Eletromagnetismo

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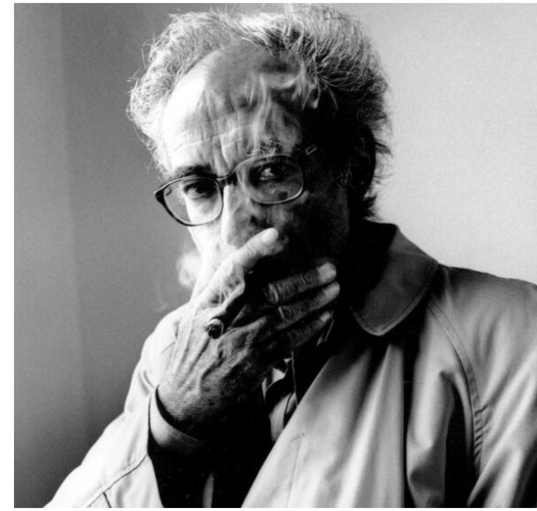
Plano do Curso

16/08	13/09	11/10	08/11
19/08	16/09	14/10	11/11
23/08	20/09 P1	18/10	15/11
26/08	23/09	21/10 P2	18/11
30/08	27/09	25/10	22/11
02/09	30/09	28/10	25/11
06/09	04/10	01/11	29/11 P3
09/09	07/10	04/11	02/12 ex
			06/12 Sub

Jean Luc Godard
(1930 -2022)



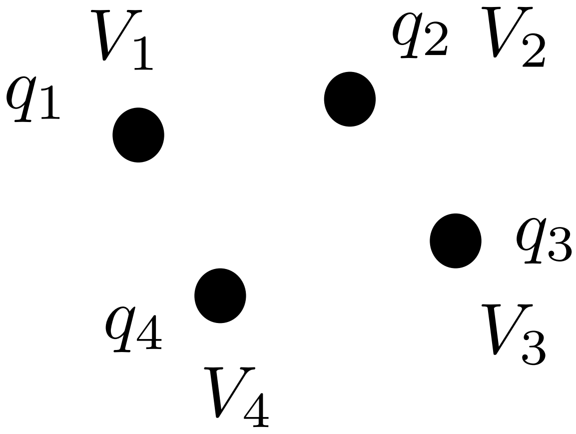
"Acossado" (1960)

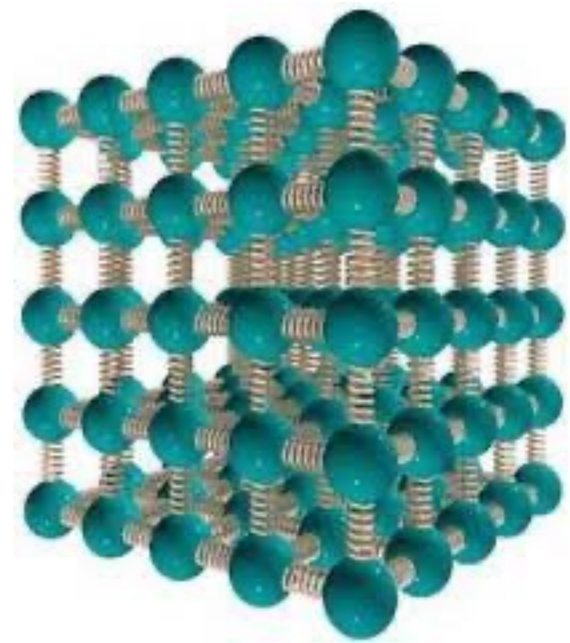
"2 ou 3 coisas que eu sei dela" (1967)

"Um filme tem que ter começo, meio e fim,
mas não necessariamente nesta ordem"

Aula 7

A energia potencial eletrostática





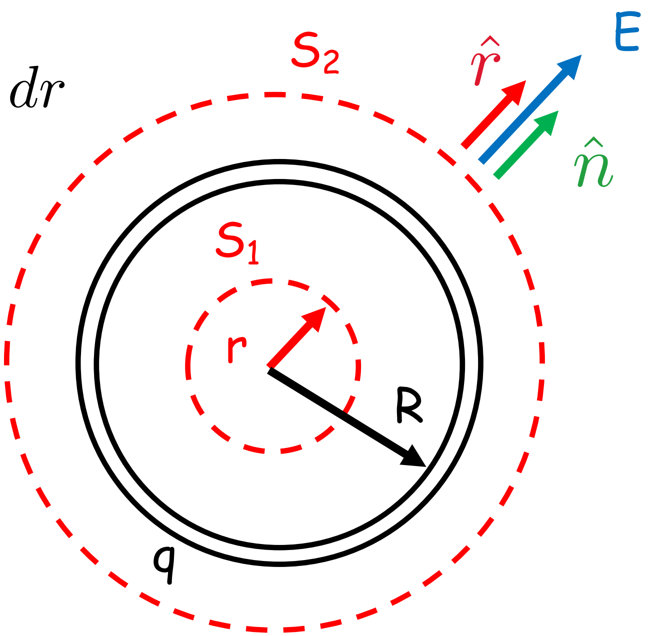
A energia gasta para vencer a repulsão e criar o sistema !

$$W = \frac{1}{2}\int \rho\, V\, d^3r'$$

$$W = \frac{\epsilon_0}{2}\int E^2\, d^3r$$

Exemplo 2.8 : calcule a energia de uma casca esférica de raio R uniformemente carregada com carga q.

Solução 1: $W = \frac{\epsilon_0}{2} \int E^2 \, d^3r \qquad d^3r = 4\,\pi\, r^2\, dr$



