Eletromagnetismo

Potencial vetor e dipolo magnético

- Se conseguimos descrever o campo elétrico de forma compacta, podemos fazer o mesmo com o campo magnético?
- Se não há monopolo magnético - até onde conseguimos ver, no concreto do dia-a-dia – o campo de dipolo é o termo mais importante neste caso.
- Quais as semelhanças e diferenças com relação ao dipolo elétrico?

Aula 14: Potencial Magnético

Temos: $\vec{E} \to$ campo conservativo

$$\vec{E} = -\nabla V$$

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

$$\oint_C \vec{E} \cdot d\vec{l} = \int_A (\nabla \times \vec{E}) \cdot d\vec{a} \Rightarrow \nabla \times \vec{E} = 0$$

Evidente, pois $\nabla \times (\nabla V) = 0$

Um ganho! $\mathbb{R}^3 \longmapsto \mathbb{R}$

E quanto a $\vec{B}$

Dado $\nabla \cdot \vec{B} = 0$, como $\nabla \cdot (\nabla \times \vec{A}) = 0$

podemos definir um Potencial Vetor $\vec{A}$.

Há alguma vantagem? $\mathbb{R}^3 \mapsto \mathbb{R}^3$

Mas ainda assim será útil quando

$\vec{B}$ e $\vec{E}$ forem acoplados!

Pelo momento, vejamos as aplicações

Lei de Biot-Savart: $\vec{B} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} dv$

Como $\nabla \left[\frac{1}{|\vec{r} - \vec{r}\,'|} \right] = - \frac{\vec{r} - \vec{r}\,'}{(\vec{r} - \vec{r}\,')^3}$, derivando em $\vec{r}$

$$\vec{B} = \frac{-\mu_0}{4\pi} \int \vec{J}(\vec{r}\,') \times \nabla \left[\frac{1}{|\vec{r} - \vec{r}\,'|} \right] dv$$

Relação: $\nabla \times (\phi \vec{A}) = (\nabla \phi) \times \vec{A} + \phi (\nabla \times \vec{A})$

$$\Rightarrow \nabla \times \left[\frac{\vec{J}}{|\vec{r} - \vec{r}\,'|} \right] = \nabla \left[\frac{1}{|\vec{r} - \vec{r}\,'|} \right] \times \vec{J} + \frac{1}{|\vec{r} - \vec{r}\,'|} \underbrace{\left[\nabla \times \vec{J}(\vec{r}\,') \right]}_{=0}$$

$$\Rightarrow \nabla \times \left[\frac{\vec{J}}{|\vec{r}-\vec{r}\,'|}\right] = \nabla\left[\frac{1}{|\vec{r}-\vec{r}\,'|}\right] \times \vec{J} + \frac{1}{|\vec{r}-\vec{r}\,'|}\underbrace{\left[\nabla \times \vec{J}(\vec{r}\,')\right]}_{=0}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\int \nabla \times \left[\frac{\vec{J}}{|\vec{r}-\vec{r}\,'|}\right] dv$$

$$= \nabla \times \left[\frac{\mu_0}{4\pi}\int \left[\frac{\vec{J}}{|\vec{r}-\vec{r}\,'|}\right] dv\right]$$

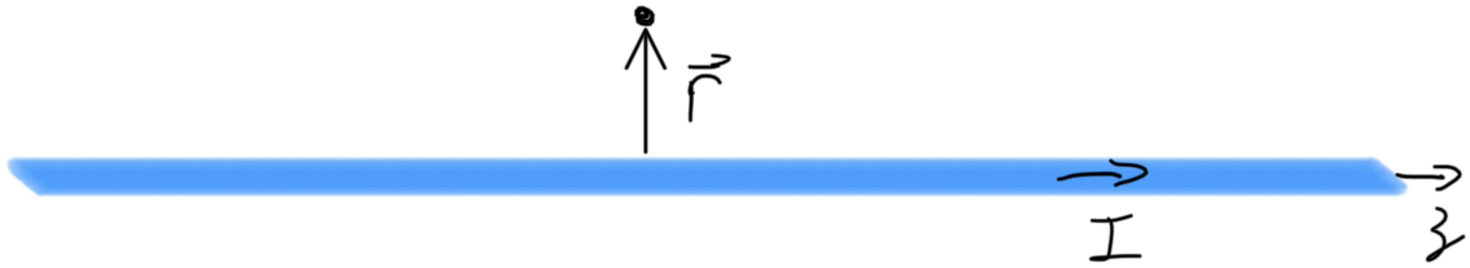
$\hookrightarrow$ integrado em $\vec{r}\,'$

$$\Rightarrow \vec{B} = \nabla \times \vec{A} \;; \quad \vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dv$$

Lembra $\quad V = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dv$

Limitações no uso:

Potencial vetor de um fio



$\vec{J}(\vec{r}') = I \cdot d\vec{l} = I \cdot dz\,\hat{k}$ → posição da fonte

Posição no espaço $\vec{r} = \rho\,\hat{\rho}$

$|\vec{r} - \vec{r}'| = \sqrt{\rho^2 + z^2}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I}{\sqrt{\rho^2+z^2}}\,dz \cdot \hat{k} = \frac{\mu_0 I \hat{k}}{4\pi}\int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2+z^2}}\,dz$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{I}{\sqrt{\rho^2+z^2}}\, dz \cdot \hat{k} = \frac{\mu_0 I \hat{k}}{4\pi} \int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2+z^2}}\, dz$$

$$z = \rho \operatorname{tg} \theta \qquad dz = \rho \sec^2\theta \, d\theta$$

$$z = l/2 \to \theta = \theta_{max}$$

$$\int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2+z^2}}\, dz = \int_{-\theta_m}^{\theta_m} \frac{\rho \sec^2\theta}{\rho \sqrt{1+\operatorname{tg}^2\theta}}\, d\theta = \int_{-\theta_m}^{\theta_m} \frac{1}{\cos\theta}\, d\theta$$

$$\frac{d}{d\theta}\left[\ln\left(\frac{1+\operatorname{sen}\theta}{\cos\theta}\right)\right] = \frac{\cos\theta}{1+\operatorname{sen}\theta}\left[\frac{\cos\theta \cdot \cos\theta - (1+\operatorname{sen}\theta)(-\operatorname{sen}\theta)}{\cos^2\theta}\right]$$

$$= \frac{\cos^2\theta + \operatorname{sen}^2\theta + \operatorname{sen}\theta}{(1+\operatorname{sen}\theta)\cos\theta} = \frac{1}{\cos\theta}$$

Limitações no uso:

Potencial vetor de um fio

$\vec{J}(\vec{r}') = I \cdot d\vec{l} = I \cdot dz\, \hat{k}$ → posição da fonte

Posição no espaço $\vec{r} = \rho\, \hat{\rho}$

$|\vec{r} - \vec{r}'| = \sqrt{\rho^2 + z^2}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{I}{\sqrt{\rho^2 + z^2}}\, dz \cdot \hat{k} = \frac{\mu_0 I \hat{k}}{4\pi} \int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2 + z^2}}\, dz$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{-l/2}^{l/2} \frac{I}{\sqrt{\rho^2+z^2}}\, dz \cdot \hat{k} = \frac{\mu_0 I \hat{k}}{4\pi}\int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2+z^2}}\, dz$$

$$z = \rho \operatorname{tg}\theta \qquad dz = \rho \sec^2\theta \, d\theta$$

$$\therefore \int_{-l/2}^{l/2} \frac{1}{\sqrt{\rho^2+z^2}}\, dz = \ln\left[\frac{1+\operatorname{sen}\theta}{\cos\theta}\right]\Bigg|_{-\theta_m}^{\theta_m} = \ln\left[\frac{1+\operatorname{sen}\theta_m}{\cos\theta_m}\cdot\frac{\cos(-\theta_m)}{1-\operatorname{sen}\theta_m}\right]$$

