REFLECTION OF ELECTRONS BY A CRYSTAL OF NICKEL

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Continuing our investigation of the interaction between a beam of electrons and a crystal of nickel (*Phys. Rev.*, **30**, 705 (1927)) we are now directing the electron beam against a {111}-face of the crystal at various angles of incidence, and are measuring the intensity of scattering in the incidence plane as a function of bombarding potential and direction.

We find that under certain conditions a sharply defined beam of scattered electrons issues from the crystal in the direction of regular reflection. This occurs whenever the speed of the incident electrons is comprised within any of certain ranges which change in location as the angle of incidence is varied. Within each of these ranges there is an optimum speed at which the intensity of the reflected beam attains a maximum.

That regular selective reflection of electrons from a crystal face would be observed under appropriate conditions was anticipated from our earlier observations on electron diffraction. The phenomenon is clearly the analogue of the regular selective reflection of x-rays on which the Bragg method of x-ray spectroscopy is based, and is, of course, to be interpreted in terms of the undulatory theory of mechanics. The incident beam of electrons of speed v is equivalent to a beam of waves of wave-length h/mv; a portion of the incident beam is regularly reflected, through the process of coherent scattering, from each of the layers of atoms lying parallel to the crystal face, and the intensity of the resultant beam is a maximum when the elementary beams proceeding from the individual layers emerge from the crystal in phase. The condition for such a maximum in the case of x-ray reflection is that the wave-length and angle of the incident beam be related to the separation between successive atom layers of the crystal through the Bragg formula $n\lambda = 2d \cos \Theta$. The condition in the case of electron reflection is somewhat different. The wave-length $\lambda (= h/mv)$ of the reflected beam at maximum intensity is not given by the Bragg formula.

These results, including the failure of the data to satisfy the Bragg formula, are in accord with those previously obtained in our experiments on electron diffraction. The reflection data fail to satisfy the Bragg relation for the same reason that the electron diffraction beams fail to coincide with their Laue beam analogues. These differences between the electron and x-ray phenomena can perhaps be accounted for by assuming, as first suggested by Eckart,[1] that the crystal is characterized by an index of refraction for electrons as it is for x-rays, and that for electrons of the speeds used in our experiments the index has values which are quite different from unity.

The present experimental arrangement is indicated in the schematic diagrams of figure 1. The face-centered cubic array of atoms in the nickel crystal (1-*a*) has been cut through at right angles to one of the cube diagonals to expose a triangular {111}-face (1-*b*). The incident beam of electrons is directed against this face and lies in the quadrant bounded

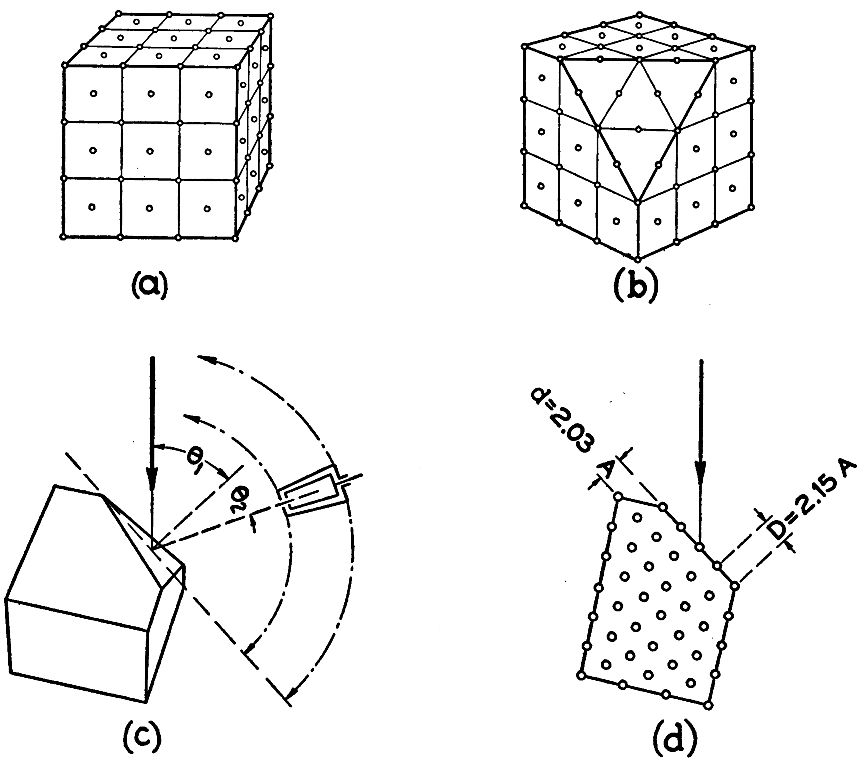

FIGURE 1

Schematic diagrams indicating the experimental arrangement for measuring the reflection of electrons.

by the normal to the crystal face and a line from the center of the triangle through one of the apexes (1-*c*). (In our previous notation the incident beam lies in one of the {111}-azimuths of the crystal.) The angle Θ_1 between the incident beam and the normal to the crystal face is adjustable, and the collector can be moved about in the plane of incidence. Unfortunately, the insulation between the parts of the collector is not so high as it should be, and we are unable for this reason to use a retarding potential to stop off the low-speed secondary electrons. We are obliged, for the present, to accept into the collector electrons of all speeds.