Vamos calcular o campo elétrico pela Lei de Gauss

$$\oint \vec{E}\,.\,\hat{n}\, da = \frac{Q_{int}}{\epsilon_0}$$

$$\vec{E}\,.\,\hat{n} = E$$

$$E\, 4\,\pi\, r^2 = \frac{Q_{int}}{\epsilon_0}$$

Interior: r < R

$$Q_{int} = 0$$

$$E_1 = 0$$

Exterior: r > R

$$Q_{int} = q$$

$$E\, 4\,\pi\, r^2 = \frac{q}{\epsilon_0}$$

$$E_2 = \frac{q}{4\,\pi\,\epsilon_0\, r^2}$$

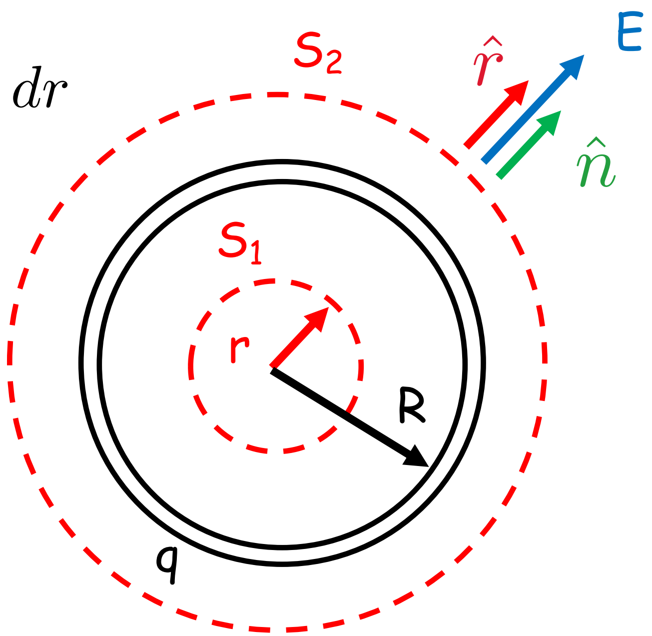
$$W = \frac{\epsilon_0}{2}\left[4\,\pi \int_0^R dr\, r^2\, E_1^2 + 4\,\pi \int_R^\infty dr\, r^2\, E_2^2\right]$$

0

$$W = \frac{\epsilon_0}{2}\, 4\,\pi \int_R^\infty dr\, r^2 \left(\frac{q}{4\,\pi\,\epsilon_0\, r^2}\right)^2$$

Exemplo 2.8 : calcule a energia de uma casca esférica de raio R uniformemente carregada com carga q.

Solução 1: $W = \frac{\epsilon_0}{2} \int E^2 \, d^3r \qquad d^3r = 4\,\pi\, r^2\, dr$



Vamos calcular o campo elétrico pela Lei de Gauss

Exterior: r > R

$$W = \frac{\epsilon_0}{2} \, 4\,\pi \int_R^\infty dr\, r^2 \left(\frac{q}{4\,\pi\,\epsilon_0\, r^2} \right)^2$$

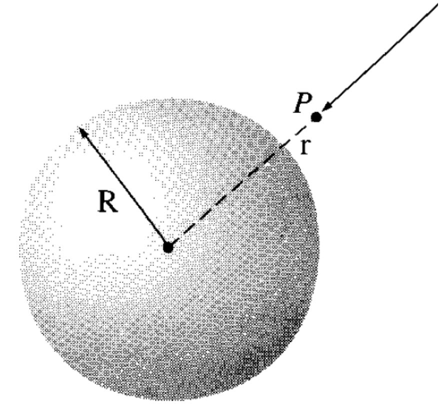
$$W = \frac{\epsilon_0}{2} \, 4\,\pi \, \frac{q^2}{16\,\pi^2\,\epsilon_0^2} \int_R^\infty dr\, r^2 \left(\frac{1}{r^4} \right) = \frac{q^2}{8\,\pi\,\epsilon_0} \int_R^\infty dr \, \frac{1}{r^2}$$

$$= \frac{q^2}{8\,\pi\,\epsilon_0} \left(-\frac{1}{r} \right)_R^\infty$$

$$\boxed{W = \frac{q^2}{8\,\pi\,\epsilon_0} \frac{1}{R}}$$

Solução 2: $W = \frac{1}{2}\int \rho\, V\, d^3r'$ $\quad W = \frac{1}{2}\int_S \sigma\, V\, da$ $\quad \sigma = \frac{q}{4\,\pi\,R^2}$

$$V(r) = -\int_\infty^r \vec{E}\,.\,d\vec{l}$$



$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$\mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr$$

Na casca r = R

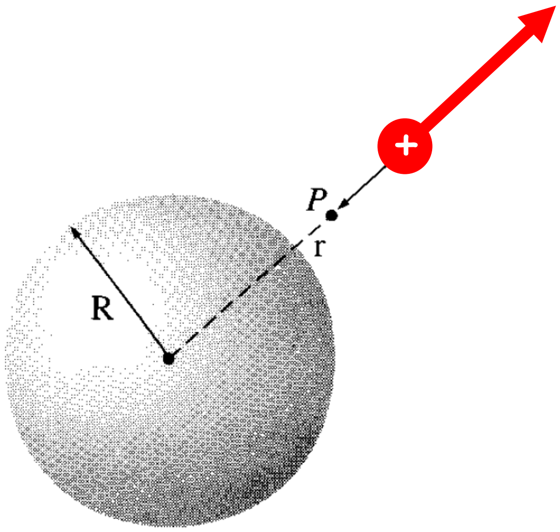
$$V(r) = \frac{-1}{4\pi\epsilon_0}\int_\infty^r \frac{q}{r'^2}\,dr' = \frac{1}{4\pi\epsilon_0}\frac{q}{r'}\Big|_\infty^r = \frac{1}{4\pi\epsilon_0}\frac{q}{r}.$$

$$V(r) = \frac{1}{4\,\pi\,\epsilon_0}\frac{q}{R}$$

$$W = \frac{1}{2}\int_S \sigma\,V\,da = \frac{1}{2}\underbrace{\frac{q}{4\,\pi\,R^2}}_{\sigma}\;\underbrace{\frac{1}{4\,\pi\,\epsilon_0}\frac{q}{R}}_{V(R)}\;\underbrace{4\,\pi\,R^2}_{A}$$

$$\boxed{W = \frac{q^2}{8\,\pi\,\epsilon_0}\frac{1}{R}}$$

Em que direção o potencial decresce ?

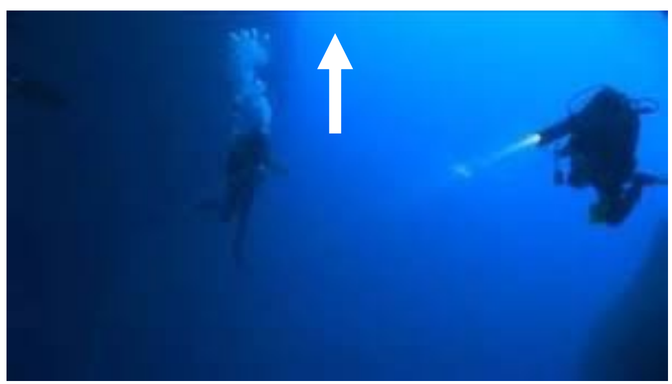


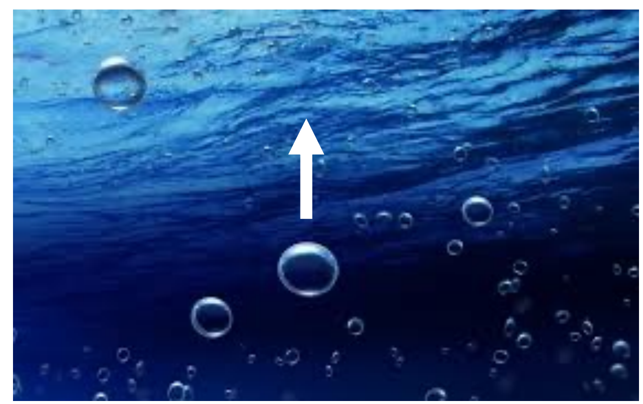
$$V(r) = -\int_{\infty}^{r} \vec{E} \,.\, d\vec{l} \qquad \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr$$

$$V(r) = \frac{-1}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{q}{r'^2} \, dr' = \frac{1}{4\pi\epsilon_0} \frac{q}{r'} \Big|_{\infty}^{r}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}.$$

