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Non-steady Geomagnetic Dynamo Models

Tsuneji Rikitake

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Summary

Bullard proved that a disk dynamo performs an oscillation of characteristic nature, but no reversal of magnetic field is achieved by that model. A system of coupled-disk dynamos (Rikitake) was proved to perform complicated oscillations with occasional reversals. The Rikitake model has been extensively examined by many workers. The model is capable of performing oscillations similar to those as revealed by palaeomagnetic study.

As the Rikitake model seems to be highly idealized, time-dependent behaviour of an Inglis model, which simulates Bullard dynamo action, was studied (Rikitake). The model seems highly stable; a field reversal occurs only when an incredibly large kick of magnetic field is given to its steady state. A small magnetic field given to the zero-field state of the model grows, eventually reaching a steady state. Analogy between the model and the actual earth suggests that the time constant of reversal amounts to a few thousand years.

A non-steady state Herzenberg dynamo model consisting of two rotating spheres embedded in a conductive medium has been studied by solving simultaneous non-linear integral equations. No great success has been achieved because of computational difficulties. Similar studies have been extended to other models though it is no easy matter to take the equation of motion into account properly.

On the basis of the time-dependent behaviour of these models, the mechanism of field reversal is discussed. A few mechanisms based on statistical fluctuations of fluid motion (e.g. Nagata) do not seem workable. It is concluded that geomagnetic polarity reversals take place as a result of complicated exchange of energy between units consisting of dynamo models.

1. Introduction

Whether a homogeneous dynamo driven by fluid motion can attain a steady state is still an open question (Bullard 1972). Since the geomagnetic field is ever changing with frequent polarity reversals of the dipole field, it is of importance and interest to examine non-steady behaviour of geomagnetic dynamo models even in case we should have no steady state at all.

Much of the nature of geomagnetic field reversals has recently become clear through palaeomagnetic studies on volcanic as well as sedimentary rocks, deep-sea sediments and geomagnetic anomalies at sea (e.g. Bullard 1968). One of the most remarkable facts brought out is that the geomagnetic field changes its polarity within a very short period of time amounting to a few thousand years.

Non-steady states of geomagnetic dynamo models have been examined from time to time in order to see to what extent those models can account for the time-dependent behaviour of the geomagnetic field (e.g. Rikitake 1966).

It is the purpose of this paper to review these studies on non-steady geomagnetic dynamo models with special reference to the mechanism of geomagnetic field reversal.

2. Non-steady disk dynamo models

2.1. The Bullard disk dynamo model

Probably, one of the simplest and best worked-out dynamo models is the Bullard disk dynamo (Bullard 1955) as shown in Fig. 1. In the presence of a magnetic field parallel to the axis, the electromotive force induced in the metallic disk drives an electric current in the coil through the brushes so that a magnetic field, which is in a direction more or less the same as that initially applied, is produced at the disk.

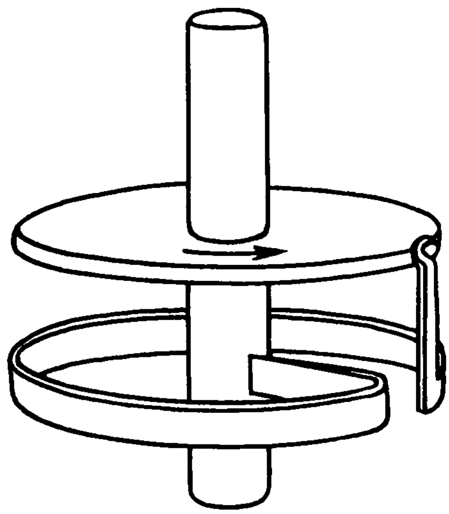

FIG. 1. Disk dynamo (Bullard 1955).

The equations that govern the electric currents and the motion of the disk are given as

$$\left.\begin{aligned} L\,dI/dt+RI &= M\omega I \\ C\,d\omega/dt &= G-MI^2 \end{aligned}\right\} \quad (1)$$

where L, R, $2\pi M$, I, ω, C and G denote respectively the self-inductance and the resistance of the circuit, the mutual inductance between the coil and the periphery of the disk, the electric current intensity, the angular velocity, the moment of inertia of the disk, and the mechanical couple driving the disk. The resistance in the disk and the friction at the axis are ignored.

