = \theta_{ct} = 0$, so from Equation (2.110) it is clear that for such a tax system, the Ramsey problem has the additional constraint

$$\frac{\beta U_{ct+1}}{U_{ct}} = \frac{1}{R_{t+1}^*}.$$

With purely residence-based taxation, $\tau_{ft} = \theta_{ft} = 0$, so from Equation (2.105) it is clear that the Ramsey problem has the additional constraint

$$F_{kt} = r_t^*.$$

Consider the Ramsey problem when both source- and residence-based taxes are available. For convenience, write the problem as

$$\max \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \lambda)$$

subject to (2.111), where $W(c_t, l_t, \lambda) = U(c_t, l_t) + \lambda\,[U_{ct}c_t + U_{lt}l_t]$. The first-order condition for capital implies that

$$F_{kt} = r_t^*, \tag{2.114}$$

while the first-order condition for consumption implies that

$$\frac{\beta W_{ct+1}}{W_{ct}} = \frac{1}{R_{t+1}^*}. \tag{2.115}$$

From Equation (2.114) it is clear that setting $\theta_{ft} = 0$ for all t is optimal. Next, note that this small economy will have a steady state only if

$$\beta R_t^* = 1 \tag{2.116}$$

for all t. Under this parameter restriction, Equation (2.115) implies that $W_{ct} = W_{ct+1}$, and thus the Ramsey allocations are constant, so in particular, $U_{ct} = U_{ct+1}$. Equations (2.110) and (2.116) imply that $\theta_{ct} = 0$ for all t.

Under a system with only source-based taxes, the Ramsey problem is to maximize $\sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \lambda)$ subject to conditions (2.111) and (2.115). If we consider a relaxed version of this problem with the constraint (2.115) dropped, the above analysis makes clear that the solution to this relaxed problem satisfies this dropped constraint and hence solves the original problem. The first-order condition for capital then implies (2.114); hence, $\theta_{ft} = 0$ for all t.

Similarly, under a system with only residence-based taxes, the Ramsey problem is to maximize $\sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \lambda)$ subject to conditions (2.111) and (2.114). If we consider a relaxed version of this problem with the constraint (2.114) dropped, the above analysis makes clear that the solution to this relaxed problem satisfies this dropped constraint and hence solves the original problem. The first-order condition for consumption in the relaxed problem is (2.115). Under the parameter restriction (2.116), $W_{ct} = W_{ct+1}$, so $U_{ct} = U_{ct+1}$. Hence, equations (2.110) and (2.116) imply that $\theta_{ct} = 0$ for all t.

In sum:

Proposition 12. *Under a system with both source- and residence-based taxes, $\theta_{ft} = \theta_{ct} = 0$ for all t. Under a system with only source-based taxes, $\theta_{ft} = 0$ for all t. Under a system with only residence-based taxes, with the additional restriction (2.116), $\theta_{ct} = 0$ for all t.*

Notice that the Ramsey allocations from the problem with both source- and residence-based taxes can be achieved with residence-based taxes alone. With the additional restriction (2.116), the allocations from the problem with both types of taxes can be achieved with source-based taxes alone. The intuition for why source-based taxes are zero is that with capital mobility, each country faces a perfectly elastic supply of capital as a factor input and therefore optimally chooses to set capital income taxes on firms to zero. The intuition for why residence-based taxes are zero is that under (2.116) the small economy instantly jumps to a steady state, and so the Chamley-type logic applies for all t.

2.4.3. Overlapping generations models

The discussion thus far has focused on models with infinitely lived agents. There is also an extensive literature on optimal policy in overlapping generations models. [See, for example, Atkinson (1971), Diamond (1973), Pestieau (1974), and Atkinson and Sandmo (1980); the surveys by Auerbach (1985) and Stiglitz (1987); and the applied work of Auerbach and Kotlikoff (1987) and Escolano (1992).] The results in this literature are much weaker than those in standard models with infinitely lived agents. One reason is that in a life cycle model, agents have very heterogeneous preferences over the infinite stream of consumption goods. For example, in a two-period overlapping generations model, an agent of generation t values consumption goods only in periods t and $t+1$.

In this subsection, we show that tax rates on capital income in a steady state are zero if certain homotheticity and separability conditions are satisfied. This result is well known. For an exposition using the dual approach, see, for example, Atkinson and Stiglitz (1980). Here we follow the primal approach used by Atkeson et al. (1999) and Garriga (1999). In this sense, the proposition we prove is more closely connected to the results on uniform commodity taxation than to the results on zero capital taxation in infinitely lived agent economies.

We briefly develop a formulation of optimal fiscal policy in an overlapping generations model. Consider a two-period overlapping generations model with a constant population normalized to 1. The resource constraint is

$$c_{1t} + c_{2t} + k_{t+1} + g = F(k_t, l_{1t}, l_{2t}) + (1-\delta)\, k_t, \tag{2.117}$$

where c_{1t} and c_{2t} denote the consumption of a representative young agent and a representative old agent in period t, l_{1t} and l_{2t} denote the corresponding labor inputs, k_t denotes the capital stock in t, and g denotes government consumption. Each young agent in t solves the problem

$$\max U(c_{1t}, l_{1t}) + \beta U(c_{2t+1}, l_{2t+1})$$

subject to

$$c_{1t} + k_{t+1} + b_{t+1} = (1-\tau_{1t})\, w_{1t} l_{1t}$$

and

$$c_{2t+1} = (1-\tau_{2t+1})\, w_{2t+1} l_{2t+1} + [1 + (1-\theta_{t+1})(r_{t+1} - \delta)]\, k_{t+1} + R_{t+1} b_{t+1},$$

where τ_{1t} and τ_{2t} are the tax rates on the two types of labor inputs and θ_t is the tax rate on capital income. The government budget constraint is

$$\tau_{1t} w_{1t} l_{1t} + \tau_{2t} w_{2t} l_{2t} + \theta_t r_t k_t + b_{t+1} = g + R_t b_t.$$

To define an optimal policy, we need to assign weights to the utility of agents in each generation. We assume that the government assigns weight λ^t to generation t with $\lambda < 1$. Then the Ramsey problem can be written as

$$\max U(c_{20}, l_{20})/\lambda + \sum_{t=0}^{\infty} \lambda^t \left[U(c_{1t}, l_{1t}) + \beta U(c_{2t+1}, l_{2t+1})\right]$$

subject to the resource constraint for each t and

$$R(c_{1t}, l_{1t}) + \beta R(c_{2t+1}, l_{2t+1}) = 0 \quad \text{for each } t, \tag{2.118}$$

where $R(c, l) \equiv cU_c(c, l) + lU_l(c, l)$ and $U(c_{20}, l_{20})$ is the utility of the initial old. There is also an implementability constraint for the initial old, which plays no role in our steady-state analysis. Constraints (2.118) are the implementability constraints associated with each generation. It is straightforward to show that if the solution to the Ramsey problem converges to a steady state with constant allocations $(c_{1t}, l_{1t}, c_{2t+1}, l_{2t+1}, k_{t+1}) = (c_1, l_1, c_2, l_2, k)$, then the Ramsey allocations satisfy

$$\frac{1}{\lambda} = F_k + 1 - \delta. \tag{2.119}$$

In a steady state, the first-order condition for capital accumulation is

$$\frac{U_c(c_1, l_1)}{\beta U_c(c_2, l_2)} = 1 + (1 - \theta)(F_K - \delta). \tag{2.120}$$

Inspecting these equations, we see that unless

$$\frac{1}{\lambda} = \frac{U_c(c_1, l_1)}{\beta U_c(c_2, l_2)} \tag{2.121}$$

the tax rate on capital income is not zero. In general, we would not expect this condition to hold. Notice the contrast with infinitely lived representative consumer models in which, in a steady state, the marginal utility of the representative consumer $U_c(c_t, l_t)$ is constant. In an overlapping generations model, we would not expect the marginal utility of a consumer to be constant over the consumer's lifetime.

If the utility function is of the form

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + V(l) \tag{2.122}$$

then we can show the following:

Proposition 13. *If the utility function is of the form (2.122), then in a steady state, the optimal tax on capital income is zero.*

Proof: To prove this, consider the first-order conditions for the Ramsey problem for consumption evaluated at a steady state:

$$U_{c1} + \alpha_t R_{c1} = \mu_t, \tag{2.123}$$

$$\beta[U_{c2} + \alpha_t R_{c2}] = \lambda\mu_t, \tag{2.124}$$

where $\lambda^t \mu_t$ and $\lambda^t \alpha_t$ are the multipliers on (2.117) and (2.118), respectively. We can easily see that α_t and μ_t are constant in a steady state. With a utility function of the form (2.122), R_c is proportional to U_c so that (2.123) and (2.124) imply (2.121). □

The key properties used in proving this result are homotheticity of the utility function over consumption and the separability of consumption and leisure. In this sense, this proposition is more closely connected to the results on uniform commodity taxation than to the results on zero capital taxation in infinitely lived agent economies.