Casca r = R $\quad V(r) = \dfrac{1}{4\,\pi\,\epsilon_0} \dfrac{q}{R}$

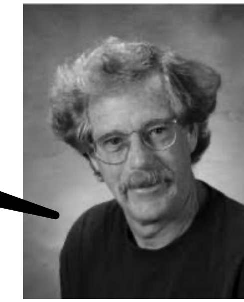




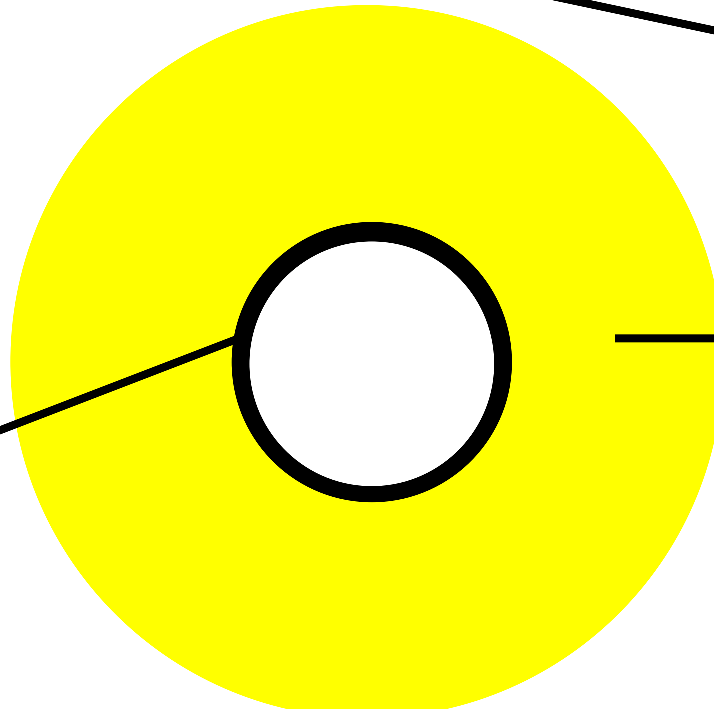
Mergulhador desorientado: siga a bolha !!!

A gente segue uma carga positiva de teste !

Onde a energia está armazenada ?



na casca de cargas ?



no campo ?

$$W = \frac{1}{2}\int \rho\, V\, d^3r'$$

$$\rho = \epsilon_0\, \vec{\nabla}.\vec{E}$$

$$W = \frac{\epsilon_0}{2}\int \vec{\nabla}\,.\,\vec{E}\ V\, d^3r'$$

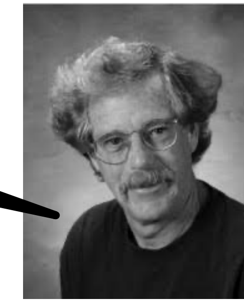
$$W = -\frac{\epsilon_0}{2}\int \vec{E}\,.\,\vec{\nabla}V\, d^3r'$$

$$\vec{\nabla}V = -\vec{E}$$

$$W = -\frac{\epsilon_0}{2}\int \vec{E}\,.\,(-\,\vec{E})\, d^3r'$$

$$W = \frac{\epsilon_0}{2}\int E^2\, d^3r$$

Onde a energia está armazenada ?



(ii) Where is the energy stored? Equations 2.43 and 2.45 offer two different ways of calculating the same thing. The first is an integral over the charge distribution; the second is an integral over the field. These can involve completely different regions. For instance, in the case of the spherical shell (Ex. 2.8) the charge is confined to the surface, whereas the electric field is present everywhere *outside* this surface. Where *is* the energy, then? Is it stored in the field, as Eq. 2.45 seems to suggest, or is it stored in the charge, as Eq. 2.43 implies? At the present level, this is simply an unanswerable question: I can tell you what the total energy is, and I can provide you with several different ways to compute it, but it is unnecessary to worry about *where* the energy is located. In the context of radiation theory (Chapter 11) it is useful (and in General Relativity it is *essential*) to regard the energy as being stored in the field, with a density

$$\frac{\epsilon_0}{2}E^2 = \text{energy per unit volume.} \tag{2.46}$$

But in electrostatics one could just as well say it is stored in the charge, with a density $\frac{1}{2}\rho V$. The difference is purely a matter of bookkeeping.

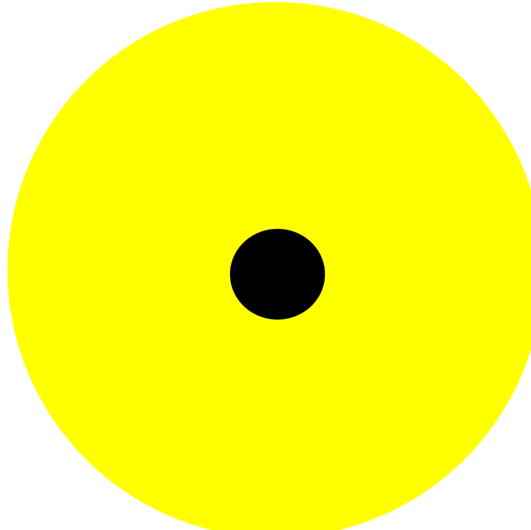
O princípio da superposição

(iii) The superposition principle. Because electrostatic energy is *quadratic* in the fields, it does *not* obey a superposition principle. The energy of a compound system is *not* the sum of the energies of its parts considered separately—there are also "cross terms":

$$\begin{aligned} W_{\text{tot}} &= \frac{\epsilon_0}{2}\int E^2\,d\tau = \frac{\epsilon_0}{2}\int (\mathbf{E}_1+\mathbf{E}_2)^2\,d\tau \\ &= \frac{\epsilon_0}{2}\int (E_1^2+E_2^2+2\mathbf{E}_1\cdot\mathbf{E}_2)\,d\tau \\ &= W_1+W_2+\epsilon_0\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau. \end{aligned} \tag{2.47}$$

For example, if you double the charge everywhere, you *quadruple* the total energy.

