The military requirements document specifies dynamic response mainly through the pole-zero requirements. These have been summarized here so that the reader may evaluate some of the controller designs described later. Much additional information covering other aspects of flying qualities is available in the requirements document, and it is essential reading for anyone with other than a casual interest in this field.

4.4 STABILITY AUGMENTATION

Most high-performance commercial and military aircraft require some form of stability augmentation. Some military aircraft are actually unstable and would be virtually impossible to fly without an automatic control system. The SAS typically uses sensors to measure the body-axes angular rates of the vehicle and feeds back processed versions of these signals to servomechanisms that drive the aerodynamic control surfaces. In this way an aerodynamic moment proportional to angular velocity and its derivatives can be generated and used to produce a damping effect on the motion. If the basic mode is unstable or if it is desired to change both damping and natural frequency independently, additional feedback signals will be required, as we will see.

Stability augmentation systems are conventionally designed separately for the longitudinal dynamics and the lateral-directional dynamics, and this is made possible by the decoupling of the aircraft dynamics in most flight conditions. In the next two subsections aircraft model dynamics will be used to describe the design of the various augmentation systems.

Pitch-Axis Stability Augmentation

The purpose of a pitch SAS is to provide satisfactory natural frequency and damping for the short-period mode. This mode involves the variables alpha and pitch rate; feedback of these variables to the elevator actuator will modify the frequency and damping. Figure 4.4-1 shows the arrangement; if the short-period mode is lightly damped but otherwise adequate, only pitch-rate feedback is required. If the frequency and damping are both unsatisfactory or the mode is unstable, alpha feedback is necessary. The phugoid mode will be largely unaffected by this feedback. Outer feedback

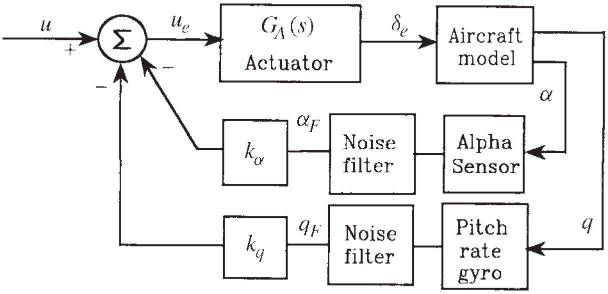

Figure 4.4-1 Pitch-axis stability augmentation.

control loops will often be closed around the pitch SAS to provide, for example, autopilot functions. Automatic adjustment of the augmentation (inner) loop feedback gains may be arranged when the outer feedback loops are engaged, so that the overall performance is optimal.

A physical understanding of the effect of alpha feedback follows from the explanation of pitch stiffness in Chapter 2. A statically unstable aircraft has a pitching moment curve with a positive slope over some range(s) of alpha. If perturbations in alpha are sensed and fed back to the elevator servo to generate a restoring pitching moment, the slope of the pitching moment curve can be made more negative in the region around the operating angle of attack. Furthermore, the overall pitching moment curve and the trimmed elevator deflection will not be affected, thus preserving the trim-drag and maneuverability characteristics that the designer built into the basic airplane design.

The angle-of-attack measurement may be obtained from the pitot-static air data system, or a small "wind vane" mounted on the side of the aircraft forebody and positioned (after much testing and calibration) to measure alpha over a wide range of flight conditions. Two sensors may be used, on opposite sides of the aircraft, to provide redundancy and possibly to average out measurement errors caused by sideslipping. In addition, it may be necessary to compute (in real time) a "true" angle of attack from the "indicated angle of attack," airspeed, and Mach number, in order to relate the freestream angle of attack of the airframe to the direction of the flowfield at the sensor position. The signal from the alpha sensor is usually noisy because of turbulence, and a noise filter is used to reduce the amount of noise injected into the control system.

Alpha feedback is avoided if possible because of the difficulty of getting an accurate, rapidly responding, noise-free measurement and because of the vulnerability of the sensor to mechanical damage. Noise from the alpha sensor can make it difficult to achieve precise pointing (e.g., for targeting), so the amount of alpha feedback is normally restricted.

