

Universidade de São Paulo  
EDM0615 – Educação Matemática  
Profa. Raquel Milani

# Sobre operações com números naturais, algoritmos e material concreto

# Sobre a adição e subtração

O que pode causar equívocos.

- Ensinaamos a tirar o menor do maior.
- Começamos da direita para a esquerda (mas escrevemos da esquerda para a direita):  
unidades, dezenas, centenas
- Calculamos cada “coluna” separadamente.

Dezenas	Unidades
4	2
2	7
<hr/>	<hr/>
6	9

Dezenas	Unidades
4	2
2	7
6	9

- “Quando o aluno resolve uma operação começando pela direita ele não tem noção do número como um todo. Ele passa a olhar cada coluna como unidade (a parte como um todo) e não como um algarismo do numeral (a parte no todo)” (p. 74).
- $42+27$  é diferente de  $4+2$  e  $2+7$

ECKHARDT, C. A. Contas de “vai um” e “pedir emprestado”: em busca de um conhecimento-empowerment. **Revista de Educação Matemática – RS**, ano 5, n. 5, p. 68-78, 2003.

- Formas de ensino reguladoras e não emancipatórias
- Romper com verdades cristalizadas
- Transformar o ensino de matemática em um conhecimento-emancipação.
- Os algoritmos precisam ser estudados como processo, com compreensão.

Poderiam as crianças ser incentivadas a construir seus próprios procedimentos e algoritmos?

( 1 1 )

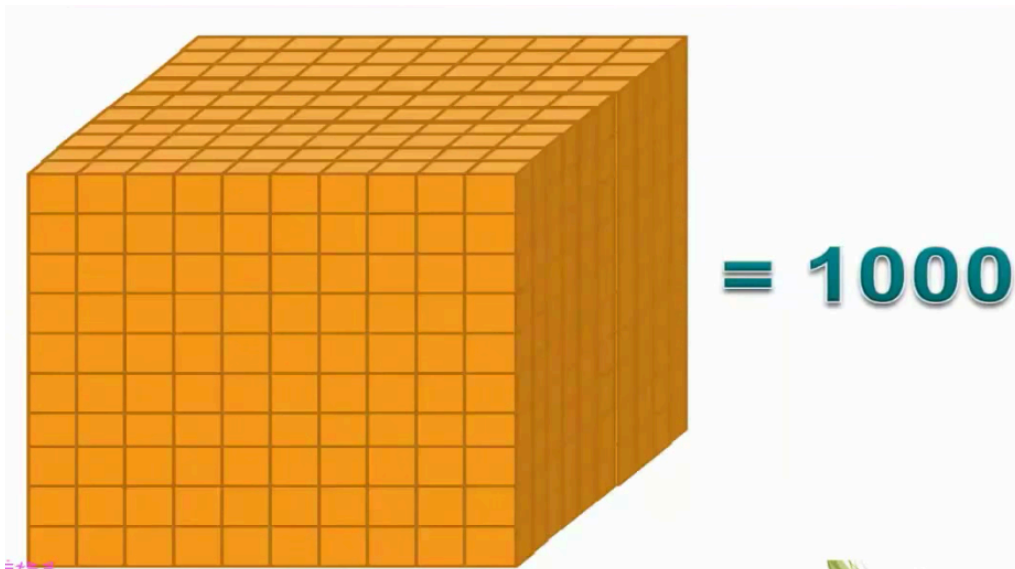
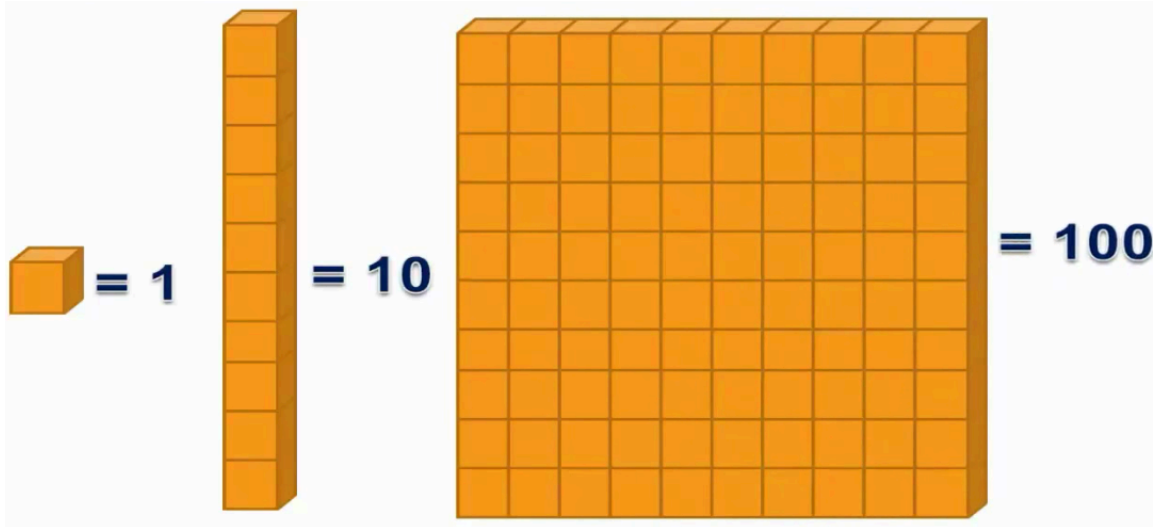
NOME: Bruno  
TURMA: B21. DATA: 29/05/03.

- 21      Pensa no 20 e tira 10.  
12      e daí tira 2 da 10 que  
09      sobrou e fica 8 e o 1 que  
         sobrou também e fica 9

+ 22      Pensa no 20 e pega a  
9      hebre e bota no lugar de  
37      daí e fica 29 e pega a  
         daí que sobrou e bota  
         junto e fica 31

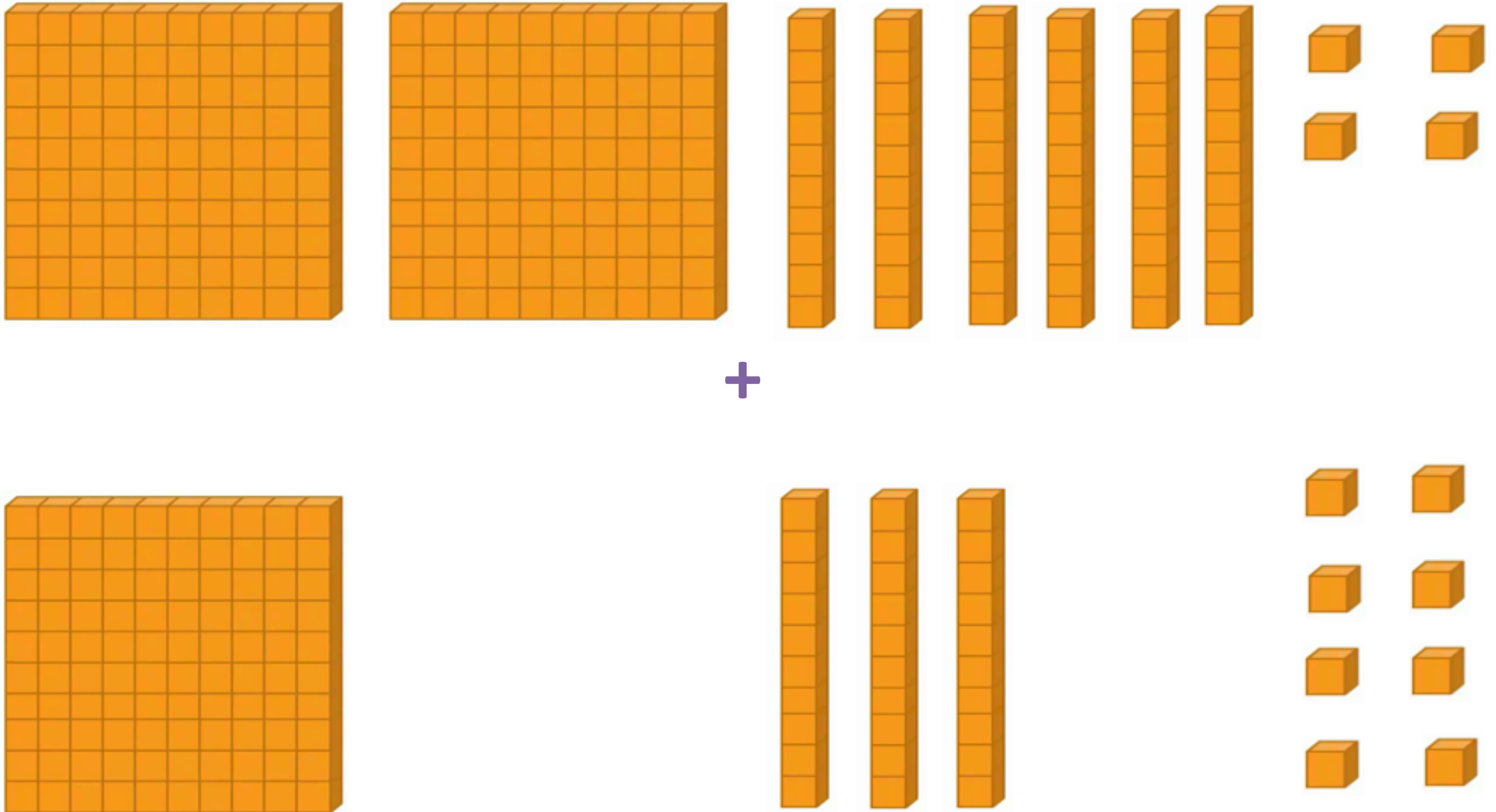
- As propriedades das operações em procedimentos distintos dos algoritmos.