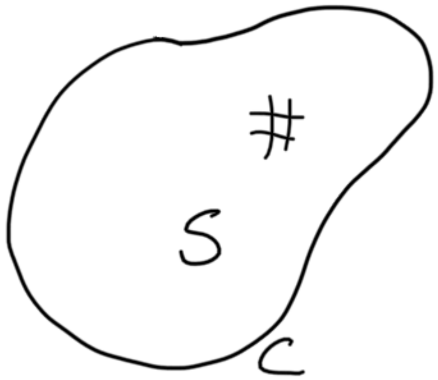
$$= \ln\left(\frac{1+\operatorname{sen}\theta_m}{1-\operatorname{sen}\theta_m}\right)$$

$$\lim_{\theta_m \to \pi/2} \ln\left(\frac{1+\operatorname{sen}\theta_m}{1-\operatorname{sen}\theta_m}\right) = \lim_{x\to 0} \ln\left(\frac{2}{x}\right) \to \infty$$

Mas até aí, o mesmo se aplica no caso do potencial V

Utilidade: Fluxo magnético ϕ_B

$$\phi_B = \int_S \vec{B} \cdot \hat{n}\, da$$



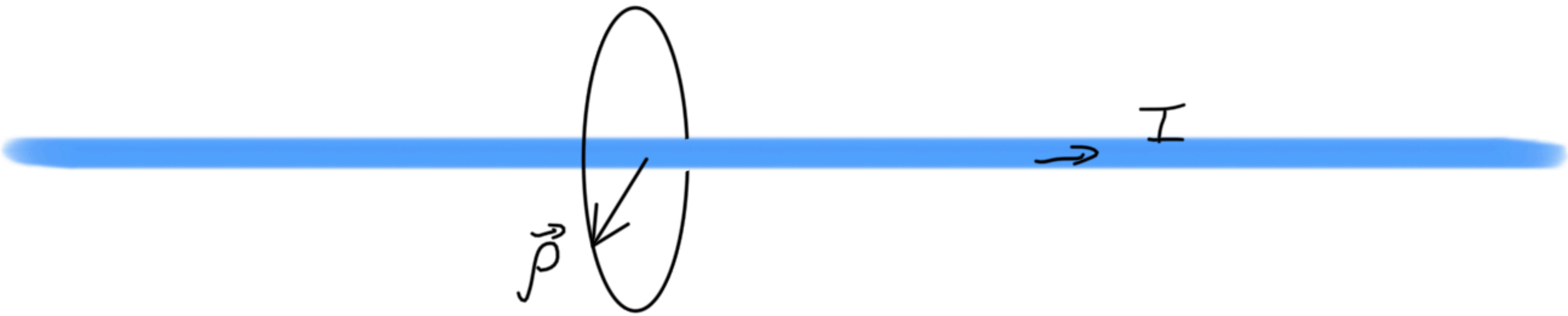
$$\vec{B} = \nabla \times \vec{A} \Rightarrow \phi_B = \int_S (\nabla \times \vec{A}) \cdot \hat{n}\, da$$

$$= \oint_C \vec{A} \cdot d\vec{\ell} \qquad \text{(teorema de Stokes)}$$

$$\int_S \vec{B} \cdot \hat{n}\, da = \oint_C \vec{A} \cdot d\vec{\ell}$$

$\rightarrow$ útil em cálculo de $\vec{A}$

Voltando ao fio: Vamos aplicar a lei de Ampère



$$\nabla \times \vec{B} = \mu_0 \cdot \vec{J}$$

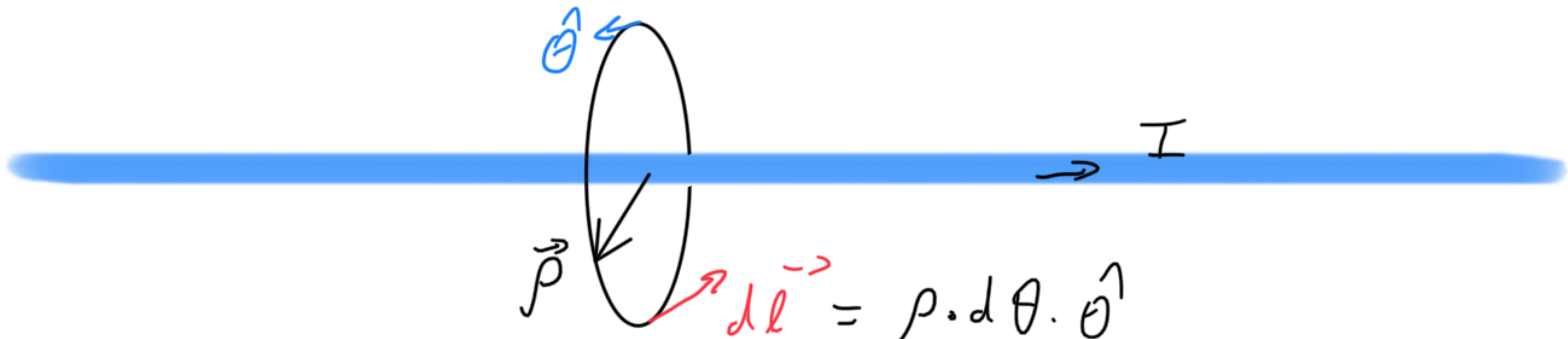
$$\int_S (\nabla \times \vec{B})\, d\vec{a} = \mu_0 \int_S \vec{J} \cdot d\vec{a}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \int_S \vec{J} \cdot d\vec{a}$$

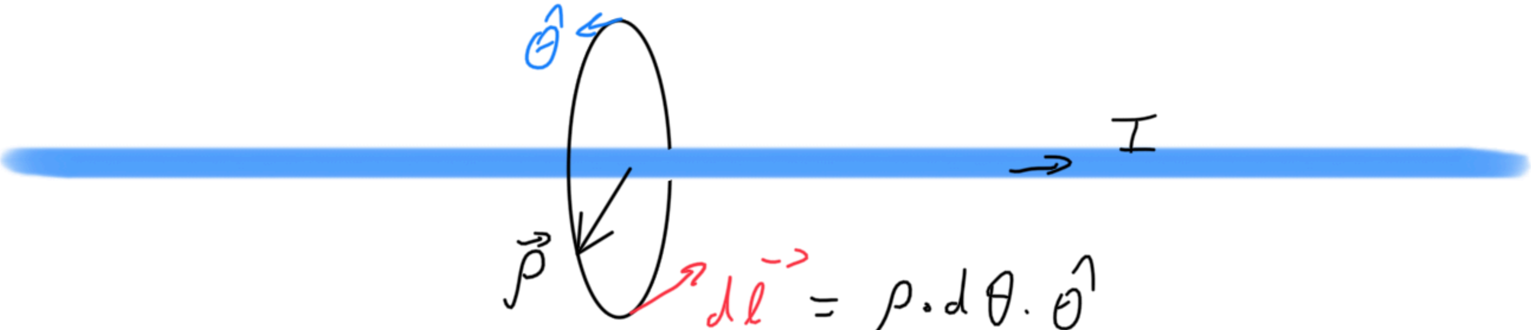
Usando a simetria do problema

→ se o fio é "infinito", $B = cte$ sobre o círculo de raio ρ

→ de $\nabla \cdot \vec{B} = 0$, sabemos que a linha de campo se fecha → $\vec{B} = B \cdot \hat{\theta}$



$$\vec{dl} = \rho \cdot d\theta \cdot \hat{\theta}$$



$$\vec{dl} = \rho \cdot d\theta \cdot \hat{\theta}$$

$$\Rightarrow \oint_C \vec{B} \cdot \vec{dl} = \int_0^{2\pi} B \cdot \hat{\theta} \cdot \rho \cdot d\theta \cdot \hat{\theta} = 2\pi \rho B$$

$$\int_S \vec{J} \, d\vec{a} = I \Rightarrow \qquad \vec{B} = \frac{\mu_0 I}{2\pi\rho} \hat{\theta}$$

