Exemplo 1 página 158

$pr(m,f) := m \cdot \cos(f) + j \cdot m \cdot \sin(f)$

$V_n := 500kV$

OBSERVAÇÕES

Bom usar formato engeneering na HP
Cuidado com a tensão, usar fase-neutro eficaz
Corrente=corrente de linha
Os parâmetros são de sequencia positiva

LT1

$z_1 := pr\left(0.4\frac{\Omega}{km}, 80deg\right)$ $\quad c_1 := 11\frac{nF}{km}$ $\quad d_1 := 270km$

LT2

$z_2 := pr\left(0.3\frac{\Omega}{km}, 76deg\right)$ $\quad c_2 := 12\frac{nF}{km}$ $\quad d_2 := 350km$

Calcular ABCD de cada linha, circuito π exato e o reator no fim da linha para Ventrada=Vsaida

Cálculo dos ABCD

$f := 60Hz \qquad \omega := 2 \cdot \pi \cdot f$

$y_1 := j \cdot \omega \cdot c_1 \qquad Zc_1 := \sqrt{\frac{z_1}{y_1}} \qquad \gamma_1 := \sqrt{z_1 \cdot y_1} \qquad Zc_1 = 309.394 - 27.068i\,\Omega \qquad \gamma_1 = 1.123\times 10^{-4} + 1.283i\times 10^{-3}\frac{1}{km}$

$A_1 := \cosh(\gamma_1 \cdot d_1) \qquad B_1 := Zc_1 \cdot \sinh(\gamma_1 \cdot d_1) \qquad C_1 := \frac{1}{Zc_1}\cdot \sinh(\gamma_1 \cdot d_1)$

$A_1 = 0.941 + 0.01i \qquad B_1 = 18.016 + 104.325i\,\Omega \qquad C_1 = -3.872\times 10^{-6} + 1.098i\times 10^{-3}S$

$y_2 := j \cdot \omega \cdot c_2 \qquad Zc_2 := \sqrt{\frac{z_2}{y_2}} \qquad \gamma_2 := \sqrt{z_2 \cdot y_2} \qquad Zc_2 = 255.597 - 31.383i\,\Omega \qquad \gamma_2 = 1.42\times 10^{-4} + 1.156i\times 10^{-3}\frac{1}{km}$

$A_2 := \cosh(\gamma_2 \cdot d_2) \qquad B_2 := Zc_2 \cdot \sinh(\gamma_2 \cdot d_2) \qquad C_2 := \frac{1}{Zc_2}\cdot \sinh(\gamma_2 \cdot d_2)$

$A_2 = 0.92 + 0.02i \qquad B_2 = 24.052 + 99.33i\,\Omega \qquad C_2 = -1.044\times 10^{-5} + 1.541i\times 10^{-3}S$

Modelo π corrigido

$Z\pi c_1 := B_1 \qquad Y\pi c_1 := \frac{A_1 - 1}{B_1} \qquad Z\pi c_1 = 18.016 + 104.325i\,\Omega \qquad Y\pi c_1 = 1.003\times 10^{-6} + 565.452i\times 10^{-6}S$

$Z\pi c_2 := B_2 \qquad Y\pi c_2 := \frac{A_2 - 1}{B_2} \qquad Z\pi c_2 = 24.052 + 99.33i\,\Omega \qquad Y\pi c_2 = 2.741\times 10^{-6} + 802.487i\times 10^{-6}S$

Modelo π nominal

$Z\pi n_1 := z_1 \cdot d_1 \qquad Y\pi n_1 := \frac{j \cdot \omega \cdot c_1 \cdot d_1}{2} \qquad Z\pi n_1 = 18.754 + 106.359i\,\Omega \qquad Y\pi n_1 = 559.832i\times 10^{-6}S$

$Z\pi n_2 := z_2 \cdot d_2 \qquad Y\pi n_2 := \frac{j \cdot \omega \cdot c_2 \cdot d_2}{2} \qquad Z\pi n_2 = 25.402 + 101.881i\,\Omega \qquad Y\pi n_2 = 791.681i\times 10^{-6}S$

Diferença dos 2 modelos

$Z\pi n_1 - Z\pi c_1 = 0.738 + 2.034i\,\Omega$ $\quad \left|\dfrac{Z\pi n_1}{Z\pi c_1}\right| = 1.0201 \quad d_1 = 270\,km$

$Z\pi n_2 - Z\pi c_2 = 1.35 + 2.551i\,\Omega$ $\quad \left|\dfrac{Z\pi n_2}{Z\pi c_2}\right| = 1.0274 \quad d_2 = 350\,km$

$Y\pi n_1 - Y\pi c_1 = -1.003\times 10^{-6} - 5.621i\times 10^{-6}\,S$ $\quad \left|\dfrac{Y\pi n_1}{Y\pi c_1}\right| = 0.9901$

$Y\pi n_2 - Y\pi c_2 = -2.741\times 10^{-6} - 1.081i\times 10^{-5}\,S$ $\quad \left|\dfrac{Y\pi n_2}{Y\pi c_2}\right| = 0.9865$

Cálculo do reator

Quadripolo equivalente

$$\begin{pmatrix} v_e \\ i_e \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\cdot\begin{pmatrix} v_i \\ i_i \end{pmatrix} \quad (1) \qquad \begin{pmatrix} v_i \\ i_i \end{pmatrix} = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}\cdot\begin{pmatrix} v_s \\ i_s \end{pmatrix}$$

Substituindo (2) em (1)

$$\begin{pmatrix} v_e \\ i_e \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\cdot\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}\cdot\begin{pmatrix} v_s \\ i_s \end{pmatrix}$$

O quadripolo equivalente é: $\quad D_1 := A_1 \qquad D_2 := A_2$

$$Q_{eq} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\cdot\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \qquad Q_{eq} = \begin{pmatrix} A_1\cdot A_2 + B_1\cdot C_2 & A_1\cdot B_2 + B_1\cdot D_2 \\ C_1\cdot A_2 + D_1\cdot C_2 & C_1\cdot B_2 + D_1\cdot D_2 \end{pmatrix}$$

$A := A_1\cdot A_2 + B_1\cdot C_2 \qquad B := A_1\cdot B_2 + B_1\cdot D_2 \qquad C := C_1\cdot A_2 + D_1\cdot C_2 \qquad D := C_1\cdot B_2 + D_1\cdot D_2$

$A = 0.705 + 0.055i \qquad B = 36.15 + 190.089i\,\Omega \qquad C = -5.074\times 10^{-5} + 2.46i\times 10^{-3}\,S \qquad D = 0.757 + 0.054i$

O quadripolo do reator é:

$$Q_r = \begin{pmatrix} 1 & 0 \\ Yr & 1 \end{pmatrix}$$

O quadripolo da associação é:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ Yr & 1 \end{pmatrix} = \begin{pmatrix} A + B\cdot Yr & B \\ C + D\cdot Yr & D \end{pmatrix}$$

Para que a tensão de saída seja igual à de entrada:

$$\begin{pmatrix} v_e \\ i_e \end{pmatrix} = \begin{pmatrix} A + B\cdot Yr & B \\ C + D\cdot Yr & D \end{pmatrix}\cdot\begin{pmatrix} v_s \\ i_s \end{pmatrix} \qquad \begin{pmatrix} v_e \\ i_e \end{pmatrix} = \begin{pmatrix} v_s\cdot A + v_s\cdot B\cdot Yr + B\cdot i_s \\ v_s\cdot C + v_s\cdot D\cdot Yr + D\cdot i_s \end{pmatrix}$$

$MW := 1000kW$

$MVAr := MW$

$MVA := MW$

$pu := 1$

$$v_e = v_s\cdot A + v_s\cdot B\cdot Yr + B\cdot i_s$$

Como a corrente de saída é nula

$$v_e = (A + B\cdot Yr)\cdot v_s$$

As tensões serão iguais se:

$$A + B\cdot Yr = 1 \qquad Yr := \frac{1 - A}{B} \qquad Xr := \frac{1}{Yr} \qquad Xr = 3.276 + 644.783i\,\Omega \qquad Qr := \left|V_n{}^2\cdot Yr\right| \qquad Qr = 387.723\,MVAr$$

Para a linha 1 somente

$$Yr1 := \frac{1 - A_1}{B_1} \qquad Xr1 := \frac{1}{Yr1} \qquad Xr1 = -3.138 + 1.768i \times 10^3\,\Omega \qquad Qr1 := \left|V_n{}^2\cdot Yr1\right| \qquad Qr1 = 141.363\,MVAr$$

Para a linha 2 somente

$$Yr2 := \frac{1 - A_2}{B_2} \qquad Xr2 := \frac{1}{Yr2} \qquad Xr2 = -4.257 + 1.246i \times 10^3\,\Omega \qquad Qr2 := \left|V_n{}^2\cdot Yr2\right| \qquad Qr2 = 200.623\,MVAr$$

$$Qr1 + Qr2 = 341.986\,MVAr$$

Exemplo 2 página 160

Considerando a LT1 do exemplo anterior.
Carga trifásica de 800 MW fp 0,9 no final. Há um capacitor série no final da LT1, antes da carga com 60 Ω.
A tensão no final da LT1 é 1,039 com fase 10 graus em pu

$$P_s := \frac{800}{3}MW \qquad \phi := acos(0.9) \qquad \phi = 25.842\,deg \qquad Q_s := P_s\cdot\tan(\phi) \qquad S_s := P_s + j\cdot Q_s \qquad S_s = 266.667 + 129.153i\,MV\!$$

monofásica

$$v_s := pr(1.039pu, -10deg) \qquad V_s := \frac{V_n}{\sqrt{3}}\cdot v_s \qquad \left|V_s\right| = 299.933\,kV \qquad arg(V_s) = -10\,deg$$

Associação linha capacitor série

$$Q_e = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\cdot\begin{pmatrix} 1 & Z_{cs} \\ 0 & 1 \end{pmatrix} \qquad Q_e = \begin{pmatrix} A_1 & A_1\cdot Z_{cs} + B_1 \\ C_1 & C_1\cdot Z_{cs} + D_1 \end{pmatrix}$$

Corrente no fim da linha (barra em cima indica conjugado)

$$S_s = V_s\cdot\overline{I_s} \qquad I_s := \overline{\frac{S_s}{V_s}} \qquad \left|I_s\right| = 0.988\,kA \qquad arg(I_s) = -35.842\,deg \qquad I_s = 0.8008 - 0.5785i\,kA$$

Cálculo da tensão e da corrente no inicio da linha

$$Z_{cs} := -60j\cdot\Omega$$

$$\begin{pmatrix} V_e \\ I_e \end{pmatrix} = \begin{pmatrix} A_1 & A_1 \cdot Z_{cs} + B_1 \\ C_1 & C_1 \cdot Z_{cs} + D_1 \end{pmatrix} \cdot \begin{pmatrix} V_s \\ I_s \end{pmatrix} \qquad \begin{pmatrix} V_e \\ I_e \end{pmatrix} = \begin{pmatrix} A_1 \cdot V_s + I_s \cdot A_1 \cdot Z_{cs} + I_s \cdot B_1 \\ C_1 \cdot V_s + I_s \cdot C_1 \cdot Z_{cs} + I_s \cdot D_1 \end{pmatrix}$$

$B_e := A_1 \cdot Z_{cs} + B_1 \quad B_e = 18.634 + 47.864i\,\Omega \quad B_1 = 18.016 + 104.325i\,\Omega \quad B_e - B_1 = 0.618 - 56.462i\,\Omega$

próximo de Zcs

$D_e := C_1 \cdot Z_{cs} + D_1 \quad D_e = 1.007 + 0.011i \quad D_1 = 0.941 + 0.01i$ muda pouco

$V_e := A_1 \cdot V_s + I_s \cdot A_1 \cdot Z_{cs} + I_s \cdot B_1 \quad V_e = 321.102 - 18.421i\,\text{kV} \quad |V_e| = 321.63\,\text{kV} \quad \arg(V_e) = -3.283\,\text{deg}$

$I_e := C_1 \cdot V_s + I_s \cdot C_1 \cdot Z_{cs} + I_s \cdot D_1 \quad I_e = 0.868 - 0.25i\,\text{kA} \quad |I_e| = 0.9036\,\text{kA} \quad \arg(I_e) = -16.036\,\text{deg}$

em pu: $\dfrac{|V_e| \cdot \sqrt{3}}{V_n} = 1.114\,\text{pu}$

Cálculo das perdas no conjunto linha capacitor série

$S_e := V_e \cdot \overline{I_e} \quad S_e = 283.451 + 64.152i\,\text{MVA} \quad \Delta S := 3(S_e - S_s) \quad \Delta S = 50.353 - 195.002i\,\text{MVA}$

Sem o capacitor série

$V_e := A_1 \cdot V_s + I_s \cdot B_1 \quad V_e = 353.268 + 27.151i\,\text{kV} \quad |V_e| = 354.31\,\text{kV} \quad \arg(V_e) = 4.395\,\text{deg} \quad \dfrac{|V_e| \cdot \sqrt{3}}{V_n} = 1.227\,\text{pu}$

$I_e := C_1 \cdot V_s + I_s \cdot D_1 \quad I_e = 0.816 - 0.212i\,\text{kA} \quad |I_e| = 0.8426\,\text{kA} \quad \arg(I_e) = -14.551\,\text{deg}$

$S_e := V_e \cdot \overline{I_e} \quad S_e = 282.361 + 96.929i\,\text{MVA} \quad \Delta S := 3(S_e - S_s) \quad \Delta S = 47.083 - 96.669i\,\text{MVA}$

Verifica-se que o capacitor série diminuiu a queda de tensão entre o início e o fim da linha LT1

O circuito π exato da associação é:

$A_e := A_1 \quad C_e := C_1$

ramo série $\quad Z_s := B_e \quad Z_s = 18.634 + 47.864i\,\Omega$

ramo shunt no início $\quad Y_e := \dfrac{D_e - 1}{B_e} \quad Y_e = 2.3954 \times 10^{-4} - 5.0511i \times 10^{-5}\,\text{S}$

ramo shunt no final $\quad Y_s := \dfrac{A_e - 1}{B_e} \quad Y_s = -2.2981 \times 10^{-4} + 0.0011i\,\text{S}$

Exemplo 3 página 162

Conectar LT1 e LT2 em série com um banco de capacitores série.
Com a linha em vazio:

$v_e := \mathrm{pr}(1.032\mathrm{pu}, 8\mathrm{deg})$ $\quad v_s := \mathrm{pr}(1.2156\mathrm{pu}, 4.144\mathrm{deg})$ $\quad v_e = 1.022 + 0.144i\,\mathrm{pu}$ $\quad v_s = 1.212 + 0.088i\,\mathrm{pu}$

Calcular a reatância do banco

$V_e := \frac{V_n}{\sqrt{3}} \cdot v_e$ $\quad |V_e| = 297.913\,\mathrm{kV}$ $\arg(V_e) = 8\,\mathrm{deg}$ $\quad V_s := \frac{V_n}{\sqrt{3}} \cdot v_s$ $\quad |V_s| = 350.913\,\mathrm{kV}$ $\arg(V_s) = 4.144\,\mathrm{deg}$

$$Q_1 = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \quad Q_r = \begin{pmatrix} 1 & Z_{cs} \\ 0 & 1 \end{pmatrix} \quad Q_2 = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$$

$$Q_e = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \cdot \begin{pmatrix} 1 & Z_{cs} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \qquad Q_e = \begin{pmatrix} A_1 & A_1 \cdot Z_{cs} + B_1 \\ C_1 & C_1 \cdot Z_{cs} + D_1 \end{pmatrix} \cdot \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$$

$$Q_e = \begin{pmatrix} A_1 \cdot A_2 + C_2 \cdot A_1 \cdot Z_{cs} + C_2 \cdot B_1 & A_1 \cdot B_2 + D_2 \cdot A_1 \cdot Z_{cs} + D_2 \cdot B_1 \\ C_1 \cdot A_2 + C_2 \cdot C_1 \cdot Z_{cs} + C_2 \cdot D_1 & C_1 \cdot B_2 + D_2 \cdot C_1 \cdot Z_{cs} + D_2 \cdot D_1 \end{pmatrix}$$

Como a linha está em vazio, $\quad I_s := 0$

$$\begin{pmatrix} V_e \\ I_e \end{pmatrix} = \begin{pmatrix} A_e & B_e \\ C_e & D_e \end{pmatrix} \cdot \begin{pmatrix} V_s \\ 0 \end{pmatrix} \qquad \begin{pmatrix} V_e \\ I_e \end{pmatrix} = \begin{pmatrix} A_e \cdot V_s \\ C_e \cdot V_s \end{pmatrix}$$

$$A_e = A_1 \cdot A_2 + C_2 \cdot A_1 \cdot Z_{cs} + C_2 \cdot B_1 \qquad V_e = \left(A_1 \cdot A_2 + C_2 \cdot A_1 \cdot Z_{cs} + C_2 \cdot B_1\right) \cdot V_s$$

Isolando Zcs $\quad Z_{cs} := \frac{-\left(-V_e + V_s \cdot A_1 \cdot A_2 + V_s \cdot C_2 \cdot B_1\right)}{C_2 \cdot A_1 \cdot V_s}$ $\quad Z_{cs} = 0.005 - 98.0121i\,\Omega$

Máxima potência transmissível para Vs=1 pu

$A_e := A_1 \cdot A_2 + C_2 \cdot A_1 \cdot Z_{cs} + C_2 \cdot B_1$ $\quad A_e = 0.847 + 0.057i$

$B_e := A_1 \cdot B_2 + D_2 \cdot A_1 \cdot Z_{cs} + D_2 \cdot B_1$ $\quad B_e = 38.889 + 105.222i\,\Omega$

$a := \arg(A_e)$ $\quad b := \arg(B_e)$ $\quad a = 3.856\,\mathrm{deg}$ $\quad b = 69.716\,\mathrm{deg}$ $\quad V_s := \frac{V_n}{\sqrt{3}}$ $\quad$ Vs = 1 pu

$$P_{smax} := 3 \cdot \left[\frac{|V_e| \cdot |V_s|}{|B_e|} - \frac{|A_e| \cdot \left(|V_s|\right)^2}{|B_e|} \cdot \cos(b - a) \right] \qquad P_{smax} = 1526.154\,\mathrm{MW}$$