The atoms in the crystal lie in lines which cut the plane of incidence as indicated in (1-*d*). The crystal may be regarded as made up of plane gratings lying parallel to the crystal face. The constant of these gratings is 2.15 Å and the separation between adjacent gratings is 2.03 Å.

The location of the incident beam in one of the principal azimuths of the crystal is of no importance, of course, so far as observations on the

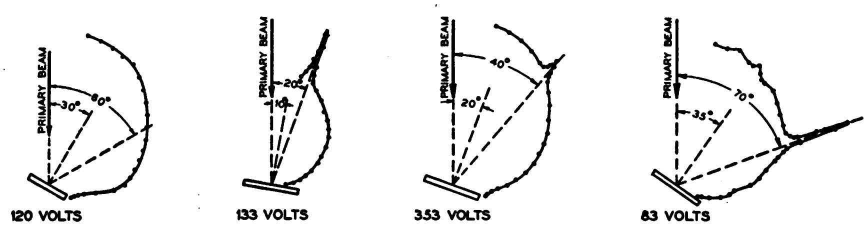

FIGURE 2

Distribution-in-angle of electrons of all speeds issuing from a [111] face of a nickel crystal for various angles of incidence and speeds of bombardment.

regularly reflected beam are concerned. This adjustment was made for the purpose of bringing some of the strong diffraction beams into the plane of incidence, and, therefore, into the range of observations. Such beams are found and have been used to calculate electron wave-lengths. The reflection beams cannot be used for this purpose.

In figure 2 we show distribution-in-angle curves for electrons of all speeds for various combinations of angle of incidence and bombarding

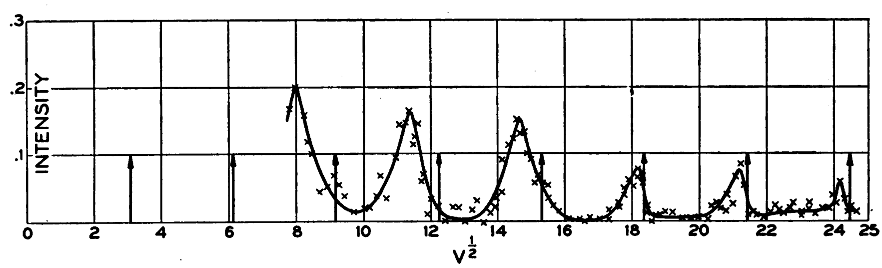

FIGURE 3

Variation of the intensity of the regularly reflected electron beam with bombarding potential, for 10° incidence—Intensity vs. $V^{1/2}$.

potential. The first curve is included to show that under certain conditions the intensity of the regularly reflected beam is immeasurably low. The sharp spurs which characterize the other curves reveal the reflected beam at one or another of its intensity maxima. The axes of these spurs appear to lie accurately in the direction of regular reflection. The angles of incidence and reflection are in all cases the same to within half a degree by our scale readings, and this is within the limit of uncertainty of the measurements.

In figure 3 we have plotted the intensity of the reflected beam for angle of incidence 10 degrees—or rather, a certain function of this intensity—against the square root of the bombarding potential. What is plotted as ordinate is one less than the ratio of the current received by the collector standing in the direction of regular reflection to the mean of the currents received in two adjacent directions, one on each side of the beam. The curve cannot be extended much below $V^{1/2} = 8$ because of limitations of the apparatus; the current of the incident beam is too small to work with for bombarding potentials much below 65 volts. By measurements made in a different way we have, however, an indication that the regularly reflected beam has another intensity maximum near $V^{1/2} = 5.3$ (bombarding potential 28 volts).

If the Bragg formula obtained, the maxima in this curve would occur at positions given by $V^{1/2} = n \times 3.06$, where n represents an integer. The wave-length $\lambda(=h/mv)$ expressed in volts V (bombarding potential) is given by $\lambda = (150/V)^{1/2}$ Å, and when this expression for λ is written into the Bragg formula we obtain

$$V^{1/2} = n \frac{(150)^{1/2}}{2d \cos \Theta}$$

which reduces to $V^{1/2} = n \times 3.06$ for $d = 2.03$ Å and $\Theta = 10$ degrees. The positions which the maxima should then occupy are indicated by arrows in figure 3.

It is evident that there is a simple correlation between the observed and calculated positions of the maxima, but whether the maximum observed at $V^{1/2} = 8.0$ is the third of the series or only the second is not yet certain.