Qual o potencial vetor? $\vec{B} = \nabla \times \vec{A}$

$$\vec{B} = B(\rho) \cdot \hat{\theta}$$

$$\nabla \times \vec{A} = \left[\frac{1}{\rho} \frac{\partial}{\partial \theta} A_z - \frac{\partial}{\partial z} A_\theta \right] \hat{\rho} + \left[\frac{\partial}{\partial z} A_\rho - \frac{\partial}{\partial \rho} A_z \right] \hat{\theta} + \left[\frac{\partial}{\partial \rho} (\rho A_\theta) - \frac{\partial}{\partial \theta} A_\rho \right] \frac{\hat{k}}{\rho}$$

$$\nabla \times \vec{A} = \left[\frac{1}{\rho}\frac{\partial}{\partial\theta}A_z - \frac{\partial}{\partial z}A_\theta\right]\hat{\rho} + \left[\frac{\partial}{\partial z}A_\rho - \frac{\partial}{\partial \rho}A_z\right]\hat{\theta} + \left[\frac{\partial}{\partial\rho}(\rho A_\theta) - \frac{\partial}{\partial\theta}A_\rho\right]\frac{\hat{k}}{\rho}$$

$= 0$ $\qquad\qquad\qquad$ $= 0$

Não há dependência em (z, θ); $\vec{B}(\vec{r}) = \vec{B}(\rho)$

$$\Rightarrow \vec{A}(\vec{r}) = \vec{A}(\rho) = A_\rho(\rho)\,\hat{\rho} + A_\theta(\rho)\,\hat{\theta} + A_z(\rho)\,\hat{k}$$

$$\nabla \times \vec{A} = -\frac{\partial}{\partial\rho}A_z\,\hat{\theta} + \frac{\partial}{\partial\rho}(\rho A_\theta)\,\hat{k}$$

$$\frac{\partial}{\partial\rho}(\rho A_\theta) = 0 \quad \rightarrow \quad A_\theta = 0$$

$$-\frac{\partial}{\partial\rho}A_z = \frac{\mu_0 I}{2\pi\rho} \quad \rightarrow \quad \frac{\partial}{\partial\rho}A_z = \left(-\frac{\mu_0 I}{2\pi}\right)\frac{1}{\rho}$$

$$A_z = K\ln\rho + C \qquad \Rightarrow \frac{\partial}{\partial z}A_z = K\frac{1}{\rho}$$

$$\nabla \times \vec{A} = \left[\frac{1}{\rho}\underbrace{\cancel{\frac{\partial}{\partial\theta}A_z}}_{=0} - \cancel{\frac{\partial}{\partial z}A_\theta}\right]\hat{\rho} + \left[\cancel{\frac{\partial}{\partial z}A_\rho} - \frac{\partial}{\partial\rho}A_z\right]\hat{\theta} + \left[\frac{\partial}{\partial\rho}(\rho A_\theta) - \underbrace{\cancel{\frac{\partial}{\partial\theta}A_\rho}}_{=0}\right]\frac{\hat{k}}{\rho}$$

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$$A_z = K\ln\rho + C \qquad \Rightarrow \frac{\partial}{\partial z}A_z = K\frac{1}{\rho}$$

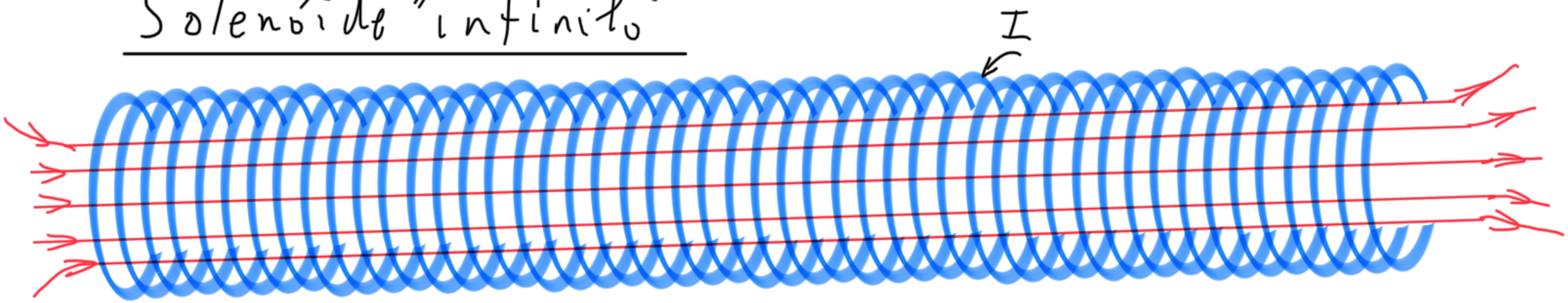
$$A_z = k \ln \rho + C \quad \Rightarrow \frac{\partial}{\partial z} A_z = k \frac{1}{\rho}$$

$$\vec{A}(\vec{r}) = -\frac{\mu_0 I}{2\pi} (\ln \rho + C)\, \hat{k}$$

Por conveniência, $\vec{A}(R) = 0$ (R = raio do fio)

$$\vec{A}(\vec{r}) = -\frac{\mu_0 I}{2\pi} (\ln \rho - \ln R)\, \hat{k} = -\frac{\mu_0 I}{2\pi} \ln\left(\frac{\rho}{R}\right) \hat{k}$$

Solenóide "infinito"



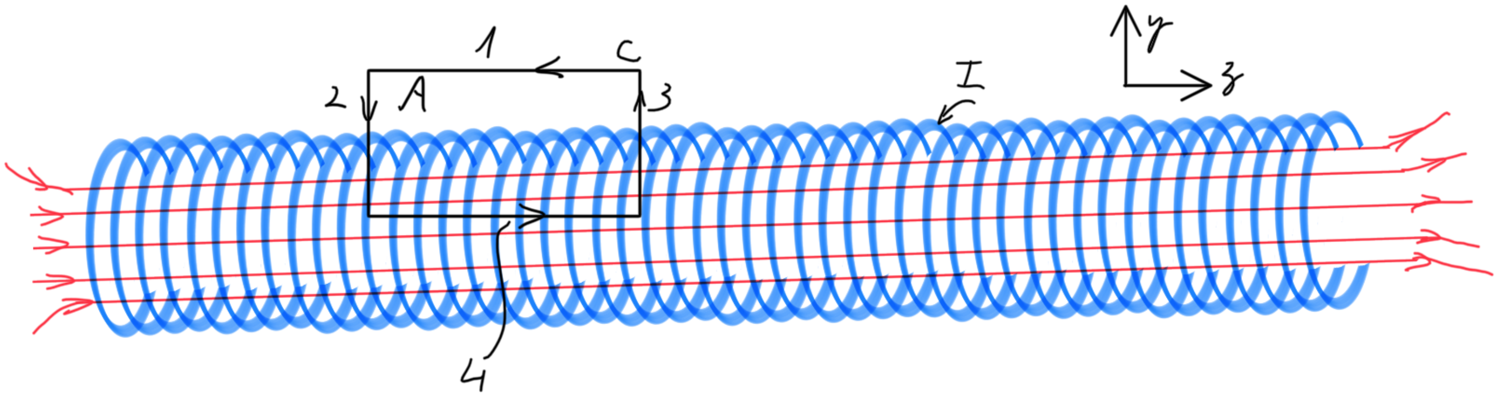
N espiras, comprimento ℓ

Fazendo a "curva amperiana" (KDM) (1234)

