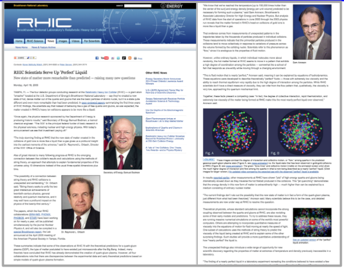
Lecture 11
Shear and bulk viscosities (part I)

2005 – "Perfect Liquid" Discovery


RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted -- raising many new questions

Paradigm Shift

inSPIRE query for all publications with

QGP, SQGP, QUARK-GLUON PLASMA, QCD PLASMA, STRONGLY COUPLED PLASMA, STRONGLY-COUPLED PLASMA, RHIC PLASMA

in their title:

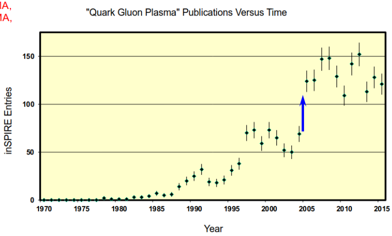
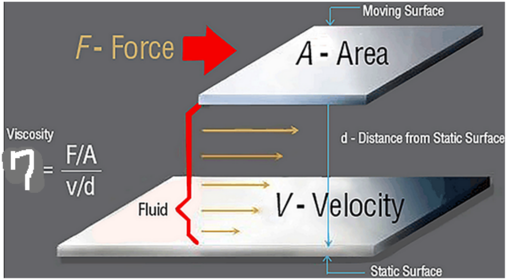


Figure by W.Zajc

How does one calculate viscosity?

Shear viscosity



Unit for viscosity: $Pa \cdot s$.
A perhaps more common unit is the centiPoise $1cP = 10^{-3} Pa \cdot .s$
Exemples:

Water $\sim 1\ cP$
Air $\sim 0.02\ cP$
Honey $\sim 2000 - 10\,000\ cP$

It is generally easy to measure, but not always.

The world longest running experiment Guinness Book of World Records

Pitch drop experiment at the University of Queensland-Australia

Started in 1927: 7 drops fell every 8 years, then air condioning was installed, the viscosity increased and drops fall about every 12-13 years.

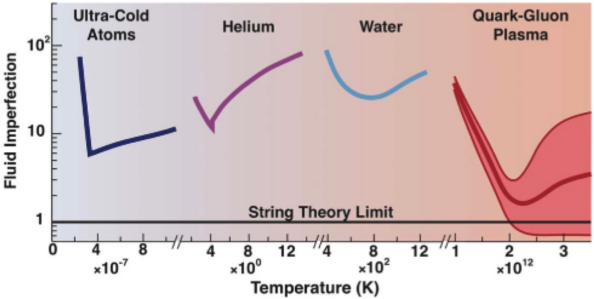
$\longrightarrow$ viscosity of pitch $\sim 2.3\ \ 10^{11}cP$

John Mainstone (custodian in 2005) and the late Thomas Parnell (started the experiment) were awarded the 2005 Ig Nobel Prize in Physics.

Quark Gluon Plasma

The most perfect fluid on earth:

$$\boxed{\eta/s = \text{few } (\hbar/k_B)1/(4\pi) = \text{few } 1/(4\pi)}$$



C.Shen & U.Heinz Nucl.Phys.News 25 (2015) 6 /arXiv:1507.01558

Figure based on a variety of data at RHIC and LHC: elliptic flow, jet quenching, etc

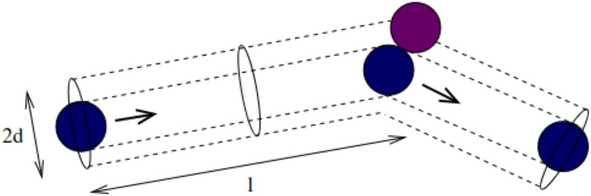
Theoretical estimates of shear viscosity

Kinetic theory

$f(v)d^v$: Boltzmann probability distribution for velocities in gas

probability for a particle to have velocity within d^3v in the neighbourhood of $\vec{v}$.