When $\lambda = \beta$ and $F(k, l_1, l_2) = F(k, l_1 + l_2)$ then one can show that for all strictly concave utility functions the optimal tax on capital income is zero in a steady state. [See Atkeson et al. (1999).]

3. Monetary policy

In this section, we study the properties of monetary policy in three monetary economies. Friedman (1969) argues that to be optimal, monetary policy should follow a rule: set nominal interest rates to zero. For a deterministic version of our economy, this would imply deflating at the rate of time preference. Phelps (1973) argues that Friedman's rule is unlikely to be optimal in an economy with no lump-sum taxes. Phelps' argument is that optimal taxation generally requires using all available taxes, including the inflation tax. Thus Phelps argues that the optimal inflation rate is higher than the Friedman rule implies. In this section, we set up a general framework that allows us to analyze Phelps' arguments.

We analyze them in three standard monetary economies with distorting taxes: a cash–credit model, a money-in-the-utility-function model, and a shopping-time model. The conditions for the optimality of the Friedman rule in the first two economies are analyzed by Chari et al. (1996), while those for the shopping-time model are extensively analyzed in the literature. [See Kimbrough (1986), Faig (1988), Woodford (1990), Guidotti and Végh (1993), and Correia and Teles (1996), as well as Chari et al. (1996).] In this section, we show that the Friedman rule is optimal when simple homotheticity and separability conditions are satisfied. These conditions are similar to the ones developed in the uniform taxation results in Section 1.

We explore the connection between the optimality of the Friedman rule and the intermediate-goods result. For all three monetary economies, when the homotheticity and separability conditions hold, the optimality of the Friedman rule follows from the intermediate-goods result. To prove this, we show that under such conditions, all three monetary economies can be reinterpreted as real intermediate-goods economies, and the optimality of the Friedman rule in the monetary economies follows directly from the intermediate-goods result in the reinterpreted real economies. In contrast, when these conditions do not hold, there is no such connection. To prove this, we show that when these conditions do not hold, there are two possibilities. First, there are monetary economies in which the Friedman rule holds which cannot be reinterpreted as real intermediate-goods economies. Second, there are monetary economies which can be reinterpreted as real intermediate-goods economies but in which the Friedman result does not hold.

Finally, we conduct some numerical exercises designed to develop quantitative features of optimal monetary policy. We find that if debt has nominal non-state-contingent returns, inflation can be used to make real returns state-contingent so that debt can serve as a shock absorber.

3.1. Three standard monetary models

3.1.1. Cash–credit

Consider a simple production economy populated by a large number of identical, infinitely lived consumers. In each period $t = 0, 1, \ldots$, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up to and including period t. The probability, as of period 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given.

In each period t, the economy has three goods: labor and two consumption goods, a cash good and a credit good. A constant returns to scale technology is available to transform labor $l(s^t)$ into output. The output can be used for private consumption of either the cash good $c_1(s^t)$ or the credit good $c_2(s^t)$ or for government consumption $g(s^t)$.

The resource constraint in this economy is thus

$$c_1(s^t) + c_2(s^t) + g(s^t) = l(s^t). \tag{3.1}$$

The preferences of each consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t)\, U(c_1(s^t), c_2(s^t), l(s^t)), \tag{3.2}$$

where the utility function U is strictly concave and satisfies the Inada conditions.

In period t, consumers trade money, assets, and goods in particular ways. At the start of period t, after observing the current state s_t, consumers trade money and assets in a centralized securities market. The assets are one-period, non-state-contingent nominal claims. Let $M(s^t)$ and $B(s^t)$ denote the money and the nominal bonds held at the end of the securities market trading. Let $R(s^t)$ denote the gross nominal return on these bonds payable in period $t+1$ in all states $s^{t+1} = (s^t, s_{t+1})$. Notice that the nominal return on debt is not state-contingent. After this trading, each consumer splits into a shopper and a worker. The shopper must use the money to purchase cash goods. To purchase credit goods, the shopper issues nominal claims, which are settled in the securities market in the next period. The worker is paid in cash at the end of each period.

This environment leads to the following constraint for the securities market:

$$\begin{aligned} M(s^t) + B(s^t) = {} & R(s^{t-1})\,B(s^{t-1}) + M(s^{t-1}) - p(s^{t-1})\,c_1(s^{t-1}) \\ & - p(s^{t-1})\,c_2(s^{t-1}) + p(s^{t-1})[1 - \tau(s^{t-1})]\,l(s^{t-1}), \end{aligned} \tag{3.3}$$

where p is the price of the consumption goods and τ is the tax rate on labor income. The real wage rate is 1 in this economy given our specification of technology. The left-hand side of Equation (3.3) is the nominal value of assets held at the end of securities market trading. The first term on the right-hand side is the value of nominal debt bought in the preceding period. The next two terms are the shopper's unspent cash. The fourth term is the payments for credit goods, and the last term is the after-tax receipts from labor services. We will assume that the holdings of real debt $B(s^t)/p(s^t)$ are bounded above and below by some arbitrarily large constants. Purchases of cash goods must satisfy the following *cash-in-advance constraint*:

$$p(s^t)\,c_1(s^t) \leqslant M(s^t). \tag{3.4}$$

We assume throughout that the cash-in-advance constraint holds with equality. We let $x(s^t) = (c_1(s^t), c_2(s^t), l(s^t), M(s^t), B(s^t))$ denote an allocation for consumers at s^t, and we let $x = (x(s^t))$ denote an allocation for all s^t. We let $q = (p(s^t), R(s^t))$ denote a price system for this economy. The initial stock of money M_{-1} and the initial stock of nominal debt B_{-1} are given.

Money is introduced into and withdrawn from the economy through open market operations in the securities market. The constraint facing the government in this market is

$$M(s^t) - M(s^{t-1}) + B(s^t) = R(s^{t-1})\,B(s^{t-1}) + p(s^{t-1})\,g(s^{t-1}) - p(s^{t-1})\,\tau(s^{t-1})\,l(s^{t-1}). \tag{3.5}$$

The terms on the left-hand side of this equation are the assets sold by the government. The first term on the right is the payments on debt incurred in the preceding period, the second term is the payment for government consumption, and the third term is tax receipts from labor income. Notice that government consumption is bought on credit. We let $\pi = (\tau(s^t))$ denote a policy for all s^t.

Given this description of an economy, we now define a competitive equilibrium. A *competitive equilibrium* is a policy π, an allocation x, and a price system q such that given the policy and the price system, the resulting allocation maximizes the representative consumer's utility and satisfies the government's budget constraint.

In this equilibrium, the consumer maximizes Equation (3.2) subject to (3.3), (3.4), and the bounds on debt. Money earns a gross nominal return of 1. If bonds earn a gross nominal return of less than 1, then the consumer can make profits by buying money and selling bonds. Thus, in any equilibrium, $R(s^t) \geqslant 1$. The consumer's first-order conditions imply that $U_1(s^t)/U_2(s^t) = R(s^t)$; thus in any equilibrium, the following constraint must hold:

$$U_1(s^t) \geqslant U_2(s^t). \tag{3.6}$$

This feature of the competitive equilibrium constrains the set of Ramsey allocations.

Consider now the policy problem faced by the government. As before, we assume that there is an institution or a commitment technology through which the government can bind itself to a particular sequence of policies once and for all in period 0, and we model this technology by having the government choose a policy $\pi = (\tau(s^t))$ at the beginning of time and then having consumers choose their allocations. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices are described by rules that associate allocations with government policies. Formally, allocation rules and price functions are sequences of functions $x(\pi) = (x(s^t \mid \pi))$ and $q(\pi) = (p(s^t \mid \pi), R(s^t \mid \pi))$ that map policies π into allocations and prices.

A *Ramsey equilibrium* is a policy π, an allocation rule $x(\cdot)$, and a price system $q(\cdot)$ that satisfy the following: (i) the policy π maximizes

$$\sum_{t,s^t} \beta^t \mu(s^t)\, U\big(c_1(s^t \mid \pi), c_2(s^t \mid \pi), l(s^t \mid \pi)\big)$$

subject to (3.5), with allocations given by $x(\pi)$, and (ii) for every π', the allocation $x(\pi')$ and the price system $q(\pi')$, together with the policy π', constitute a competitive equilibrium.

As is well known, if the initial stock of nominal assets held by consumers is positive, then welfare is maximized in the Ramsey problem by increasing the initial price level to infinity. If the initial stock is negative, then welfare is maximized by setting the initial price level so low that the government raises all the revenue it needs without levying any distorting taxes. To make the problem interesting, we set the initial sum of nominal assets of consumers $M_{-1} + R_{-1}B_{-1}$ to zero. For convenience, let $U_i(s^t)$ for $i = 1, 2, 3$ denote the marginal utilities at state s^t. Using standard techniques [for example, from Lucas and Stokey (1983), Chari et al. (1991), and Section 1], we can establish the implementability constraint:

Proposition 14. *The consumption and labor allocations in a competitive equilibrium satisfy conditions (3.1), (3.6), and the implementability constraint*

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[c_1(s^t)\, U_1(s^t) + c_2(s^t)\, U_2(s^t) + l(s^t)\, U_3(s^t)\right] = 0. \tag{3.7}$$

Furthermore, allocations that satisfy (3.1), (3.6), and (3.7) can be decentralized as a competitive equilibrium.

The Ramsey problem is to maximize consumer utility subject to conditions (3.1), (3.6), and (3.7). Consider utility functions of the form

$$U(c_1, c_2, l) = V(w(c_1, c_2), l), \tag{3.8}$$

where w is homothetic. We then have

Proposition 15. *For utility functions of the form (3.8), the Ramsey equilibrium has $R(s^t) = 1$ for all s^t.*

Proof: Consider for a moment the Ramsey allocation problem with constraint (3.6) dropped. We will show that under (3.8), constraint (3.6) is satisfied. Let λ denote the Lagrange multiplier on (3.7) and $\beta^t \mu(s^t)\, \gamma(s^t)$ denote the Lagrange multiplier on (3.1). The first-order conditions for $c_i(s^t)$ for $i = 1, 2$ in this problem are

$$(1+\lambda)\, U_i(s^t) + \lambda \left[\sum_{j=1}^{2} c_j(s^t)\, U_{ji}(s^t) + l(s^t)\, U_{3i}(s^t) \right] = \gamma(s^t). \tag{3.9}$$

Recall from Section 1 that a utility function which satisfies (3.8) also satisfies

$$\sum_{j=1}^{2} \frac{c_j(s^t)\, U_{j1}(s^t)}{U_1(s^t)} = \sum_{j=1}^{2} \frac{c_j(s^t)\, U_{j2}(s^t)}{U_2(s^t)}. \tag{3.10}$$

Next, dividing Equation (3.9) by U_i and noting that $U_{3i}/U_i = V_{12}/V_1$ for $i = 1, 2$, we have that

$$(1+\lambda) + \lambda \left[\sum_{j=1}^{2} \frac{c_j(s^t)\, U_{ji}(s^t)}{U_i(s^t)} + l(s^t) \frac{V_{12}(s^t)}{V_1(s^t)} \right] = \frac{\gamma(s^t)}{U_i(s^t)}. \tag{3.11}$$

Using Equation (3.10), we have that the left-hand side of (3.11) has the same value for $i = 1$ and for $i = 2$. Therefore, $U_1(s^t)/U_2(s^t) = 1$. Since the solution to the less-constrained problem satisfies (3.6), it is also a solution to the Ramsey allocation problem. From the consumer's first-order condition, we have that $U_1(s^t)/U_2(s^t) = R(s^t)$ and thus that $R(s^t) = 1$. □

Now let us relate our results to Phelps' (1973) arguments for taxing liquidity services. Phelps (1973, p. 82) argues that "if, as is often maintained, the demand for money is highly interest-inelastic, then liquidity is an attractive candidate for heavy taxation at least from the standpoint of monetary and fiscal efficiency". Our results suggest that the connection between the interest elasticity of money demand and the desirability of taxing liquidity services is, at best, tenuous.

To see this, suppose that the utility function is of the form

$$U(c_1, c_2, l) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} + V(l). \tag{3.12}$$

Then the consumer's first-order condition $U_1/U_2 = R$ becomes

$$\frac{m^{-\sigma}}{(c-m)^{-\sigma}} = R, \tag{3.13}$$

where m is real money balances and $c = c_1 + c_2$. The implied interest elasticity of money demand η is given by

$$\eta = \frac{1}{\sigma} \frac{R^{1/\sigma}}{1 + R^{1/\sigma - 1}}. \tag{3.14}$$

Evaluating this elasticity at $R = 1$ gives $\eta = 1/2\sigma$, and thus the elasticity of money demand can range from zero to infinity. Nevertheless, all preferences in this class satisfy our homotheticity and separability conditions; hence the Friedman rule is optimal. Phelps' partial equilibrium intuition does not hold up for reasons we saw in Section 1. As we noted there, in general equilibrium, it is not necessarily true that inelastically demanded commodities should be taxed heavily.

The homotheticity and separability conditions are equivalent to the requirement that the consumption elasticity of money demand is unity. To see this, consider a standard money demand specification:

$$\log m = \alpha_0 + \alpha_1 \log c + f(R),$$

where $f(R)$ is some invertible function of the interest rate. If $\alpha_1 = 1$, so the consumption elasticity of money demand is unity, this formulation implies that $m/c = e^{\alpha_0 + f(R)}$, or that there is some function h such that $h(m/c) = R$. The consumer's first-order condition is $U_1/U_2 = R$. Thus U_1/U_2 must be homogeneous of degree 0 in m and c if the consumption elasticity of money demand is unity. This formulation immediately implies the homotheticity and separability conditions.

Note two points about the generality of the result. First, restricting w to be homogeneous of degree 1 does not reduce the generality of the result, since we can write $w(\cdot) = g(f(\cdot))$, where g is monotone and f is homogeneous of degree 1, and simply reinterpret V accordingly. Second, the proof can be easily extended to economies with more general production technologies, including those with capital accumulation. To see how, consider modifying the resource constraint (3.1) to

$$f\left(c_1(s^t), c_2(s^t), g(s^t), l(s^t), k(s^t), k(s^{t-1})\right) = 0, \tag{3.15}$$

where k is the capital stock and f is a constant returns to scale function, and modifying the consumer's and the government's budget constraints appropriately. Let capital income net of depreciation be taxed at rate $\theta(s^t)$, and let capital be a credit good, although the result holds if capital is a cash good. For this economy, combining the consumer's and the firm's first-order conditions gives

$$\frac{U_1(s^t)}{U_2(s^t)} = R(s^t)\frac{f_1(s^t)}{f_2(s^t)}.$$

Thus the optimality of the Friedman rule requires that $U_1(s^t)/U_2(s^t) = f_1(s^t)/f_2(s^t)$. The constraint requiring that $R(s^t) \geqslant 1$ now implies that

$$\frac{U_1(s^t)}{U_2(s^t)} \geqslant \frac{f_1(s^t)}{f_2(s^t)}, \tag{3.16}$$

and the implementability constraint (3.7) now reads

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left[U_1(s^t)\, c_1(s^t) + U_2(s^t)\, c_2(s^t) + U_3(s^t)\, l(s^t)\right] \\ = U_c(s_0) \left\{[1 - \theta(s_0)] \left[f_k(s_0) - \delta\right]\right\} k_{-1}, \tag{3.17}$$

where k_{-1} is the initial capital stock. Since the tax on initial capital $\theta(s_0)$ acts like a lump-sum tax, setting it as high as possible is optimal. To make the problem interesting, we follow the standard procedure of fixing it exogenously. The Ramsey allocation problem is to choose allocations to maximize utility subject to conditions (3.15), (3.16), and (3.17). For preferences of the form (3.8), the analog of Equation (3.11) has the right-hand side multiplied by $f_i(s^t)$ for $i = 1, 2$. This analog implies that $U_1(s^t)/U_2(s^t) = f_1(s^t)/f_2(s^t)$, and thus the Friedman rule holds.

We now develop the connection between the optimality of the Friedman rule and the uniform taxation result. In this economy, the tax on labor income implicitly taxes consumption of the cash good and the credit good at the same rate. In Section 1, we showed that if the utility function is separable in leisure and the subutility function over consumption goods is homothetic, then the optimal policy is to tax all consumption goods at the same rate. If $R(s^t) > 1$, the cash good is effectively taxed at a higher rate than the credit good, since cash goods must be paid for immediately, but credit goods are paid for with a one-period lag. Thus, with such preferences, efficiency requires that $R(s^t) = 1$ and therefore that monetary policy follow the Friedman rule.

To make this intuition precise, consider a real barter economy with the same preferences (3.2) and resource constraint (3.1) as the monetary economy and with commodity taxes on the two consumption goods. Consider a period-0 representation of the budget constraints. The consumer's budget constraint is

$$\sum_t \sum_{s^t} q(s^t) \left\{[1 + \tau_1(s^t)]\, c_1(s^t) + [1 + \tau_2(s^t)]\, c_2(s^t)\right\} = \sum q(s^t)\, l(s^t), \tag{3.18}$$

and the government's budget constraint is

$$\sum_t \sum_{s^t} q(s^t)\, g(s^t) = \sum_t \sum_{s^t} q(s^t) \left[\tau_1(s^t)\, c_1(s^t) + \tau_2(s^t)\, c_2(s^t)\right], \tag{3.19}$$

where $q(s^t)$ is the price of goods in period t and at state s^t. A Ramsey equilibrium for this economy is defined in the obvious fashion. The Ramsey allocation problem for this barter economy is similar to that in the monetary economy, except that the barter economy has no constraint (3.6).

The consumer's first-order conditions imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = \frac{1+\tau_1(s^t)}{1+\tau_2(s^t)}.$$

Thus Ramsey taxes satisfy $\tau_1(s^t) = \tau_2(s^t)$ if and only if in the Ramsey allocation problem of maximizing Equation (3.2) subject to (3.1) and (3.7), the solution has $U_1(s^t)/U_2(s^t) = 1$. Recall from Proposition 3 in Section 1 that for utility functions of the form (3.8), the Ramsey equilibrium has $\tau_1(s^t) = \tau_2(s^t)$ for all s^t.

Thus, with homotheticity and separability in the period utility function, the optimal taxes on the two consumption goods are equal at each state. Notice that this proposition does not imply that commodity taxes are equal across states. [That is, $\tau_i(s^t)$ may not equal $\tau_j(s^r)$ for $t \neq r$ and for $i, j = 1, 2$.]

We have shown that if the conditions for uniform commodity taxation are satisfied in the barter economy, then in the associated monetary economy, the Friedman rule is optimal. Of course, since the allocations in the monetary economy must satisfy condition (3.6) while those in the barter economy need not, there are situations in which uniform commodity taxation is not optimal in the barter economy but in which the Friedman rule is optimal in the monetary economy. To see this, consider the following.

Example. Let preferences have the form

$$U(c_1, c_2, l) = \frac{c_1^{1-\sigma_1}}{1-\sigma_1} + \frac{c_2^{1-\sigma_2}}{1-\sigma_2} + V(l). \tag{3.20}$$

The first-order conditions for the Ramsey problem in the barter economy imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = \frac{c_1(s^t)^{-\sigma_1}}{c_2(s^t)^{-\sigma_2}} = \frac{1+\lambda(1-\sigma_2)}{1+\lambda(1-\sigma_1)}. \tag{3.21}$$

Clearly, $U_1(s^t) \geqslant U_2(s^t)$ if and only if $\sigma_1 \geqslant \sigma_2$. For cases in which $\sigma_1 = \sigma_2$, these preferences satisfy condition (3.6), and both uniform commodity taxation and the Friedman rule are optimal. If $\sigma_1 > \sigma_2$, then neither uniform commodity taxation nor the Friedman rule is optimal. What is optimal is to tax good 1 at a higher rate than good 2. In the barter economy, this higher taxation is accomplished by setting $\tau_1(s^t) > \tau_2(s^t)$, while in the monetary economy, it is accomplished by setting $R(s^t) > 1$. More interestingly, when $\sigma_1 < \sigma_2$, uniform commodity taxation is not optimal, but the Friedman rule is. To see this, note that when $\sigma_1 < \sigma_2$, the solution in the monetary economy that ignores the constraint $U_1(s^t) \geqslant U_2(s^t)$ violates this constraint. Thus this constraint must bind at the optimum, and in the monetary economy, $U_1(s^t) = U_2(s^t)$. Thus, in the barter economy, taxing good 1 at a lower rate than good 2 is optimal, and this is accomplished by setting $\tau_1(s^t) < \tau_2(s^t)$. In the monetary economy, taxing good 1 at a lower rate than good 2 is not feasible, since $R(s^t) \geqslant 1$, and the best feasible solution is to set $R(s^t) = 1$.

In this subsection, we have focused on the Lucas and Stokey (1983) cash–credit version of the cash-in-advance model. It turns out that in the simpler cash-in-advance model without credit goods, the inflation rate and the labor tax rate are indeterminate. The first-order conditions for a deterministic version of that model are the cash-in-advance constraint, the budget constraint, and

$$-\frac{U_{1t}}{U_{2t}} = \frac{R_t}{1-\tau_t}, \qquad \frac{1}{\beta}\frac{U_{1t}}{U_{2t}} = \frac{R_t p_t}{p_{t+1}},$$

where the period utility function is $U(c_t, l_t)$ and R_t is the nominal interest rate from period t to period $t+1$. Here, only the products $R_t/(1-\tau_t)$ and $R_t p_t/p_{t+1}$ are pinned down by the allocations. Thus the nominal interest rate, the tax rate, and the inflation rate are not separately determined. The Ramsey allocation can be decentralized in a variety of ways. In particular, trivially, both the Friedman rule and arbitrarily high rates of inflation are optimal.

3.1.2. *Money-in-the-utility-function*

In this section, we prove that the Friedman rule is optimal for a money-in-the-utility-function economy under homotheticity and separability conditions similar to those above.

Consider the following monetary economy. In this economy, labor is transformed into consumption goods according to

$$c(s^t) + g(s^t) = l(s^t). \tag{3.22}$$

(We use the same notation here as in the last subsection.) The preferences of the representative consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t)\, U(M(s^t)/p(s^t), c(s^t), l(s^t)), \tag{3.23}$$

where the utility function has the usual monotonicity and concavity properties and satisfies the Inada conditions. In period t, the consumer's budget constraint is

$$p(s^t)\,c(s^t) + M(s^t) + B(s^t) = M(s^{t-1}) + R(s^{t-1})\,B(s^{t-1}) + p(s^t)[1-\tau(s^t)]\,l(s^t). \tag{3.24}$$

The holdings of real debt $B(s^t)/p(s^t)$ are bounded below by some arbitrarily large constant, and the holdings of money are bounded below by zero. Let M_{-1} and

$R_{-1}B_{-1}$ denote the initial asset holdings of the consumer. The budget constraint of the government is given by

$$B(s^t) = R(s^{t-1})\,B(s^{t-1}) + p(s^t)\,g(s^t) - [M(s^t) - M(s^{t-1})] - p(s^t)[1 - \tau(s^t)]l(s^t). \tag{3.25}$$

A Ramsey equilibrium for this economy is defined in the obvious fashion. We set the initial stock of assets to zero for reasons similar to those given in the preceding section. Let $m(s^t) = M(s^t)/p(s^t)$ denote the real balances in the Ramsey equilibrium. Using logic similar to that in Proposition 14, we can show that the consumption and labor allocations and the real money balances in the Ramsey equilibrium solve the Ramsey allocation problem

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t)\, U(m(s^t), c(s^t), l(s^t)) \tag{3.26}$$

subject to the resource constraint (3.22) and the implementability constraint

$$\sum \beta^t \left[m(s^t)\,U_1(s^t) + c(s^t)\,U_2(s^t) + l(s^t)\,U_3(s^t)\right] = 0. \tag{3.27}$$

These two constraints, (3.22) and (3.27), completely characterize the set of competitive equilibrium allocations.

We are interested in finding conditions under which the Friedman rule is optimal. Now the consumer's first-order conditions imply that

$$\frac{U_1(s^t)}{U_2(s^t)} = 1 - \frac{1}{R(s^t)}. \tag{3.28}$$

Thus, for the Friedman rule to hold, namely, for $R(s^t) = 1$, it must be true that

$$\frac{U_1(s^t)}{U_2(s^t)} = 0. \tag{3.29}$$

Since the marginal utility of consumption goods is finite, condition (3.29) will hold only if $U_1(s^t) = 0$, that is, if the marginal utility of real money balances is zero. Intuitively, we can say that under the Friedman rule, satiating the economy with real money balances is optimal.

We are interested in economies for which preferences are not satiated with any finite level of money balances and for which the marginal utility of real money balances converges to zero as the level of real money balances converges to infinity. That is, for each c and l, $\lim_{m\to\infty} U_1(m,c,l) = 0$ and $\lim_{m\to\infty} U_2(m,c,l) > 0$. Intuitively, in such economies, the Friedman rule holds exactly only if the value of real money balances is infinite, and for such economies, the Ramsey allocation problem has no solution. To get around this technicality, we consider an economy in which the level of real money balances is exogenously bounded by a constant. We will say that the

Friedman rule is optimal if, as this bound on real money balances increases, the associated nominal interest rates in the Ramsey equilibrium converge to one.

With this in mind, we modify the Ramsey allocation problem to include the constraint

$$m(s^t) \leqslant \bar{m}, \tag{3.30}$$

where $\bar{m}$ is a finite bound. Consider preferences of the form

$$U(m, c, l) = V(w(m, c), l), \tag{3.31}$$

where w is homothetic. We then have

Proposition 16. *If the utility function is of the form (3.31), then the Friedman rule is optimal.*

Proof: The Ramsey allocation problem is to maximize Equation (3.23) subject to (3.22), (3.27), and (3.30). Consider a less-constrained version of this problem in which constraint (3.30) is dropped. Let $\beta^t \mu(s^t)\gamma(s^t)$ and λ denote the Lagrange multipliers on constraints (3.22) and (3.27). The first-order conditions for real money balances and consumption are

$$(1+\lambda)\, U_1(s^t) + \lambda\left[m(s^t)\, U_{11}(s^t) + c(s^t)\, U_{21}(s^t) + l(s^t)\, U_{31}(s^t)\right] = 0 \tag{3.32}$$

and

$$(1+\lambda)\, U_2(s^t) + \lambda\left[m(s^t)\, U_{12}(s^t) + c(s^t)\, U_{22}(s^t) + l(s^t)\, U_{32}(s^t)\right] = \gamma(s^t). \tag{3.33}$$

Since the utility function satisfies condition (3.31), it follows (as in Section 1) that

$$\frac{m(s^t)\, U_{11}(s^t) + c(s^t)\, U_{21}(s^t)}{U_1(s^t)} = \frac{m(s^t)\, U_{12} + c(s^t)\, U_{22}(s^t)}{U_2(s^t)}. \tag{3.34}$$

Using the form of Equation (3.31), we can rewrite conditions (3.32) and (3.33) as

$$(1+\lambda) + \lambda\left[\frac{m(s^t)\, U_{11}(s^t) + c(s^t)\, U_{21}(s^t)}{U_1(s^t)} + l(s^t)\frac{V_{21}(s^t)}{V_1(s^t)}\right] = 0 \tag{3.35}$$

and

$$(1+\lambda) + \lambda\left[\frac{m(s^t)\, U_{12}(s^t) + c(s^t)\, U_{22}(s^t)}{U_2(s^t)} + l(s^t)\frac{V_{21}(s^t)}{V_1(s^t)}\right] = \frac{\gamma(s^t)}{U_2(s^t)}. \tag{3.36}$$

From Equation (3.34), we know that

$$\frac{\gamma(s^t)}{U_2(s^t)} = 0 \tag{3.37}$$

in the less-constrained problem. Hence the associated $m(s^t)$ is arbitrarily large, and thus for any finite bound $\bar{m}$, the constraint (3.30) binds in the original problem. The result then follows from (3.28). □

Again, restricting w to be homogeneous does not reduce the generality of the result.

Clearly, the Friedman rule is optimal for some preferences which do not satisfy condition (3.31). Consider

$$U(m, c, l) = \frac{m^{1-\sigma_1}}{1-\sigma_1} + \frac{c^{1-\sigma_2}}{1-\sigma_2} + V(l). \tag{3.38}$$

Note that for cases in which $\sigma_1 \neq \sigma_2$, Equation (3.38) does not satisfy condition (3.31). The first-order condition for the Ramsey problem for money balances $m(s^t)$, when the upper bound on money balances is ignored, is

$$[1 + \lambda(1 - \sigma_1)]\, m(s^t)^{-\sigma_1} = 0. \tag{3.39}$$

Unless the endogenous Lagrange multiplier λ just happens to equal $(\sigma_1 - 1)^{-1}$, Equation (3.38) implies that the Friedman rule is optimal.

In related work, Woodford (1990) considers the optimality of the Friedman rule within the restricted class of competitive equilibria with constant allocations and policies. Woodford shows that if consumption and real balances are gross substitutes, then the Friedman rule is not optimal. Of course, there are functions that satisfy our homotheticity and separability assumptions which are gross substitutes, for example,

$$U(m, c, l) = \frac{m^{1-\sigma}}{1-\sigma} + \frac{c^{1-\sigma}}{1-\sigma} + V(l).$$

The reason for the difference in the results arises from the difference in the implementability constraints. Woodford's problem is

$$\max U(m, c, l) \tag{3.40}$$

subject to

$$c + g \leqslant l, \tag{3.41}$$

$$U_1 m + U_2 c + U_3 l = (1 - \beta)\, U_1, \tag{3.42}$$

where (3.42) is the implementability constraint associated with a competitive equilibrium with constant allocations. The first-order conditions for our problem are similar to those for Woodford's problem, except that his include derivatives of the right-hand side of condition (3.42). Notice that in Woodford's problem, if $\beta = 1$ and preferences satisfy our homotheticity and separability conditions, then the Friedman rule is optimal.

Notice, too, that if the model had state variables, such as capital, then constant policies would not typically imply constant allocations. To analyze the optimal constant monetary policy for such an economy, we would analyze a problem similar to that in Equation (3.26) with extra constraints on allocations that capture these restrictions. [These restrictions would be similar in spirit to those in (2.47).]

3.1.3. Shopping-time

In this subsection, we prove the optimality of the Friedman rule in a shopping-time monetary economy under appropriate homotheticity and separability conditions.

Consider a monetary economy along the lines of Kimbrough (1986). Labor is transformed into consumption goods according to

$$c(s^t) + g(s^t) \leqslant l(s^t). \tag{3.43}$$

The preferences of the representative consumer are given by

$$\sum_t \sum_{s^t} \beta^t \mu(s^t)\, U\big(c(s^t), l(s^t) + \phi(c(s^t), M(s^t)/p(s^t))\big), \tag{3.44}$$

where U is concave, $U_1 > 0$, $U_2 < 0$, $\phi_1 > 0$, and $\phi_2 < 0$. The function $\phi(c_1, M/p)$ describes the amount of time needed to obtain c units of the consumption good when the consumer has M/p units of real money balances. We assume that $\phi_1 > 0$ so that with the same amount of money, more time is needed to obtain more consumption goods. We also assume that $\phi_2 < 0$ so that with more money, less time is needed to obtain the same amount of consumption goods. The budget constraints of the consumer and the government are the same as (3.24) and (3.25).

The Ramsey equilibrium is defined in the obvious fashion. Let $m(s^t) = M(s^t)/p(s^t)$ and set the initial nominal assets to zero; we can then show that the consumption and labor allocations and the real money balances in the Ramsey equilibrium solve the problem

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t)\, U\big(c(s^t), l(s^t) + \phi(c(s^t), m(s^t))\big)$$

subject to condition (3.43) and

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) \left\{ c(s^t) \left[U_1(s^t) + \phi_1(s^t)\, U_2(s^t) \right] + l(s^t)\, U_2(s^t) + m(s^t)\, \phi_2(s^t)\, U_2(s^t) \right\} = 0. \tag{3.45}$$

From the consumer's first-order conditions, we know that $R(s^t) = 1$ if and only if $\phi_2 = 0$. We then have

Proposition 17. *If ϕ is homogeneous of degree k and $k \geqslant 1$, then the Friedman rule is optimal.*

Proof: The first-order conditions for the Ramsey problem with respect to $m(s^t)$ and $l(s^t)$ are given by

$$U_2\phi_2 + \lambda\left[cU_{12}\phi_2 + U_{22}\phi_2(\phi_1 c + \phi_2 m + l) + U_2\phi_2 + U_2(\phi_{12}c + \phi_{22}m)\right] = 0 \tag{3.46}$$

and

$$U_2 + \lambda\left[cU_{12} + U_{22}(\phi_1 c + \phi_2 m + l) + U_2\right] + \gamma = 0, \tag{3.47}$$

where γ is the multiplier on the resource constraint and we have dropped reference to s^t.

Suppose first that $\phi_2 \neq 0$ so that the optimal policy does not follow the Friedman rule. Then, from Equations (3.46) and (3.47), we have that

$$-\frac{\lambda U_2(\phi_{12}c + \phi_{22}m)}{\phi_2} + \gamma = 0. \tag{3.48}$$

Now, under the condition that $\phi(c, m)$ is homogeneous of degree k and $k \geqslant 1$, we have that $\phi_2(ac, \gamma m) = a^{k-1}\phi_2(c, m)$. Differentiating with respect to a and evaluating at $a = 1$, we have that $c\phi_{12} + m\phi_{22} = (k-1)\phi_2$, and thus

$$\frac{c\phi_{12} + m\phi_{22}}{\phi_2} \geqslant 0. \tag{3.49}$$

Since $\lambda \geqslant 0$, $U_2 < 0$, and $\gamma > 0$, conditions (3.48) and (3.49) contradict each other. □

Note that this proof does not go through if $\phi(c, m)$ is homogeneous of degree less than 1. Using the dual approach, however, Correia and Teles (1996) prove that the Friedman rule is optimal for this shopping-time economy when $\phi(c, m)$ is homogeneous of any degree.

3.2. From monetary to real

In this subsection, we examine the relationship between the optimality of the Friedman rule and the intermediate-goods result developed in Section 1. The relationship is the following. First, if the homotheticity and separability conditions hold, then in the three monetary models we have studied, the optimality of the Friedman rule follows from the intermediate-goods result. Second, if these conditions do not hold, then in all three economies, the optimality of the Friedman rule and the intermediate-goods result are not connected.

To establish these results, we proceed as follows. We begin by setting up the notation for a simple real intermediate-goods economy and review the intermediate-goods result for that economy. We then show that when our homotheticity and separability conditions hold, the cash–credit goods and the money-in-the-utility-function economies can be reinterpreted as real economies with intermediate goods. For these two monetary economies, we establish that the optimality of the Friedman rule in the monetary economy follows from the intermediate-goods result in the reinterpreted real economy. It is easy to establish a similar result for the shopping-time economy. This proves the first result.

Next, we consider monetary economies which do not satisfy our conditions. We establish our second result with a couple of examples. We start with an example in which the monetary economy can be reinterpreted as a real intermediate-goods economy but in which the Friedman rule does not hold in the monetary economy. We then give an example of a monetary economy in which the Friedman rule does hold, but this economy cannot be reinterpreted as a real intermediate-goods economy.

The cash–credit economy can be reinterpreted as a real production economy with intermediate goods. Under our homotheticity and separability assumptions, the period utility is $U(w(c_{1t}, c_{2t}), l_t)$ and the resource constraint is

$$c_{1t} + c_{2t} + g_t = l_t. \tag{3.50}$$

Since the gross nominal interest rate cannot be less than unity, the allocations in the monetary economy must satisfy

$$w_1(c_{1t}, c_{2t}) \geqslant w_2(c_{1t}, c_{2t}). \tag{3.51}$$

The reinterpreted economy is an infinite sequence of real static economies. In each period, the economy has two intermediate goods z_{1t} and z_{2t}, a final private consumption good x_t, labor l_t, and government consumption g_t. The intermediate goods z_{1t} and z_{2t} in the real economy correspond to the final consumption goods c_{1t} and c_{2t} in the monetary economy. The period utility function is $U(x_t, l_t)$. The technology set for producing the final good x_t is given by

$$f^1(x_t, z_{1t}, z_{2t}, l_t) = x_t - w(z_{1t}, z_{2t}) \leqslant 0, \tag{3.52}$$

$$f^2(x_t, z_{1t}, z_{2t}, l_t) = w_2(z_{1t}, z_{2t}) - w_1(z_{1t}, z_{2t}) \leqslant 0, \tag{3.53}$$

while the technology for producing the intermediate goods and government consumption is given by

$$h(z_{1t}, z_{2t}, g_t, l_t) = z_{1t} + z_{2t} + g_t - l_t \leqslant 0. \tag{3.54}$$

The real economy and the monetary economy are obviously equivalent. The intermediate-goods result for the real economy is that the Ramsey allocations satisfy production efficiency. For this economy, because the marginal rate of transformation between z_1 and z_2 is 1 in the intermediate-goods technology, production efficiency requires that

$$\frac{w_1}{w_2} = 1. \tag{3.55}$$

Recall that in the monetary economy, the Friedman rule is optimal when Equation (3.55) holds. Thus the intermediate-goods result in the real economy implies the optimality of the Friedman rule in the monetary economy.

Does this implication hold more generally? Whenever the monetary economy can be reinterpreted as an intermediate-goods economy, is the Friedman rule optimal in the monetary economy? No. Suppose that the utility function $U(c_1, c_2, l)$ is of the separable form $V(w(c_1, c_2), l)$, but that it does not have a representation in which w exhibits constant returns to scale. Suppose that w instead exhibits decreasing returns. For example, suppose that $w(c_1, c_2) = (c_1 + k)^{\alpha} c_2^{1-\alpha}$, where k is a constant. In the intermediate-goods reinterpretation, the constant k can be thought of as a scarce factor inelastically supplied by the consumer. The intermediate-goods result holds, provided that the returns to the scarce factor are fully taxed away. If the returns to the scarce factor cannot be taxed, then the intermediate-goods result does not hold. It is easy to show that the Friedman rule is not optimal in the monetary economy. In a sense, the Friedman rule is not optimal because in the monetary economy, there is no sensible interpretation under which the parameter k can be taxed.

Next, one might ask, Is it true that whenever the Friedman rule is optimal in the monetary economy, there exists an analogous intermediate-goods economy? Again, no. Consider, for example, Ramsey allocation problems in which the constraint $U_1 \geqslant U_2$ binds, but in which the utility function is not separable in consumption and leisure. The Friedman rule is optimal, but the monetary economy cannot be reinterpreted as an intermediate-goods economy.

In this subsection, we have shown that under our homotheticity and separability assumptions, the optimality of the Friedman rule follows from the optimality of uniform commodity taxation. We have also shown that the optimality of the Friedman rule follows from the intermediate-goods result. These findings are not inconsistent because the uniform taxation result actually follows from the intermediate-goods result. (See Section 1.)

The construction of the intermediate-goods economy for the money-in-the-utility-function economy is straightforward. Recall that in the monetary economy, under our homotheticity and separability conditions, the period utility function is $U(w(m_t, c_t), l_t)$ and the resource constraint is $c_t + g_t = l_t$. The reinterpreted economy is again an infinite sequence of real static economies. In each period, the economy has two intermediate goods z_{1t} and z_{2t}, a final private consumption good x_t, labor l_t, and government consumption g_t. The intermediate goods z_{1t} and z_{2t} correspond to money m_t and the consumption good c_t in the monetary economy, respectively. The technology set for producing the final good x_t is given by

$$f(x_t, z_{1t}, z_{2t}, l_t) = x_t - w(z_{1t}, z_{2t}) \leqslant 0.$$

The technology set for producing intermediate goods and consumption is given by

$$h(x_t, z_{1t}, z_{2t}, l_t) = z_{2t} + g_t - l_t \leqslant 0.$$

The real and monetary economies are obviously equivalent. Production efficiency in the intermediate-goods economy requires that the marginal rates of transformation between z_1 and z_2 in the two technologies be equated. Since the marginal rate of transformation between z_1 and z_2 in the intermediate-goods technology is zero ($h_2/h_3 = 0$), we have $w_1/w_2 = 0$. Thus production efficiency in the intermediate-goods economy implies optimality of the Friedman rule in the monetary economy.

3.3. Cyclical properties

We turn now to some quantitative exercises which examine the cyclical properties of optimal monetary policy in our cash–credit goods model. For some related work, see Cooley and Hansen (1989, 1992).

In these exercises, we consider preferences of the form

$$U(c, l) = \{[c^{1-\gamma}(L-l)^{\gamma}]^{\psi} - 1\}/\psi,$$

where L is the endowment of labor and

$$c = \left[(1-\sigma)\, c_1^{\nu} + \sigma c_2^{\nu}\right]^{1/\nu}.$$

The technology shock z and government consumption both follow the same symmetric two-state Markov chains as in the model in Section 2.

In the *baseline* model, for preferences, we set the discount factor $\beta = 0.97$; we set $\psi = 0$, which implies logarithmic preferences between the composite consumption good and leisure; and we set $\gamma = 0.80$. These values are the same as those in Christiano and Eichenbaum (1992). The parameters σ and ν are not available in the literature, so we estimate them using the consumer's first-order conditions. These conditions imply that $U_{1t}/U_{2t} = R_t$. For our specification of preferences, this condition can be manipulated to be

$$\frac{c_{2t}}{c_{1t}} = \left(\frac{\sigma}{1-\sigma}\right)^{1/(1-\nu)} R_t^{1/(1-\nu)}. \tag{3.56}$$

With a binding cash-in-advance constraint, c_1 is real money balances and c_2 is aggregate consumption less real money balances. We measure all the variables with US data: real money balances by the monetary base, R_t by the return on three-month Treasury bills, and consumption by consumption expenditures. Taking logs in Equation (3.56) and running a regression using quarterly data for the period 1959–1989 gives $\sigma = 0.57$ and $\nu = 0.83$.

Our regression turns out to be similar to those used in the money demand literature. To see this, note that Equation (3.56) implies that

$$\frac{c_{1t}}{c_{1t}+c_{2t}} = \left[1 + \left(\frac{\sigma}{1-\sigma}\right)^{1/(1-\nu)} R_t^{1/(1-\nu)}\right]^{-1}. \tag{3.57}$$

Taking logs in Equation (3.57) and then taking a Taylor's expansion yields a money demand equation with consumption in the place of output and with the restriction that the coefficient on consumption is 1. Our estimates imply that the interest elasticity of money demand is 4.94. This estimate is somewhat smaller than estimates obtained when money balances are measured by M1 instead of the base.

Table 3
Properties of the cash–credit goods monetary models

Rates	Percentage in models		
	Baseline	High risk aversion	I.I.D.
Labor income tax			
Mean	20.05	20.18	20.05
Standard deviation	0.11	0.06	0.11
Autocorrelation	0.89	0.89	0.00
Correlation with shocks			
Government consumption	0.93	−0.93	0.93
Technology	−0.36	0.35	−0.36
Output	0.03	−0.06	0.02
Inflation			
Mean	−0.44	4.78	−2.39
Standard deviation	19.93	60.37	9.83
Autocorrelation	0.02	0.06	−0.41
Correlation with shocks			
Government consumption	0.37	0.26	0.43
Technology	−0.21	−0.21	−0.70
Output	−0.05	−0.08	−0.48
Money growth			
Mean	−0.70	4.03	−2.78
Standard deviation	18.00	54.43	3.74
Autocorrelation	0.04	0.07	0.00
Correlation with shocks			
Government consumption	0.40	0.28	0.92
Technology	−0.17	−0.20	−0.36
Output	0.00	−0.07	0.02

We set the initial real claims on the government so that, in the resulting stationary equilibrium, the ratio of debt to output is 44 percent. This is approximately the ratio of US federal government debt to GNP in 1989.

For the second parametrization, we set $\psi = -8$, which implies a relatively *high degree of risk aversion*. For the third, we set $\psi = 0$ and make both technology shocks and government consumption *i.i.d.*

In Table 3, we report the properties of the labor tax rate, the inflation rate, and the money growth rate for these three parametrizations of our cash–credit goods model. In all three, the labor tax rate inherits the persistence properties of the underlying shocks (as it did in Subsection 2.3.1).

Consider the inflation rate and the money growth rate. Recall that for these cash–credit goods monetary models, the nominal interest rate is identically zero. Table 3 shows that the average inflation rate and the money growth rate are roughly zero. This result may, at first glance, be puzzling to readers familiar with the implications of the Friedman rule in deterministic economies. If government consumption and the technology shock were constant, then the price level and the money stock would fall at the rate of time preference, which is 3 percent per year. In a stochastic economy, the inflation rate and the money growth rate vary with consumption. Therefore, the mean inflation rate depends not only on the rate of time preference, but also on the covariance of the inflation rate and the intertemporal marginal rate of substitution. Specifically, the consumer's first-order conditions imply that

$$1 = \beta E_t[U_1(s^{t+1})/U_1(s^t)]\, R(s^t)\, p(s^t)/p(s^{t+1}), \tag{3.58}$$

where E_t is the expectation conditional on s^t.

Under the Friedman rule, $R(s^t) = 1$. Using the familiar relationship that the expectation of a product of two random variables is the sum of the product of the expectations of these variables and their covariance in Equation (3.58) and rearranging, we obtain

$$E_t[p(s^t)/p(s^{t+1})] = \frac{1/\beta - \mathrm{cov}_t(p(s^t)/p(s^{t+1}),\, U_1(s^{t+1})/U_1(s^t))}{E_t[U_1(s^{t+1})/U_1(s^t)]}. \tag{3.59}$$

In a stationary deterministic economy, Equation (3.59) reduces to $p_t/p_{t+1} = 1/\beta$ so that following the Friedman rule is equivalent to deflating at the rate of time preference. In our stochastic economy, periods of higher-than-average consumption (and hence lower-than-average marginal utility) are also periods of lower-than-average inflation (and hence higher-than-average $p(s^t)/p(s^{t+1})$). Thus the covariance term in Equation (3.59) is negative. Taking unconditional expectations on both sides of Equation (3.59), we have that following the Friedman rule implies that $E[p(s^t)/p(s^{t+1})] > 1/\beta$.

For all three parametrizations, the autocorrelation of the inflation rate is small or negative. Thus, in each, the inflation rate is far from a random walk. The correlations of inflation with government consumption and with the technology shock have the expected signs. Notice that these correlations have opposite signs, and in the baseline and high risk aversion models, this leads to inflation having essentially no correlation with output. The most striking feature of the inflation rates is their volatility. In the baseline model, for example, if the inflation rate were normally distributed, it would be higher than 20 percent or lower than –20 percent approximately a third of the time. The inflation rates for the high risk aversion model are even more volatile. The money growth rate has essentially the same properties as the inflation rate. The inflation rates in these economies serve to make the real return on debt state-contingent. In this sense,

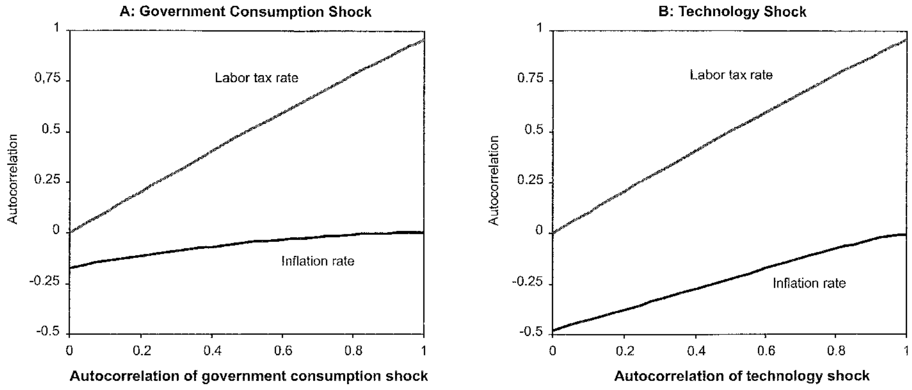

Fig. 1. Persistence plots of inflation rates and labor tax rates versus shocks to government consumption and technology: (a) government consumption shock; (b) technology shock.

debt, together with appropriately chosen monetary policy, acts as a shock absorber. The inflation rates are volatile in these economies because we have not allowed for any other shock absorbers.

The results for the high risk aversion model are basically similar to those for the baseline model, with two exceptions. First, the correlation of the labor tax rate with the shocks has opposite signs from the baseline model. Changing the risk aversion changes the response of the marginal rate of substitution of consumption and leisure to the shocks. This change in the response alters the sign of the correlation. Second, and more significantly, the inflation rate in the high risk aversion model is substantially more variable and has a higher mean than the inflation rate in the baseline model. The reason for the difference is that the higher variability in the inflation rate increases the covariance term in Equation (3.59) and thus increases the average inflation rate.

The results for the i.i.d. model are similar to those for the baseline model, with two exceptions. In the i.i.d. model, the autocorrelation of the labor tax rate and the autocorrelation of the inflation rate are quite different from their values in the baseline model. The labor tax rate has basically the same persistence properties as the underlying shocks – and so does the price level. A standard result is that if a random variable is i.i.d., its first difference has an autocorrelation of -0.5. The inflation rate is approximately the first difference of the log of the price level. Thus, in our i.i.d. model, the autocorrelation of the inflation rate is close to -0.5.

We investigated the autocorrelation properties of the labor tax rate and the inflation rate as we varied the autocorrelation (or persistence) of the underlying shocks. We found that the autocorrelation of both the labor tax rate and the inflation rate increased as we increased the persistence of the underlying shocks. Specifically, we set one shock at its mean value and varied the persistence of the other shock. In Figure 1A, we plot the autocorrelations of the labor tax rate and the inflation rate as functions of the autocorrelation of government consumption. In Figure 1B, we plot the autocorrelations

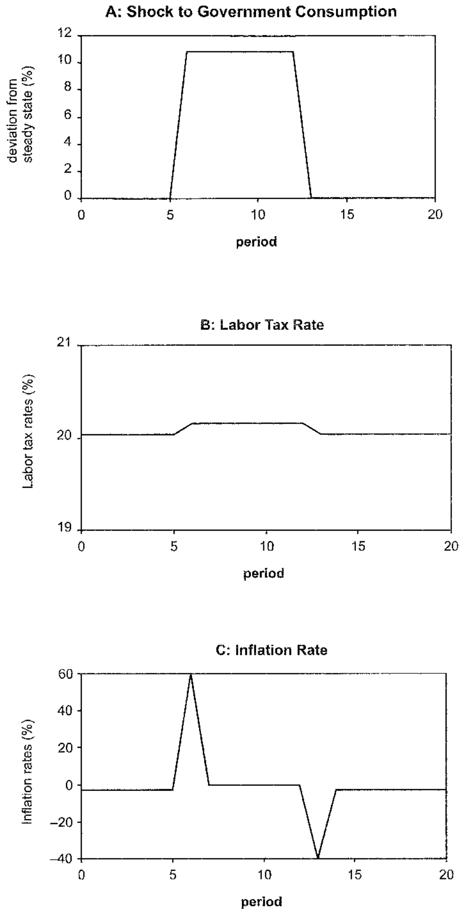

Fig. 2. Responses to government consumption shock: (a) the shock to government consumption; (b) labor tax rate; (c) inflation rate.

of these rates as functions of the autocorrelation of the technology shock. In both of these figures, the autocorrelations of the rates increase as the autocorrelations of the shocks increase.

The inflation rate and money growth rate are close to i.i.d. These rates are positively correlated with government consumption and negatively correlated with the technology shock. As with the labor tax rate, these shocks have opposing effects on inflation and similar effects on output, implying that the correlation of inflation and money growth with output is roughly zero.

To gain some intuition for the labor tax rates and the inflation rates, we simulated a version of the baseline model in which technology shocks were set equal to their mean levels so that the only source of uncertainty is government consumption. In Figure 2, we report a 20-period segment of our realizations. In Figure 2A, we see the shock to government consumption: this variable is constant at a low level from period 0 to period 5, is then high from period 6 to period 12, and returns to its low level from period 13 to period 20. In Figure 2B, we plot the optimal labor tax rates. These tax rates follow the same pattern: they are constant between periods 0 and 5, when government consumption is low; are slightly higher between periods 6 and 12, when government consumption is higher; and return to their low level between periods 13 and 20, when government consumption returns to its low level. The striking feature is that labor tax rates hardly fluctuate in response to the shocks. In Figure 2C, we plot the optimal inflation rate. There is a large inflation rate from period 5 to period 6, when government consumption rises to its higher level, and a large deflation rate from period 12 to period 13, when government consumption falls. In periods without a change in government consumption, the inflation rate is roughly zero.

To gain an appreciation of the magnitude of the shock absorber role of inflation, it is useful to trace through the effects of shocks on government debt, revenues, and expenditures. Using the analog of Proposition 7 for this economy, we can show that the allocations $c(s^t), l(s^t)$, real money balances $m(s^t)$, and real debt $B(s^t)/p(s^t)$ depend only on the current state s_t, while the change in the price level $p(s^t)/p(s^{t-1})$ depends on s_{t-1} and s_t. We write these functions as $c(s_t)$, $l(s_t)$, $m(s_t)$, $b(s_t)$, and $\pi(s_{t-1}, s_t)$.

Consider now the government's budget constraint under the assumption that the economy in period $t-1$ is at the mean level of government consumption and the mean level of the technology shock. Denote this state by $\bar{s}$. Consider two scenarios. Suppose first that the economy in period t stays at $\bar{s}$. We can rearrange the government's budget constraint to obtain

$$b(\bar{s}) = \frac{R(\bar{s})\, b(\bar{s})}{\pi(\bar{s},\bar{s})} + \frac{1}{\pi(\bar{s},\bar{s})}\left[g(\bar{s}) - \tau(\bar{s})\, z(\bar{s})\, l(\bar{s})\right] - \left[m(\bar{s}) - \frac{m(\bar{s})}{\pi(\bar{s},\bar{s})}\right]. \tag{3.60}$$

Suppose next that the economy in period t switches to state s', where g is higher and the technology shock is at its average level. The government budget constraint can then be written as

$$b(s') = \frac{R(\bar{s})\, b(\bar{s})}{\pi(\bar{s},s')} + \frac{1}{\pi(\bar{s},s')}\left[g(\bar{s}) - \tau(\bar{s})\, z(\bar{s})\, l(\bar{s})\right] - \left[m(s') - \frac{m(\bar{s})}{\pi(\bar{s},s')}\right]. \tag{3.61}$$

In both (3.60) and (3.61), the term on the left is the new debt. The first term on the right is the inherited debt obligations net of the inflation tax. The second term on the right is the inflation-adjusted government deficit from period $t-1$. The inflation adjustment reflects that both government consumption and tax revenues are credit goods that are paid for with a one-period lag. The last term on the right is the seigniorage. Subtracting Equation (3.60) from (3.61) gives the accounting identity

$$\underset{(-23)}{\Delta\,\text{New debt}} \equiv \underset{(-19)}{\Delta\,\text{Value of old debt}} + \underset{(+1)}{\Delta\,\text{Tanzi effect}} - \underset{(+5)}{\Delta\,\text{Seigniorage}}, \tag{3.62}$$

where the Tanzi effect is the difference in the inflation-adjusted deficit. [See Tanzi (1977).] (The numbers in parentheses are discussed below.)

We can use our simulation to calculate the terms in Equation (3.62). We normalize the economy so that mean output is 100 units of the consumption good. We consider an innovation in government consumption of 1 unit of this consumption good. This innovation leads to an increase in the present value of government consumption of 28 units of the consumption good. The numbers in parentheses below the terms in Equation (3.62) are the changes in the relevant terms in units of the consumption good. The value of the old debt falls by 19 units because the sharp rise in inflation acts as a tax on inherited nominal debt. In our economy, the government debt is positive when the shocks are at their mean values. The government runs a surplus to pay the interest on the debt. A rise in the inflation rate erodes the value of the nominal surplus, leading to a Tanzi effect of 1 unit. The large inflation rate is, of course, due to a sharp rise in the money growth rate. The government collects 5 units of additional seigniorage by printing this money. Thus the new debt falls by 23 units. Since the present value of government consumption rises by 28 units, the present value of labor tax revenues needs to rise by only 5 units. This result implies that labor tax rates need to change by only a small amount.

In this economy, the volatile inflation rate acts as a shock absorber, allowing the labor tax rate to be smooth. In essence, the government pays for 82 percent (23/28) of the increase in the present value of government spending by increasing the price level sharply, which taxes inherited nominal claims, and for only 18 percent (5/28) by increasing the present value of labor taxes.

Note that our autocorrelation results are quite different from those of Mankiw (1987). Using a partial equilibrium model, he argues that optimal policy implies that both labor taxes and inflation should follow a random walk. It might be worth investigating whether there are any general equilibrium settings that rationalize Mankiw's argument.

In the models considered in this subsection, nominal asset markets are incomplete because returns on nominal debt are not state-contingent. The government, however, can insure itself against adverse shocks by varying the ex post inflation rate appropriately. These variations impose no welfare costs because private agents care only about the expected inflation rate and not about the ex post inflation rate. A useful extension might be to consider models in which ex post inflation imposes welfare costs. An open question is whether optimal inflation rates will be roughly a random walk if the welfare costs are high enough.

4. Conclusion

In this chapter we have analyzed how the primal approach can be used to answer a fundamental question in macroeconomics: How should fiscal and monetary policy be set over the long run and over the business cycle? We use this approach to draw a number of substantive lessons for policymaking. Obviously, these lessons depend on the details of the specific models considered. By and large we have considered environments without imperfections in private markets, such as externalities and missing markets. In models with such imperfections, optimal policy not only must be responsive to the efficiency considerations we have emphasized, but also must attempt to cure the private market imperfections.

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