Consideramos que o campo sobre 1 é muito menor que em 4

$$\int_A (\nabla \times \vec{B})\, d\vec{a} = \mu_0 \int_A \vec{J} \cdot d\vec{a}$$

$$\int_A (\vec{\nabla} \times \vec{B})\, d\vec{a} = \oint_C \vec{B} \cdot d\vec{l}$$



$$\oint \vec{B}\cdot d\vec{l} = \int_4 \vec{B}\cdot d\vec{l} + \int_3 \vec{B}\, d\vec{l} + \int_2 \vec{B}\cdot d\vec{l} + \int_1 \vec{B}\cdot d\vec{l}$$

$$= \int_4 B\cdot\hat{k}\cdot dz\cdot\hat{k} + \int_3 B\cdot\underbrace{\hat{k}\cdot dy\,\hat{j}}_{0} + \int_2 \vec{B}\,\underbrace{\hat{k}\, dy\cdot(-\hat{j})}_{0} + \underbrace{\int_1 B'\hat{k}\cdot dz(-\hat{k})}_{B'<<B}$$

$$= B\cdot l$$

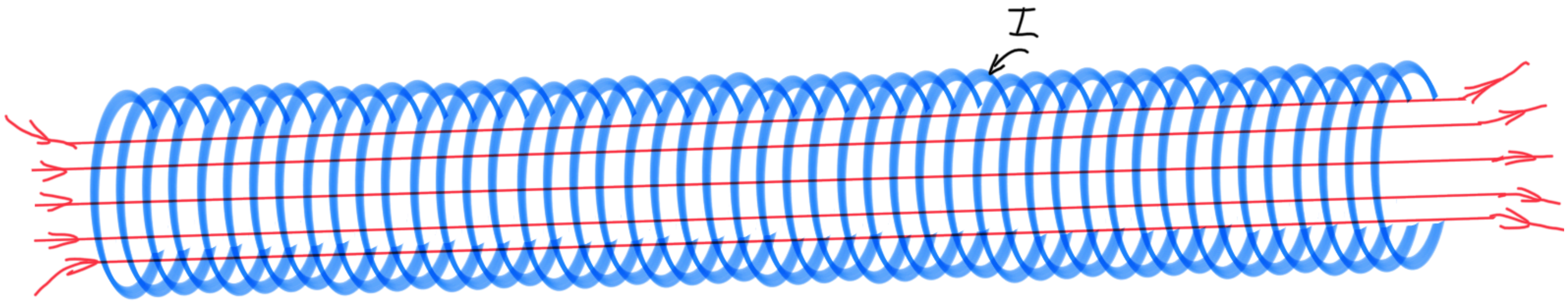
$$\int \vec{J}\cdot d\vec{l} = N\cdot I$$ (N espiras, corrente I)

$$\Rightarrow \quad B\cdot l = \mu_0 N I \quad \Rightarrow \quad \boxed{\vec{B} = \mu_0 \left(\frac{N}{l}\right) I\,\hat{k}}$$

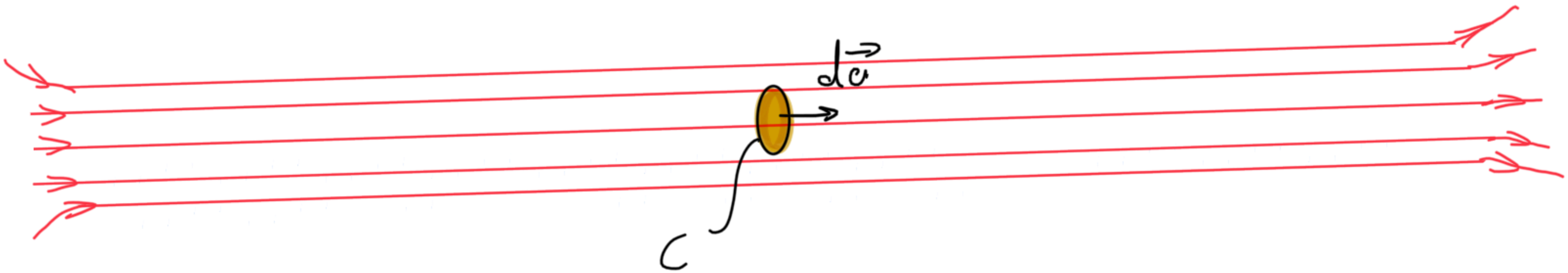
<u>Uniforme dentro do solenóide</u>

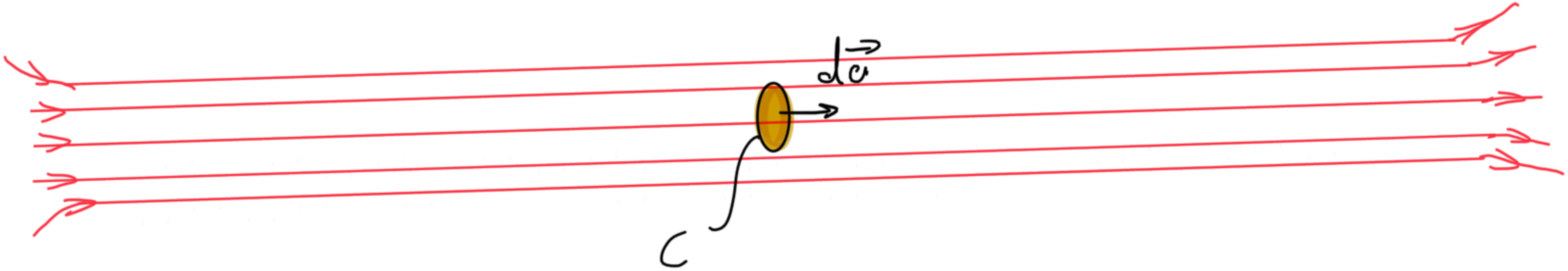
Qual o valor de $\vec{A}$?