A energia de uma única carga



$$W = \frac{\epsilon_0}{2} \int E^2 \, d^3 r$$

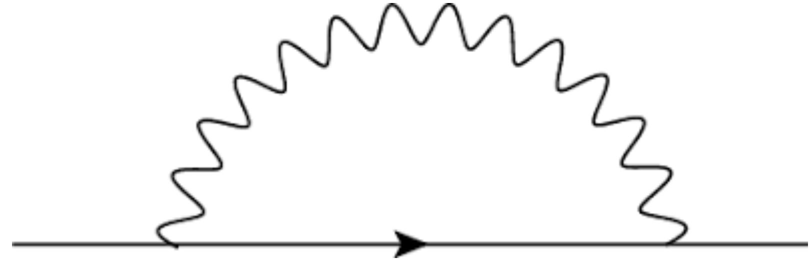
$$E = \frac{1}{4\,\pi\,\epsilon_0} \frac{q}{r^2}$$

$$W = \frac{\epsilon_0}{2(4\pi\epsilon_0)^2} \int \left(\frac{q^2}{r^4} \right) (r^2 \underbrace{\sin\theta \, dr \, d\theta \, d\phi)}_{4\,\pi} = \frac{q^2}{8\pi\epsilon_0} \int_0^\infty \frac{1}{r^2} \, dr = \infty$$

Especulação: $W = \dfrac{q^2}{8\,\pi\,\epsilon_0} \displaystyle\int_{r_e}^\infty dr \, \frac{1}{r^2} = \frac{q^2}{8\,\pi\,\epsilon_0\, r_e} = m_e \, c^2$

Uma parte da massa do elétron vem da interação eletromagnética ?

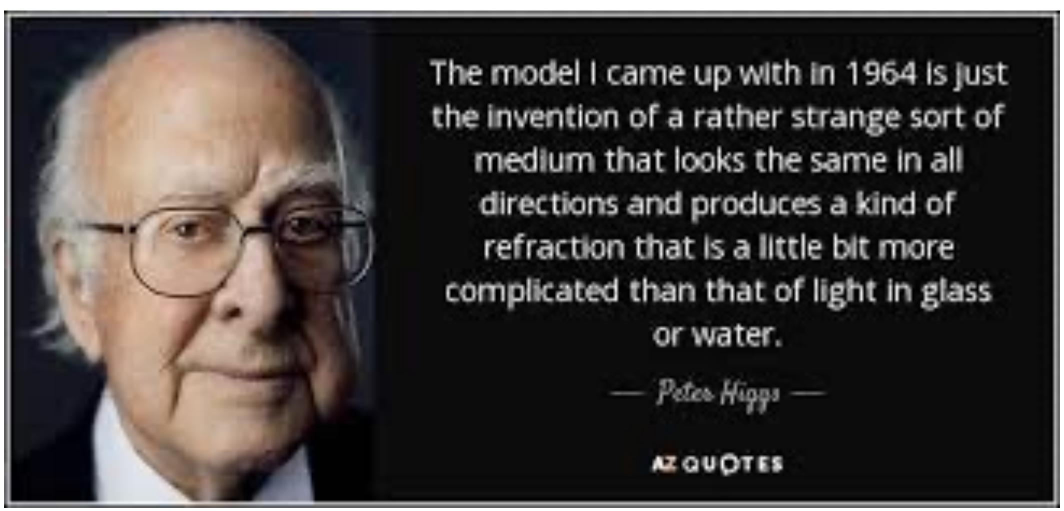
Problema: limite quântico !



Problema: Higgs !

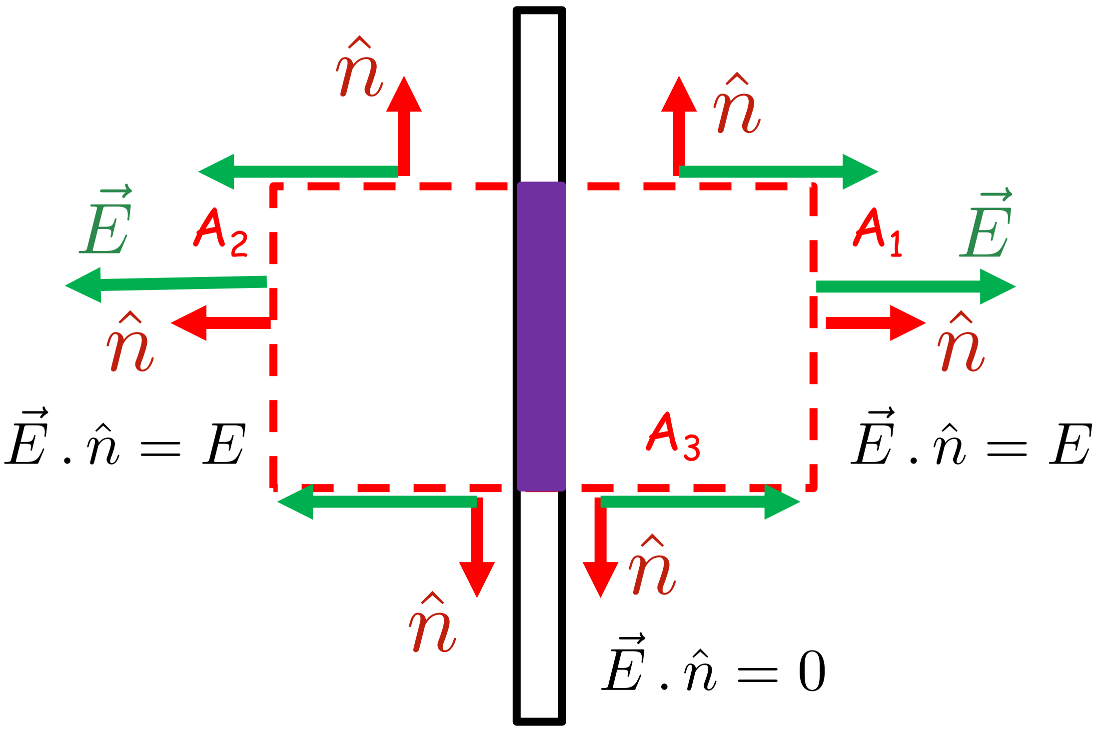
(~1962)

(~2012)



Prêmio Nobel de 2013

Treinamento funcional: campo elétrico criado pela placa infinita carregada :



