Eletromagnetismo

4300372

F.S. Navarra

navarra@if.usp.br

Guilherme Germano (monitor)

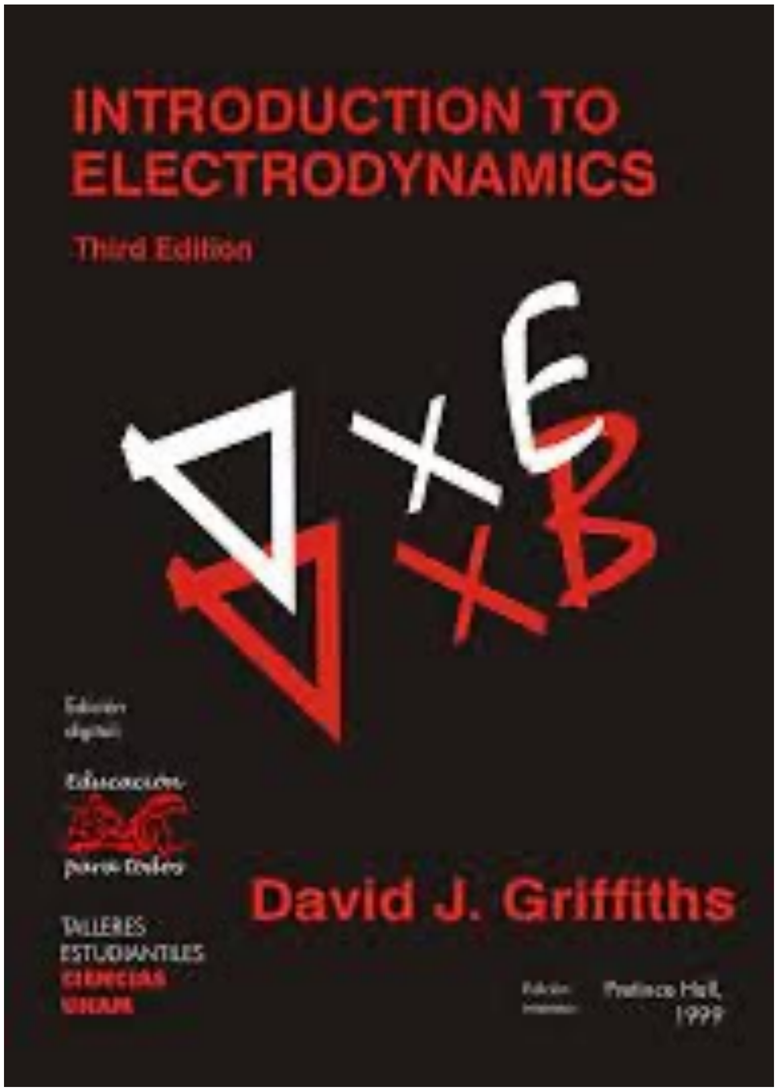
guilherme.germano@usp.br

edisciplinas.if.usp.br

Plano do Curso

16/08	13/09	11/10	08/11
19/08	16/09	14/10	11/11
23/08	20/09 P1	18/10	15/11
26/08	23/09	21/10 P2	18/11
30/08	27/09	25/10	22/11
02/09	30/09	28/10	25/11
06/09	04/10	01/11	29/11 P3
09/09	07/10	04/11	02/12 ex
			06/12 Sub

Bibliografia



Capítulo 2 : eletrostática

Capítulo 5 : magnetostática

Capítulo 7 : eletrodinâmica

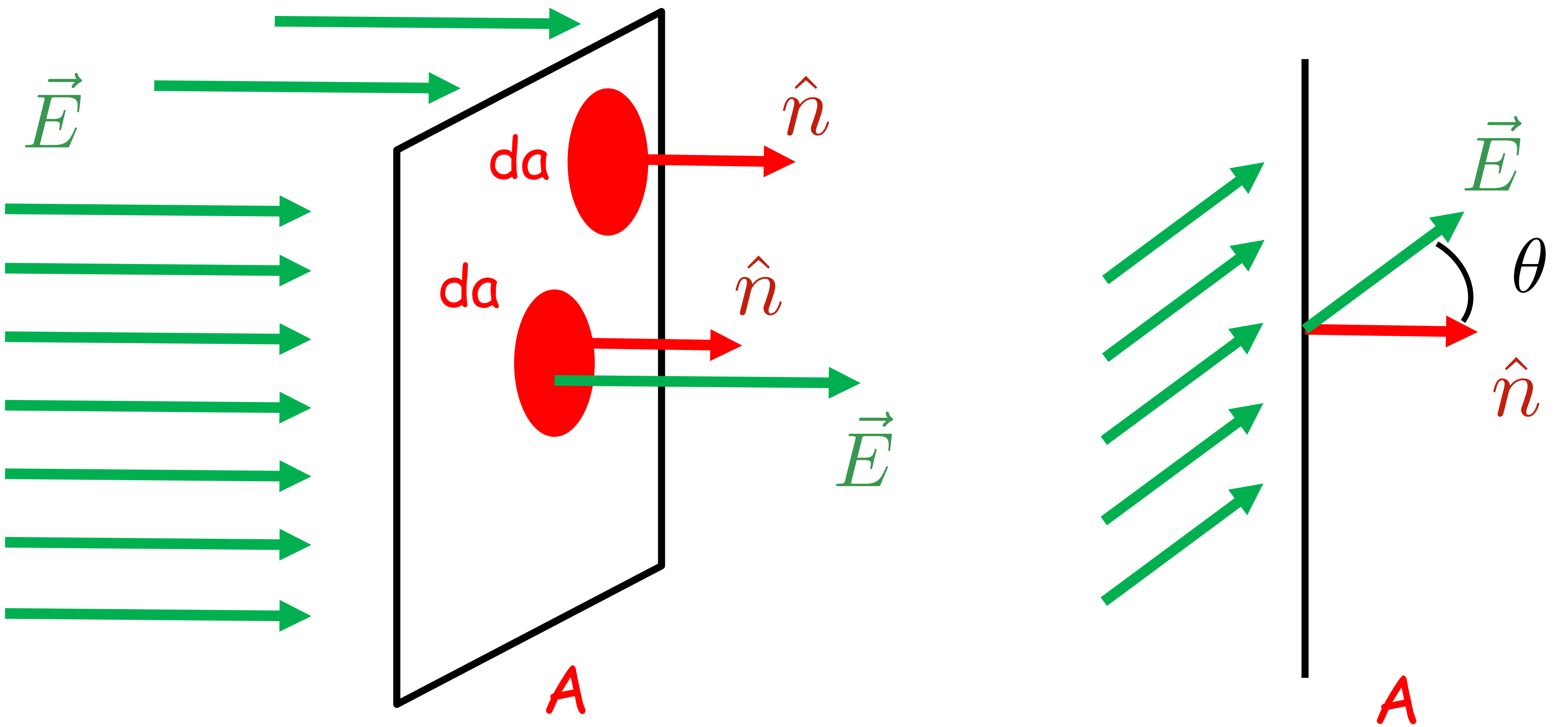
Capítulo 8 : leis de conservação

Capítulo 9 : ondas eletromagnéticas

Capítulo 10 : campos e potenciais

Capítulo 11 : radiação

Fluxo de campo elétrico através de uma superfície :



Fluxo elétrico :

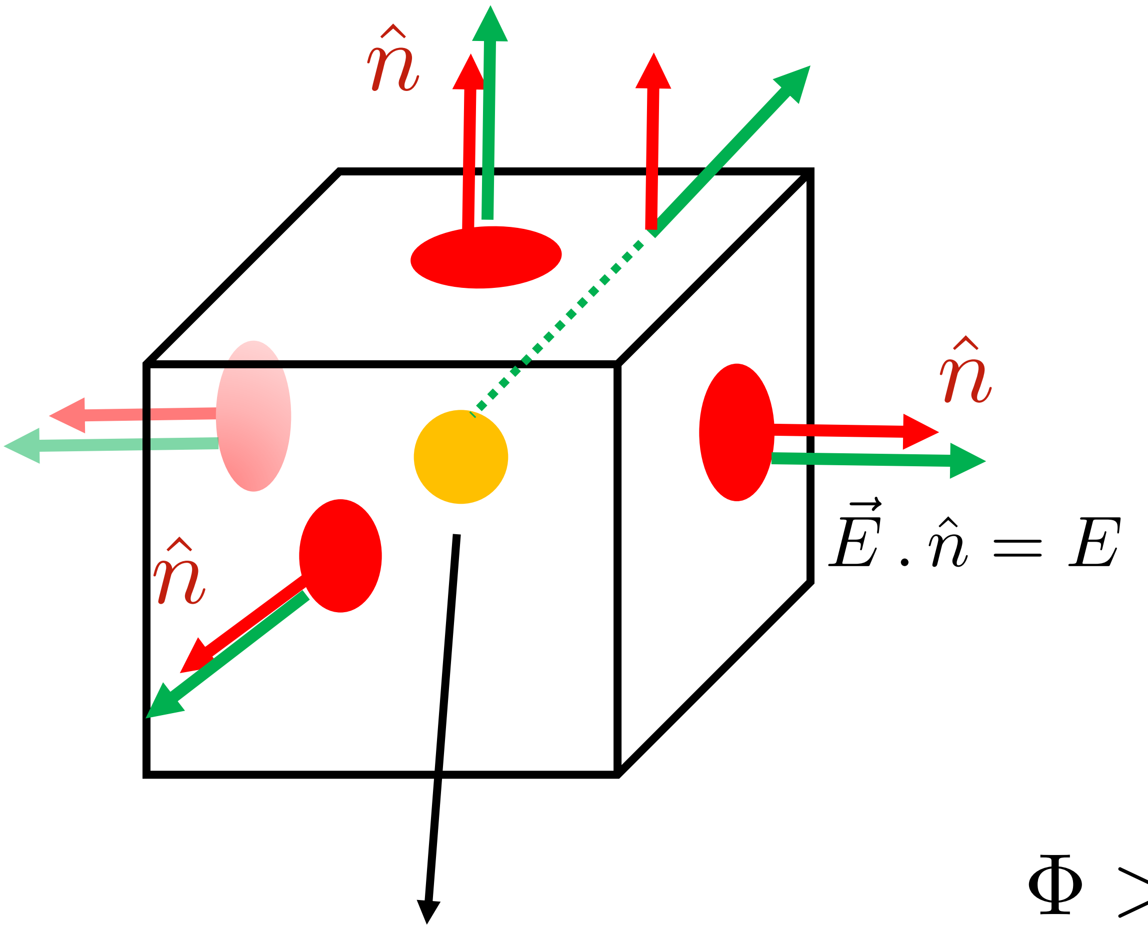
$$\Phi = \int_A \vec{E} \, . \, \hat{n} \, da$$

$$\vec{E} \, . \, \hat{n} = E \, cos\theta$$

É uma medida de quanto a superfície é "furada" pelas flechas

Se a superfície for fechada:

$$\Phi = \oint_A \vec{E}\,.\,\hat{n}\,da$$



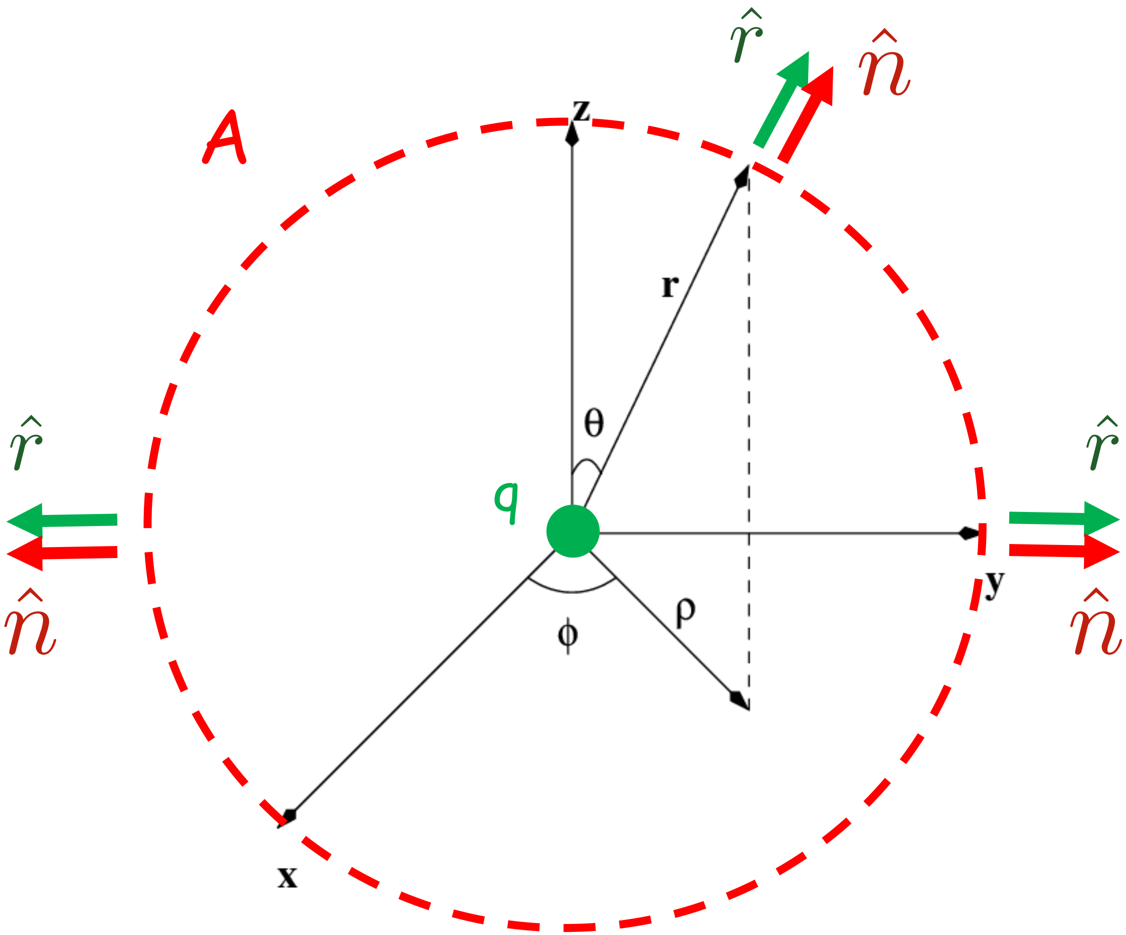
$$\Phi > 0$$

Lei de Gauss

Não é dedução...

É sedução...

Em caso de dúvida volte sempre para este exemplo-mãe :



$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

$$\hat{n} = \hat{r}$$

$$\vec{E}.\hat{n} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}.\hat{r}$$

$$\vec{E}.\hat{n} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}$$

$$da = r\,d\theta\;r\,sen\,\theta\,d\,\phi$$

$$\Phi = \oint_A \vec{E}\,.\,\hat{n}\,da \;=\; \underbrace{\int_0^{2\pi} d\phi}_{2\,\pi}\;\underbrace{\int_0^{\pi} sen\,\theta\,d\theta}_{2}\;\cancel{r^2}\quad \frac{q}{4\pi\epsilon_0}\frac{1}{\cancel{r^2}} \;=\; \frac{q}{\epsilon_0}$$

Lei de Gauss

Fazemos uma generalização corajosa e despudorada !

Forma intergal

$$\oint_A \vec{E} \, . \, \hat{n} \, da = \oint_A \vec{E} \, . \, d\vec{a} = \frac{Q_e}{\epsilon_0}$$

Forma diferencial

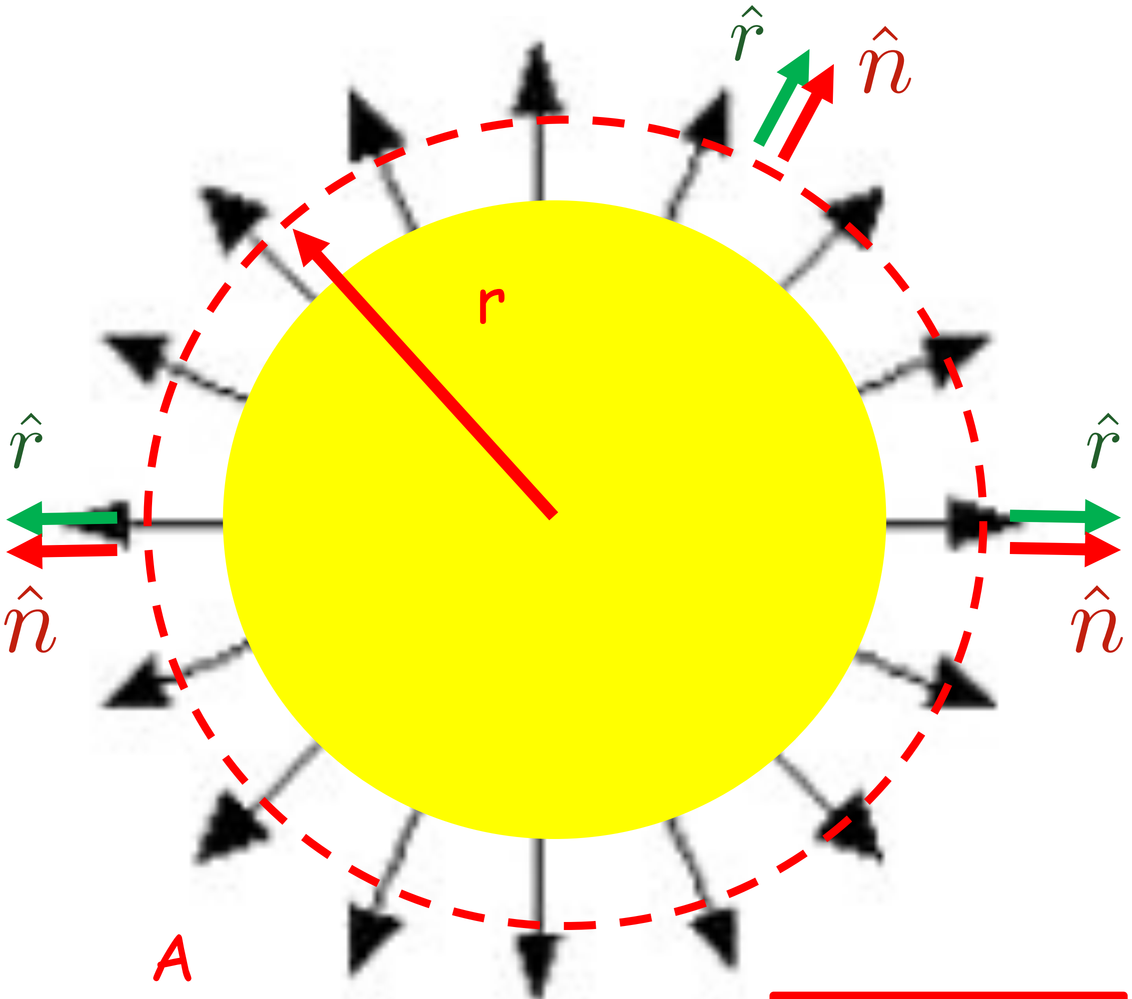
$$\vec{\nabla} \, . \, \vec{E} = \frac{\rho}{\epsilon_0}$$

Aula 4

Aplicações da Lei de Gauss

Esfera uniform. carreg. com raio R e carga Q :

Simetria esférica: o campo elétrico é radial !



$r > R$

$$E = \frac{Q}{4\,\pi\,\epsilon_0}\,\frac{1}{r^2}$$

$$\vec{E} = E\,\hat{r}$$

$$\hat{n} = \hat{r}$$

$$\vec{E}\,.\,\hat{n} = E\,\hat{r}\,.\,\hat{r} = E$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \oint_A E\,da$$

E é constante na sup. *A*

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E \oint_A da$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E\,4\,\pi\,r^2$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0} \quad \text{(Gauss)}$$

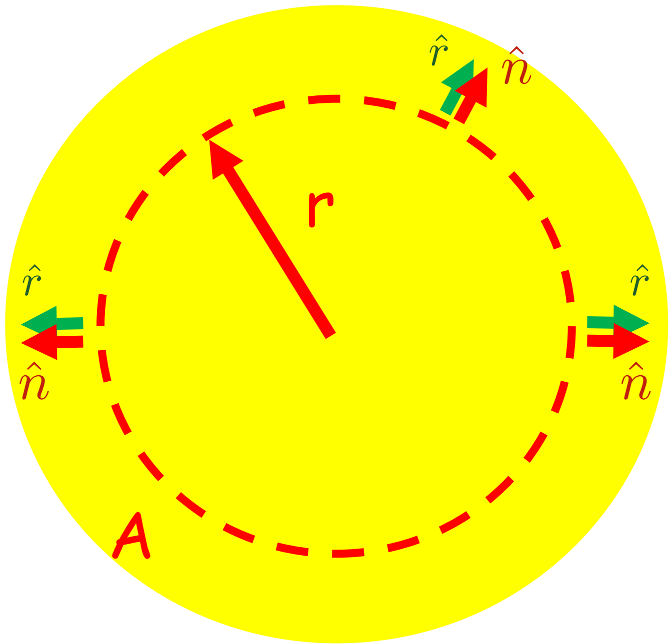
$$E\,4\,\pi\,r^2 = \frac{Q}{\epsilon_0}$$

Esfera uniform. carreg. com raio R e carga Q :

$$\vec{E}\,.\,\hat{n} = E\,\hat{r}\,.\,\hat{r} = E$$

Simetria esférica: o campo elétrico é radial !

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \oint_A E\,da$$



E é constante na sup. *A*

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E \oint_A da$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E\,4\,\pi\,r^2$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0} \quad \text{(Gauss)}$$

$$\rho = \frac{Q}{\frac{4}{3}\,\pi\,R^3}$$

$$E\,4\,\pi\,r^2 = \frac{4\,\pi\,r^3}{3\,\epsilon_0}\,\rho$$

$$Q_e = \rho\,\frac{4}{3}\,\pi\,r^3$$

$$r < R$$

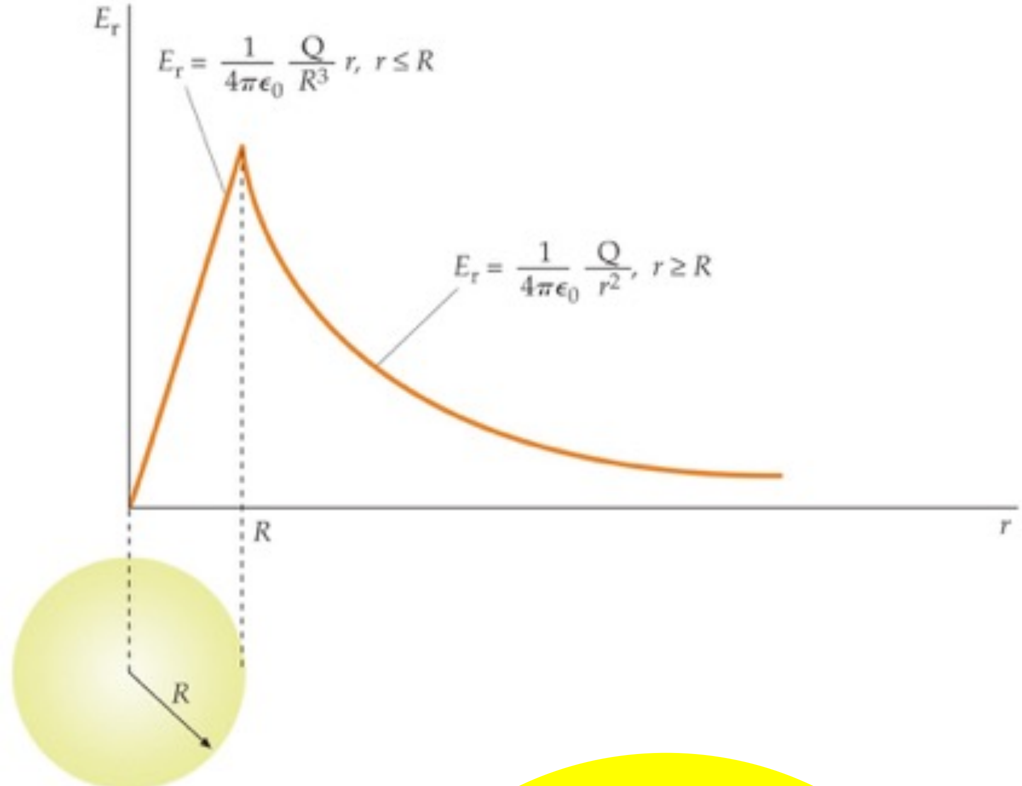
$$\boxed{E = \frac{\rho}{3\,\epsilon_0}\,r}$$

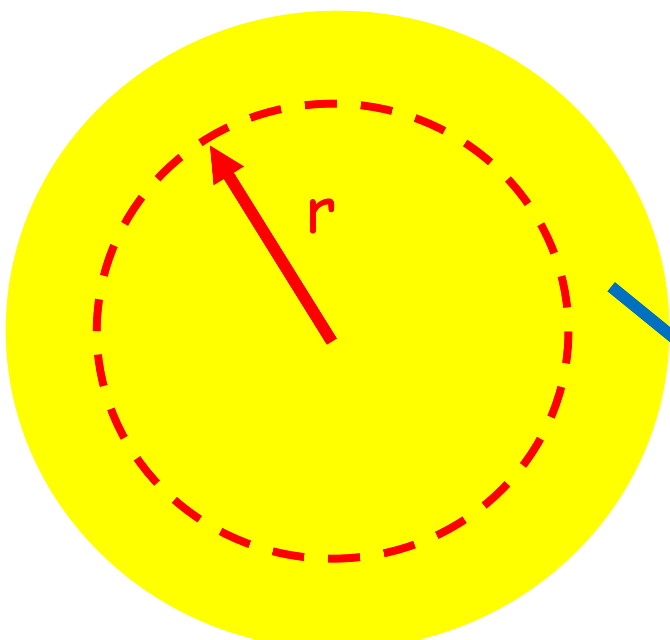
O gráfico é importante !

$$E = \frac{\rho}{3\,\epsilon_0}\,r$$

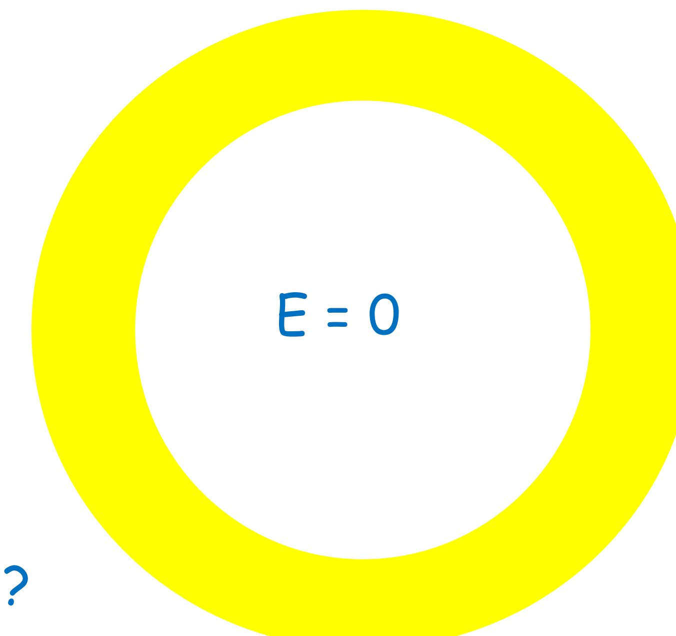
$$\rho = \frac{Q}{\frac{4}{3}\,\pi\,R^3}$$

$$E = \frac{Q}{4\,\pi\,\epsilon_0}\,\frac{r}{R^3}$$

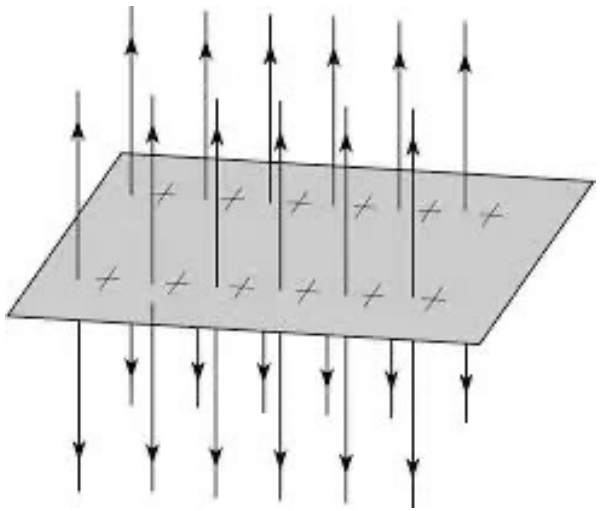




E a contribuição
da casca externa ?

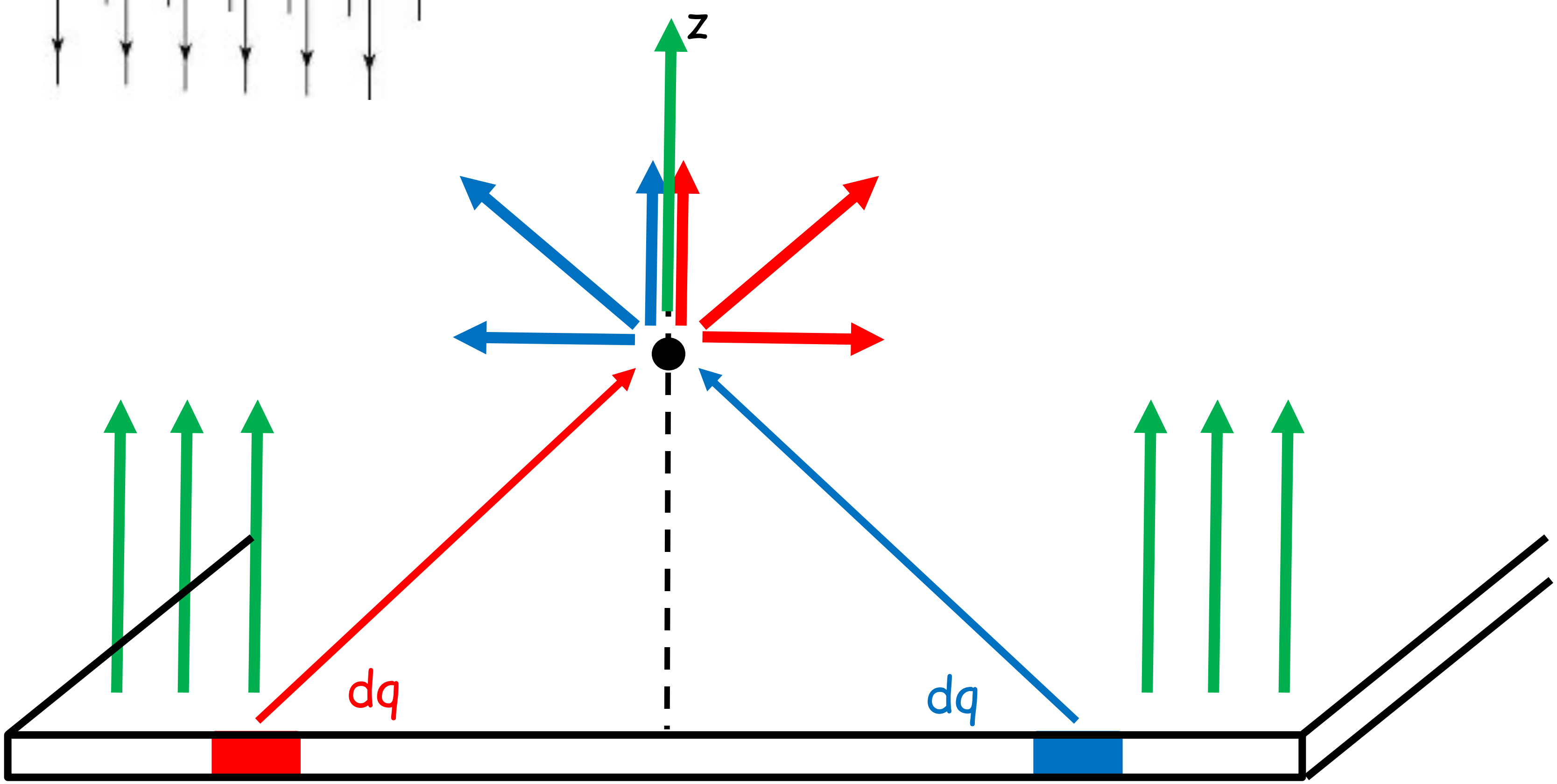


Plano infinito carregado com densidade superficial de carga σ

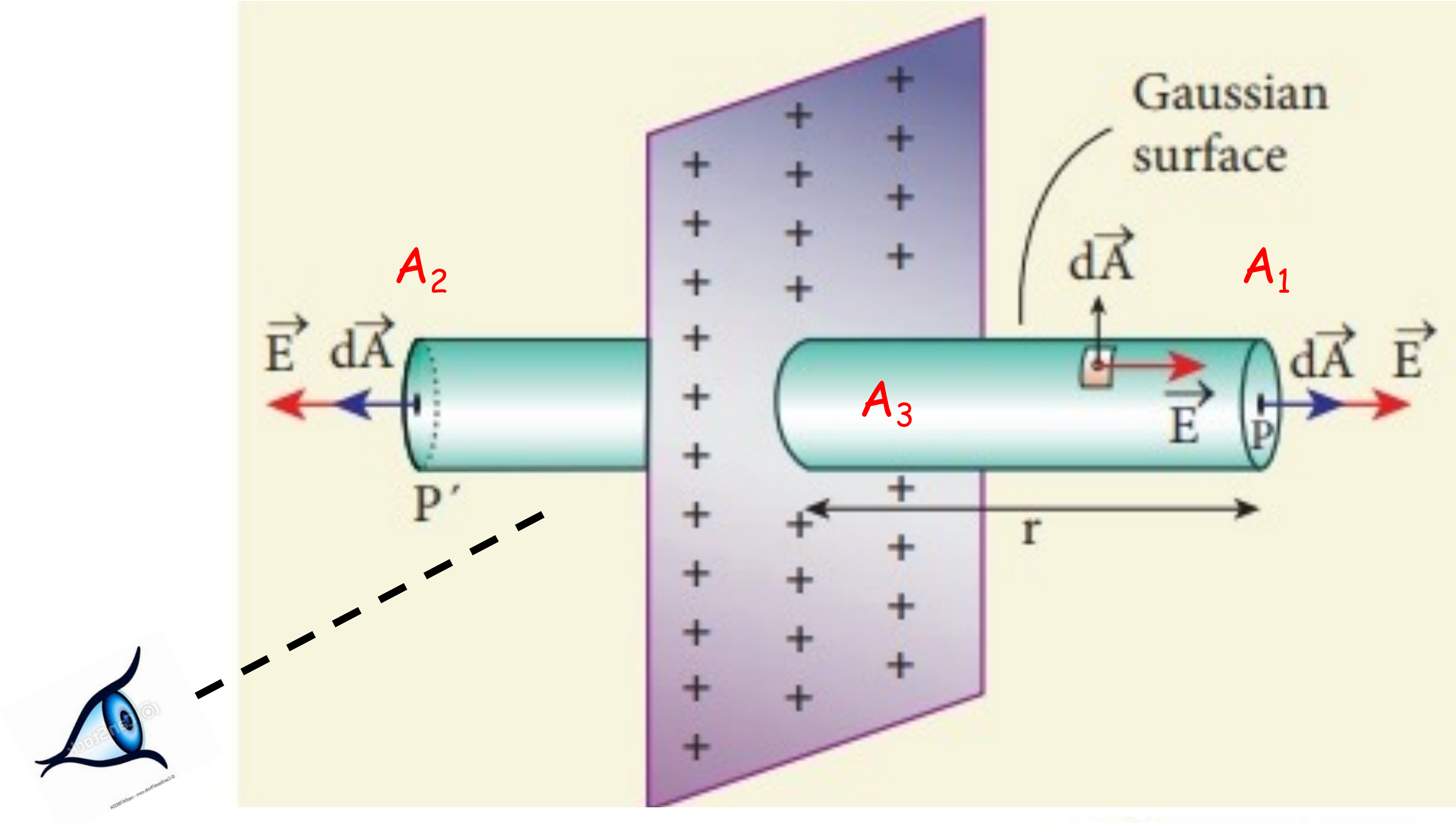


Componentes horizontais se cancelam !

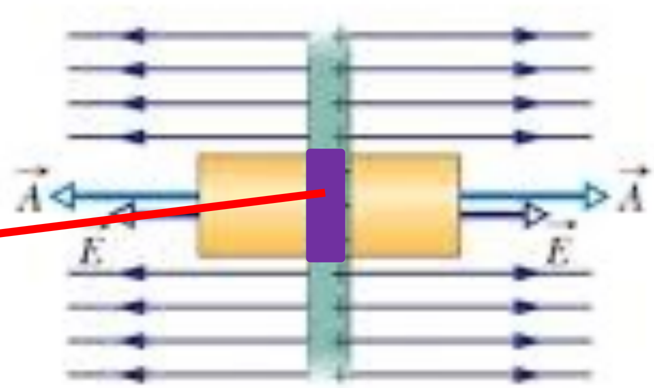
Componentes verticais se somam !



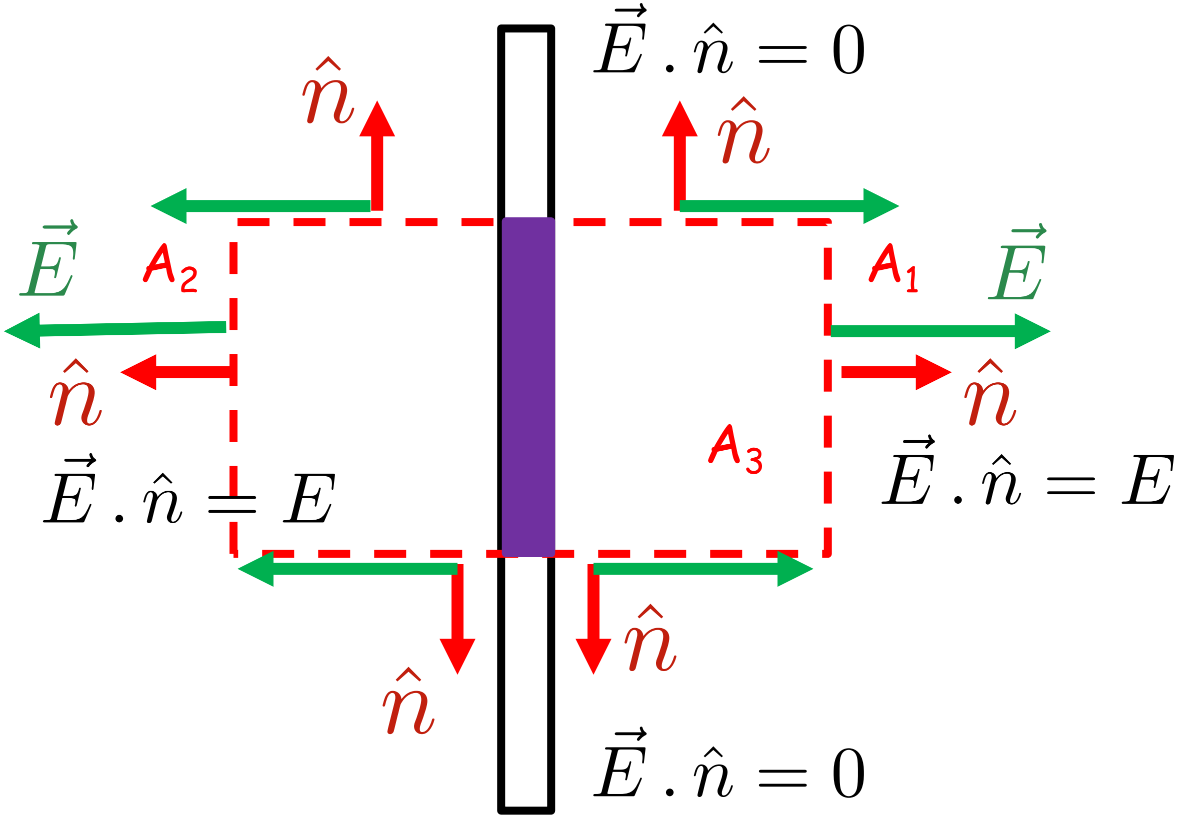
Plano infinito carregado com densidade superficial de carga σ



Superfície Gaussiana é um cilindro !



Carga envolvida



$$\oint_A \vec{E}\,.\,\hat{n}\,da = \frac{Q_e}{\epsilon_0}$$

$$2\,E\,\cancel{A} = \frac{\sigma}{\epsilon_0}\,\cancel{A}$$

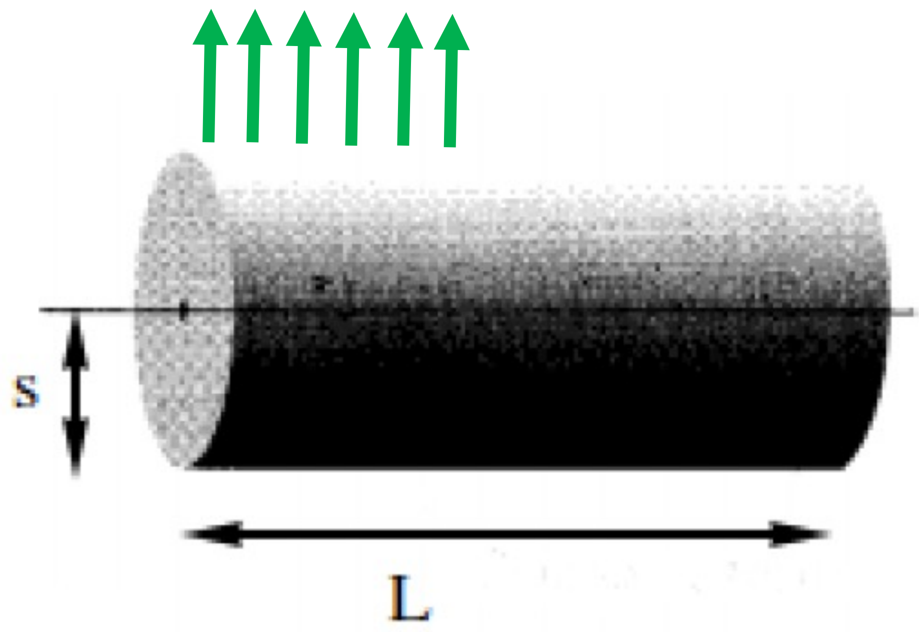
$$\boxed{E = \frac{\sigma}{2\,\epsilon_0}}$$

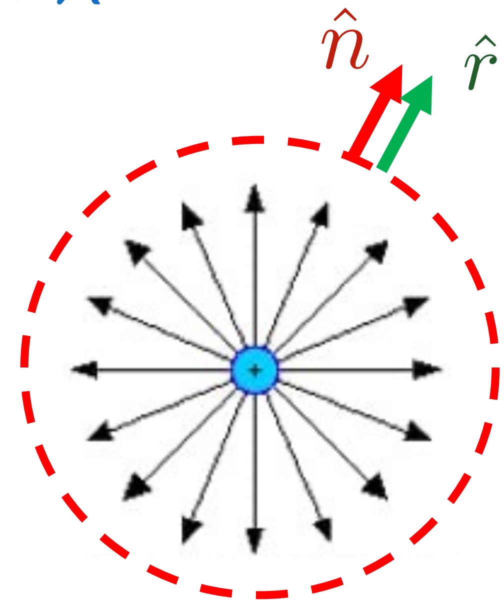
$$Q_e = \sigma\,A$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = \int_{A_1} \underbrace{\vec{E}\,.\,\hat{n}}_{E}\,da + \int_{A_2} \underbrace{\vec{E}\,.\,\hat{n}}_{E}\,da + \int_{A_3} \underbrace{\vec{E}\,.\,\hat{n}}_{0}\,da$$

$$\oint_A \vec{E}\,.\,\hat{n}\,da = E\int_{A_1} da + E\int_{A_2} da = 2\,E\,A$$

Fio infinito carregado com densidade linear de carga λ





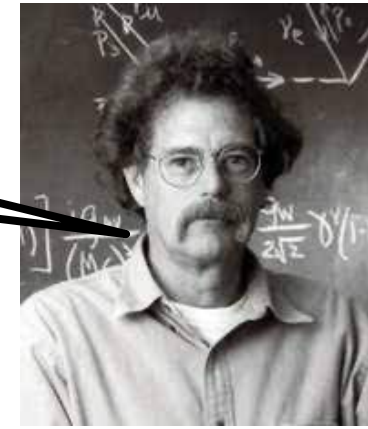
Superfície gaussiana
é um cilindro

$$\oint_A \vec{E} \,.\, \hat{n}\, da = \frac{Q_e}{\epsilon_0}$$

$$E \oint da = E 2\pi s L = \frac{Q_{in}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\boxed{\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}.}$$

Vamos dar um passeio
pela matemática !



D.J. Griffiths

Depois do nabla: $$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

Depois do gradiente de um vetor : $$\nabla T = \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) T$$

Depois do divergente de um vetor:

$$\nabla \cdot \mathbf{v} = \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right) \cdot (v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

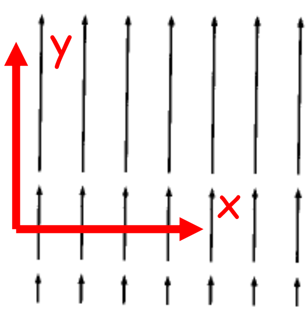
Conheça o Rotacional !

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

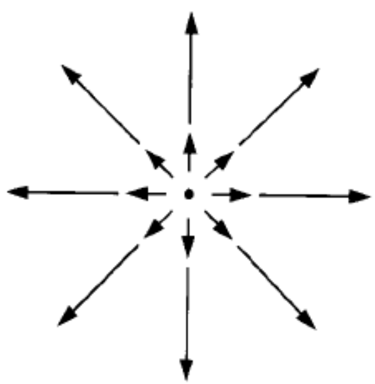
$$\mathbf{v} \;=\; v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}$$

$$\nabla \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$

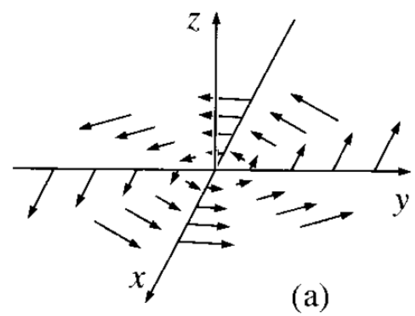
$$= \hat{\mathbf{x}}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$



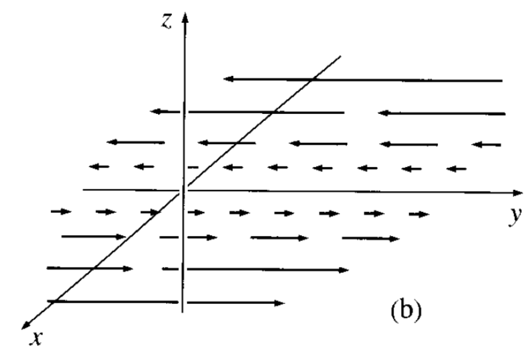
$\vec{\nabla} \times \vec{v} = 0$



$\vec{\nabla} \times \vec{v} = 0$



$\vec{\nabla} \times \vec{v} > 0$



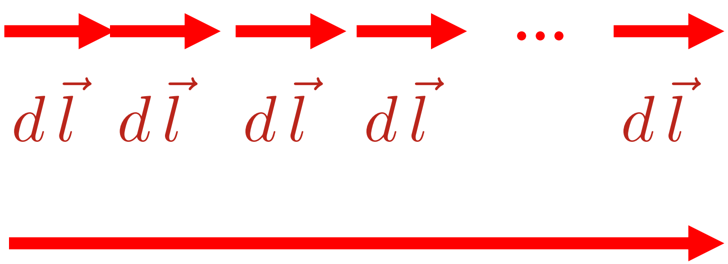
$\vec{\nabla} \times \vec{v} > 0$

Conheça a Integral de linha !

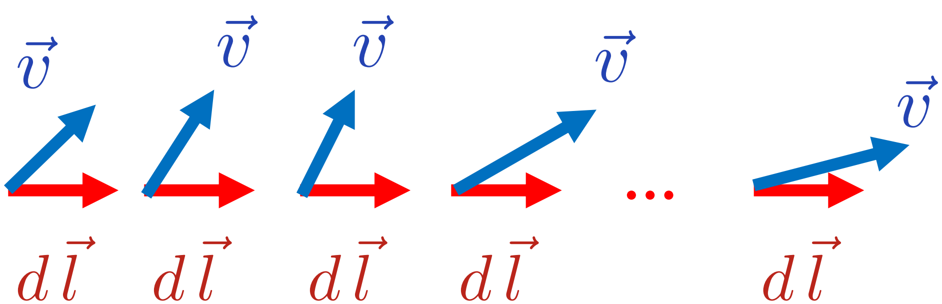
Integral usual : soma de segmentos infinitesimais

$$\sum dx \rightarrow \int dx = L$$

Integral de linha : soma de vetores infinitesimais

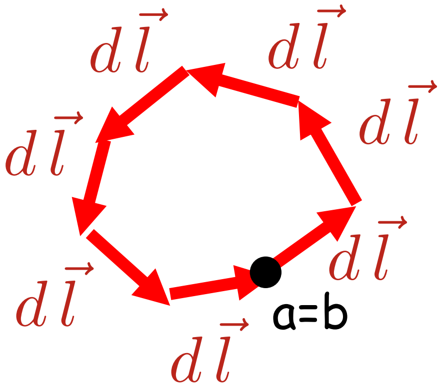


$$\sum d\vec{l} \rightarrow \int d\vec{l} = \vec{L}$$

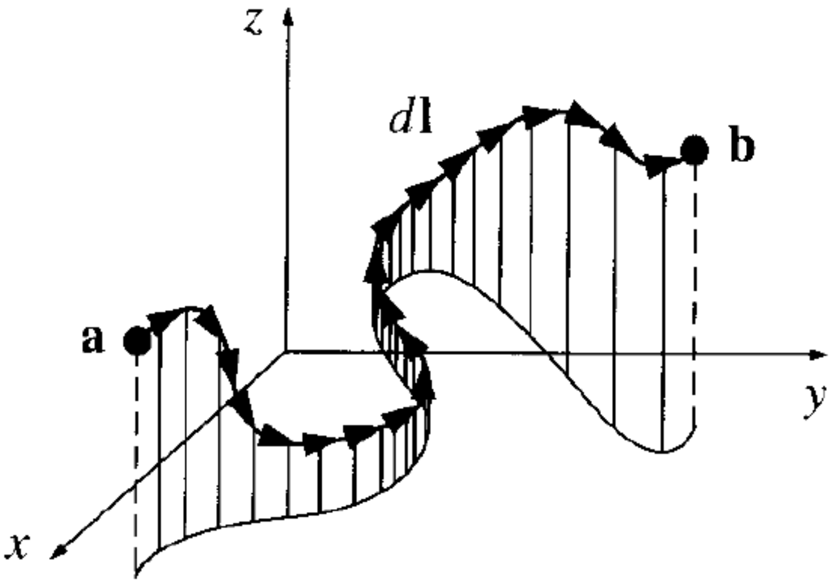


$$\sum \vec{v}\,.\,d\vec{l} \rightarrow \int \vec{v}\,.\,d\vec{l}$$

Integral de linha fechada



$$\oint d\vec{l}$$



$$\int_{\mathbf{a}\,\mathcal{P}}^{\mathbf{b}} \mathbf{v} \cdot d\mathbf{l}$$

$\mathcal{P}$ é o caminho que vai de a a b

Agora vamos juntar estas duas coisas:

Teorema de Stokes

Estudei em Cambridge !
Sou da sociedade real
e o escambau !

Deduzi a equação de
Navier-Stokes !

George Stokes
1819-1903

$$\int_{\mathcal{S}} (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}$$