Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3210 - Mecânica dos Sólidos I

Aula #20

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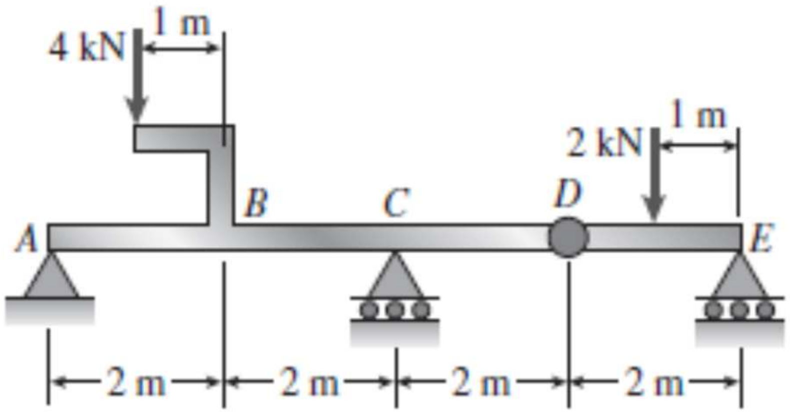
24/06/2022

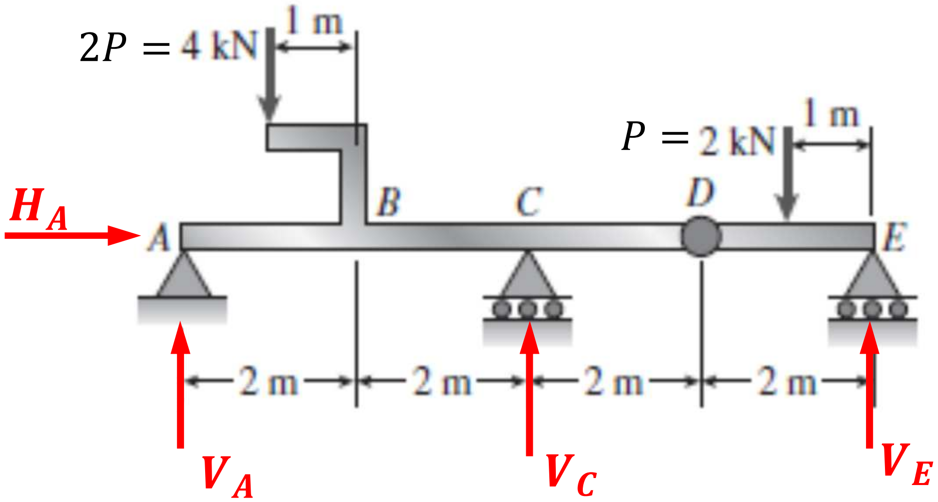
Problem 4.5-30 The compound beam *ABCDE* shown in the figure consists of two beams (*AD* and *DE*) joined by a hinged connection at *D*. The hinge can transmit a shear force but not a bending moment. The loads on the beam consist of a 4-kN force at the end of a bracket attached at point *B* and a 2-kN force at the midpoint of beam *DE*.

Draw the shear-force and bending-moment diagrams for this compound beam.



Solução:

1) D.C.L.:



$L = 1\ m$

$P = 2\ kN$

2) Grau de hiperestaticidade estrutural: $g = 4 - 4 = 0$ (estrutura isostática)

3) Equações de equilíbrio estático:

$H_A = 0$

$V_E.2L = PL$

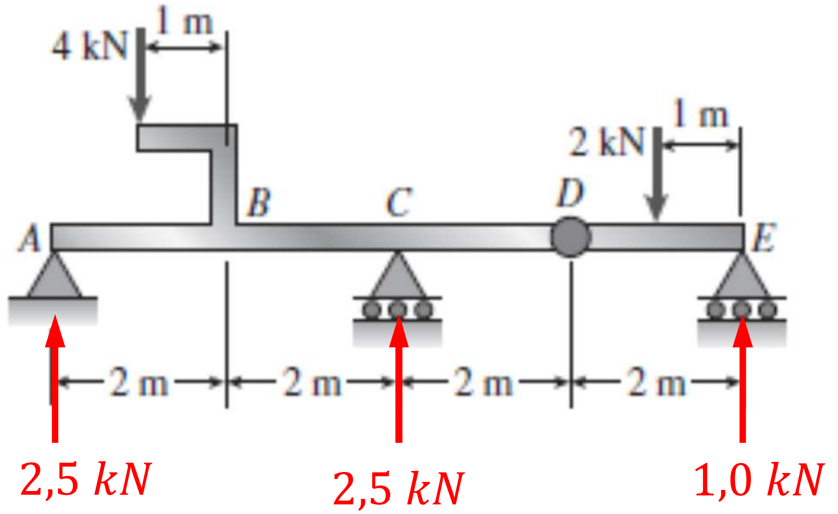
$V_A.4L + P.3L = V_E.4L + 2P.3L$

$$V_E = \frac{P}{2} = 1\ kN$$

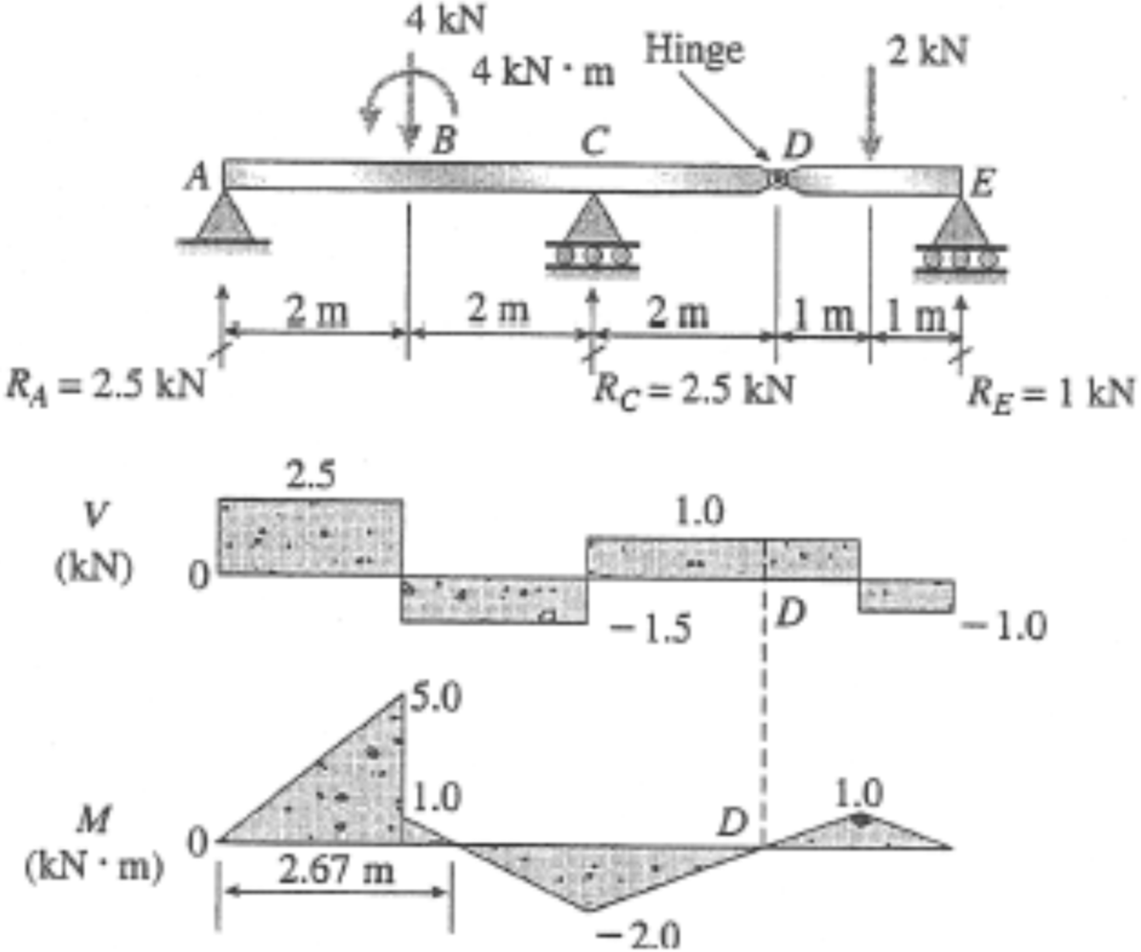
$$V_A = \frac{5P}{4} = 2{,}5\ kN$$

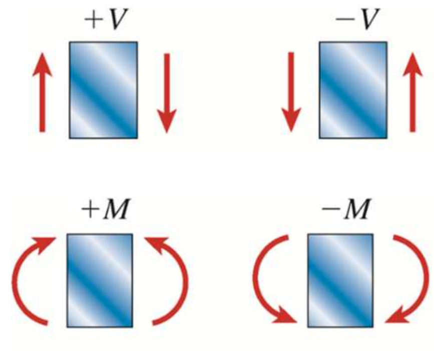
Finalmente: $V_A + V_C + V_E = 3P \Leftrightarrow \frac{5P}{4} + V_C + \frac{P}{2} = 3P$

$$V_C = \frac{5P}{4} = 2{,}5\ kN$$



Diagramas de forças cortantes e de momentos fletores:

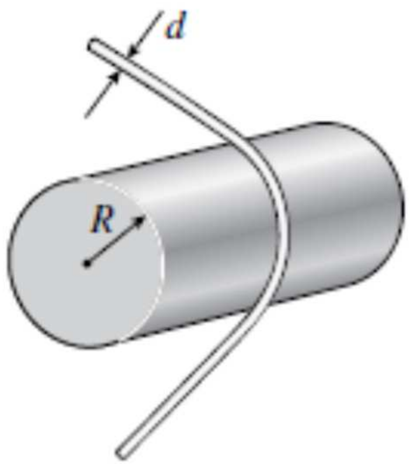




$V\ (kN)$

$M\ (kN.m)$

Problem 5.4-1 Determine the maximum normal strain ε_{max} produced in a steel wire of diameter $d = 1/16$ in. when it is bent around a cylindrical drum of radius $R = 24$ in. (see figure).



Solução:

O alongamento depende da distância (y) do ponto da seção transversal (mais afastado da linha neutra) até a linha neutra e da curvatura imposta ao eixo central do fio. Em valor absoluto:

$$|\varepsilon|_{máx} = |\kappa . y_{máx}|$$

No caso:

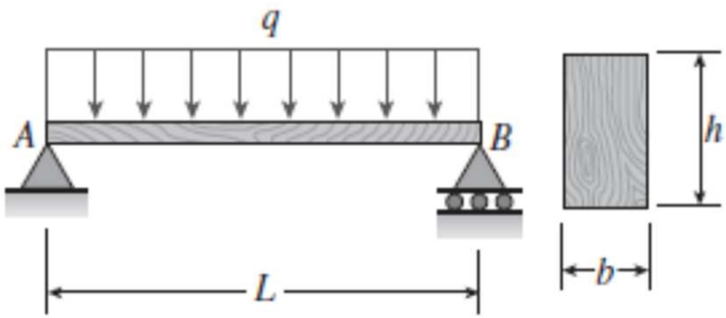
$$|\kappa| = \frac{1}{\rho} = \frac{1}{R + 0{,}5d} = \frac{1}{(24 + 1/32)in} \cong 0{,}0416125 \; in^{-1}$$

$$|y_{máx}| = \frac{d}{2} = \frac{1}{32} in = 0{,}03125 \; in$$

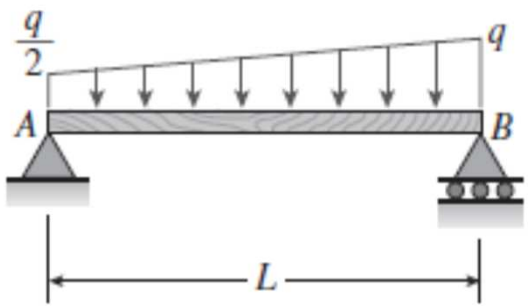
Logo: $$|\varepsilon|_{máx} = \frac{0{,}5d}{R + 0{,}5d} = \frac{d}{2R + d} \cong 0{,}0013$$

Problem 5.5-4 A simply supported wood beam AB with span length $L = 4$ m carries a uniform load of intensity $q = 5.8$ kN/m (see figure).

(a) Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 140$ mm and height $h = 240$ mm.
(b) Repeat (a) but use the trapezoidal distuibuted load shown in the figure part (b).



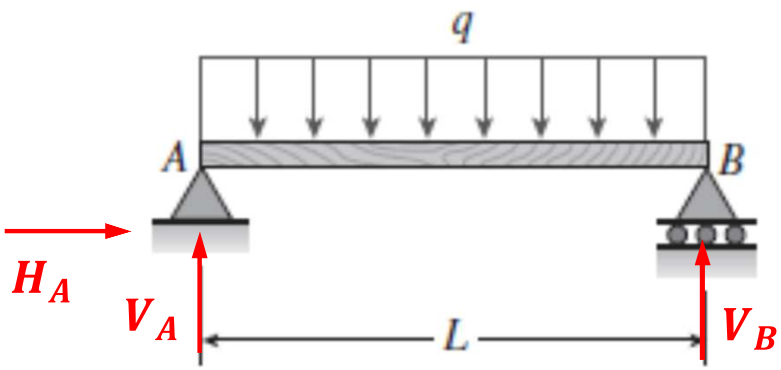
(a)



(b)