$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0}$$

$$2\,E\,\cancel{A} = \frac{\sigma}{\epsilon_0}\,\cancel{A}$$

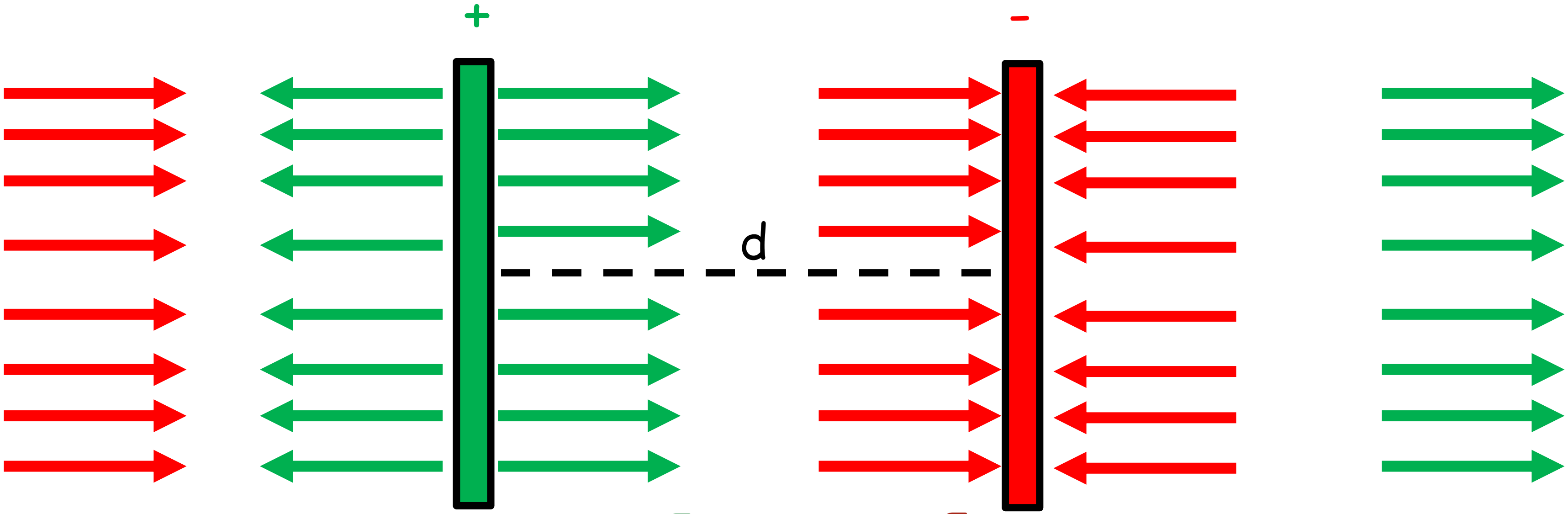
$$\boxed{E = \frac{\sigma}{2\,\epsilon_0}}$$

$$Q_e = \sigma\,A$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \int_{A_1} \overbrace{\vec{E}\,.\,\hat{n}}^{E}\,da + \int_{A_2} \overbrace{\vec{E}\,.\,\hat{n}}^{E}\,da + \int_{A_3} \overbrace{\vec{E}\,.\,\hat{n}}^{0}\,da$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E\int_{A_1} da + E\int_{A_2} da \quad = 2\,E\,A$$

Energia por unidade de área armazenada num capacitor de placas paralelas (infinitas) com densidade de carga sigma

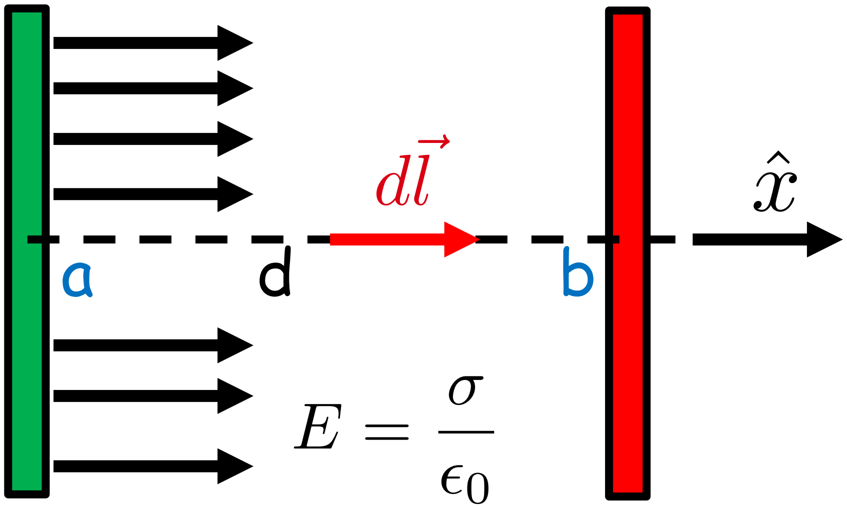


$E = 0$ $\quad E = \frac{\sigma}{2\,\epsilon_0}$ $\quad E = \frac{\sigma}{2\,\epsilon_0}$ $\quad E = 0$

$$E = \frac{\sigma}{\epsilon_0}$$

$$W = \frac{\epsilon_0}{2} \int E^2\, d^3 r \longrightarrow W = \frac{\epsilon_0}{2} E^2\, A\, d \longrightarrow \frac{W}{A} = \frac{\sigma^2\, d}{2\,\epsilon_0}$$

Agora usando o potencial :



$$E = \frac{\sigma}{\epsilon_0}$$

Vamos calcular o potencial :

$$V(b) - V(a) = -\int_a^b \vec{E} \, . \, d\vec{l}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} \qquad d\vec{l} = dx \, \hat{x} \qquad \vec{E} \, . \, d\vec{l} = \frac{\sigma}{\epsilon_0} \, dx$$

$$V(b) - V(a) = -(b - a) \frac{\sigma}{\epsilon_0}$$

$$V(a) = 0 \qquad V(b) = -d \frac{\sigma}{\epsilon_0}$$

$$W = \frac{1}{2} \int_S \sigma \, V \, da$$

$$W = \frac{1}{2} \left(\sigma_+ V(a) \, A + \sigma_- V(b) \, A\right)$$

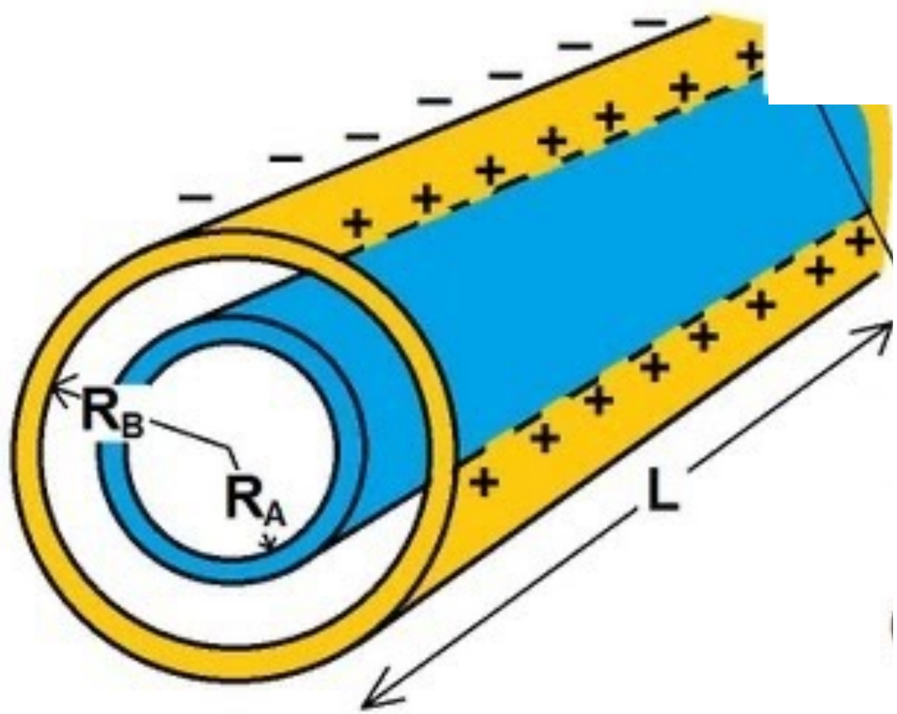
(0)