The pitch-rate sensor is normally a mechanical gyroscopic device arranged to measure the (inertial) angular rate around the pitch axis. The location of the gyro must be chosen very carefully to avoid picking up the vibrations of the aircraft structure. At a node of an idealized structural oscillation there is angular motion but no displacement, and at an antinode the converse is true. Thus, the first choice for the rate gyro location is an antinode corresponding to the most important structural mode. Flight tests must then be used to adjust the position of the gyros. A bad choice of gyro locations can adversely affect handling qualities or, in extreme cases, cause oscillations in the flight control systems (AFWAL-TR-84-3105, 1984). The gyro filter shown in Figure 4.4-1 is usually necessary to remove noise and/or cancel structural-mode vibrations.

The sign convention that has been adopted in this book (see Chapter 3) means that a positive elevator deflection leads to a negative pitching moment. Therefore, for convenience, a phase reversal will be included between the elevator actuator and the control surface in each example, so that the positive-gain root-locus algorithm can be used for design.

Example 4.4-1: The Effects of Pitch Rate and Alpha Feedback The longitudinal (four-state) Jacobian matrices for the F-16 model in the nominal flight condition in Table 3.6-3 are

$$A = \begin{array}{c} \begin{array}{cccc} \quad v_T \quad & \quad \alpha \quad & \quad \theta \quad & \quad q \quad \end{array} \\ \begin{bmatrix} -1.9311E-02 & 8.8157E+00 & -3.2170E+01 & -5.7499E-01 \\ -2.5389E-04 & -1.0189E+00 & 0.0000E+00 & 9.0506E-01 \\ 0.0000E+00 & 0.0000E+00 & 0.0000E+00 & 1.0000E+00 \\ 2.9465E-12 & 8.2225E-01 & 0.0000E+00 & -1.0774E+00 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \delta_e \\ \begin{bmatrix} 1.7370E-01 \\ -2.1499E-03 \\ 0.0000E+00 \\ -1.7555E-01 \end{bmatrix} \end{array} \tag{1}$$

$$C = \begin{bmatrix} 0.000000E+00 & 5.729578E+01 & 0.000000E+00 & 0.000000E+00 \\ 0.000000E+00 & 0.000000E+00 & 0.000000E+00 & 5.729578E+01 \end{bmatrix} \begin{array}{c} \alpha \\ q \end{array}$$

The single input is the elevator deflection, δ_e, in degrees, and the two outputs are the appropriate feedback signals: alpha and pitch rate. The entries in the C-matrix are the conversions to units of degrees for consistency with the input.

Either of the two SISO transfer functions obtained from the coefficient matrices will exhibit the dynamic modes for this flight condition; the elevator-to-alpha transfer function is

$$\frac{\alpha}{\delta_e} = \frac{-0.1232(s+75.00)(s+0.009820 \pm j0.09379)}{(s-0.09755)(s+1.912)(s+0.1507 \pm j0.1153)} \tag{2}$$

Unlike the transfer functions for stable cg positions (e.g., $x_{CG} = 0.3\,\bar{c}$) in Chapter 3, this transfer function does not exhibit the usual phugoid and short-period poles. The pole at $s \approx .098$ indicates an unstable exponential mode with a time constant of about 10 s. The complex pole pair corresponds to an oscillatory mode with a period of 33 s and damping ratio of 0.79; this is like a phugoid period with a short-period damping ratio. This mode is the “third oscillatory mode” of the statically unstable airplane (see Section Aircraft Rigid-Body Modes).

The modes described above obviously do not satisfy the requirements for good handling qualities, and providing continuous control of the unstable mode would be a very demanding job for a pilot. We will now show that alpha and pitch-rate feedback together will restore stability and provide virtually complete control of the position of the short-period poles.