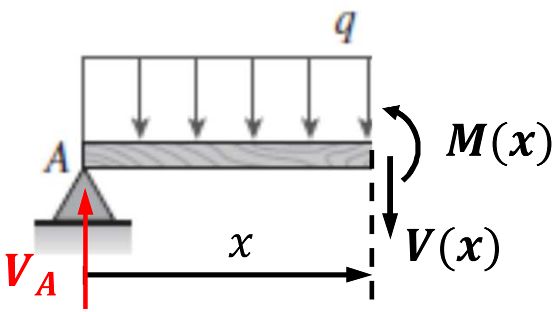
Solução:

a) D.C.L.:



Pelas equações da estática, encontramos:

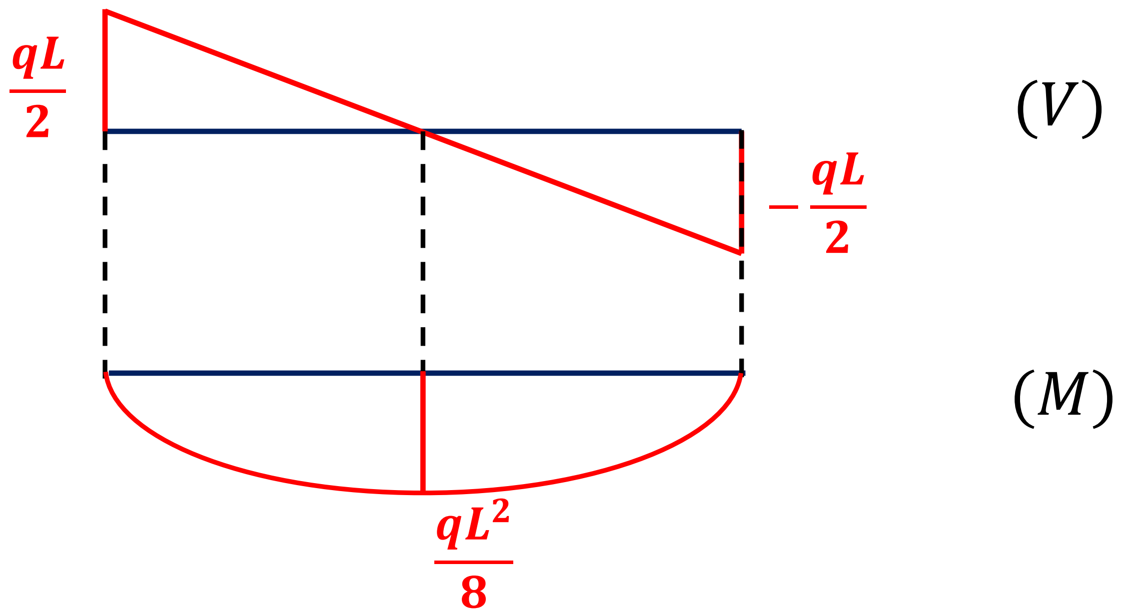
$$V_A = V_B = \frac{qL}{2} = \frac{(5{,}8\, kN/m).(4m)}{2} = 11{,}6\ kN$$



$$V(x) + qx = V_A = \frac{qL}{2} \quad \Leftrightarrow \quad V(x) = q\left(\frac{L}{2} - x\right)$$

$$M(x) + \frac{qx^2}{2} = V_A x = \frac{qLx}{2} \quad \Leftrightarrow \quad M(x) = \frac{qx(L-x)}{2}$$

Diagramas:



Note que o ponto onde a força cortante é nula é um ponto crítico do momento fletor, podendo ser um ponto de máximo local ou mínimo local, pois, como vimos:

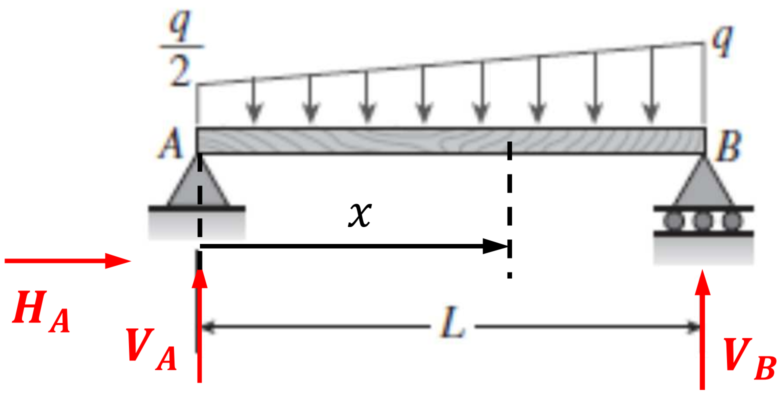
$$V(x) = \frac{dM(x)}{dx}$$

Do exposto: $M_{máx} = \dfrac{qL^2}{8}$

$$|\sigma_{máx}| = \frac{|M_{máx}|}{I}|y_{máx}| = \frac{|M_{máx}|}{bh^3/12}.\frac{h}{2} = \frac{|M_{máx}|}{bh^2/6} = \frac{6}{bh^2}\frac{qL^2}{8} = \frac{3}{4}\frac{q}{b}\left(\frac{L}{h}\right)^2$$

$$|\sigma_{máx}| = \frac{3}{4}\frac{(5{,}8\,N/mm)}{140\,mm}\left(\frac{4000}{240}\right)^2 = \frac{3}{4}(0{,}04143\,MPa).(277{,}78) \cong 8{,}63\,MPa$$

b) D.C.L.:



$$q(x) = \frac{q}{2}\left(1 + \frac{x}{L}\right)$$

Pelas equações da estática, encontramos:

$$V_B L = \int_0^L q(x) x dx = \int_0^L \frac{q}{2}\left(x + \frac{x^2}{L}\right) dx = \frac{q}{2}\left(\frac{L^2}{2} + \frac{L^2}{3}\right) = \frac{5}{12} q L^2 \Leftrightarrow V_B = \frac{5}{12} qL$$

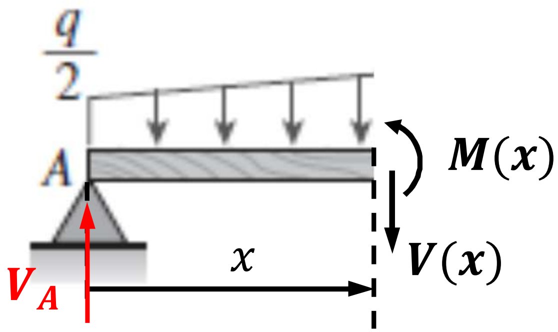
$$V_A + V_B = \int_0^L q(x) dx = \int_0^L \frac{q}{2}\left(1 + \frac{x}{L}\right) dx = \frac{q}{2}\left(L + \frac{L}{2}\right) = \frac{3}{4} qL \quad \Leftrightarrow \quad V_A = \frac{qL}{3}$$

Utilizando as relações entre $q(x), V(x)$ e $M(x)$, teremos:

$$-\frac{dV(x)}{dx} = q(x) = \frac{q}{2}\left(1 + \frac{x}{L}\right) \Longleftrightarrow V(x) = V_A - \int_0^x q(x)dx = V_A - \int_0^x \frac{q}{2}\left(1 + \frac{x}{L}\right)dx$$

$$V(x) = \frac{qL}{3} - \frac{q}{2}\left(x + \frac{x^2}{2L}\right)$$

$$\frac{dM(x)}{dx} = V(x) \Longleftrightarrow M(x) = M(0) + \int_0^x V(x)dx = \int_0^x \left[\frac{qL}{3} - \frac{q}{2}\left(x + \frac{x^2}{2L}\right)\right]dx$$



$$M(x) = \frac{qLx}{3} - \frac{q}{2}\left(\frac{x^2}{2} + \frac{x^3}{6L}\right)$$

Determinação dos pontos críticos da função $M(x)$:

$$\frac{dM(x)}{dx} = V(x) = 0 \quad \Leftrightarrow \quad \frac{qL}{3} - \frac{q}{2}\left(x + \frac{x^2}{2L}\right) = 0$$

$$3x^2 + 6Lx - 4L^2 = 0$$

$$x = \frac{-6L}{6} \pm \frac{\sqrt{36+48}}{6}L$$

A única raiz possível no domínio $0 \leq x \leq L$ é: $\qquad x \cong 0{,}52753L$

$$x = 0{,}52753L \quad \Rightarrow \quad M = M_{máx} \cong 0{,}0940376qL^2$$

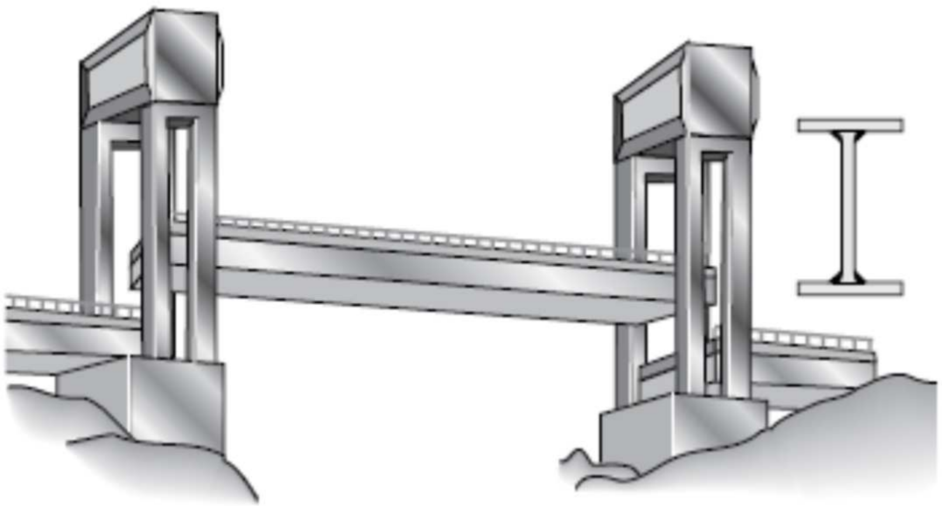
Logo:

$$|\sigma_{máx}| = \frac{|M_{máx}|}{I}|y_{máx}| = \frac{|M_{máx}|}{bh^3/12}.\frac{h}{2} = \frac{|M_{máx}|}{bh^2/6} = 0,5642255\,\frac{q}{b}\left(\frac{L}{h}\right)^2$$

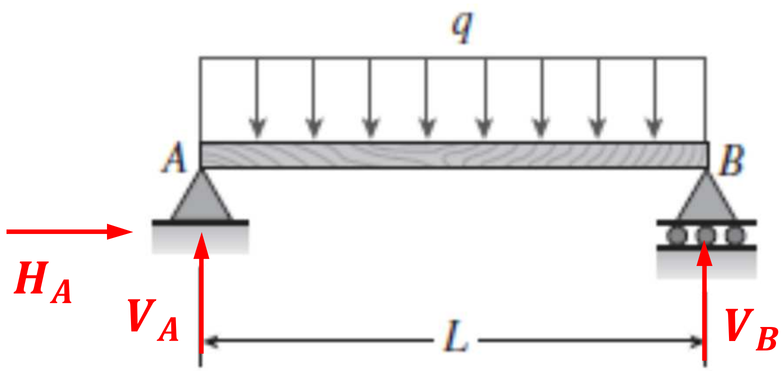
$$|\sigma_{máx}| = 0,5642255\,\frac{(5,8\,N/mm)}{140\,mm}\left(\frac{4000}{240}\right)^2 \cong 6,49\,MPa$$

Problem 5.5-5 Each girder of the lift bridge (see figure) is 180 ft long and simply supported at the ends. The design load for each girder is a uniform load of intensity 1.6 k/ft. The girders are fabricated by welding tree steel plates so as to form an I-shaped cross section (see figure) having section modulus $S = 3600$ in.^{3}.

What is the maximum bending stress σ_{max} in a girder due to the uniform load?



Solução: O modelo de cálculo é o de uma viga simplesmente apoiada nas extremidades e submetida a um carregamento uniformemente distribuído de intensidade q como ilustra a figura:



Como vimos no problema anterior: $M_{máx} = \frac{qL^2}{8}$ (em $x = L/2$)

$$M_{máx} = \frac{1600\left(\frac{lbf}{ft}\right).\left(\frac{1\,ft}{12\,in}\right).(180 \times 12\,in)^2}{8} = 7{,}776 \times 10^7 lbf.in$$

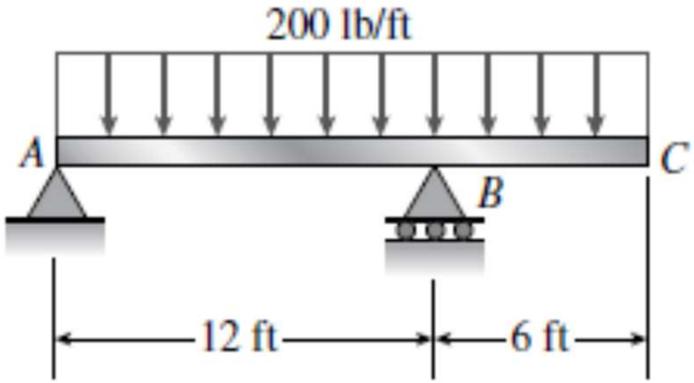
A tensão máxima será:

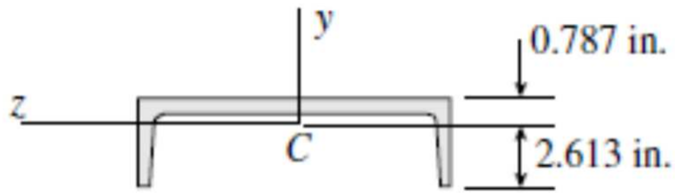
$$|\sigma_{máx}| = \frac{|M_{máx}|}{I}|y_{máx}| = \frac{|M_{máx}|}{S} = \frac{7{,}776 \times 10^7 lbf.in}{3600\ in^3}$$

$$|\sigma_{máx}| = 21600\ psi = 21{,}6\ ksi$$

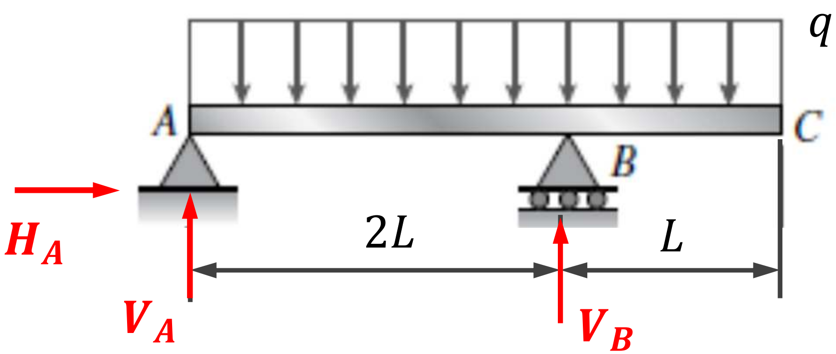
Problem 5.5-19 A beam *ABC* with an overhang from *B* to *C* supports a uniform load of 200 lb/ft throughout its length (see figure). The beam is a channel section with dimensions as shown in the figure. The moment of inertia about the z axis (the neutral axis) equals 8.13 in.^{4}

Calculate the maximum tensile stress σ_t and maximum compressive stress σ_c due to the uniform load.





Solução:



$$L = 6\ ft = 72\ in$$

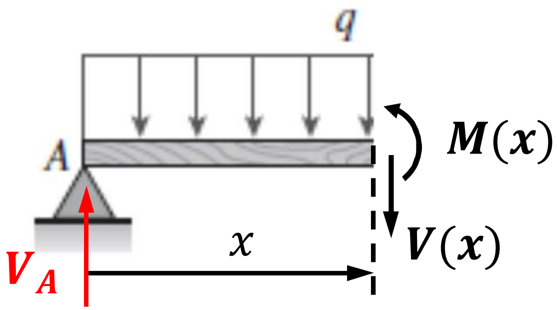
$$q = 200\frac{lbf}{ft} = 16{,}667\frac{lbf}{in}$$

$$V_B(2L) = \frac{q(3L)^2}{2} \quad \Leftrightarrow \quad V_B = \frac{9}{4}qL = 2{,}25qL$$

$$V_A + V_B = 3qL \quad \Leftrightarrow \quad V_A = \frac{3}{4}qL = 0{,}75qL$$

Como os momentos fletores são nulos nas extremidades A e C da viga, e como a distribuição de momentos é contínua (dado que não há momentos concentrados aplicados ao longo da viga), basta analisarmos o trecho AB.

A distribuição de força cortante no trecho AB será:



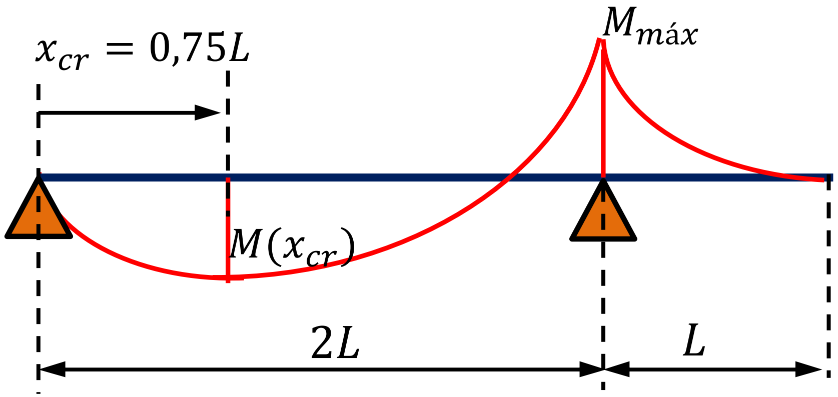
$$V(x) + qx = V_A = \frac{3}{4}qL \Leftrightarrow \quad V(x) = q\left(\frac{3L}{4} - x\right)$$

$$V(x) = \frac{dM(x)}{dx} = 0 \Leftrightarrow \; x_{cr} = \frac{3L}{4}$$

$$M(x) = V_A x - \frac{qx^2}{2} = \frac{3}{4}qLx - \frac{qx^2}{2}$$

$$x_{cr} = \frac{3L}{4}: \qquad M(x_{cr}) = \frac{9}{32}qL^2 = 24300\; lbf.in$$

Além disso: $M_{máx} = |M_B| = qL^2/2 = 43200\, lbf.in$

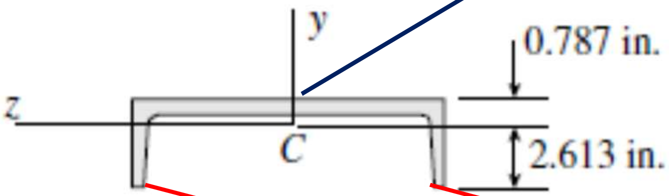


Como a seção não é simétrica em relação ao eixo de flexão, é preciso analisar as duas seções transversais: aquela para a qual $M = M(x_{cr})$ e aquela para a qual $M = M_{máx}$.

Para a seção referente à $x = x_{cr}$: $\quad M(x_{cr}) = 24300\ lbf.in$

Temos:

Máxima tensão de compressão (na topo)



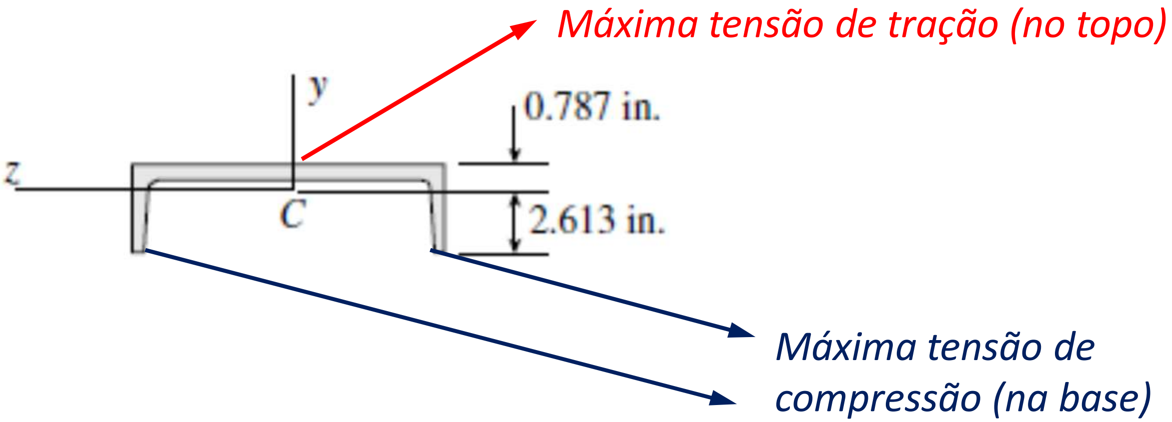
Máxima tensão de tração (na base)

$$\left|\sigma_{máx,t}\right| = \frac{|M(x_{cr})|}{I}\left|y_{máx,t}\right| = \frac{24300\ lbf.in}{8{,}13\ in^4}.2{,}613\ in \cong 7810\ psi$$

$$\left|\sigma_{máx,c}\right| = \frac{|M(x_{cr})|}{I}\left|y_{máx,c}\right| = \frac{24300\ lbf.in}{8{,}13\ in^4}.0{,}787\ in \cong 2352\ psi$$

Para a seção referente à $x = x_B$: $\quad M = M_{máx} = |M_B| = qL^2/2 = 43200\ lbf.in$

Temos:

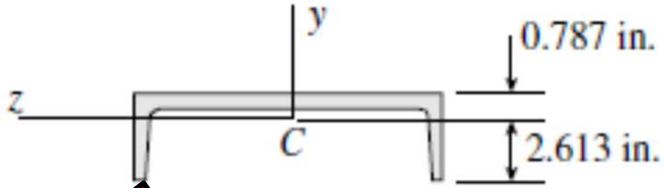


$$\left|\sigma_{máx,t}\right| = \frac{|M_{máx}|}{I}\left|y_{máx,t}\right| = \frac{43200\ lbf.in}{8{,}13\ in^4}.0{,}787\ in \cong 4182\ psi$$

$$\left|\sigma_{máx,c}\right| = \frac{|M_{máx}|}{I}\left|y_{máx,c}\right| = \frac{43200\ lbf.in}{8{,}13\ in^4}.2{,}613\ in \cong 13885\ psi$$

Logo, a máxima tensão de tração ocorre na seção em que $x = x_{cr} = 0{,}75L$ e vale:

$$\left|\sigma_{máx,t}\right| \cong 7810\, psi$$



E a máxima tensão de compressão ocorre na seção em que $x = x_B = 2L$ e vale:

$$\left|\sigma_{máx,c}\right| \cong 13885\, psi$$