$$W = \frac{1}{2} \sigma_- V(b) \, A$$

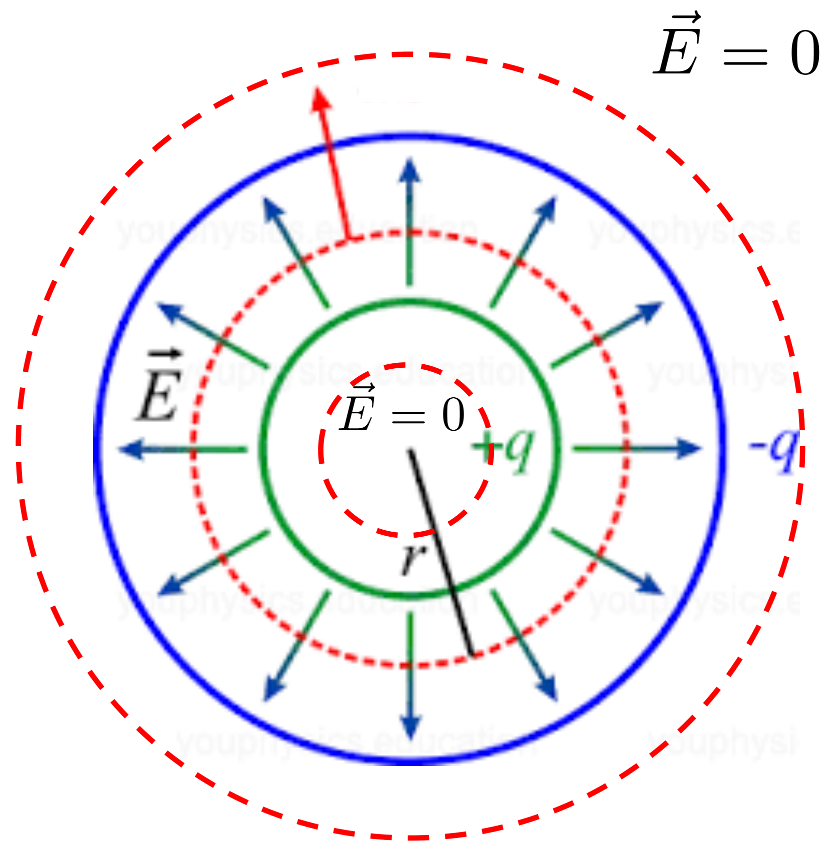
$$W = \frac{1}{2} (-\sigma) \left(-\frac{d \, \sigma}{\epsilon_0}\right) A$$

$$\frac{W}{A} = \frac{\sigma^2 \, d}{2 \, \epsilon_0}$$

Capacitor cilíndrico



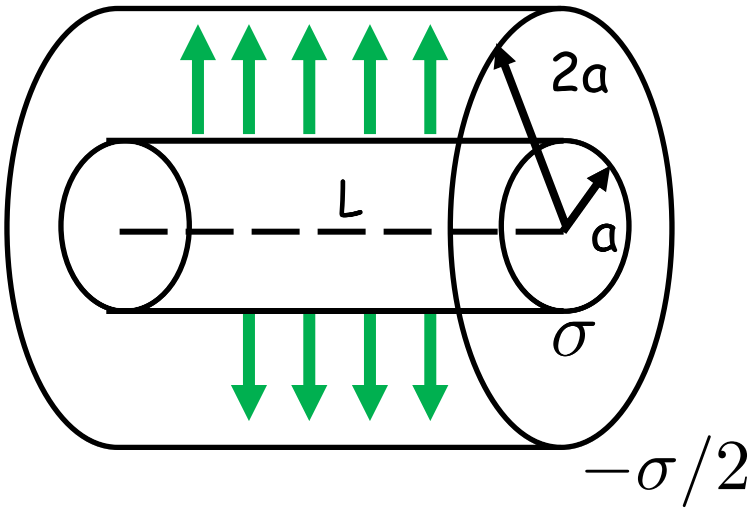
Carga total positiva interna
é igual à
carga total negativa externa

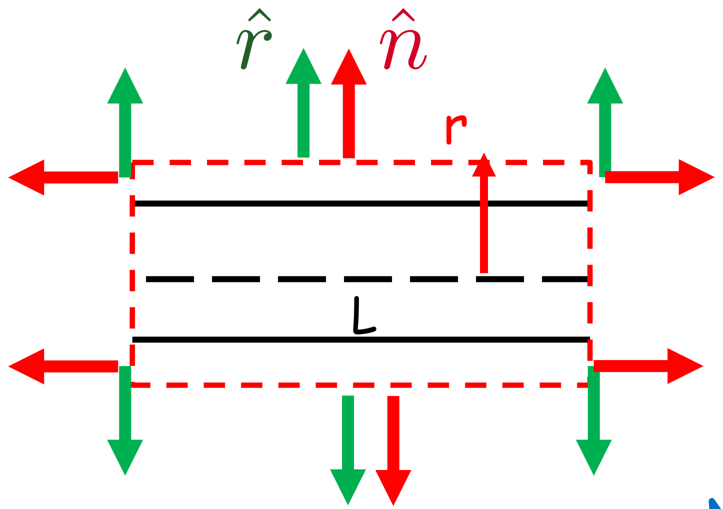


O campo elétrico
no interior da casca menor
e
no exterior da casca maior
é zero !

Capacitor cilíndrico de raios a e 2a

Campo elétrico entre as cascas:





$$\vec{E} = E\,\hat{r}$$

$$\hat{n} = \hat{r}$$

$$\vec{E}\,.\,\hat{n} = E$$

Nas tampas
o fluxo é zero !

