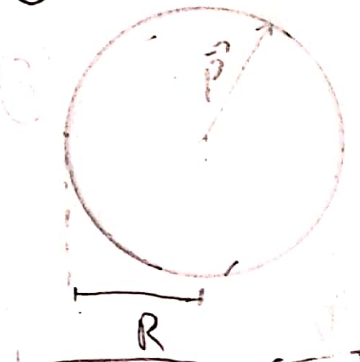
(4) $\vec{p} = k\vec{r}$



(a) $\sigma_b = \vec{p}\cdot\hat{n}$; $\rho_b = -\nabla\cdot p$

$\sigma_b = k r \hat{r}\cdot\hat{n}\Big|_{sup} = k r \hat{r}\cdot\hat{r}\Big|_{sup} = k r\Big|_{sup} = \underline{kR}$ //

$\rho_b = -\nabla\cdot p = -\frac{\partial}{\partial r} k r = \underline{-k}$ //

~~PARA OBTER O CAMPO [illegible] O POTENCIAL~~

~~$V(r<R) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) = V_2(r)$ — CONTINUIDADE DO POT.~~

~~$\to V_1(R) = V_2(R)$~~

~~$\to B_l = R^{2l+1} A_l$~~

~~$V(r>R) = \sum_{l=0} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$~~

(B)

$r<R$: $\nabla\cdot E = \frac{\rho_b}{\varepsilon_0} \Rightarrow \oint_{sup} \vec{E}\, d\vec{s} = \iiint \frac{\rho}{\varepsilon_0} dv_{ol}$: $E\, 4\pi r^2 = \frac{\rho_b}{\varepsilon_0}\frac{4}{3}\pi r^3 \Rightarrow$

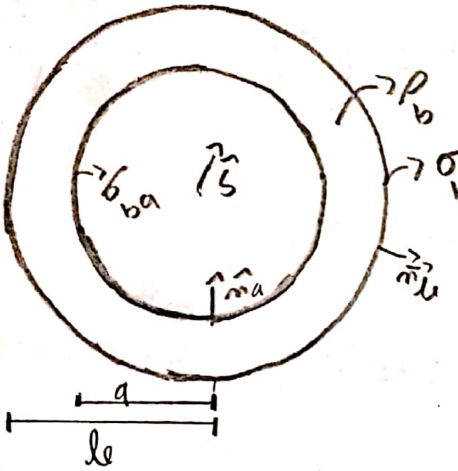
$\hookrightarrow Q_{in}$ $\quad$ $\hookrightarrow -k$

$$\boxed{\vec{E} = \hat{r}\,\frac{-k}{\varepsilon_0}\frac{r}{3}}$$ //

$r>R$: $E\, 4\pi r^2 = \left[\frac{\rho_b}{\varepsilon_0}\frac{4}{3}\pi R^3 + \frac{1}{\varepsilon_0}\sigma_b 4\pi R^2\right] = \left[\frac{-k}{\varepsilon_0}\frac{4}{3}\pi R^3 + \frac{k}{\varepsilon_0}4\pi R^3\right]$

$$E = \frac{1}{4\pi r^2}\left[\frac{-k}{3} + k\right]\frac{1}{\varepsilon_0}4\pi R^3 = \frac{1}{\varepsilon_0}\frac{R^3}{r^2}\left[\frac{2}{3}k\right] \Rightarrow \boxed{\vec{E} = \frac{2}{3\varepsilon_0}\frac{kR^3}{r^3}\vec{r}}$$ //

(6)



$\vec{P} = ks^2\hat{s}$; DESPREZAMOS EFEITOS DE BORDA

(A) SABEMOS QUE AS DENSIDADES DE CARGA DEVIDO A POLARIZAÇÃO SÃO DADAS POR:

$$\sigma_b = \vec{P}\cdot\hat{n}\Big|_{sup} \quad e \quad \rho_b = -\nabla\cdot\vec{P}$$

SENDO ASSIM TEMOS:

NA SUPERFÍCIE DE RAIO a: $\sigma_{ba} = \vec{P}\cdot\hat{n}_a = \vec{P}\cdot(-\hat{s})\Big|_{sup} = -ks^2\Big|_{sup} = -ka^2$

NA SUPERFÍCIE DE RAIO b: $\sigma_{bb} = \vec{P}\cdot\hat{n}_b = \vec{P}(\hat{s})\Big|_{sup} = +ks^2\Big|_{sup} = kb^2$

$$\rho_b = -\nabla\cdot\vec{P} = -\frac{1}{s}\frac{\partial}{\partial s}(s\vec{P}\cdot\hat{s}) = -\frac{1}{s}\frac{\partial}{\partial s}ks^3 = -3\frac{ks^2}{s} = -3ks$$

(b) PELA LEI DE GAUSS PODEMOS OBTER O CAMPO.

USANDO DE UMA SUPERFÍCIE GAUSSEANA CILÍNDRICA DE COMPRIMENTO "L" E RAIO "s" TEMOS:

PARA s<a: $\oiint_{sup}\vec{E}\cdot d\vec{s} = \iiint\frac{\rho}{\varepsilon_0}dVol = \frac{Q_{in}}{\varepsilon_0} \Rightarrow 2\pi sLE = 0 \Rightarrow$

$\vec{E} = 0$

PARA $s \in [a,b]$: $2\pi sLE = \left[\int_0^L\int_0^{2\pi}\sigma_{ba}\,a\,d\theta\,dz + \int_0^L\int_0^{2\pi}\int_0^s\rho_b\,s'ds'd\theta dz\right]\frac{1}{\varepsilon_0} \Rightarrow$

$$E = \frac{1}{2\pi sL\varepsilon_0}\left[2\pi La(-ka^2) + 2\pi L\left(-3k\frac{s^3}{3}\right)\Bigg|_a^s\right]$$

$$= \frac{k}{\varepsilon_0}\left[-\frac{a^3}{s} - \left(\frac{s^3}{s} + \frac{a^3}{s}\right)\right] = -\frac{ks^2}{\varepsilon_0}$$

PARA $s > l_e$: $2\pi L s E = \frac{1}{\varepsilon_0}\left[2\pi a L \sigma_{ba} + 2\pi l_e L \sigma_{ble} + \int_0^L\int_0^{2\pi}\int_a^{l_e} \rho_b(s') s' ds' d\phi dz\right] \Rightarrow$

$$E = \frac{1}{2\pi L s \varepsilon_0}\left[2\pi L(-ka^3) + 2\pi L k l_e^3 + 2\pi L\left(-3k\frac{s^3}{3}\right)\Big|_a^{l_e}\right]$$

$$= \frac{k}{\varepsilon_0}\left[\frac{-a^3}{s} + \frac{l_e^3}{s} - \left(l_e^3 - a^3\right)\frac{1}{s}\right] = 0$$

ASSIM TEMOS, POR SIMETRIA E PELA GEOMETRIA DO PROBLEMA

$\vec{E} = 0 \quad , s < a$

$\vec{E} = -\frac{k s^2 \vec{s}}{\varepsilon_0} = \frac{-1}{\varepsilon_0}\vec{P} \quad , s \in [a, l_e)$

$\vec{E} = 0 \quad , s > l_e$