• Mean free path l: average distance travelled by a particle of diameter d between each collision



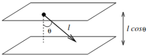
A particle of radius $d/2$ travels, on average, a length l between each collision. this time it sweeps out a volume $\pi d^2 l$.

$$\pi d^2 l = V/N \Rightarrow l = 1/(n\pi d^2)$$

- Viscosity:

Direction of plate $\longrightarrow x$
Perpendicular to plate $\longrightarrow z$
$F/A = \eta(du_x/dz)$ with u flow velocity

nb. of particles per unit time per unit area passing through slab in z-direction: $= n\, v_z\, f(v) d^3 v$
momentum picked up by particle at z from collisions from particles above and below: $\Delta p = m[u_x(z+\Delta z) - u_x(z)] \sim m du_x/dz \Delta z$ with $\Delta z = l \cos\theta$



Force per unit area acting on any z slice is given by rate of change of momentum $F/A = (-1/A)\Delta p/\Delta t$

negative sign due to F force we apply, $\Delta p/\Delta t$ force of fluid pushing back
$F/A = -n \int d^3 v \Delta p v_z\, f(v) = -mn(du_x/dz) \int d^3 v v_z\, f(\vec{v} l \cos\theta$
$= (mnl/3)(du_x/dz) \int dv 4\pi v^3 f(v) = (mnl/3)(du_x/dz) < v > =$
$\eta(du_x/dz)$

$$\boxed{\eta = mnl < v > /3}$$

Not intuitive: large cross section σ ($l \propto 1/\pi d^2 \sim 1/\sigma$) $\Rightarrow$ small η
small cross section $\Rightarrow$ large η

In fact there is a lower limit for η:

P.Danielewicz & M.Gyulassy Phys. Rev. D31 (1985) 53

Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i. First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \,, \tag{3.3}$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

Uncertainty principle: $l < p > \geq \hbar$

$\Rightarrow \eta \geq n/3$

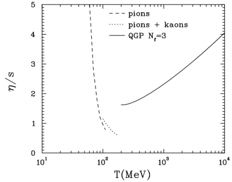
For gluons and massless quarks (at $\mu_b = 0$): $s_g + s_q/(n_g + n_q) \sim 4$

$\Rightarrow \eta/s \geq 1/(3 \times 4) \sim 1/(4\pi)$

pQCD

At leading logarithmic order for $N_f = 3$: $\frac{\eta}{s} \sim \frac{5.12}{g(T)^4 \ln[1.42/g(T)]}$

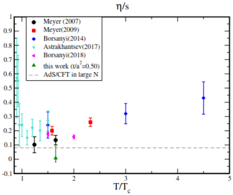
P. Arnold, G. D. Moore and L. G. Yaffe, JHEP 05, 051 (2003)



L. P. Csernai, J. I. Kapusta, L. D. McLerran Phys.Rev.Lett.97 (2006) 152303

Note: $\eta \nearrow$ when $T \nearrow$ (smaller coupling constant)

Lattice QCD



Minimum values are around $0.1 \sim 1/(4\pi)$

Gauge/gravity correspondence Maldacena '97

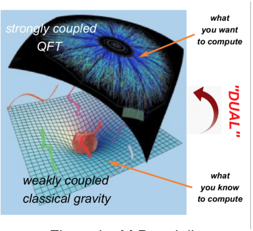


Figure by M.Baggioli

A quantum field theory in d flat dimensions can be mapped into a quantum gravity theory in (d+1) (or more) dimensions.

Main appeal for us:

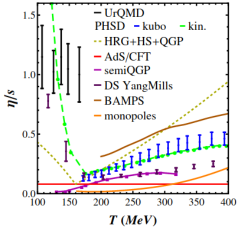
When one side of the duality is strongly coupled the other is weakly coupled and vice-versa.

$\longrightarrow$ Use a quantum gravity weakly coupled theory to learn about a strongly coupled quantum field theory.

Drawback: so far, quantum field theories for which the corresponding dual theory of gravity is known are not QCD, but close to it.

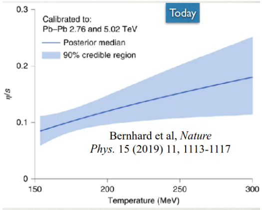
For a large class of gauge theories which accept a description at strongly coupling by a dual gravitational theory: $\eta/s = 1/(4\pi)$

Summary of theoretical estimates



J.Noronha-Hostler arXiv:1512.06315

Best values to explain data (Bayesian analysis)



Homework

Estimate the force that must be applied to spread a 1 mm layer of cream on 1 cm^2 of your face, without rushing (fingger moving 3 cm in 1/10th second). Assume the shear viscosity of the cream is 1 $Pa \cdot s$