The more general form of the Bragg formula is

$$n\lambda = 2d(\mu^2 - \sin^2\Theta)^{1/2}$$

where μ represents the refractive index of the crystal. Writing in $(150/V)^{1/2}$ for λ and solving for μ one obtains

$$\mu = \left[\frac{150\, n^2}{4Vd^2} + \sin^2\Theta\right]^{1/2}.$$

In the table below we give values of μ corresponding to the two possible assignments of orders.

TABLE 1

$V^{1/2}$	FIRST ASSIGNMENT n	μ	SECOND ASSIGNMENT n	μ
[5.3]	1	[0.60]	2	[1.15]
8.0	2	0.77	3	1.14
11.4	3	0.81	4	1.07
14.7	4	0.84	5	1.04
18.1	5	0.85	6	1.02
21.2	6	0.87	7	1.01
24.2	7	0.89	8	1.01

Further observations at other angles of incidence will, we believe, indicate whether the first or the second of these assignments is the correct one, and whether or not the diffraction phenomena can be adequately accounted for by writing an index of refraction into the Bragg formula.

The widths of the peaks shown in figure 3 are determined presumably by the rate of extinction of the equivalent electron radiation in the crystal. It should be possible to calculate coefficients of extinction for electrons of different speeds from such data. In a similar manner it is possible to calculate from the widths of the spurs shown in figure 2 a lower limit to the mean size of facet on the surface of the crystal. These calculations will be carried out as soon as precise data are available.

Several diffraction beams, as distinguished from reflection beams, have so far been observed in the present investigation. It may be shown that the wave-lengths of such beams will satisfy the plane grating formula

$$n\lambda = D(\sin \Theta_2 - \sin \Theta_1)$$

with respect to the atomic plane grating lying parallel to the surface of the crystal, no matter what value the index of refraction of the crystal may have. This is the formula used in calculating wave-lengths in our previous experiments, and it is used now to calculate the wave-lengths of two diffraction beams for which the data are exceptionally precise. The calculated wave-lengths are in excellent agreement with the theoretical values of h/mv as shown in the accompanying table.

TABLE 2

D = grating constant = 2.15Å;
V = bombarding potential;
Θ_1 = angle of incidence;
Θ_2 = angle of diffraction (angle between diffraction beam and normal to crystal face, the sign being chosen so that $\Theta_1 + \Theta_2$ represents the angle between incident beam and diffraction beam);
n = order of beam;
λ (obs.) = $D(\sin \Theta_2 - \sin \Theta_1)$
λ (cal.) = h/mv = $(150/V)^{1/2}$Å.

V (VOLTS)	Θ_1	Θ_2	N	λ(OBS.)	λ(CAL.)	$\frac{\lambda(\text{OBS.})}{\lambda(\text{CAL.})} - 1$
165	0.0	+62.8	+2	0.956 Å	0.953 Å	+0.003
130	44.5	+11.9	−1	1.064	1.074	−0.01

Our present apparatus, regarded as a spectrometer, is superior to the one employed in our earlier experiments. The positions of beams can be determined with an uncertainty of less than half a degree of arc, and as a consequence the values of observed wave-length given above should be in error by not more than about one per cent. The data obtained in our previous experiments yielded values of observed wave-length which, in a few cases, differed from the calculated values by more than fifteen per cent (loc. cit., Fig. 17). That we were justified in attributing these large discrepancies to instrumental error seems now quite certain.

It is interesting to note that the first of the diffraction beams for which data are given above is one that was observed under similar conditions (primary beam incident normally) with our first and less precise apparatus. The values found for the critical potential and position of the beam in the earlier experiments were $V = 160$ volts and $\Theta_2 = 60$ degrees.

[1] Eckart, *Proc. Nat. Acad. Sci.*, **13**, 460 (1927).

THE LIMITS OF ACCURACY IN PHYSICAL MEASUREMENTS

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1. *Heisenberg's Uncertainty Principle and the Motion of Free Particles.*—Heisenberg[1] has called attention to an important feature of our physical measurements. If we determine the value of a coördinate q for a specified value of the time, or possibly of some other parameter, the conjugate momentum p is altered during the measurement. Conversely, if we determine p, then q is changed by the process of measurement. In discussing these alterations, we shall neglect errors caused by the imperfections of the observer. In a measurement of q, let Δq be the uncertainty introduced by the finite "q-resolving power" of the apparatus, and Δp the uncertainty of the conjugate momentum at the same instant. According to Heisenberg,

$$\Delta q \cdot \Delta p \sim h. \tag{1}$$

Using the concepts of the probability interpretation of quantum mechanics, Darwin[2] justified this relation for certain general types of dynamical systems.[3]

The uncertainty discussed by Heisenberg, due to modification of the measured object by the measuring device, is quite distinct from that ordinarily considered, which arises from the dimensions and imperfections of the measuring device and the observer. It is present even if the observer obtains what we may call "ideal performance" from his apparatus. To illustrate what is meant, we might say that "ideal performance" is attained in a length measurement with a microscope if the precision is of the order of the wave-length of the light employed. To sum up, the content of equation (1) may be restated as follows: