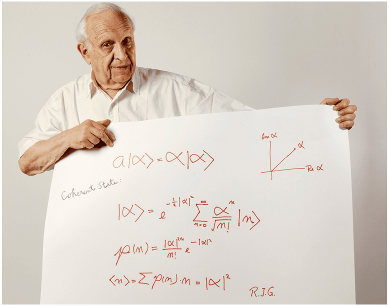
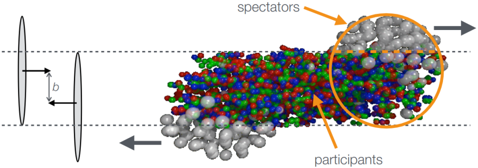
Lecture 4
Geometry: Glauber model



Nobel prize in 2005 for his work on quantum optics

but he also worked on high-energy collisions.

Participants and spectators



- N_{part}: number of nucleons which underwent at least one inelastic collision
- N_{coll}: number of inelastic nucleon-nucleon collisions

In elastic collisions, the colliding particles glance each other, same particles are in initial and final states. In ineslatic collisions, new particles appear after the collisions. In both cases, 4-momentum is conserved (component by component).

Ojectives of the Glauber model

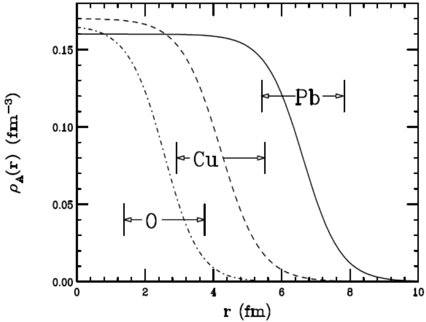
- Glauber model gives estimates for N_{part} and N_{coll}
- It shows up in many models: HIJET, HIJING, VENUS, RQMD, etc.
- It contains approximations: independent linear trajectories of the constituent nucleons, constant value of the inelastic nucleon-nucleon cross section, etc.

Nuclear density

- Distribution of matter (or charge) inside a nucleus often assumed to be of a Woods-Saxon type:

$$\rho_A(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}}$$

ρ_0 is obtained from the normalization $\int \rho(r) d^3r = A$. Other parameters are obtained from charge density measurements. Typical values are: $R = 1.12A^{1/3}$, $a = 0.54\ fm$, $\rho_0 = 0.17\ fm^{-3}$



It works well for nuclei with $A > 16$

- To do analytical calculations, we can use the hard sphere approximation:

$$\boxed{\rho_A(r) = \frac{3}{4\pi r_0^3} \equiv \rho_{hs}} \text{ for all } r \leq R_A \text{ and } R_A = r_0 A^{1/3}$$

so $\rho_A(r)$ is constant inside the nucleus and has the same value for all nuclei.

Exercise:

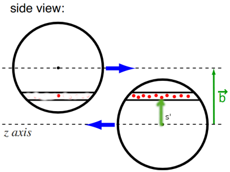
Let us check that $\rho_A(r)$ obeys the correct the normalisation $\int d^3 r \rho_A(r) = \rho_{hs} \int d^3 r = \frac{3}{4\pi r_0^3} \frac{4\pi}{3} R_A^3 = A$

- For non spherical nuclei, one can generalize the Woods-Saxon formula:
  $R \longrightarrow R(\theta,\phi) =$
  $R\{1 + \beta_2[\cos\gamma Y_2^0(\theta,\phi) + \frac{2}{\sqrt{2}}\sin\gamma Re(Y_2^2(\theta,\phi))] + \beta_3 Y_3^0(\theta,\phi)\}$
  β_2 allows for ellipsoidal shape, γ for triaxiality (all axes of ellipsoid non equal) and β_3 for pear shape.

  This is what we do when we study Ruthenium and Zirconium in view of the STAR data Phys. Rev. C 105 (2022) 014901

Spherical nuclei assumed in the following for simplicity

Nuclear thickness function



Let us start with a single nucleon from B. It drills a cylindrically-shaped tube when colliding in A.

Nuclear thickness function: integral of the nuclear density over the longitudinal direction (= how much matter there is in the tube per unit area)

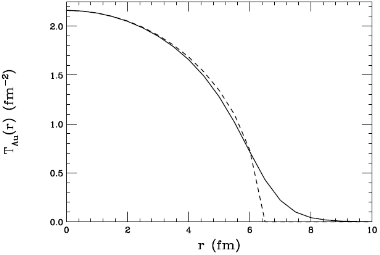
$$T_A(s') = \int dz \rho_A(z, s')$$

s' is the distance of the tube with respect to the the center of nucleus A.

Exercise:

Compute the nuclear thickness in the hard sphere approximation

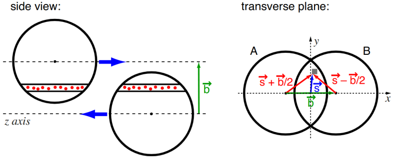
$$T_A(s') = \int dz \rho_A(s', z) = \rho_{hs} \int_{-\sqrt{R_A^2 - s'^2}}^{\sqrt{R_A^2 - s'^2}} dz = 2\rho_{hs}\sqrt{R_A^2 - s'^2}$$



Comparison of the nuclear thickness for Au, computed with a Woods-Saxon and a hard sphere nuclear density (R.Vogt).

Note that $\int d^2s' \, T_A(s') = 4\pi\rho_{hs} \int_0^{R_A} s' \, ds' \sqrt{R_A^2 - s'^2} = \frac{4\pi\rho_{hs}}{3} R_A^3 = A$

Nuclear overlap function



Consider row-on-row collisions

Number of possible nucleon-nucleon enconters per unit transverse area:

$$dT_{AB} = T_A(|\vec{s} + \vec{b}/2|)\, T_B(|\vec{s} - \vec{b}/2|) d^2s$$

Nuclear overlap function:

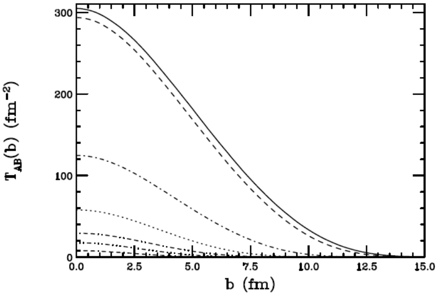
$$\boxed{T_{AB}(b) = \int T_A(|\vec{s} + \vec{b}/2|)\, T_B(|\vec{s} - \vec{b}/2|) d^2s}$$

Integral is on overlap area

Number of collisions

$$N_{coll}(b) = T_{AB}(b)\,\sigma_{inel}^{NN}$$

σ_{inel}^{NN} is the inelastic collision cross section and it depends on $\sqrt{s}$: 32 mb at 20 GeV, 42 mb at 200 GeV and 60 mb at 5.5 TeV. It is supposed to be constant for all the collisions a nucleon is undergoing.



Comparison of the nucleus-nucleus thickness functions top to bottom for a Woods-Saxon distribution: Pb+Pb, Au+Au, I+I, Cu+Cu, Ca+Ca, Si+SI, O+O. (R.Vogt)

Exercise:

Compute the nuclear overlap function at $b = 0$ for the hard sphere approximation for Au+Au and Pb+Pb. Verify it agrees with the previous figure.

$$T_A(s) = 2\rho_{hs}\sqrt{R_A^2 - s^2} \Rightarrow T_{AB}(0) = \int T_A(s)^2 d^2 s =$$
$$4\rho_{hs}^2 \int (R_A^2 - s^2) d^2 s = 4\rho_{hs}^2\, 2\pi (R_A^2 \int_0^{R_A} s ds - \int_0^{R_A} s^3 ds) =$$
$$4\rho_{hs}^2\, 2\pi R_A^4/4 = 2\pi \left(\frac{3}{4\pi r_0^3}\right)^2 (r_0 A^{1/3})^4 \sim 0.25 A^{4/3} \text{ for } r_0 \sim 1.2\, fm.$$

So $T_{Au+Au}(0) \sim 285\, fm^{-2}$ and $T_{Pb+Pb}(0) \sim 305\, fm^{-2}$, which agree reasonably with the figure.

Probability for an inelastic A+B collision

$\hat{T}_A(s') = T_A(s')/A$: proba. per unit transverse area of a given nucleon being located in the tube in A

$d\hat{T}_{AB}(|\vec{s} \pm \vec{b}/2|) = dT_{AB}(|\vec{s} \pm \vec{b}/2|)/AB$= proba. per unit transverse area of nucleons being located in the respective overlapping target and projectile tubes

Then: $N_{coll}(b) = AB\hat{T}_{AB}(b)\,\sigma_{inel}^{NN}$

$\Rightarrow$ proba. for a nucleon from A to collide with a nucleon from B:

$$\boxed{p_{NN} = \hat{T}_{AB}(b)\,\sigma_{inel}^{NN}}$$

Proba for k collisions at impact parameter *b* out of possible AB:

$$P(k,b) = \binom{AB}{k} p_{NN}^k (1 - p_{NN})^{AB-k}$$

Probability for k = 0 is $(1 - p_{NN})^{AB}$. Thus:

$$\boxed{p_{inel}^{AB}(b) = 1 - [1 - \hat{T}_{AB}(b)\sigma_{inel}^{NN}]^{AB}}$$

This can be integrated on d^2b to get σ_{inel}^{AB}

Number of participants at a given impact parameter

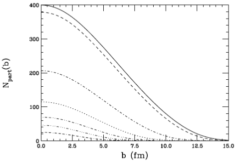
So the probability for a collision when a nucleon from A collides with nucleus B is:

$p^B_{inel}(|\vec{s}-\vec{b}/2|) = 1 - [1 - \hat{T}_B(|\vec{s}-\vec{b}/2|)\sigma^{NN}_{inel}]^B$

and the number of such hadrons is $T_A(|\vec{s}+\vec{b}/2|)d^2s$

Similarly for nucleons from B colliding with A. So

$$
\begin{aligned}
N_{part}(b) &= N^A_{part}(b) + N^B_{part}(b) \\
&= \int d^2s\, T_A(|\vec{s}+\vec{b}/2|)\{1 - [1 - \hat{T}_B(|\vec{s}-\vec{b}/2|)\sigma^{NN}_{inel}]^B\} \\
&+ \int d^2s\, T_B(|\vec{s}-\vec{b}/2|)\{1 - [1 - \hat{T}_A(|\vec{s}+\vec{b}/2|)\sigma^{NN}_{inel}]^A\}
\end{aligned}
$$



Number of participants from top to bottom for a Woods-Saxon distribution: Pb+Pb, Au+Au, I+I, Cu+Cu, Ca+Ca, Si+SI, O+O. (R.Vogt, dependence in σ^{NN}_{inel} weak)

Exercise:

Compute the number of participants at $b = 0$ in the hard sphere approximation for an A+A collision

$N_{part}(0) = 2\int d^2s\, T_A(s)\{1 - [1 - \hat{T}_A(s)\sigma_{inel}^{NN}]^A\} \sim$
$2\int d^2s\, T_A(s)\{1 - exp(-T_A(s)\sigma_{inel}^{NN})\}$ if $T_A(s)\sigma_{inel}^{NN} << A$
and if $T_A(s)\sigma_{inel}^{NN} >> 1$: $N_{part}(0) \sim 2\int d^2s\, T_A(s) = 2A$ (Integral is on overlap region, i.e. a disk of radius R_A)

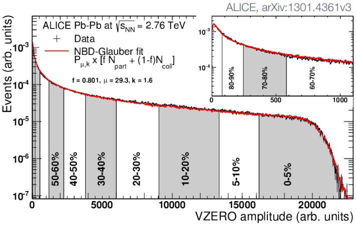
Centrality

It is usual to classify collisions with their impact parameter in centrality bins

$$c = \frac{\pi b^2}{\sigma^{AB}_{inel}}$$

So $c = 0 \Leftrightarrow b = 0$ corresponds to central collisions (total overlap of the colliding nuclei).

Experimentally it is not possible to measure b, so collisions are classified in centrality using multiplicity, energy in some forward detector (ZDC), etc.



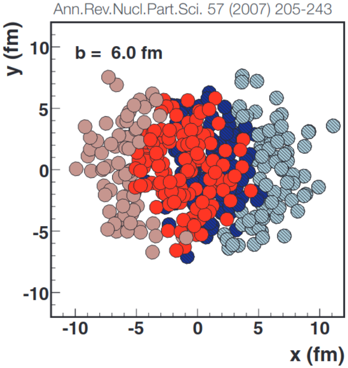
Exercise:

Show that in the hard shere approximation $\sigma_{inel}^{AA} = \pi\,(2R_A)^2$ and $c = [b/(2R_A)]^2$

$p_{inel}^{AA}(b) = 1 - [1 - \hat{T}_{AA}(b)\sigma_{inel}^{NN}]^{AA} \sim 1 - exp(-T_{AA}(b)\sigma_{inel}^{NN})$ if $T_{AA}(b)\sigma_{inel}^{NN} << A^2$
$\sigma_{inel}^{AA} = \int d^2bp_{inel}^{AA}(b) = \int d^2b[1 - exp(-T_{AA}(b)\sigma_{inel}^{NN})] \sim \int d^2b$ if $T_{AA}(b)\sigma_{inel}^{NN} >> 1$
So $\sigma_{inel}^{AA} = \pi(2R_A)^2$ (as expected: the hard spheres can collide as long as their impact parameter is smaller than $2R_A$)

and $c = [b/(2R_A)]^2$

Glauber Monte Carlo Approach



- Randomly select impact parameter b
- Distribute nucleons of two nuclei according to nuclear density distribution
- Consider all pairs with one nucleon from nucleus A and the other from B
- Count pair as inel. n-n collision if distance d in x-y plane satisfies:

$$d < \sqrt{\sigma_{\text{inel}}^{\text{NN}}/\pi}$$

- Repeat many times: $\langle N_{\text{part}}\rangle(b)$ $\langle N_{\text{coll}}\rangle(b)$

Challenge

Use information from the lecture to estimate the number of participants in the centrality classes 0-5%, 5-10%,10-20% for Pb+Pb at 2.76 TeV.

Homework

In the hard sphere approximation, compute for Pb+Pb: $T_{Pb}(0)$, $T_{PbPb}(0)$, $N_{part}(0)$, σ_{inel}^{PbPb} and the impact parameter interval corresponding to the centrality bin 0-5%.

Other references on this topic

- Michael L. Miller, Klaus Reygers, Stephen J. Sanders and Peter Steinberg "Glauber Modeling in High-Energy Nuclear Collisions" Annu. Rev. Nucl. Part. Sci. 57 (2007) 205
- R. Vogt, Ultrarelativistic Heavy-ion Collisions, Elsevier, 2007
- W. Florkowski, Phenomenology of Ultra-Relativistic Heavy-Ion Collisions, World Scientific, 2010
- C.Y. Wong, Introduction to High-Energy Heavy-Ion Collisions, World Scientific, 1994
- `https://www.physi.uni-heidelberg.de/~reygers/lectures/2019/qgp/qgp2019_02_kinematics.pdf`