Conditions $dI/dt = 0$ and $d\omega/dt = 0$ result in

$$\omega_0 = R/M, \qquad I_0 = \pm(G/M)^{1/2} \quad (2)$$

which represent the angular velocity and the electric current intensity for the steady states. Attention should be drawn to the fact that there are two steady directions of electric current or magnetic field for a critical angular velocity.

Eliminating ω from (1), we obtain

$$d^2(\log I)/dt^2 = (GM/CL)(1-MG^{-1}I^2). \quad (3)$$

Putting

$$\tau = (2GM/CL)^{1/2}t, \quad y = \log(MI^2/G) \tag{4}$$

and introducing them into (3), we obtain

$$d^2y/d\tau^2 = 1 - e^y \tag{5}$$

which can be solved by an elementary method.

The solutions of (5) lead to an oscillation of electric current intensity around one of the steady states. The oscillation is characterized by bursts of electric current during a short period of time, while the current intensity becomes very small for remaining periods. Changes in the angular velocity are gradual for the latter period, but a rapid change occurs during the bursts of electric current (Bullard 1955). It has been proved, however, that the current never changes its sign, so that a time-dependent disk dynamo does not provide a model that can account for a magnetic field reversal.

2.2. *The Rikitake coupled-disk dynamo model*

On the supposition that there must be a number of eddy motions in the Earth's core which are electromagnetically coupled to each other, Rikitake (1958) presented a dynamo model consisting of two disks coupled to one another as shown in Fig. 2.

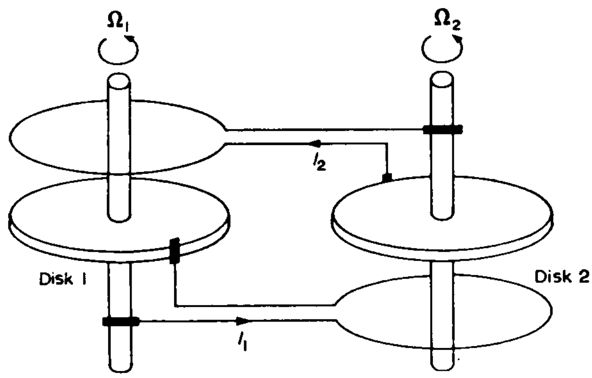

FIG. 2. The Rikitake coupled-disk dynamo (Rikitake 1958).

It is apparent that the time-dependent behaviour of such a coupled-disk dynamo model is governed by two differential equations for the electric currents flowing in the two coils and the equations of motion of the two disks. In the case of two similar disks and electric circuits, these equations, written in dimensionless forms, reduce to

$$\left.\begin{aligned} dx_1/d\tau + \mu x_1 &= y_1 x_2 \\ dx_2/d\tau + \mu x_2 &= y_2 x_1 \\ dy_1/d\tau = dy_2/d\tau &= 1 - x_1 x_2 \end{aligned}\right\} \tag{6}$$

in which x_1 and x_2 denote the dimensionless electric currents flowing respectively in circuits 1 and 2, while y_1 and y_2 are the dimensionless angular velocities for disks 1 and 2. τ and μ denote the dimensionless time and the square root of the ratio of a mechanical time scale of the system to its electrical time constant.

Rikitake (1958) studied numerically oscillations of the system showing that the magnetic field can perform a reversal at least initially. The Rikitake model was later integrated very extensively with the aid of analogue (Mathews & Gardner 1963) and digital computers (Allan 1958, 1962; Cook & Roberts 1970). Fig. 3 shows a