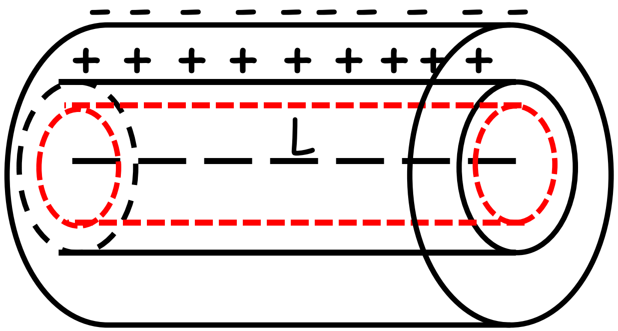
$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0}$$

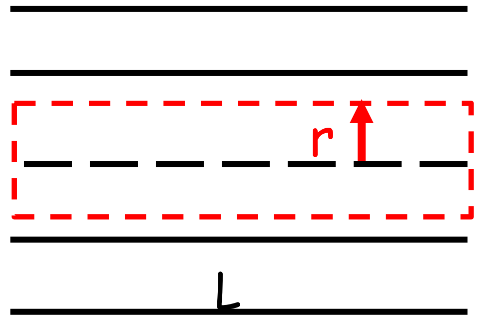
$$\int_{A_3} E\,da = \frac{\sigma\,A_c}{\epsilon_0}$$

$$\cancel{2\pi}\,r\,\cancel{L}\,E = \frac{\cancel{2\pi}\,a\,\cancel{L}\,\sigma}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\frac{a}{r}\hat{r}}$$

Campo elétrico: $r < a$

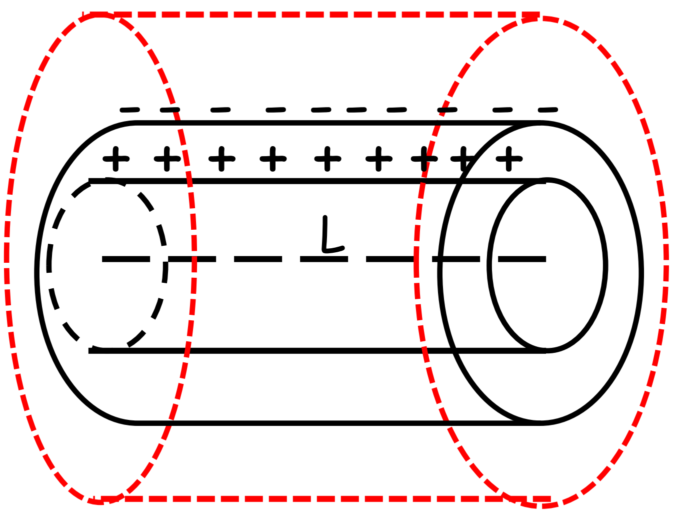


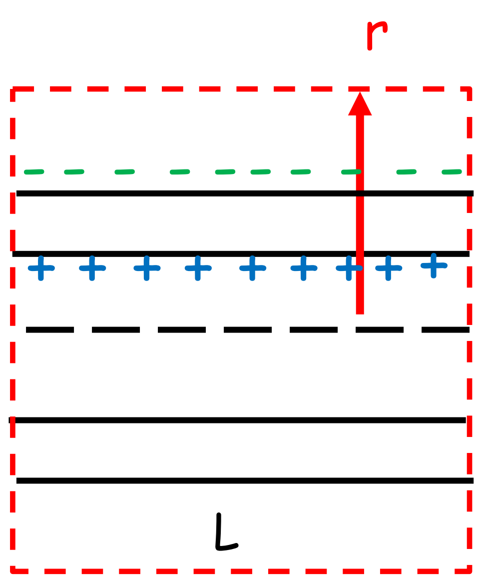


Carga envolvida pela sup. Gaussiana é zero
o campo elétrico é zero !

$$\vec{E} = 0$$

Campo elétrico: $r > 2\,a$





Carga envolvida pela sup. Gaussiana é zero
o campo elétrico é zero !

$$\vec{E} = 0$$

Cálculo da energia :

$$W = \frac{\epsilon_0}{2} \int E^2 \, d^3 r \qquad \vec{E} = \frac{\sigma}{\epsilon_0} \frac{a}{r} \hat{r}$$

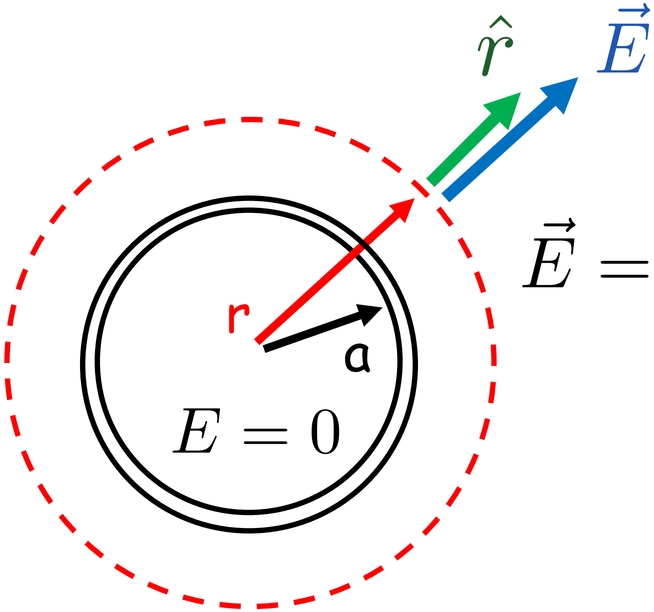
$$E^2 = \frac{\sigma^2 \, a^2}{\epsilon_0^2 \, r^2} \qquad d^3 r = 2 \, \pi \, r \, L \, dr$$

$$W = \frac{\epsilon_0}{2} \frac{\sigma^2 \, a^2}{\epsilon_0^2} 2 \, \pi \, L \int_a^{2a} \frac{1}{r} dr \qquad \int_a^{2a} \frac{1}{r} \, dr = ln(\frac{2 \, a}{a})$$

$$\boxed{W = \frac{\sigma^2 \, a^2}{\epsilon_0} \pi \, L \, ln \, 2}$$

Fim

Potencial de uma casca cilíndrica de raio a com densid. sup. de carga σ



$$\vec{E} = \frac{\sigma}{\epsilon_0}\frac{a}{r}\hat{r}$$

$$V(b) - V(a) = -\int_a^b \vec{E}\,.\,d\vec{l}$$

$$d\vec{l} = d\vec{r} \qquad d\vec{r} = dr\,\hat{r}$$

Ponto de referência: r = a onde V = 0

$$a = a$$

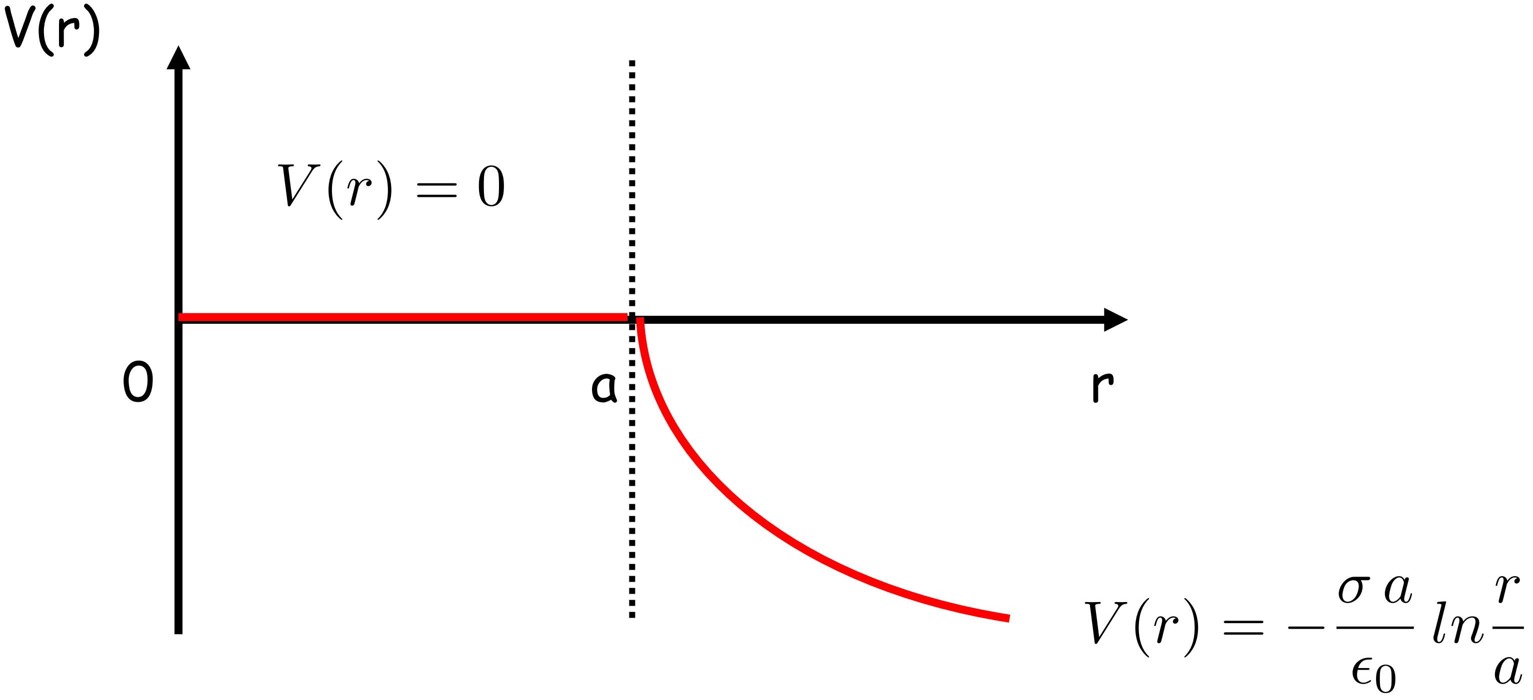
Ponto de interesse: r = r onde V(r)

$$b = r$$

$$V(r) = -\int_a^r \frac{\sigma\,a}{\epsilon_0\,r}\hat{r}\,.\,\hat{r}\,dr = -\frac{\sigma\,a}{\epsilon_0}\int_a^r \frac{1}{r}\,dr = -\frac{\sigma\,a}{\epsilon_0}(lnr - lna)$$

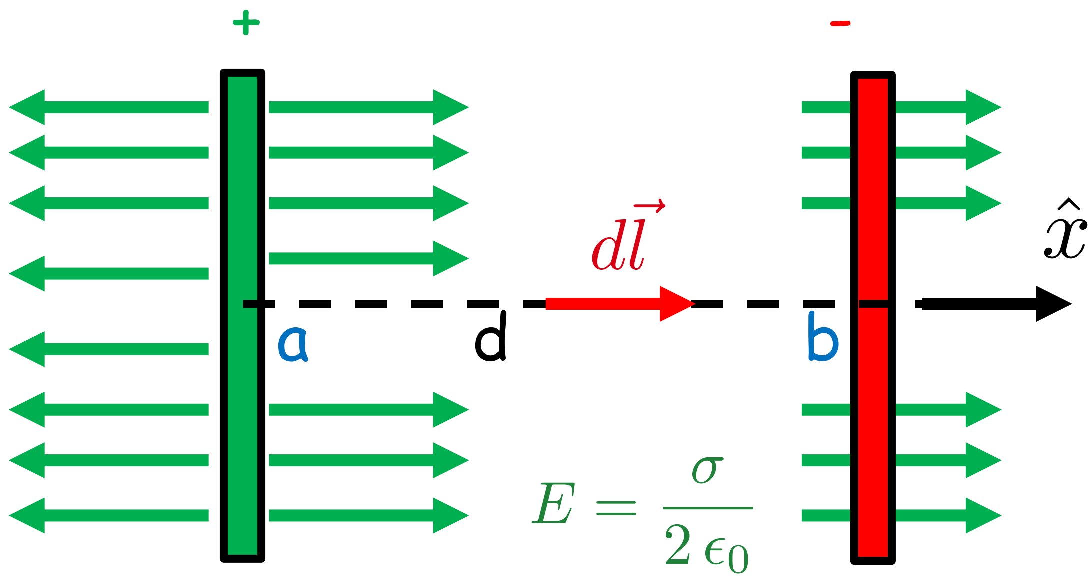
$$V(r) = -\frac{\sigma\,a}{\epsilon_0}\,ln\frac{r}{a}$$

O gráfico



Agora usando a definição : $W = q\,V$

$$W = q\,V$$



$$W = q_+\,V_- = q_+\,V(a)$$

ou

$$W = q_-\,V_+ = q_-\,V(b)$$

Vamos calcular o potencial na placa vermelha :

$$V(b) - V(a) = -\int_a^b \vec{E}\,.\,d\vec{l} \qquad \vec{E} = \frac{\sigma}{2\,\epsilon_0}\hat{x} \qquad d\vec{l} = dx\,\hat{x} \qquad \vec{E}\,.\,d\vec{l} = \frac{\sigma}{2\,\epsilon_0}\,dx$$

$$V(b) - V(a) = -\,(b-a)\,\frac{\sigma}{2\,\epsilon_0} \qquad V(a) = 0 \qquad V(b) = -\,d\,\frac{\sigma}{2\,\epsilon_0}$$

$$W = q_-\,V(b) = -\sigma\,A\,(-\,\frac{\sigma\,d}{2\,\epsilon_0}) \qquad \frac{W}{A} = \frac{\sigma^2\,d}{2\,\epsilon_0}$$