# Material Dourado

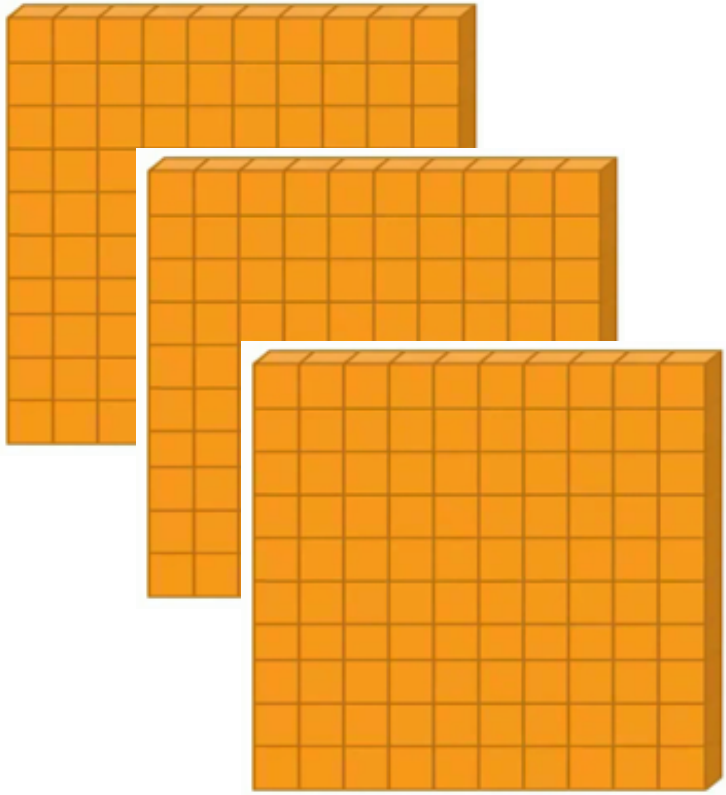


<https://www.youtube.com/watch?v=3IWUVs2UMns>

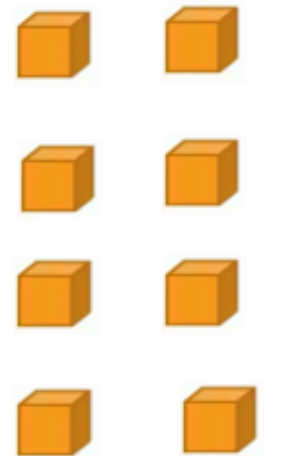
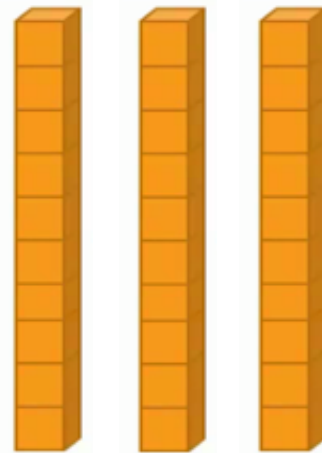
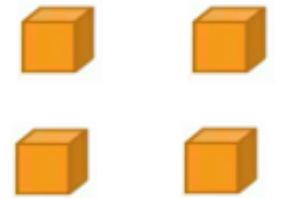
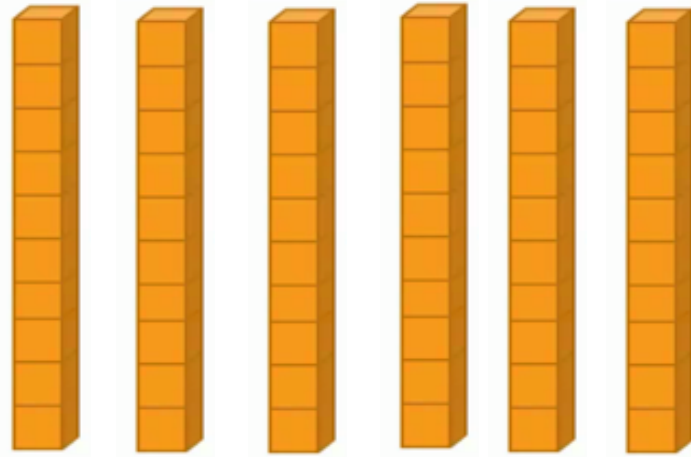
$$264 + 138$$



$$264 + 138$$

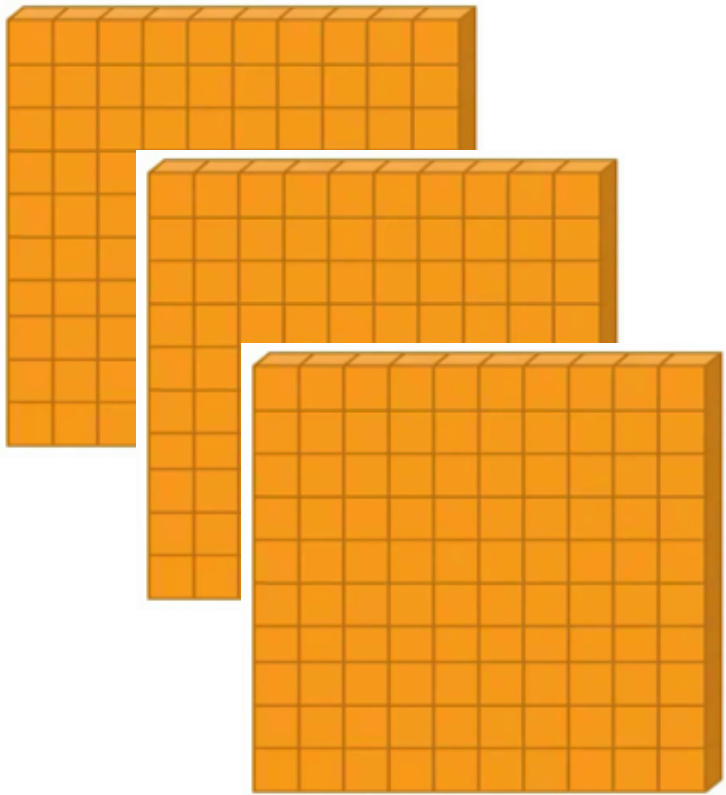


3 centenas

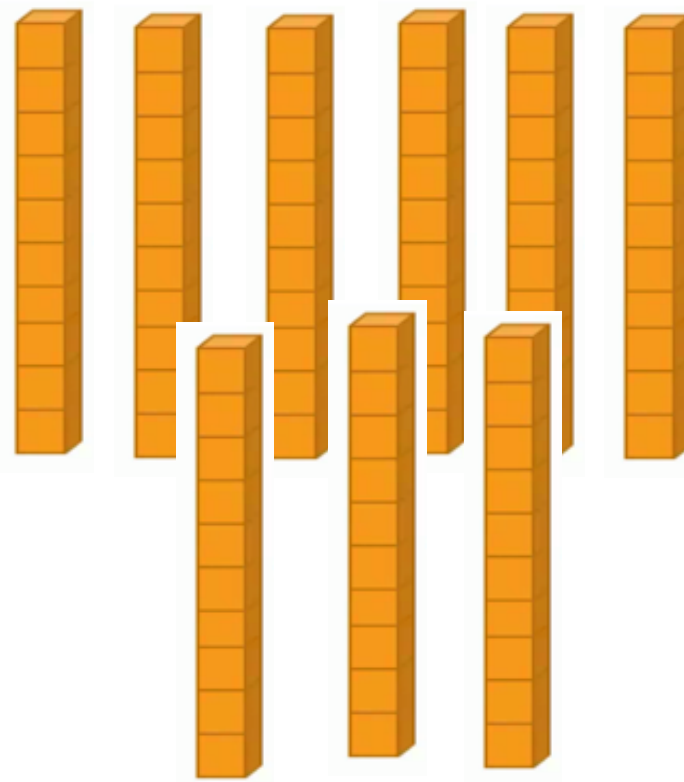




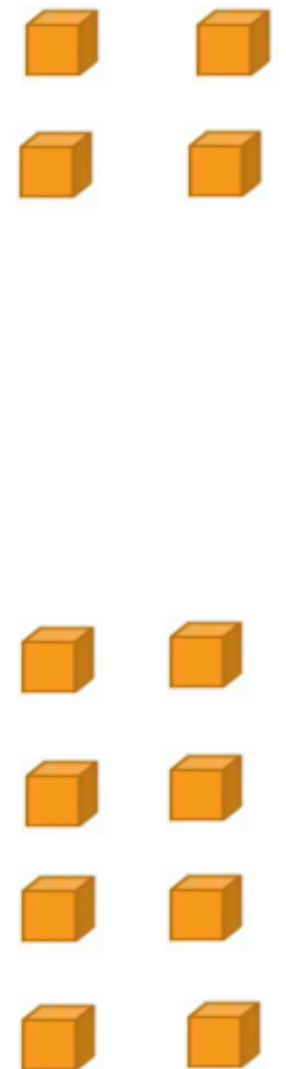
$$264 + 138$$



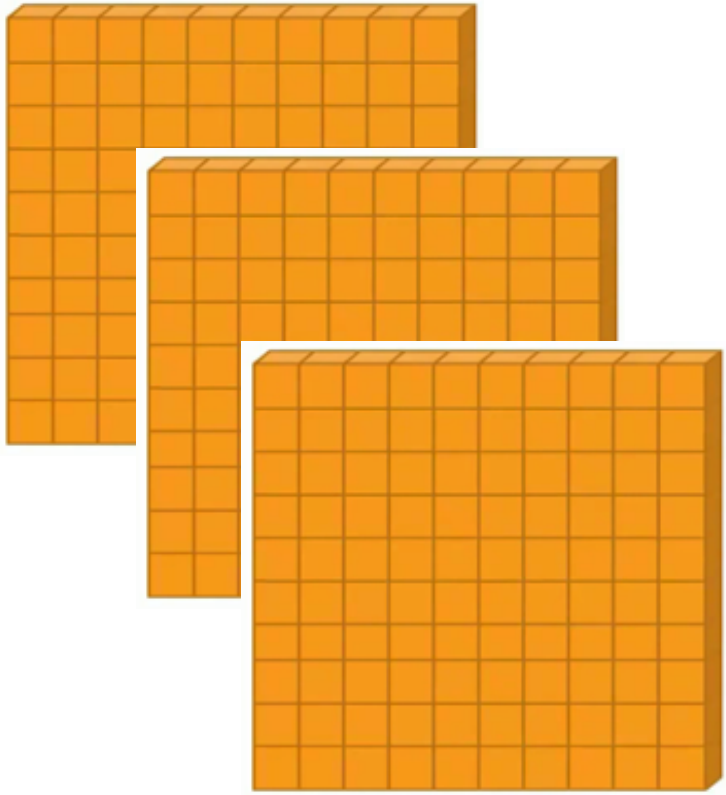
3 centenas



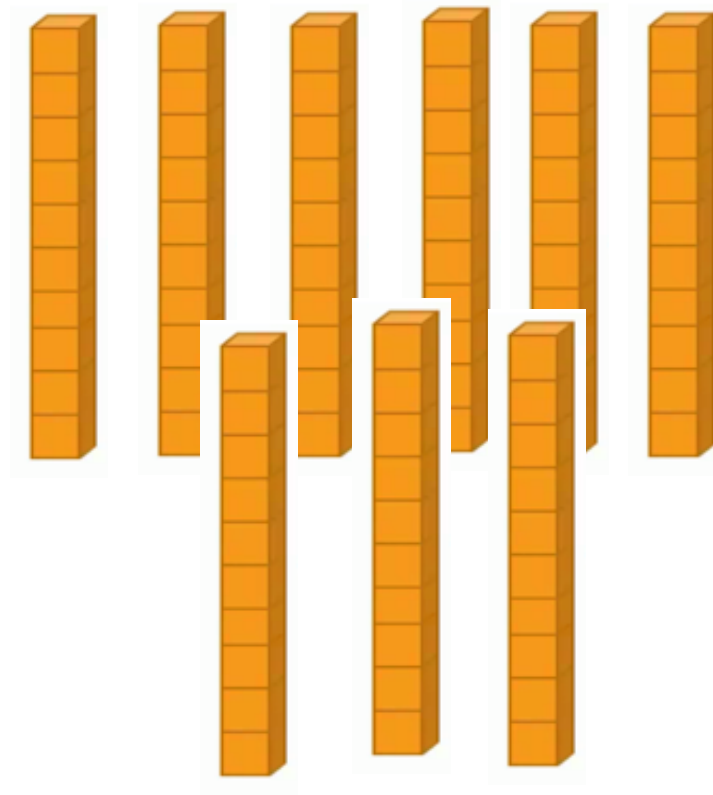
9 dezenas



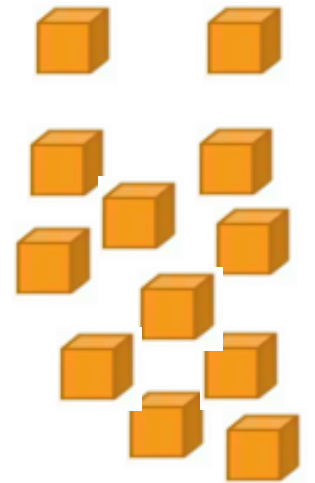
$$264 + 138$$



3 centenas

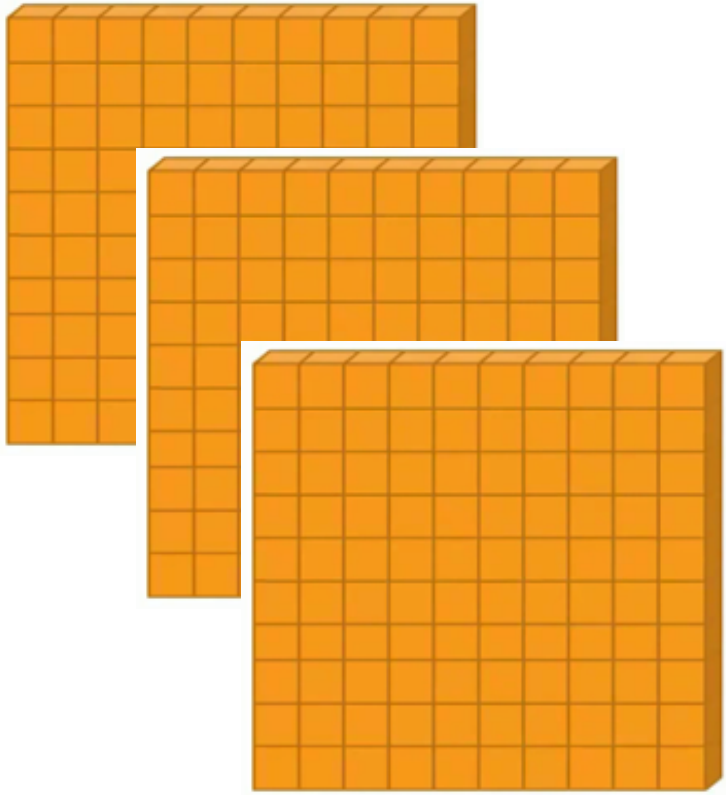


9 dezenas

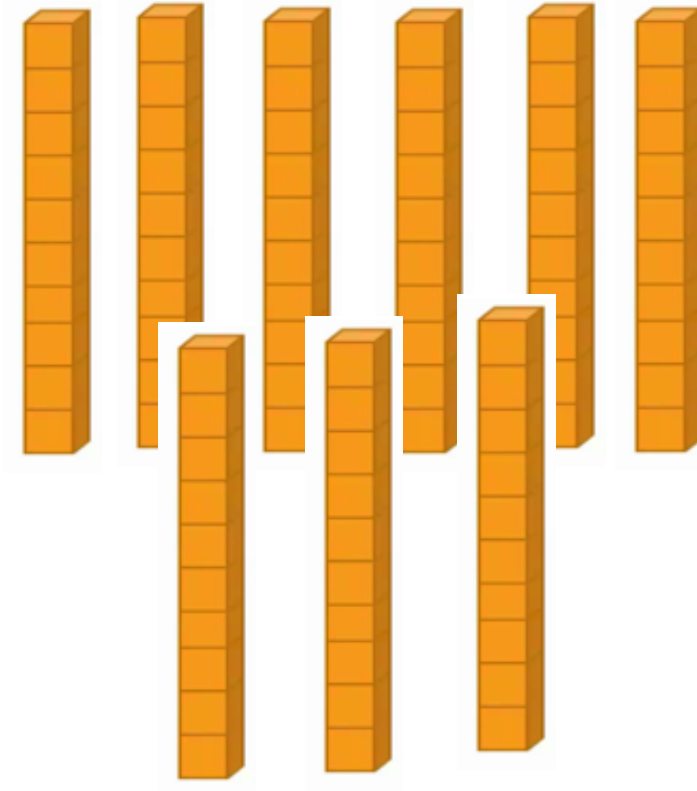


12 unidades

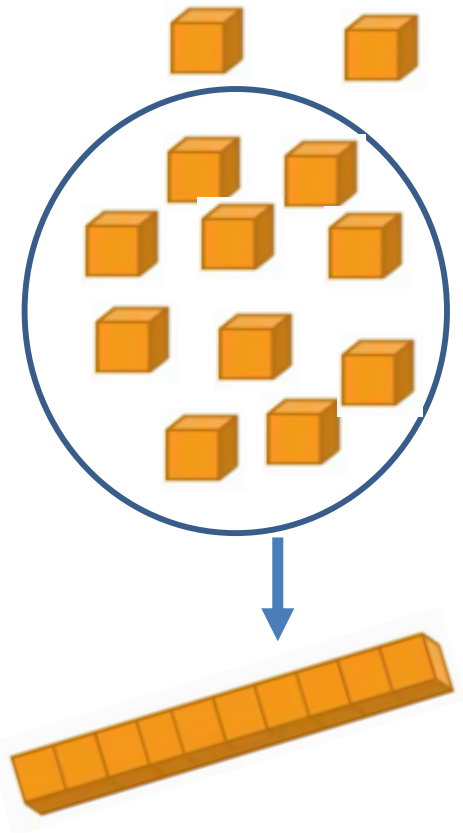
$$264 + 138$$



**3 centenas**

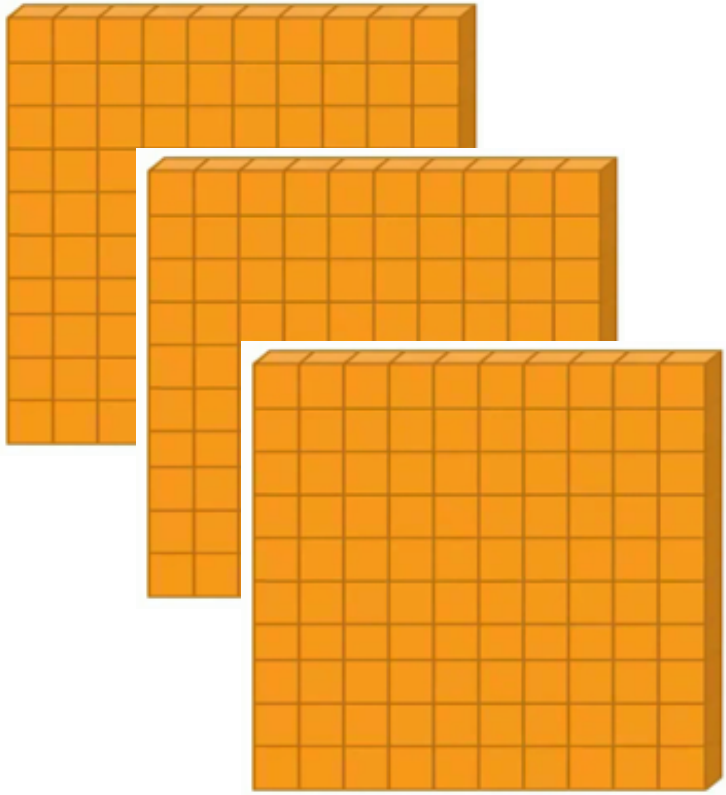


**9 dezenas**

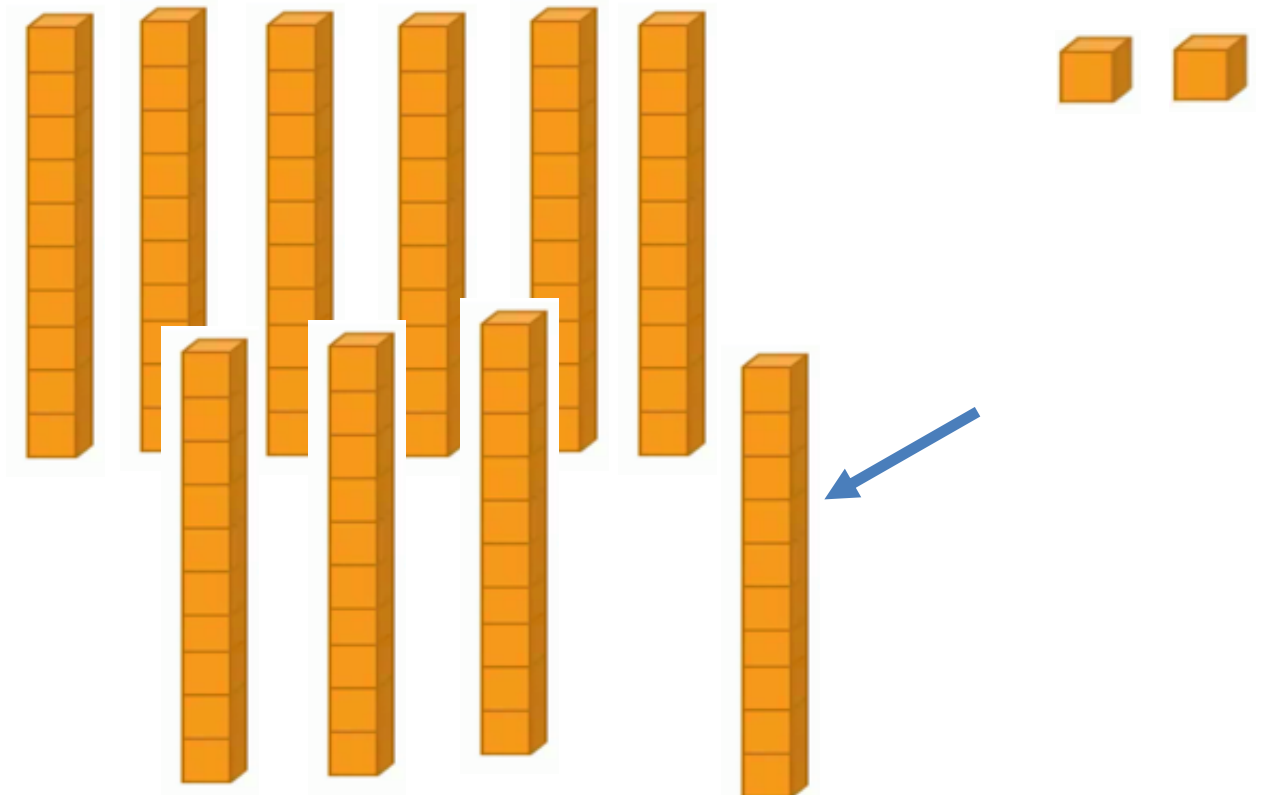


**12 unidades =  
10 unidades + 2 unidades**

$$264 + 138$$



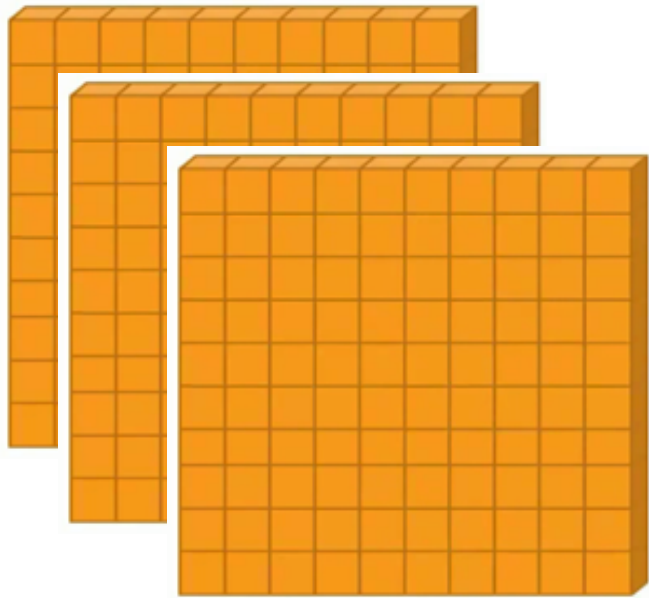
**3 centenas**



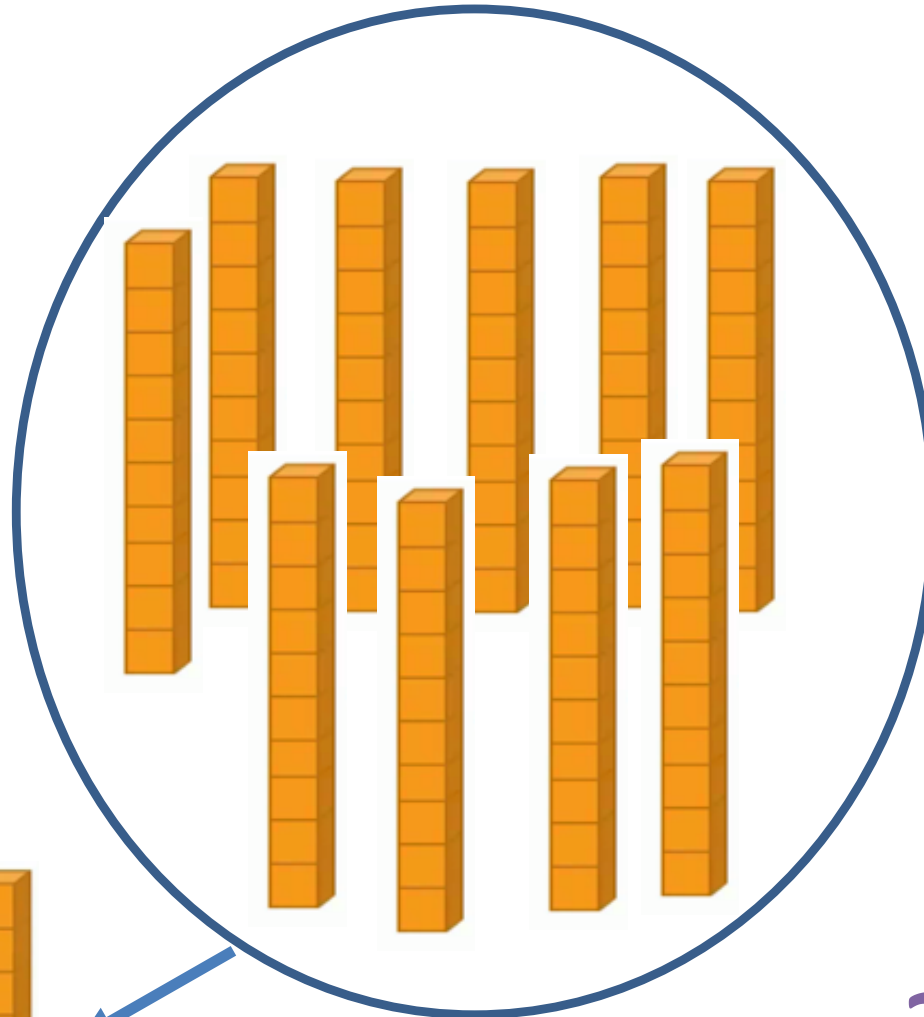
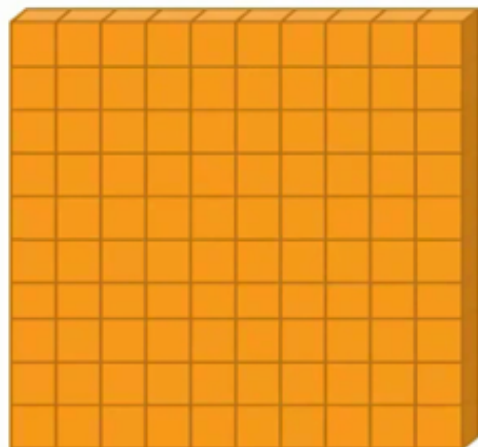
9 dezenas + 1 dezena =  
10 dezenas

**2 unidades**

$$264 + 138$$



**3 centenas**

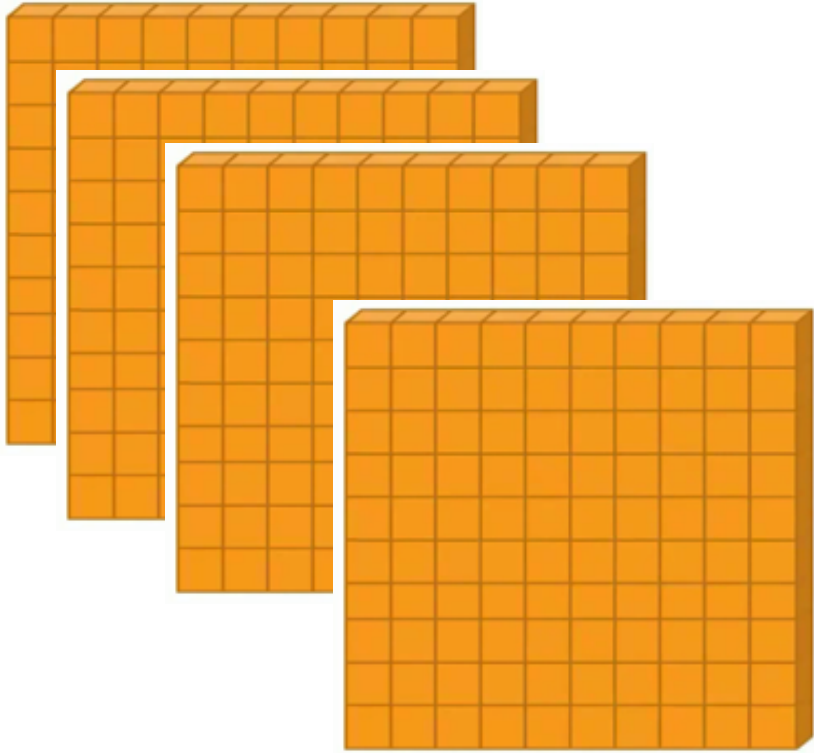


9 dezenas + 1 dezena =  
10 dezenas = 1 centena



**2 unidades**

$$264 + 138$$

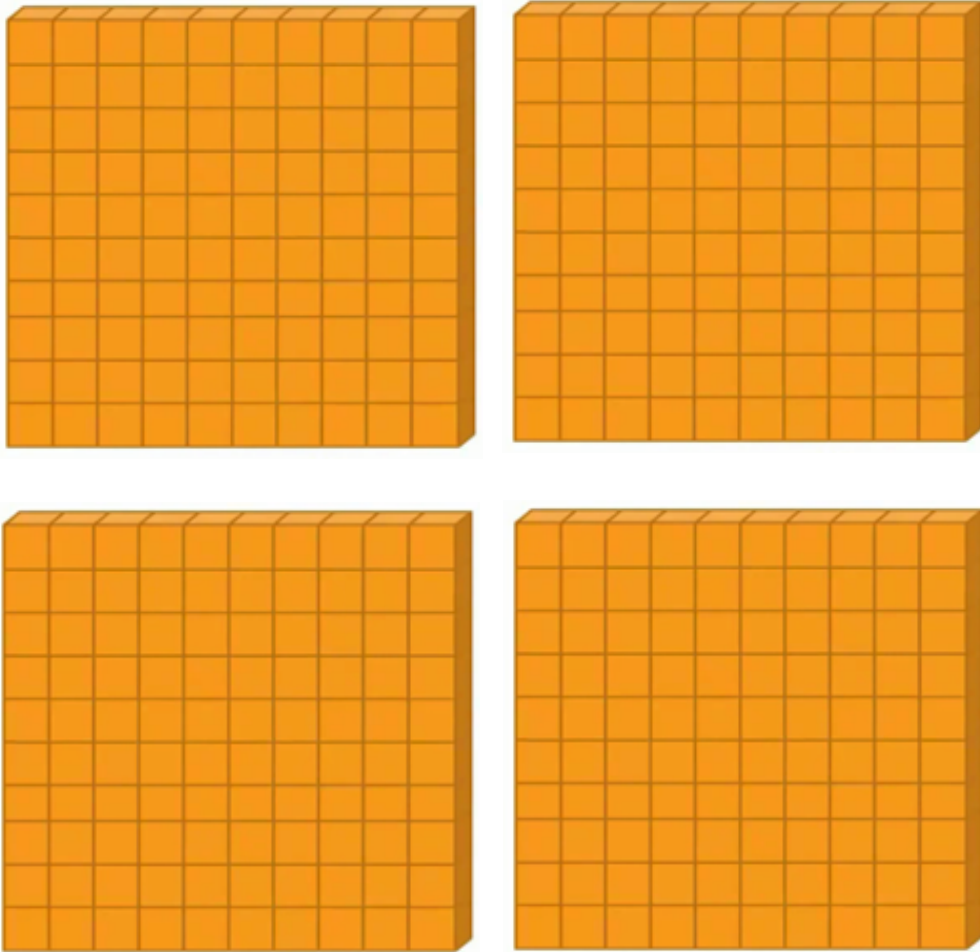


3 centenas + 1 centena =  
4 centenas



2 unidades

$$264 + 138$$



4 centenas



2 unidades

$$264 + 138 = 402$$

$$\begin{array}{r} 264 \\ + 138 \\ \hline \end{array}$$

Passos:

$$2C+1C=3C$$

$$6D+3D=9D$$

$$4U+8u=12u$$

$$12U=10U+2U=1D+2U$$

$$9D+1D=10D=1C$$

$$3C+1C=4C$$

$$264+138 = 402$$



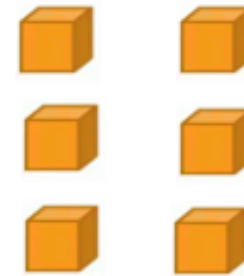
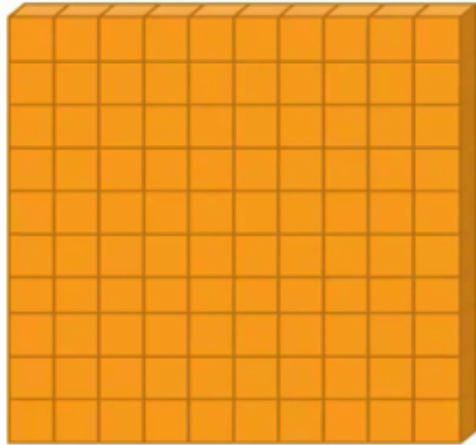
C	D	U
2	6	4
+ 1	+ 3	+ 8
3	9	<del>12</del>
	1	2
	<del>10</del>	
1	0	
4	0	2

C	D	U
2	6	4
+ 1	+ 3	+ 8
3	9	<del>12</del>
	<del>10</del>	2
4	0	2

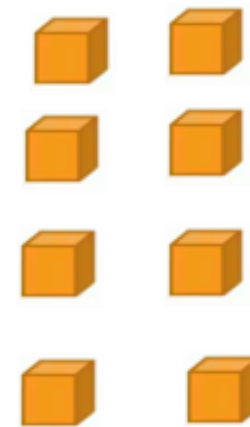
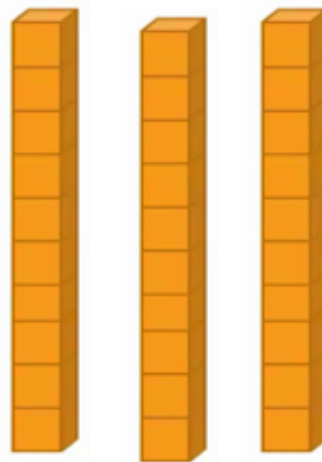
2	6	4
+ 1	+ 3	+ 8
4	0	2

O que ganhamos e perdemos com estes algoritmos?

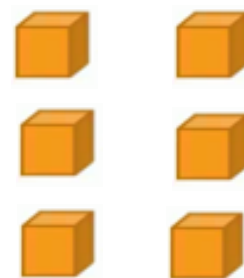
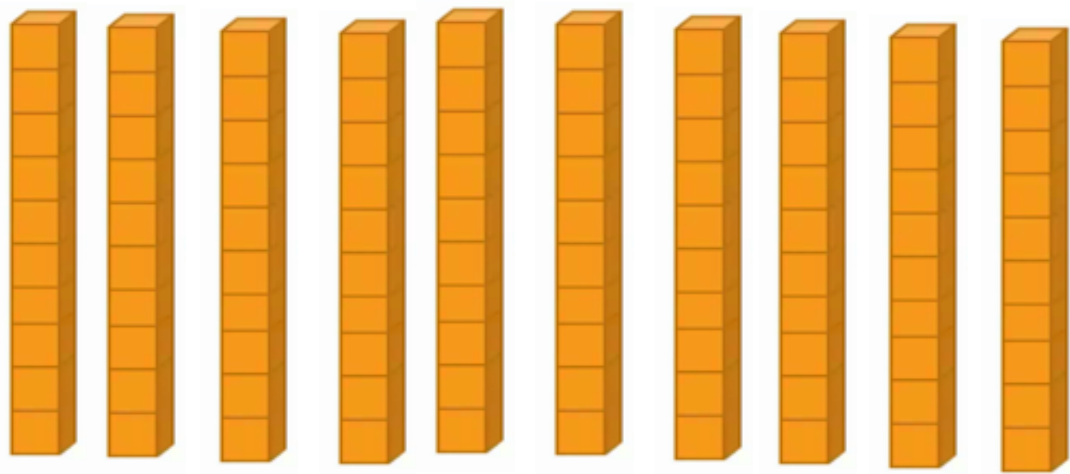
$$106 - 38$$



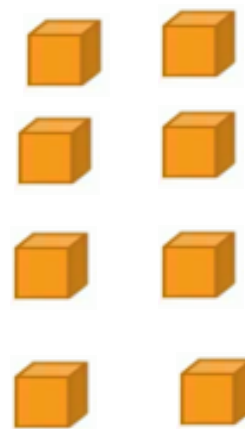
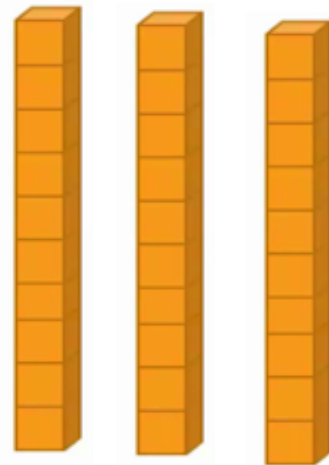
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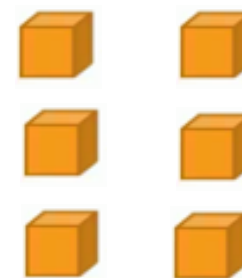
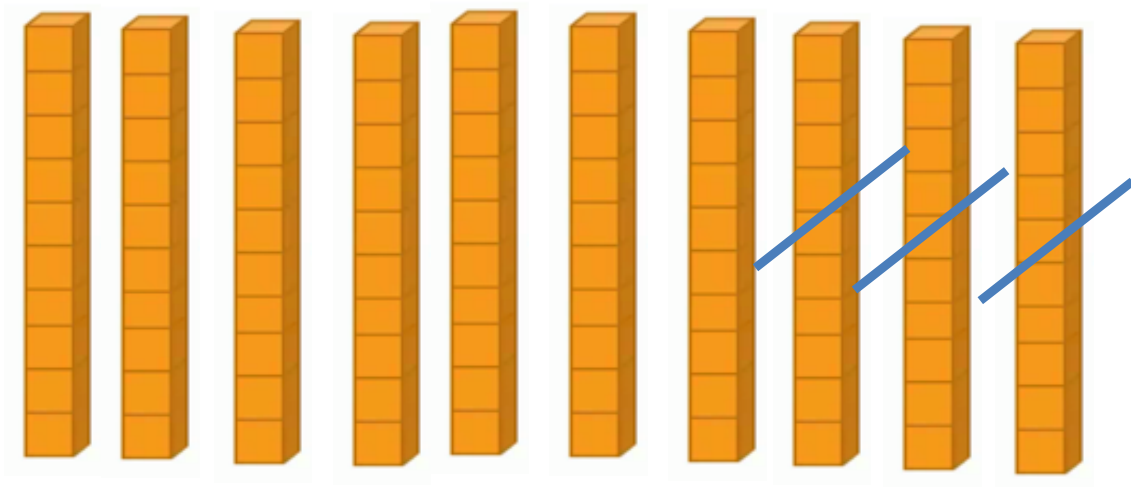
106 - 38



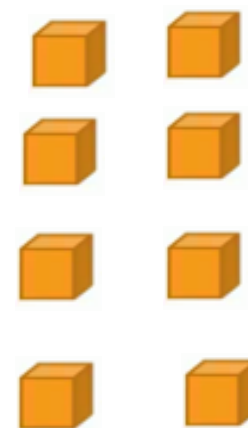
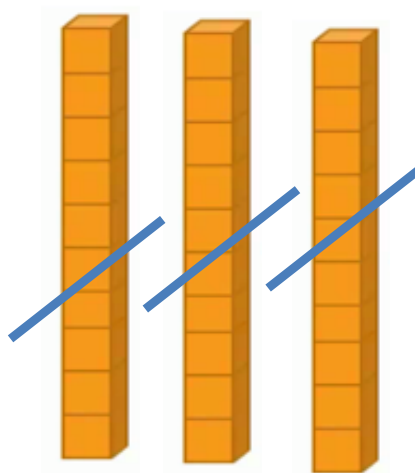
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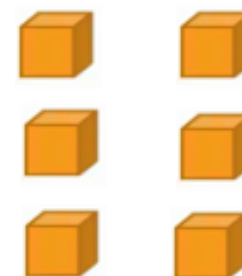
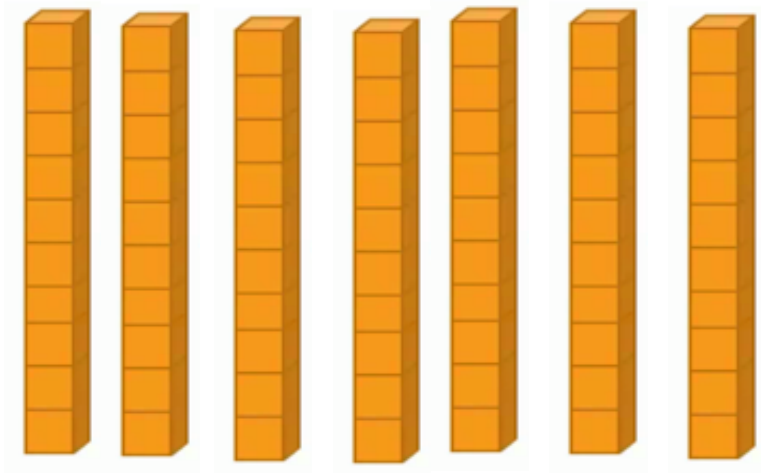
106 - 38



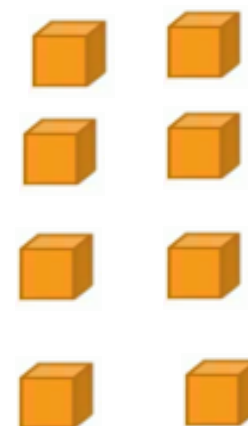
-



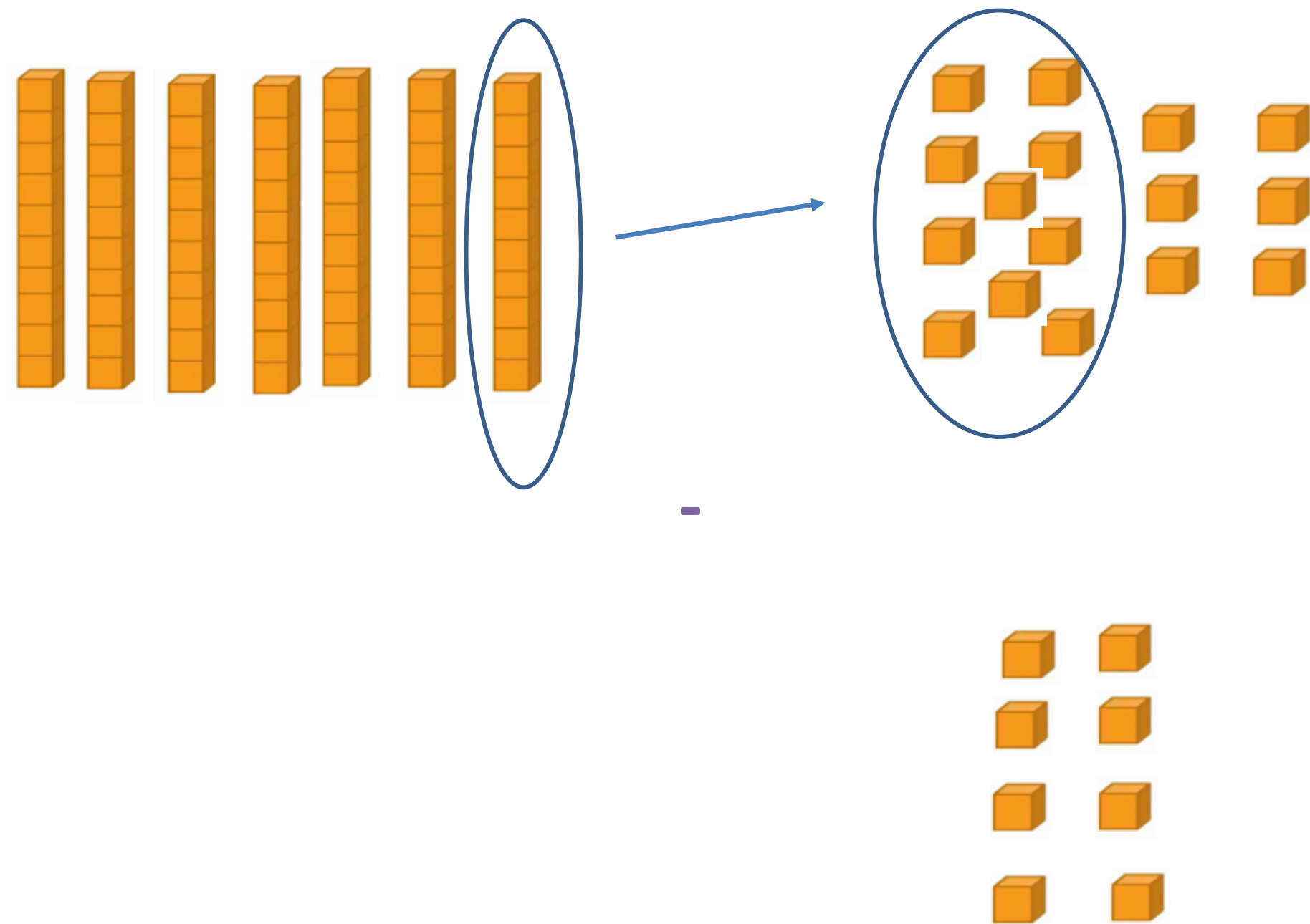
106 - 38



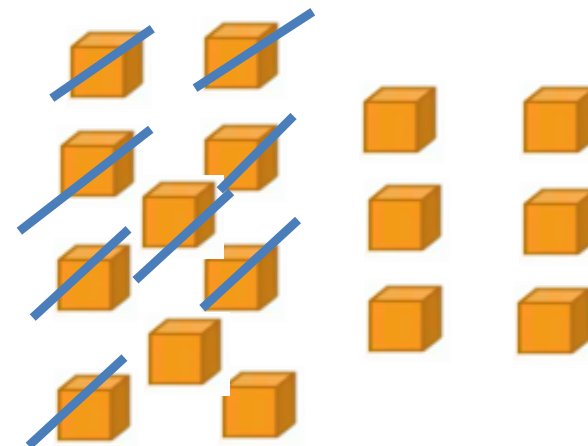
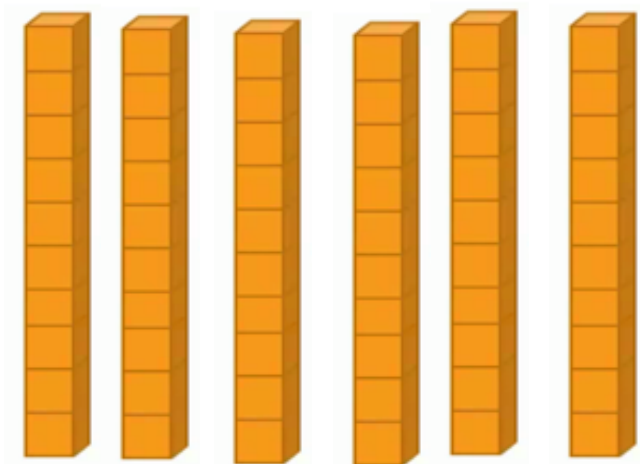
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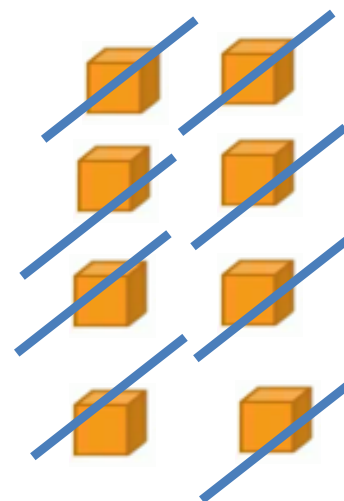
106 - 38



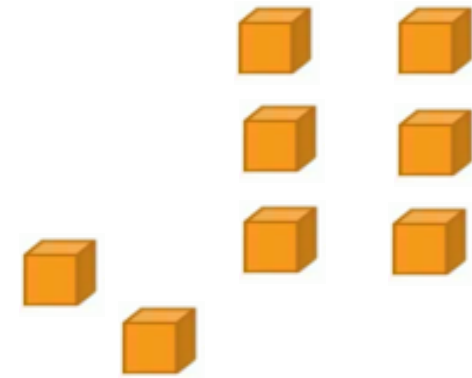
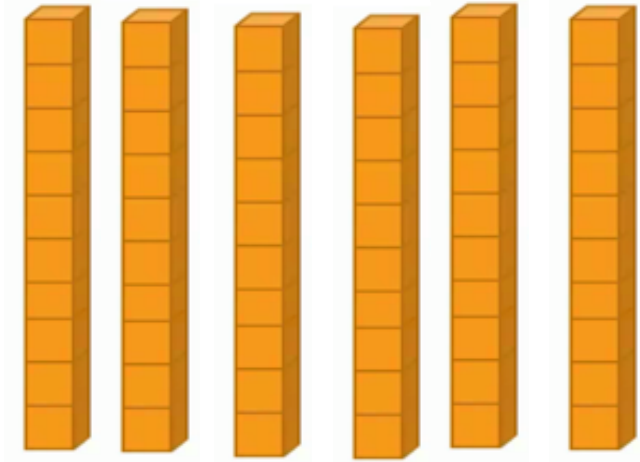
106 - 38



-



$$106 - 38$$



-

$$106 - 38 = 68$$



$$\begin{array}{r} 1 \ 0 \ 6 \\ - \ 3 \ 8 \\ \hline \end{array}$$

Passos:

$$1C=10D$$

$$10D-3D=7D$$

$$7D=6D+1D=6D+10U$$

$$10U+6U=16U$$




$$16U-8U=8U$$

$$106-38=68$$

C	D	U
<del>1</del>	0	6
-	3	8
	10	6
-	3	8
	<del>7</del>	6
-		8
	6	16
-		8
	6	8

C	D	U
<del>1</del>	0	6
-	3	8
	<del>10</del>	16
-	3	8
	9	16
-	3	8
	6	8

C	D	U
<del>1</del>	<sup>9</sup> <del>10</del>	<sup>1</sup> 6
-	3	8
	6	8

<b>C</b> = X 100	<b>D</b> = X 10	<b>U</b> = X 1
		

- Cuidados ao associar QVL e material dourado

**C** = X 100

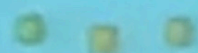
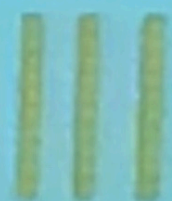
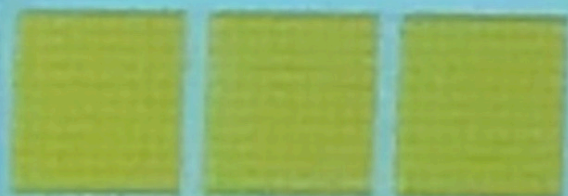
**D** = X 10

**U** = X 1

**3**

**3**

**3**



CENTENA

DEZENA

UNIDADE



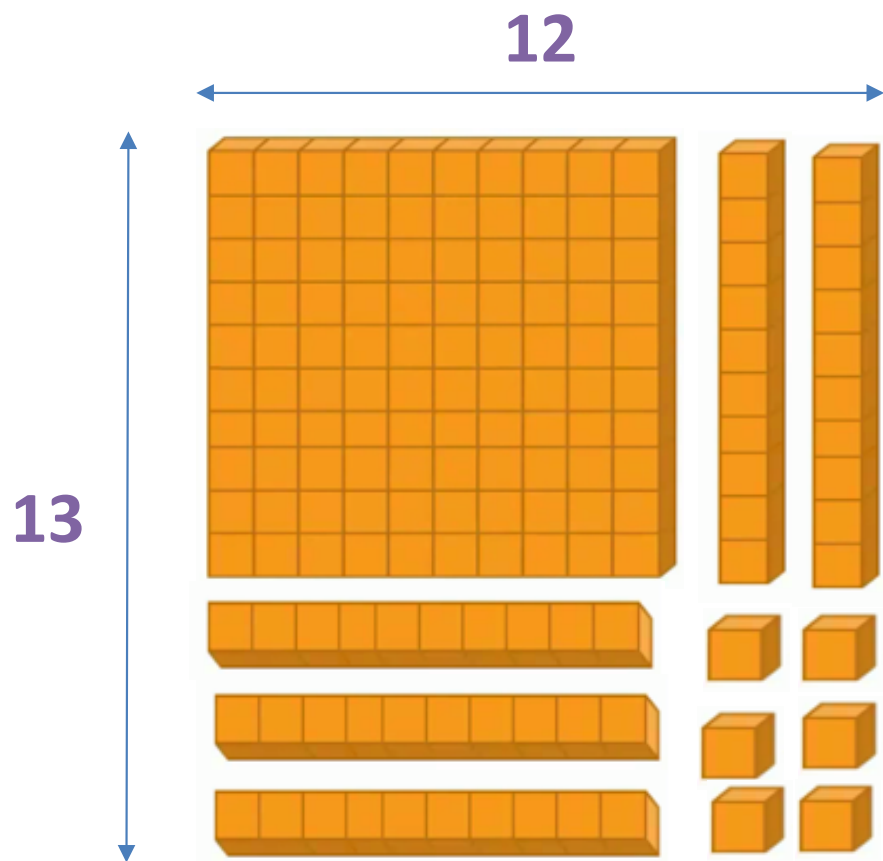
# Sobre a operação de multiplicação

Multiplicação como soma de parcelas iguais.

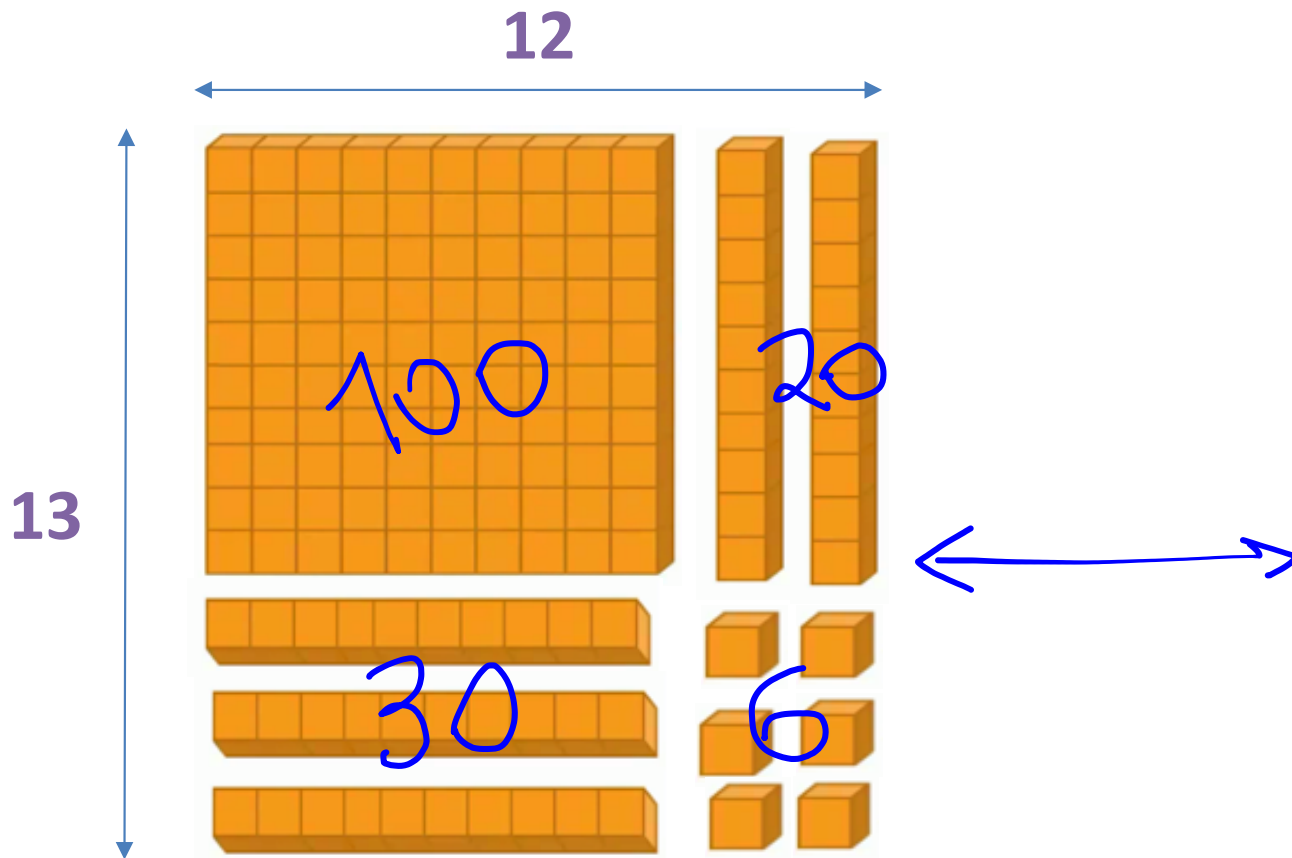
$$12 \times 13 = 13+13+13+13+13+13+13+13+13+13+13+13$$

- Utilizando o material dourado, construa um retângulo (com a região interna) de lados medindo 12 e 13 com o menor número possível de peças.
- Usando o algoritmo da multiplicação, resolva  $12 \times 13$ .
- Estabeleça relações entre as multiplicações parciais do algoritmo e as peças do material dourado utilizadas para construir o retângulo.

$12 \times 13$



# 12 x 13



**D U**

$$\begin{array}{r} 12 = 10 + 2 \\ \times 13 = 10 + 3 \\ \hline \end{array}$$

Handwritten calculations:

$$\begin{array}{r} 36 \\ + 20 \\ \hline 156 \end{array}$$



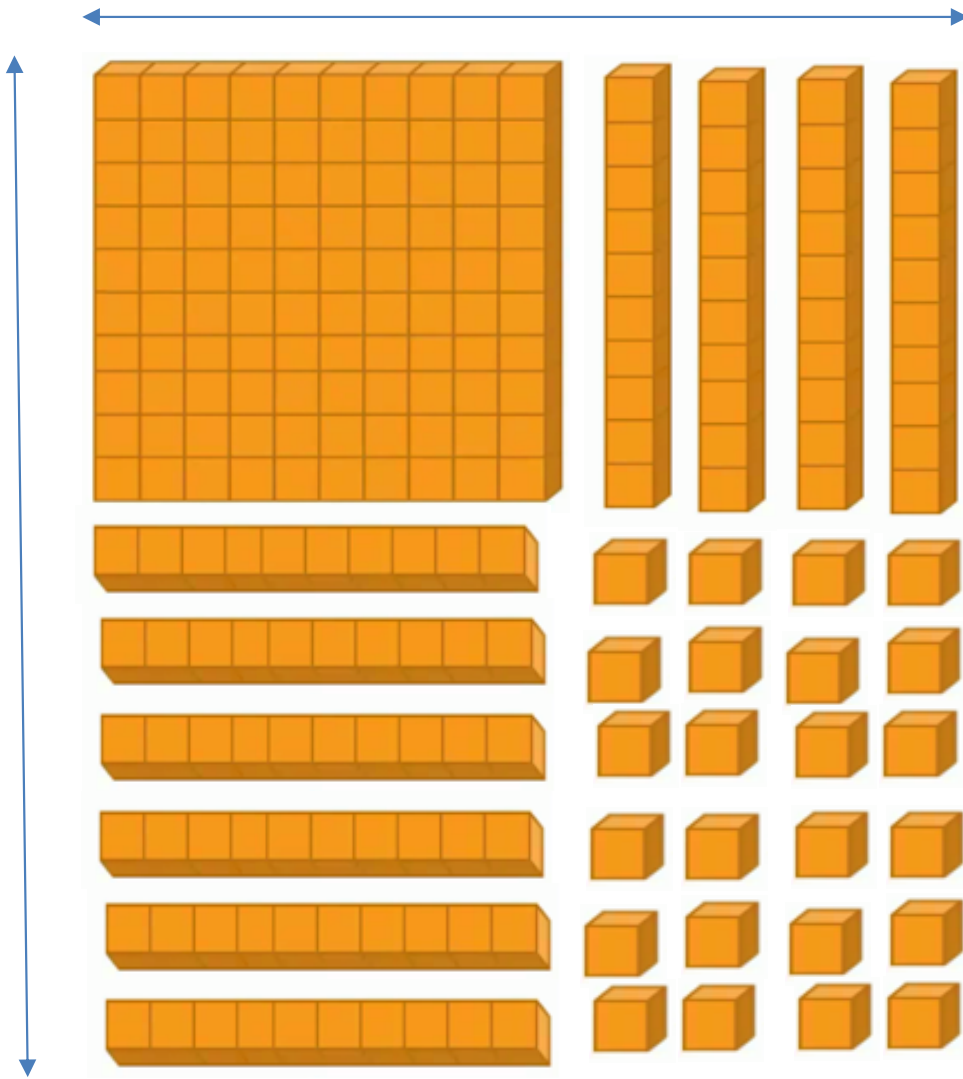
# Sobre a operação de multiplicação

- Represente, no papel quadriculado, o retângulo formado com o material dourado, identificando as peças e valores.
- Repita os passos acima para as operações: **14x16**, 19x15 e 21x14.

14 x 16

14

16

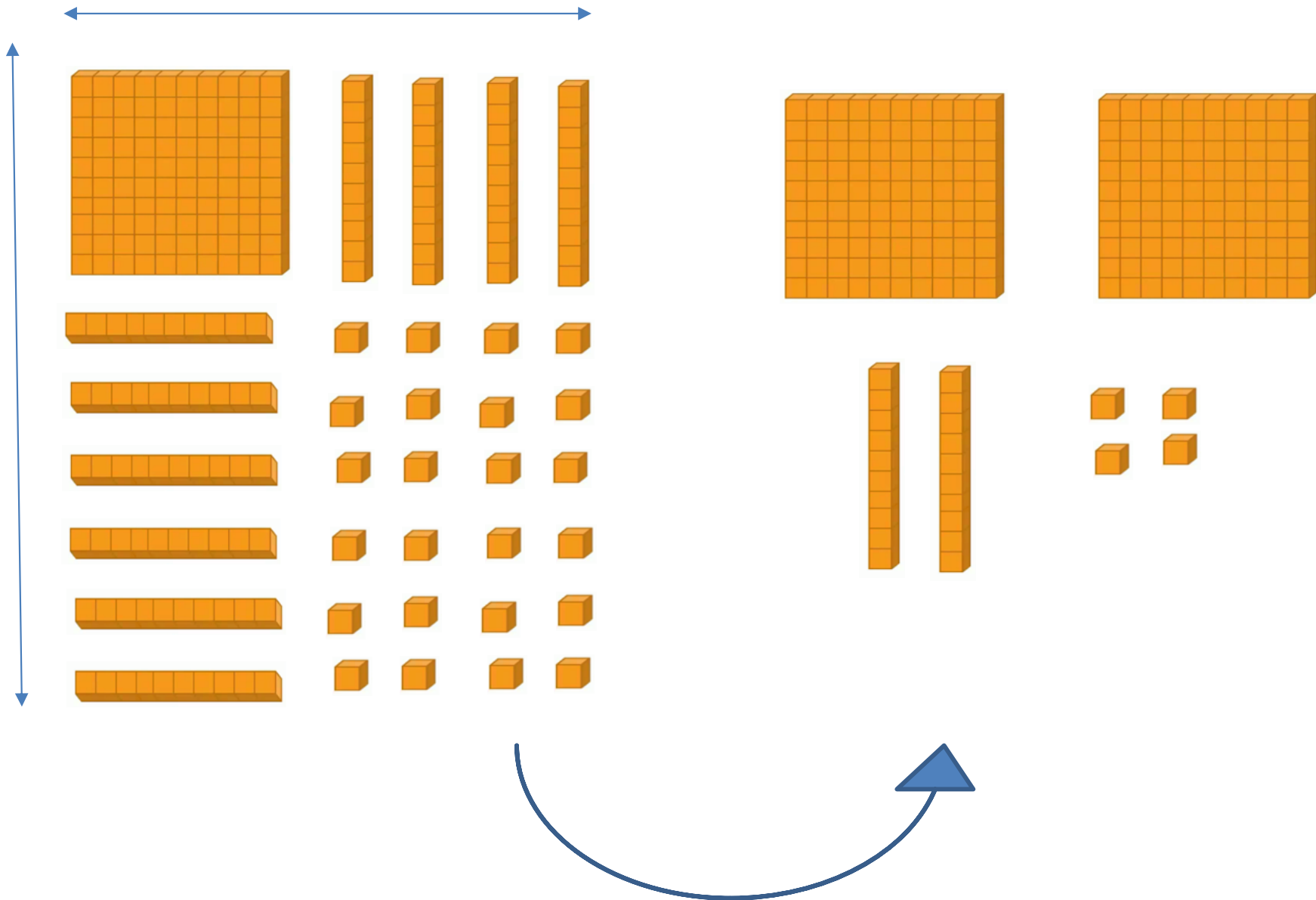


Temos  
retângulo, mas  
não o menor  
número de  
peças.  
Se fôssemos  
fazer as trocas...