The configuration shown in Figure 4.4-1 will be used with an alpha filter but, for simplicity, no pitch-rate filter. The actuator and alpha filter models are taken from the original F-16 model report (Nguyen et al., 1979) and are both simple-lag filters with time constants $\tau_a = 1/20.2$ s and $\tau_F = 0.1$ s, respectively. The aircraft state-space model (1) augmented with these models, is

$$\dot{x} = \left[\begin{array}{cccccccc} & & & & : & & : & 0 \\ & & & & : & & : & 0 \\ & \multicolumn{2}{c}{A} & & : & -B & : & 0 \\ & & & & : & & : & 0 \\ \hline 0 & 0 & 0 & 0 & : & -20.2 & : & 0 \\ 0 & 10.0 & 0 & 0 & : & 0 & : & -10.0 \end{array}\right] \left[\begin{array}{c} v_T \\ \alpha \\ \theta \\ q \\ \hline x_a \\ x_F \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hline 20.2 \\ 0 \end{array}\right] u_e \qquad (3a)$$

$$y = \left[\begin{array}{c} \alpha \\ q \\ \hline \alpha_F \end{array}\right] = \left[\begin{array}{cccccc} & \multicolumn{2}{c}{C} & & : \; 0 & 0 \\ & & & & : \; 0 & 0 \\ \hline 0 & 0 & 0 & 0 & : \; 0 & 57.29578 \end{array}\right] x \qquad (3b)$$

Notice that the original state equations are still satisfied and the original δ_e input is now connected to the actuator state x_a through the phase reversal. The actuator is driven by a new input, u_e. Also, the α filter is driven by the α state of the aircraft dynamics, and an additional output has been created so that the filtered signal α_F is available for feedback. These state equations could also have been created by simulating the filters as part of the aircraft model and running the linearization program again. In the rest of this chapter the augmented matrices will be created by the MATLAB "series" command, as used in Chapter 3.

The state equations (3) can now be used to obtain the loop transfer functions needed for root-locus design. In the case of the innermost (alpha) loop, we already know that the α-loop transfer function will consist of Equation (2) with the two lag filters in cascade, and the effect of the feedback k_α can be anticipated using a sketch of the pole and zero positions. The goal of the alpha feedback is to pull the unstable pole, at $s = 0.098$, back into the left-half s-plane. Let the augmented coefficient matrices in Equation (3) be denoted by *aa*, *ba,* and *ca*. Then the following MATLAB commands can be used to obtain the root locus:

```
k= logspace(-2,1,2000);
r= rlocus(aa,ba,ca(3,:),0,k);     % 3rd row of C
plot(r)
grid on
axis([-20,1,-10,10])
```

Figures 4.4-2*a* and *b* show the root-locus plot for the inner loop on two different scales. The expanded scale near the origin (Figure 4.4-2*b*) shows that the effect of the alpha feedback is to make the loci from the third-mode poles come together on the real axis (near $s = -0.2$). The branch going to the right then meets the locus coming from the unstable pole, and they leave the real axis to terminate on the complex zeros near the origin. This provides a pair of closed-loop poles that correspond to a

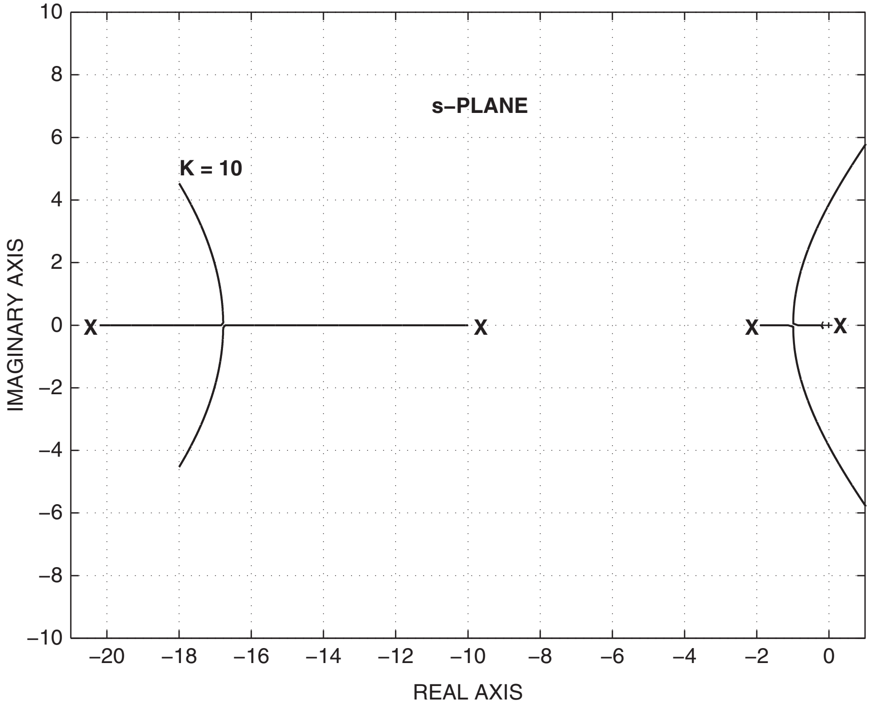

Figure 4.4-2*a* Inner-loop root-locus plot for pitch SAS.

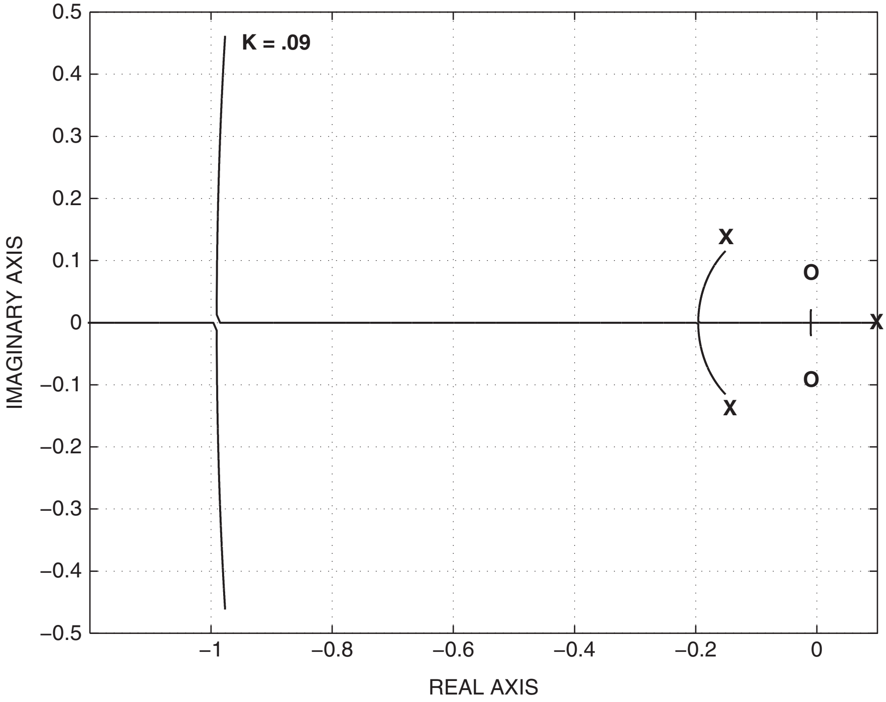

Figure 4.4-2*b* Expanded inner-loop root-locus plot for pitch SAS.

phugoid mode. The left branch from the third-mode poles meets the locus from the pole at $s = -1.9$, and they leave the axis near $s = -1$ to form a short-period mode. Alpha feedback has therefore produced the anticipated effect: The aircraft is stable with conventional longitudinal modes.

The larger-scale plot (Figure 4.4-2*a*) shows that as the magnitude of the alpha feedback is increased, the frequency of the new short-period poles increases and they move toward the right-half plane. The movement toward the right-half plane is in accordance with the constant net damping rule and the filter and actuator poles moving left. A slower (less expensive) actuator would place the actuator pole closer to the origin and cause the short-period poles to have a lower frequency at a given damping ratio. The position of the short-period poles for $k_\alpha = 0.5$ is $-070 \pm j2.0$. At this position the natural frequency is about 2.2 rad/s, which is acceptable according to the flying qualities requirements, but the damping ratio ($\zeta = 0.33$) is quite low.

A root-locus plot will now show the effect of varying k_q, with k_α fixed at 0.5. The following MATLAB commands can be used:

```
acl= aa- ba*kα*ca(3,:);              % Choose kα
%[z,p,k]= ss2zp(acl,ba,ca(2,:),0)    % q/u transf. fn
r= rlocus(acl,ba,ca(2,:),0);
plot(r)
```

The *q/u* transfer function with $k_\alpha = 0.5$ and $k_q = 0$ is

$$\frac{q}{u} = \frac{203.2s(s+10.0)(s+1.027)(s+0.02174)}{(s+20.01)(s+10.89)(s+0.6990 \pm j2.030)(s+0.008458 \pm j0.08269)} \tag{4}$$

Note that the zeros of this transfer function are the $1/T_{\theta_1}$ and $1/T_{\theta_2}$ unaugmented open-loop zeros, with the addition of a zero at $s = -10$. This zero has appeared because of the MIMO dynamics (two outputs, one input). It originally canceled the alpha filter pole out of the pitch-rate transfer function, but the inner-loop feedback has now moved the alpha filter pole to $s = -10.89$.

Figure 4.4-3 shows the root-locus plot for variable k_q. The phugoid poles move very slightly but are not visible on the plot. The short-period poles follow a circular arc around $s = -1$ (roughly constant natural frequency) as the pitch-rate feedback is increased. The poles become real for quite low values of k_q and, with larger values, a new higher-frequency oscillatory mode is created by the filter and actuator poles. Such a mode would be objectionable to the pilot, and we look for lower values of k_q that make the short-period poles match the flying qualities requirements, with no additional oscillatory mode. The value $k_q = 0.25$ places the short-period poles at $s = -2.02 \pm j1.94$. This corresponds to a natural frequency of 2.8 rad/s and a damping ratio of $\zeta = 0.72$. The corresponding closed-loop transfer function for pitch rate is given by

$$\frac{q}{u} = \frac{203.2s(s+10.0)(s+1.027)(s+0.02174)}{(s+16.39)(s+11.88)(s+2.018 \pm j1.945)(s+0.008781 \pm j0.06681)} \tag{5}$$

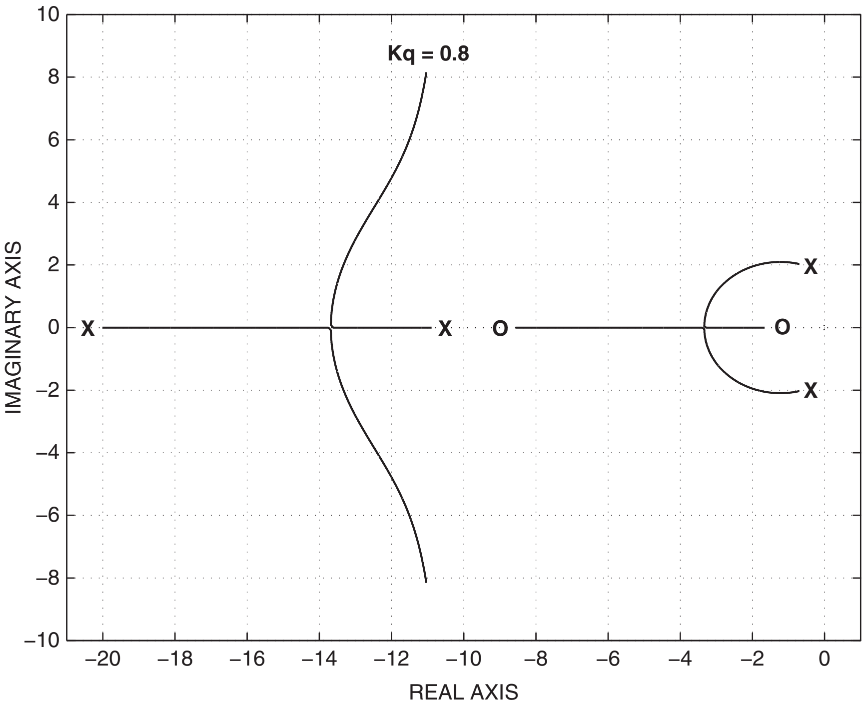

Figure 4.4-3 Outer-loop root-locus plot for pitch SAS.

The original actuator pole has moved from $s = -20.2$ to $s = -16.39$, and the α-filter pole has moved from $s = -10$ to $s = -11.88$. Apart from these factors, this transfer function is very similar to the stable-cg transfer function in Example 3.8-3 but with improved short-period pole positions. ■

Example 4.4-1 shows that alpha feedback stabilizes the unstable short-period mode and determines its natural frequency, while the pitch-rate feedback mainly determines the damping. The amount of alpha feedback needed to get a satisfactory natural frequency was 0.5° of elevator deflection per degree of alpha. The alpha signal is noisy and sometimes unreliable, and this large amount of alpha feedback is preferably avoided. In the second root-locus plot it can be seen that, as the pitch-rate feedback is varied, the locus of the short-period poles circles around the $1/T_{\theta_2}$ zero. Therefore, by moving the zero to the left, a higher natural frequency can be achieved, or the same natural frequency can be achieved with less alpha feedback. This will be demonstrated in the next example.

Example 4.4-2: A Pitch-SAS Design The coefficient matrices *aa*, *bb*, *cc* from Example 4.4-1 are used again here, and the alpha feedback gain will be reduced to $k_\alpha = 0.1$. A lag compensator with a pole at $s = -1$ and a zero at $s = -3$ will be cascaded with the plant to effectively move the $1/T_{\theta_2}$ zero to $s = -3$. The MATLAB commands are

```
acl= aa - ba*0.1*ca(3,:);         % Close alpha loop, Kα=.1
qfb= ss(acl,ba,ca(2,:),0);        % SISO system for q f.b.
z=3; p=1;
lag= ss(-p,1,z-p,1);              % Lag compensator
csys= series(lag,qfb);            % Cascade Comp. before plant
[a,b,c,d]= ssdata(csys);
k= logspace(-2,0,2000);
r= rlocus(a,b,c,d,k);
plot(r)
grid on
axis([-20,1,-10,10])
```

The root-locus plot is the same shape as Figure 4.4-3, and when the pitch-rate feedback gain is $k_q = 0.2$, the closed-loop transfer function is

$$\frac{q}{u} = \frac{203.2s(s+10.0)(s+1.027)(s+0.0217)(s+3)}{(s+18.02)(s+10.3)(s+1.025)(s+1.98\pm j2.01)(s+0.0107\pm j0.0093)} \tag{1}$$

When the pole and zero close to $s = -1$ are canceled out, this transfer function is essentially the same as in Example 4.4-1 except that there is a zero at $s = -3$ instead of $s = -1$. This zero can be replaced by a zero at $s = -1$ once again, by placing the lag compensator in the feedback path. However, a zero at $s = -1$ produces a much bigger overshoot in the step response than the zero at $s = -3$. Therefore the flying qualities requirements on T_{θ_2} should be checked (see Section The Handling Qualities Requirements) to obtain some guidance on the position of the zero.

This example shows that the same short-period mode, as in Example 4.4-1, can be achieved with much less alpha feedback and less pitch-rate feedback. Also, the transfer function (1) shows that no additional modes are introduced. A dynamic compensator is the price paid for this. Section 4.3 shows that the $1/T_{\theta_2}$ zero will move with flight conditions, and so the compensator parameters may have to be changed with flight conditions. ■

Lateral-Directional Stability Augmentation/Yaw Damper

Figure 4.4-4 shows the most basic augmentation system for the lateral-directional dynamics. Body-axis roll rate is fed back to the ailerons to modify the roll subsidence mode, and yaw rate is fed back to the rudder to modify the dutch roll mode (yaw damper feedback). The lateral (rolling) motion is not, in general, decoupled from the yawing and sideslipping (directional) motions. Therefore, the augmentation systems will be analyzed with the aid of the multivariable state equations (two inputs, ailerons and rudder, and two or more outputs), as implied by the figure. This analysis will be restricted to the simple feedback scheme shown in the figure; in a later section additional feedback couplings will be introduced between the roll and yaw channels.