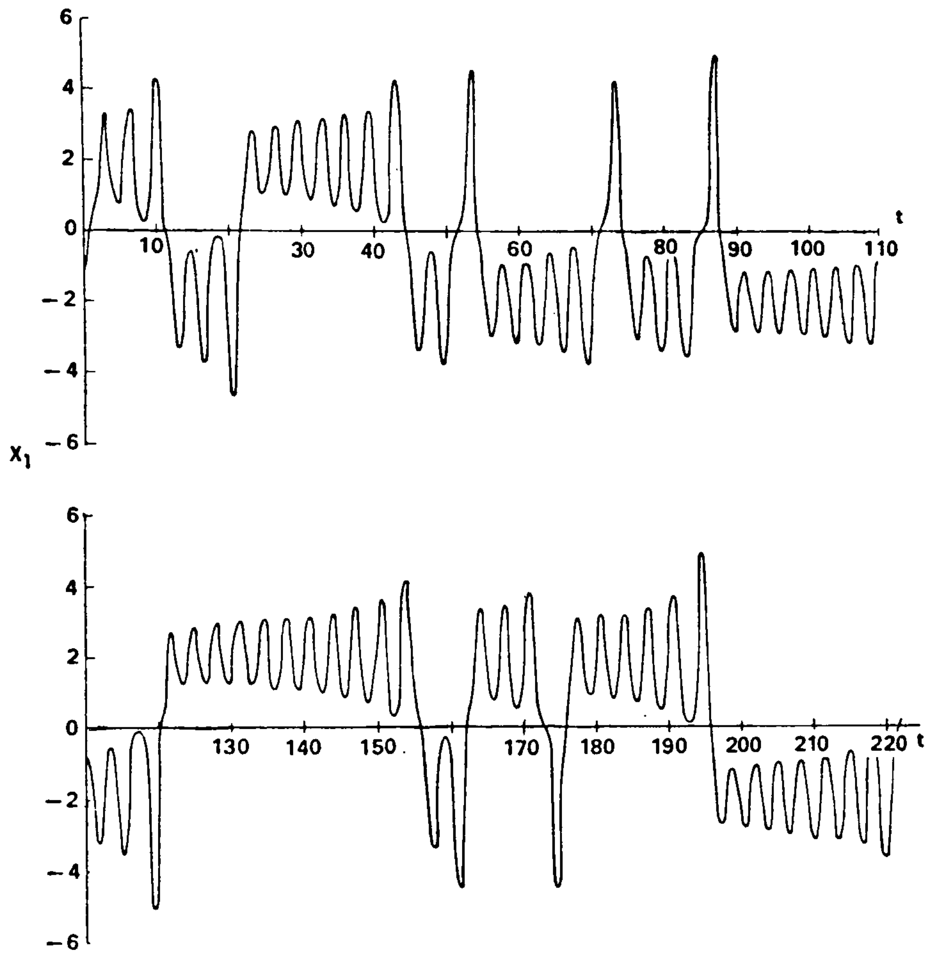

FIG. 3. Typical oscillations of the coupled-disk dynamo (Cook & Roberts 1970).

typical feature of oscillations of the model. The electric current flowing in one of the coils performs an oscillation around one of its steady states, the amplitude getting larger with time. Then, the oscillation suddenly switches over to that around another steady state of different sign. Reversals of the sign of electric current occasionally occur.

The model is so idealized that detailed discussion of the oscillation period, the time required for completing a reversal, or the like cannot be made in regard to the actual Earth. It is important that the model can provide a series of field reversals similar to those that have been suggested for the Earth's dipole field from palaeomagnetic studies.

2.3. *The Inglis multi-disk dynamo model*

Inglis (1955) put forward a visual aid for understanding a dynamo process such as the Bullard one (Bullard 1949; Bullard & Gellman 1954). The Inglis model consists of a few stages in which electromagnetic induction by rotating disks and wheels play an important role. It was originally proposed in order to simulate a steady dynamo action in the Earth's core. Rikitake (1966, pp. 67–71) set out a set of non-linear equations that govern a non-steady Inglis model. He found out that a field reversal can take place when a large disturbance is given to its steady state. It was also proved that a small magnetic field given to the zero-field state of the model can grow eventually reaching a steady state after performing complicated oscillations as shown in Fig. 4. If we think of an Inglis model, which is analogous to a dynamo prevailing in the Earth's core, it was proved that a field reversal occurs within a period of the order of 10^3 years.

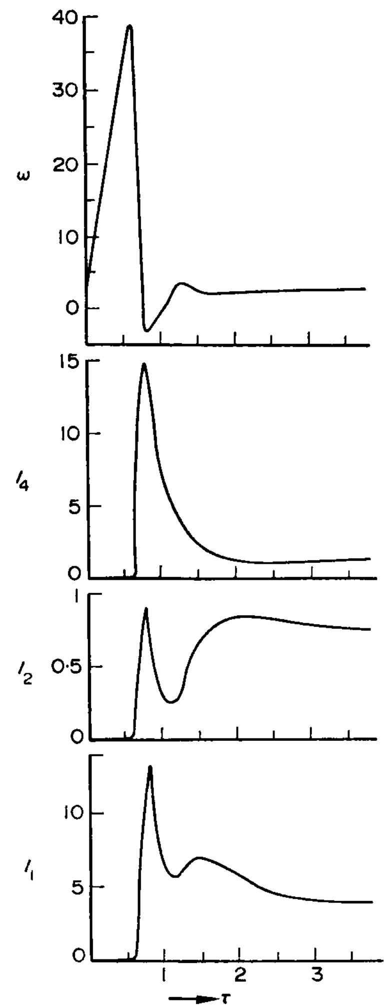

FIG. 4. Changes in the angular velocity and electric currents with time when a small magnetic field is given to the zero-field state of the Inglis model. The top curve denotes the changes in the angular velocity. Other curves are those for electric currents in a few circuits. I_4 is the current that produces the dipole field. All quantities are dimensionless (Rikitake 1966).

3. Non-steady state of other dynamo models

3.1. The Herzenberg two-sphere model

Time-dependent behaviour of a Herzenberg dynamo model (Herzenberg 1958) consisting of two rigid rotating spheres embedded in a conductor was approached by Rikitake & Hagiwara (1966). Solving a set of simultaneous non-linear integral equations, they showed that the model performs an oscillation similar to that of a disk dynamo. Integration over a long period of time was difficult to perform because of computational difficulties. A Herzenberg model in an infinite medium seems likely to be unstable.

3.2. Kinematic models

Both the equations of motion and those governing electric current (and consequently magnetic field) are taken into account in studying the dynamo models in the above. However, those models are far from realistic because they all rely on rigid rotators. No rigorous treatment for solving time-dependent dynamo problems, in which the equation of motion of the core's fluid is properly taken into account, has ever been put forward. Studies of non-steady state of homogeneous fluid dynamos have so far been made mostly ignoring the reaction of the mechanical force due to electromagnetic origin.

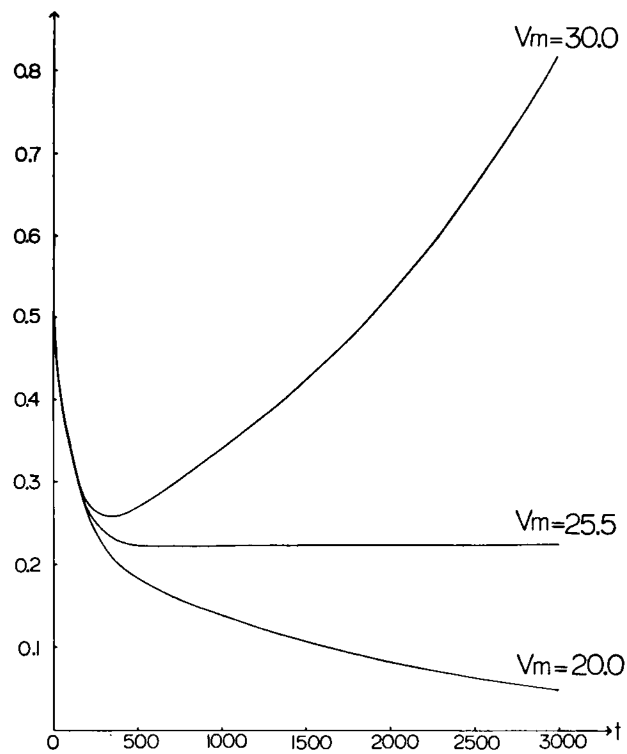

FIG. 5. Changes in the dipole field at the core's surface for three values of the measure of fluid velocity when an initial field is given to the zero-field state of a Bullard–Gellman–Lilley dynamo. $V_m = 25{\cdot}50$ corresponds to the steady state. The time is dimensionless (Uno 1972).

Rikitake & Hagiwara (1968a, b) studied how a dipole field given to the zero-field state of the Bullard–Gellman dynamo model changes with time. The fluid motion is assumed to be the same as that for the steady state of the dynamo. Uno (1972) made a similar study of the Bullard–Gellman–Lilley model (Lilley 1970), a model improved by adding a more complicated fluid motion to that of the Bullard–Gellman model. As can be seen in Fig. 5, the dipole field given to the zero-field state model tends to decrease in the beginning eventually reaching a steady value when the velocity takes on a critical value which corresponds to the steady state.

The initial decrease thus brought out is believed to be caused by the fact that some magnetic energy is needed to create toroidal fields. In Uno's study only four types of magnetic field are taken into account. If magnetic fields of much higher degree and order in spherical harmonic expression are taken into account, it is expected that the steady value of the dipole field, that will be finally reached, would become somewhat smaller.

One of the important results deduced from this sort of study is the point that about 6000 years is required for a Bullard–Gellman–Lilley dynamo to achieve its steady state when an initial field is given to the zero-field state.

Rikitake (1971) presented a pilot study on time-dependent behaviour of a homogeneous fluid dynamo in an entirely numerical way. A conducting fluid confined in a cubic volume is studied. Although no exact proof of existence or non-existence of a steady state could have been obtained because of computational difficulties, it was proved that the magnetic field can be kept constant over a fairly long period of time.

Studies of non-steady kinematic dynamos, which may possibly lead us to erroneous results when the magnetic field reaches a large value because of the neglect of the equation of motion, may have some bearing on arguments of geomagnetic reversal as we shall see in the following section.

4. Geomagnetic field reversal

Takeuchi (1956) and Nagata (1969, 1970) presented speculations that the magnetic field of a homogeneous fluid dynamo would tend to decay when the velocity of fluid motion becomes slightly lower than the critical value or loses its asymmetry and that, when the motion should recover its original state, the magnetic field would grow from an accidental field with equal probability either in the same or opposite directions.

Theories of non-steady kinematic dynamo models do not seem to support such a theory of field reversal due to statistical fluctuations of the fluid motion as pointed out by Rikitake (1972). First of all, the field once established would not vanish within a period of time comparable to that of free decay for the dipole field, 10^4 years or so say (e.g. Yukutake 1968), because the model still have an appreciable power for creating the dipole field even when the fluid motion slightly deviates from its steady state.

In the second place, it has been shown in the last section that the field given to the zero-field state does not grow. In order to expect a growth of the field, we need much kinematic energy that can be converted to magnetic energy as we have seen in the case of the Inglis model.

It appears to the writer that the mechanisms of geomagnetic field reversal as suggested by Takeuchi and Nagata do not seem to work although he cannot say anything about Cox's mechanism (Cox 1968, 1969) which also depends on some statistical causes.

In order to account for a field reversal completed within a period of a few thousand years as suggested by palaeomagnetic studies, it seems necessary to think of a negative dynamo action which destroys the already established field (Parker 1969). Such an action is achieved by conversion between kinetic and magnetic energies in the case of a coupled-disk system.

5. Concluding remarks

The present status of studies on non-steady states of a number of geomagnetic dynamo models is briefly reviewed in this paper. Although no detailed study could be carried out on realistic models which stimulate the dynamo action prevailing in the Earth's core, possibility of geomagnetic polarity reversal has been suggested on the basis of complicated exchange of energy between units consisting of dynamo models in the case of the Rikitake coupled-disk dynamo model.

Discussion based on kinematic models led us to a conclusion that a few mechanisms hitherto proposed for accounting for field reversal do not seem to work.

In conclusion, it should be emphasized that the studies on non-steady dynamo models so far made are so crude that no concrete discussion about the time required for completing a reversal, the length of polarity interval and the like can be made. Much more intensive studies on the subject should be encouraged in the future although we shall need an extremely large computer for that purpose.

Earthquake Research Institute,
University of Tokyo

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