$$\vec{B} = \nabla \times \vec{A}$$



Usando mais uma vez $\int_S \vec{B} \cdot \hat{n}\, da = \oint_C \vec{A} \cdot d\vec{l}$





$$\int_S \vec{B}\cdot\hat{k}\cdot da = B\cdot\Delta A = B\cdot\pi\rho^2$$

$$\oint_c \vec{A}\cdot d\vec{l} = \, ?$$

/ $\vec{A}$ ortogonal a $\vec{B}$
paralelo a $d\vec{l}$ $\Rightarrow$ $\vec{A} = A(\rho)\,\hat{\theta}$

$$\oint_C \vec{A}\cdot d\vec{l} = \int_0^{2\pi} A(\rho)\,\hat{\theta}\cdot\rho\cdot d\theta\,\hat{\theta} = A(\rho)\cdot\rho\cdot 2\pi$$

$$B\,\pi\rho^2 = A(\rho)\,\rho\cdot 2\pi$$

$$A(\rho) = \frac{B}{2}\rho = \mu_0\left(\frac{N}{\ell}\right)\frac{I\rho}{2} \Rightarrow \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right)I\rho\,\hat{\theta}$$

para $\rho < R$ (raio do solenóide)

para $\rho > R$

$$\oint_c \vec{A}\cdot d\vec{\ell} = A(\rho)\cdot\rho\,2\pi \quad ; \quad \int_s \vec{B}\cdot\hat{k}\,da = B\pi R^2$$

$$A(\rho) = \frac{B}{2}\frac{R^2}{\rho} \qquad \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right)I\,\frac{R^2}{\rho}\,\hat{\theta}$$

Verificando: $\vec{B} = \nabla\times\vec{A}$; $\vec{A} = A_\theta(\rho)\,\hat{\theta}$

$$\nabla\times\vec{A} = \Big[\underbrace{\frac{1}{\rho}\frac{\partial}{\partial\theta}A_z}_{=0} - \underbrace{\frac{\partial}{\partial z}A_\theta}_{=0}\Big]\hat{\rho} + \Big[\underbrace{\frac{\partial}{\partial z}A_\rho - \frac{\partial}{\partial\rho}A_z}_{=0}\Big]\hat{\theta} + \Big[\frac{\partial}{\partial\rho}(\rho A_\theta) - \underbrace{\frac{\partial}{\partial\theta}A_\rho}_{=0}\Big]\frac{\hat{k}}{\rho}$$

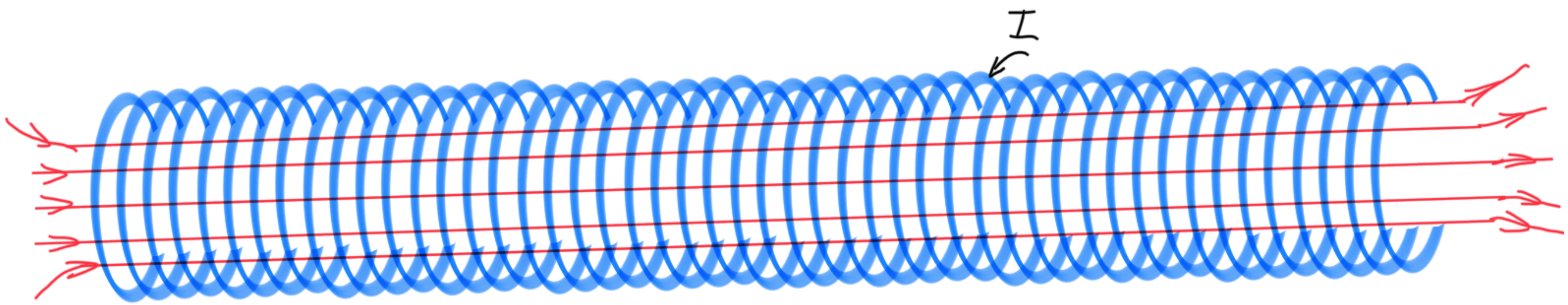
$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\theta)\,\hat{k}$$

$$\rho < R \Rightarrow \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I \rho \, \hat{\theta} \, ;$$

$$\nabla \times \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\frac{\mu_0}{2} \frac{N}{\ell} I \rho^2\right) \hat{k} = \frac{1}{\rho} \cdot \mu_0 \frac{N}{\ell} I \cdot \rho \, \hat{k} \Rightarrow \boxed{\vec{B} = \mu_0 \frac{N}{\ell} I \, \hat{k}}$$

$$\rho > R \Rightarrow \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I \frac{R^2}{\rho} \, \hat{\theta}$$

$$\nabla \times \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I R^2\right) \hat{\theta} = \boxed{0} \; !$$



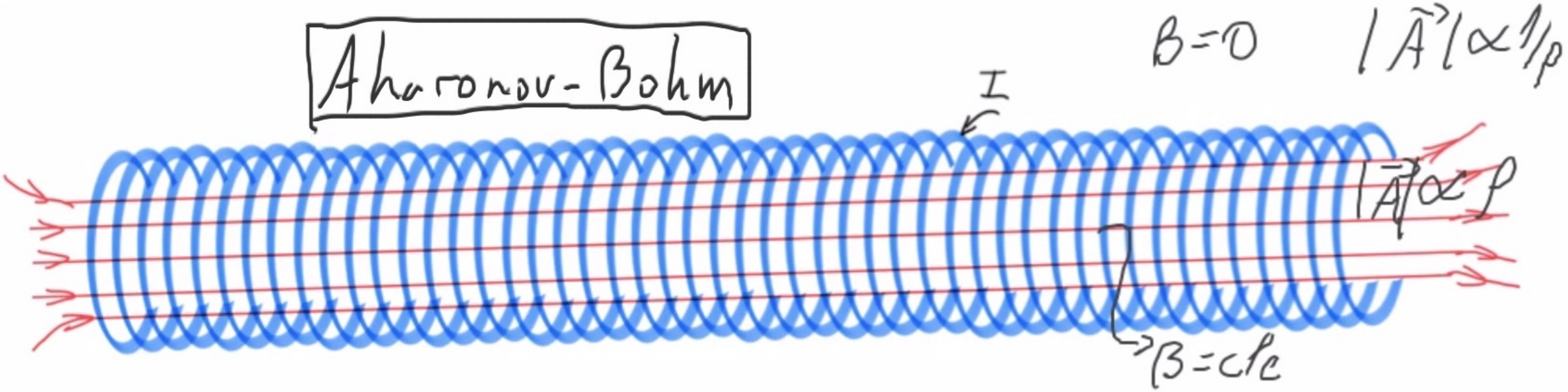
$$\rho < R \Rightarrow \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I \rho \, \hat{\theta};$$

$$\nabla \times \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\frac{\mu_0}{2}\frac{N}{\ell} I \rho^2\right)\hat{k} = \frac{1}{\rho} \cdot \mu_0 \frac{N}{\ell} I \cdot \rho \, \hat{k} \Rightarrow \boxed{\vec{B} = \mu_0 \frac{N}{\ell} I \, \hat{k}}$$

$$\rho > R \Rightarrow \vec{A} = \frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I \frac{R^2}{\rho} \hat{\theta}$$

$$\nabla \times \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\frac{\mu_0}{2}\left(\frac{N}{\ell}\right) I R^2\right) \hat{\theta} = \boxed{0} \; !$$

Aharonov-Bohm



Importante: temos uma liberdade na definição de $\vec{A}$

Assim como $V(\vec{r})$ e $V(\vec{r})+C$ levam ao mesmo campo $\vec{E}$

Podemos usar $\vec{A}(\vec{r})$ ou $\vec{A}(\vec{r})+\nabla\psi(\vec{r})$

$$\vec{B}=\nabla\times\left[\vec{A}(\vec{r})+\nabla\psi(\vec{r})\right]=\nabla\times\vec{A}+\underbrace{\nabla\times(\nabla\psi)}_{=0}$$

(lembra de $\nabla\times\vec{E}$?)

Não tão importante: se $\nabla \times \vec{B} = 0$ $(\vec{J} = 0)$

podemos usar uma função ϕ, tal que $\nabla \times \nabla \phi = 0$

$\Rightarrow \vec{B} = \nabla \phi \rightarrow$ potencial escalar magnético

→ Não é geral

→ Não é mensurável diretamente, como $V(\vec{r})$

Não tão importante: se $\nabla \times \vec{B} = 0$ $(\vec{J} = 0)$

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$\Rightarrow \vec{B} = \nabla\phi \rightarrow$ potencial escalar magnético

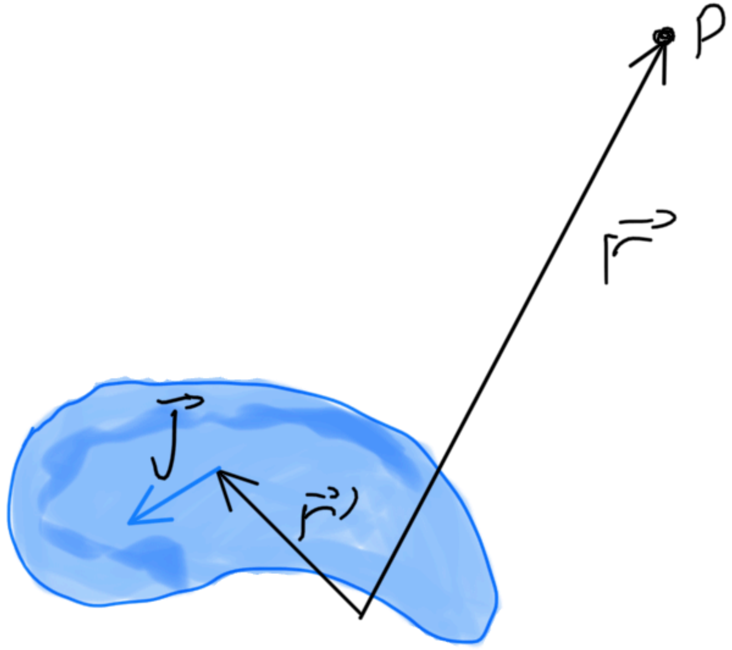
$\rightarrow$ Não é geral

$\rightarrow$ Não é mensurável diretamente, como $V(\vec{r})$

$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \nabla^2 \phi = 0$$

Laplace

Aula 14: Dipolo magnético



Potencial vetor gerado por um conjunto de correntes $\vec{J}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv$$

Podemos expandir $\dfrac{1}{|\vec{r} - \vec{r}'|} = \dfrac{1}{\sqrt{r^2 + r'^2 - 2\vec{r} \cdot \vec{r}'}} =$

$$= \frac{1}{r} \left[1 - 2 \frac{\vec{r} \cdot \vec{r}'}{r^2} + \frac{r'^2}{r^2} \right]^{-1/2}$$

Podemos expandir $\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{\sqrt{r^2+r'^2-2\vec{r}\cdot\vec{r}'}} =$

$$= \frac{1}{r}\left[1-2\frac{\vec{r}\cdot\vec{r}'}{r^2}+\frac{r'^2}{r^2}\right]^{-1/2}$$

$$\frac{1}{\sqrt{1+\alpha}} \simeq 1-\frac{1}{2}\alpha+\frac{3}{2}\alpha^2+\dots$$

$$\Rightarrow \frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}-\frac{1}{2}\frac{1}{r}\left[-2\frac{\vec{r}\cdot\vec{r}'}{r^2}+\frac{r'^2}{r^2}\right]+\frac{3}{2}\frac{1}{r}\left[-2\frac{\vec{r}\cdot\vec{r}'}{r^2}+\frac{r'^2}{r^2}\right]^2+\dots$$

$$\propto\left(\frac{r'}{r}\right) \qquad \propto\left(\frac{r'}{r}\right)^2 \qquad \propto\left(\frac{r'}{r}\right)^2;\ \left(\frac{r'}{r}\right)^3;\ \left(\frac{r'}{r}\right)^4$$

Mantendo apenas os dois primeiros termos

$$\frac{1}{|\vec{r}-\vec{r}'|} \simeq \frac{1}{r}+\frac{\vec{r}'\cdot\vec{r}}{r^3}$$

Assim, longe da fonte, prevalecem dois termos

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi r}\int_V \vec{J}(\vec{r}\,')\,dv + \frac{\mu_0}{4\pi r^3}\int_V (\vec{r}\cdot\vec{r}\,')\,\vec{J}(\vec{r}\,')\,dv$$

integrado em $\vec{r}\,'$

$$\int_V \vec{J}(\vec{r}\,')\,dv = 0$$ → dedução matemática na bibliografia

→ Mas lembrando que isto é a soma da corrente em um circuito <u>fechado</u>, tem que se anular

→ Quando expandimos o potencial elétrico em multipolos, este termo nos dava o potencial de monopolo.

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Sabemos que ele não aparece aqui, para o campo magnético

Resta: $\vec{A}(\vec{r}) \cong \frac{\mu_0}{4\pi r^3} \int_V (\vec{r} \cdot \vec{r}') \vec{J}(\vec{r}') dv$

$$= \frac{\mu_0}{4\pi r^2} \int (\vec{r}' \cdot \hat{r}) \vec{J}(\vec{r}') dv$$

(KDM → tensor de Levi-Civitta, Griffiths → unlovely manipulation)

$$\vec{A}(\vec{r}) \simeq \frac{\mu_0}{4\pi r^2} \int \vec{J}(\vec{r}\,') \, r' \cos\theta \, r'^2 dr' d\theta d\varphi$$

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}} \qquad \boxed{\vec{m} = \frac{1}{2} \int_V [\vec{r}\,' \times \vec{J}(\vec{r}\,')] dv}$$

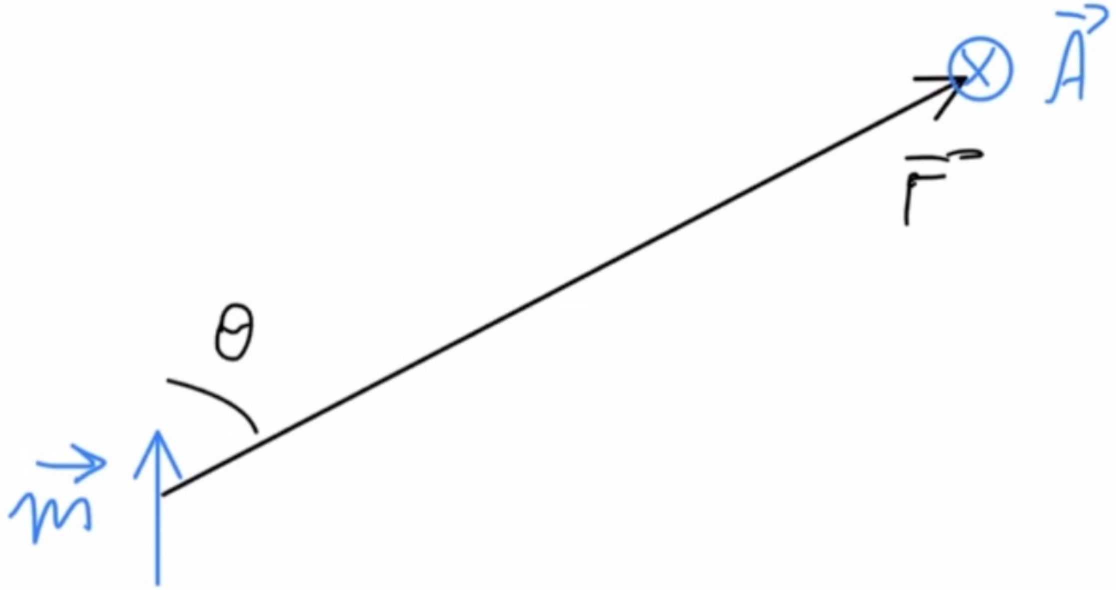
Potencial vetor de dipolo magnético

Momento de dipolo magnético

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}} \qquad \boxed{\vec{p} = \int \vec{r}\,' \rho(\vec{r}\,') dv}$$

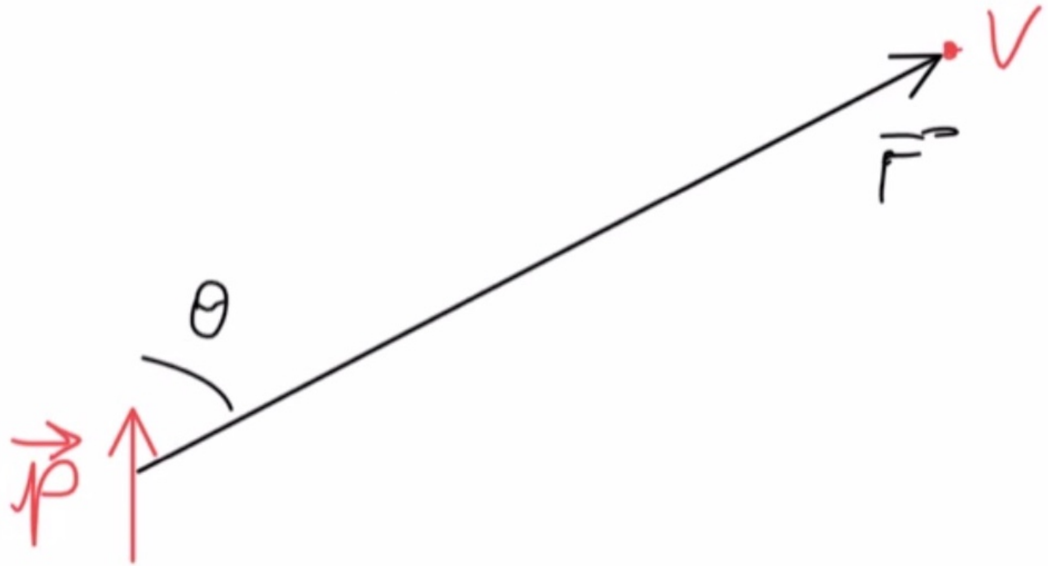
Potencial de dipolo elétrico

Momento de dipolo elétrico

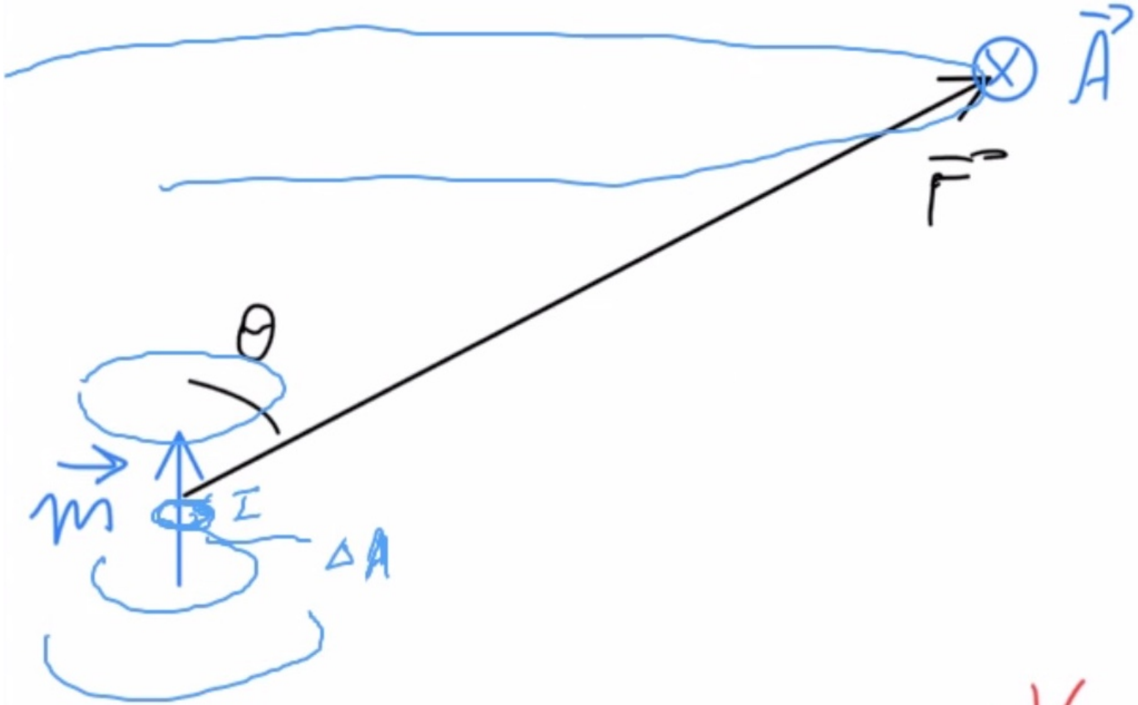


$$A = \frac{\mu_0 m}{4\pi r^2} \operatorname{sen}\theta$$

(canéis)

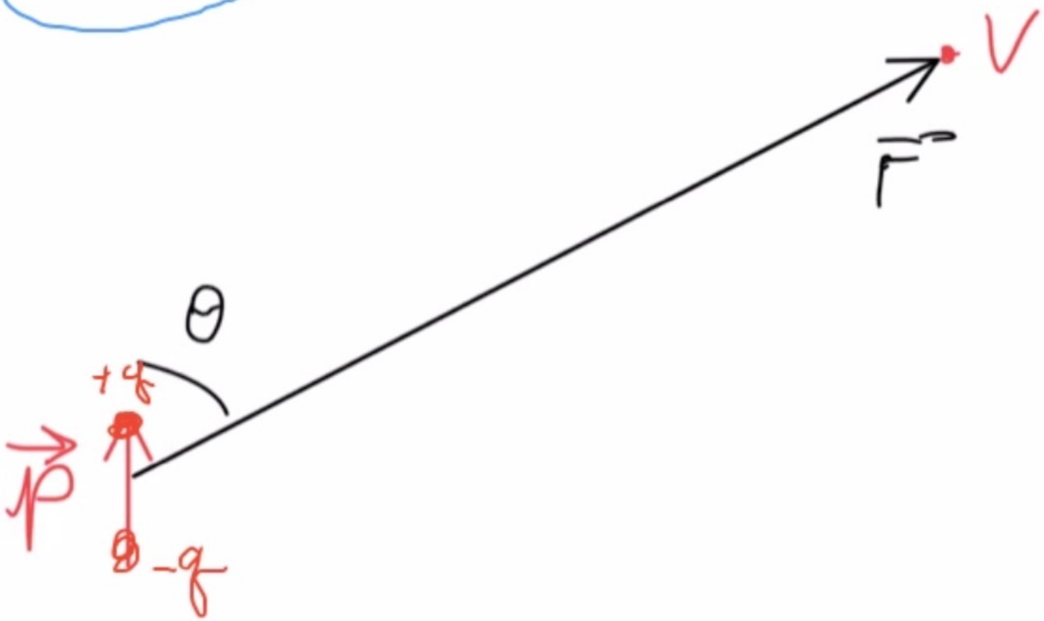


$$V = \frac{p}{4\pi \varepsilon_0 r^2} \cos\theta$$



$$A = \frac{\mu_0 m}{4\pi r^2} \operatorname{sen}\theta$$

(anéis)



$$V = \frac{p}{4\pi \varepsilon_0 r^2} \cos\theta$$

Se a visualização do potencial vetor é difícil, o campo é mais simples: $\vec{B} = \nabla \times \vec{A}$

$$\vec{B} = \frac{\mu_0}{4\pi} \nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

Mais uma vez: $\nabla \times (\vec{A} \times \vec{B}) = (\nabla \cdot \vec{B})\vec{A} - (\nabla \cdot \vec{A})\vec{B} + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B}$

$$\vec{A} \to \vec{m} \qquad \vec{B} \to \frac{\vec{r}}{r^3}$$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right) = \left[\nabla \cdot \left(\frac{\vec{r}}{r^3} \right) \right] \vec{m} - \underbrace{(\nabla \cdot \vec{m})}_{=0} \frac{\vec{r}}{r^3} + \underbrace{\left[\left(\frac{\vec{r}}{r^3} \right) \cdot \nabla \right] \vec{m}}_{=0} - (\vec{m} \cdot \nabla) \left(\frac{\vec{r}}{r^3} \right)$$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3}\right) = \left[\nabla \cdot \left(\frac{\vec{r}}{r^3}\right)\right]\vec{m} - \underbrace{(\nabla \cdot \vec{m})}_{=0}\frac{\vec{r}}{r^3} + \underbrace{\left[\left(\frac{\vec{r}}{r^3}\right) \cdot \nabla\right]\vec{m}}_{=0} - (\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)$$

$$\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = \nabla \cdot \nabla\left(\frac{1}{r}\right) = 4\pi\,\delta(\vec{r})$$

como $|\vec{r}| >> |\vec{r}\,'|$ $\rightarrow$ $|\vec{r}| \neq 0$ $\Rightarrow$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3}\right) = -(\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)$$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3}\right) = \left[\nabla \cdot \left(\frac{\vec{r}}{r^3}\right)\right]\vec{m} - (\nabla \cdot \vec{m})\frac{\vec{r}}{r^3} + \left[\left(\frac{\vec{r}}{r^3}\right) \cdot \nabla\right]\vec{m} - (\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)$$

$$0 \qquad\qquad = 0 \qquad\qquad = 0$$

$$\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = \nabla \cdot \nabla\left(\frac{1}{r}\right) = 4\pi\,\delta(\vec{r})$$

como $|\vec{r}| >> |\vec{r}'| \to |\vec{r}| \neq 0 \Rightarrow \delta(\vec{r}) = 0$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3}\right) = -(\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)$$

$$\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right) = -(\vec{m} \cdot \nabla) \left(\frac{\vec{r}}{r^3} \right)$$

$$(\vec{m} \cdot \nabla) = (m_x \hat{i} + m_y \hat{j} + m_z \hat{k}) \cdot \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right)$$

$$= \left(m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right)$$

$$(\vec{m} \cdot \nabla) \left(\frac{\vec{r}}{r^3} \right) = (\vec{m} \cdot \nabla) \frac{x \hat{i} + y \hat{j} + z \hat{k}}{[x^2 + y^2 + z^2]^{3/2}}$$

$$\left[(\vec{m} \cdot \nabla) \left(\frac{\vec{r}}{r^3} \right) \right]_x = \hat{i} \left(m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right) \left(\frac{x}{[x^2 + y^2 + z^2]^{3/2}} \right)$$

$$(\vec{m}\cdot\nabla)\left(\frac{\vec{r}}{r^3}\right) = (\vec{m}\cdot\nabla)\,\frac{x\,\hat{i} + y\,\hat{j} + z\,\hat{k}}{\left[x^2+y^2+z^2\right]^{3/2}}$$

$$\left[(\vec{m}\cdot\nabla)\left(\frac{\vec{r}}{r^3}\right)\right]_x = \hat{i}\left(m_x\frac{\partial}{\partial x} + m_y\frac{\partial}{\partial y} + m_z\frac{\partial}{\partial z}\right)\left(\frac{x}{\left[x^2+y^2+z^2\right]^{3/2}}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{\partial}{\partial x}x\cdot\frac{1}{r^3} + x\,\frac{\partial}{\partial r}\left(\frac{1}{r^3}\right)\cdot\frac{\partial r}{\partial x} \quad ; \quad r = \sqrt{x^2+y^2+z^2}$$

$$= \frac{1}{r^3} - 3x\,\frac{1}{r^4}\cdot\frac{1}{2}\,\frac{1}{r}\cdot 2x = \frac{1}{r^3} - 3\,\frac{x^2}{r^5}$$

$$\frac{\partial}{\partial y}\left(\frac{x}{r^3}\right) = x\,\frac{\partial}{\partial r}\,\frac{1}{r^3}\cdot\frac{\partial r}{\partial y} = -3\,\frac{xy}{r^5} \quad ; \quad \frac{\partial}{\partial z}\left(\frac{x}{r^3}\right) = -3\,\frac{xz}{r^5}$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{\partial}{\partial x} x \cdot \frac{1}{r^3} + x \frac{\partial}{\partial r}\left(\frac{1}{r^3}\right) \cdot \frac{\partial r}{\partial x} \quad ; \quad r = \sqrt{x^2+y^2+z^2}$$

$$= \frac{1}{r^3} - 3x \frac{1}{r^4} \cdot \frac{1}{2} \frac{1}{r} \cdot 2x = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

$$\frac{\partial}{\partial y}\left(\frac{x}{r^3}\right) = x \frac{\partial}{\partial r} \frac{1}{r^3} \cdot \frac{\partial r}{\partial y} = -3x \frac{y}{r^5} \quad ; \quad \frac{\partial}{\partial z}\left(\frac{x}{r^3}\right) = -3x \frac{z}{r^5}$$

$$\Rightarrow \left[(\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)\right]_x = \hat{i}\left[\frac{m_x}{r^3} - \frac{3x}{r^5}(m_x \cdot x + m_y \cdot y + m_z \cdot z)\right]$$

$$= \hat{i}\left[\frac{m_x}{r^3} - 3\frac{x}{r^5}(\vec{m} \cdot \vec{r})\right]$$

$$\Rightarrow \left[(\vec{m} \cdot \nabla)\left(\frac{\vec{r}}{r^3}\right)\right] = \frac{\vec{m}}{r^3} - 3\left(\frac{\vec{m} \cdot \vec{r}}{r^5}\right) \cdot \vec{r}$$

$$\Rightarrow \left[(\vec{m}\cdot\nabla)\left(\frac{\vec{r}}{r^3}\right)\right] = \frac{\vec{m}}{r^3} - 3\left(\frac{\vec{m}\cdot\vec{r}}{r^5}\right)\cdot\vec{r}$$

$$\Rightarrow \boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\,\nabla\times\left[\vec{m}\times\left(\frac{\vec{r}}{r^3}\right)\right] = \frac{\mu_0}{4\pi r^3}\left[3\,(\vec{m}\cdot\hat{r})\,\hat{r} - \vec{m}\right]}$$

$$\Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{-1}{4\pi\varepsilon_0}\,\nabla\cdot\left[\vec{p}\cdot\left(\frac{\vec{r}}{r^3}\right)\right] = \frac{1}{4\pi\varepsilon_0\, r^3}\left[3\,(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}\right]}$$