14 x 16

14

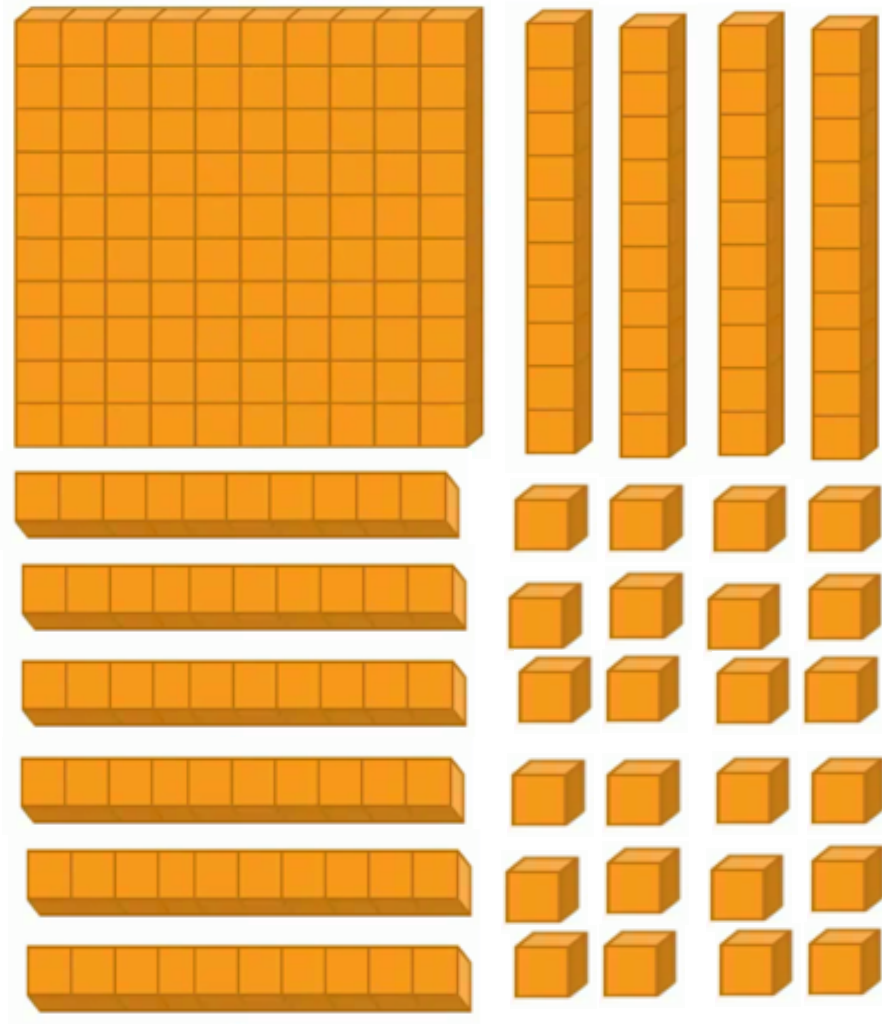
16



14 x 16

14

16

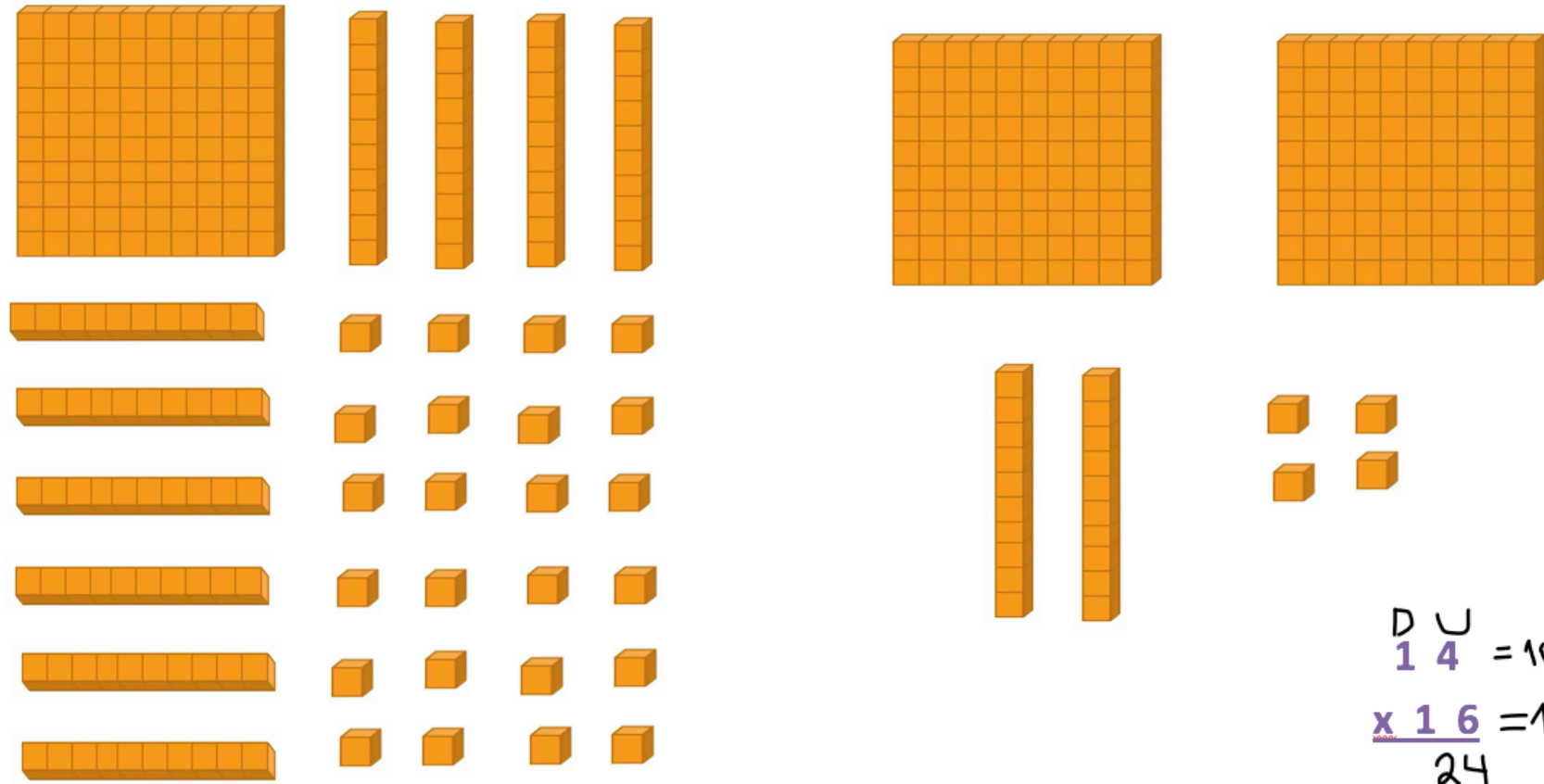


$$\begin{array}{r} 14 \\ \times 16 \\ \hline \end{array}$$

# 14 x 16

14

16



$$\begin{array}{r} \text{D U} \\ 14 = 10 + 4 \\ \times 16 = 10 + 6 \\ \hline 24 \\ 60 \\ + \\ 40 \\ 100 \\ \hline 224 \end{array}$$

# Sobre a operação de divisão

Duas ideias associadas:

1) Se temos 42 azulejos para fazer um quadro retangular e colocarmos 6 azulejos em cada fila, quantas filas poderão ser feitas?

Formar um grupo de 6. Quantos grupos de 6 fazemos em 42? Quantas vezes 6 cabe em 42? Usar 6 como unidade de medida para 42.

Divisão ligada à ação de medir.

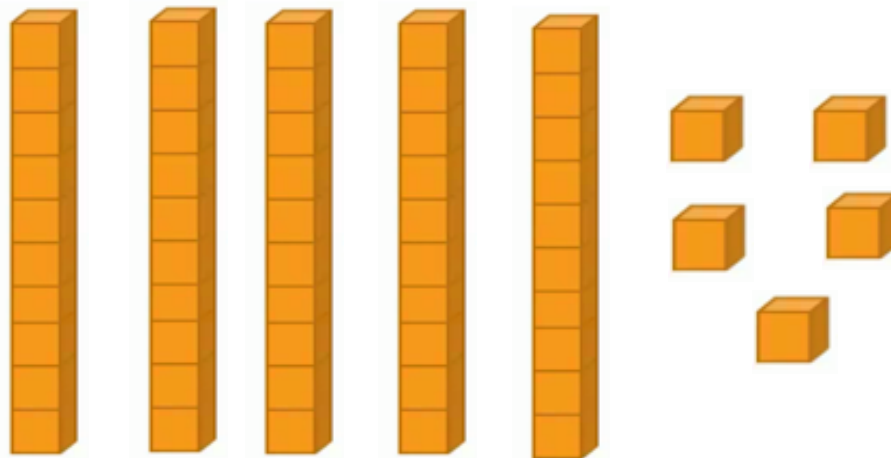
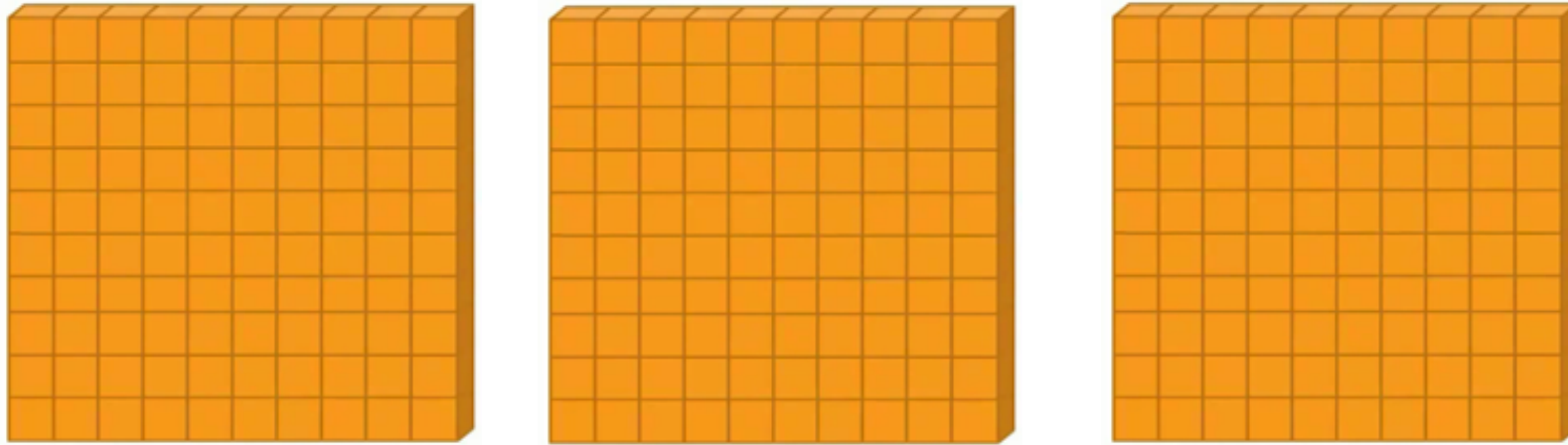
2) Um vendedor de vinho quer colocar 1872 garrafas em 104 caixas. Quantas garrafas terá que colocar em cada caixa?

Garrafas e caixas. Distribuir 1872 garrafas em 104 caixas.

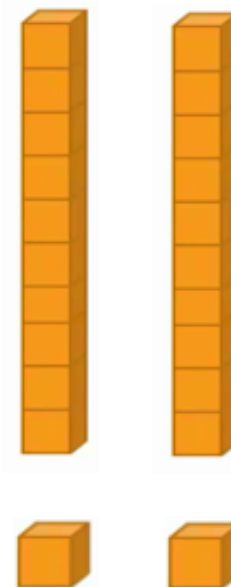
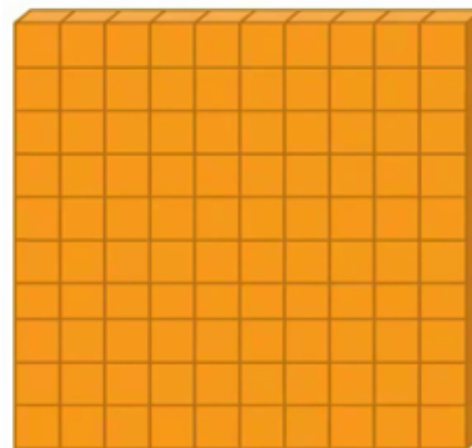
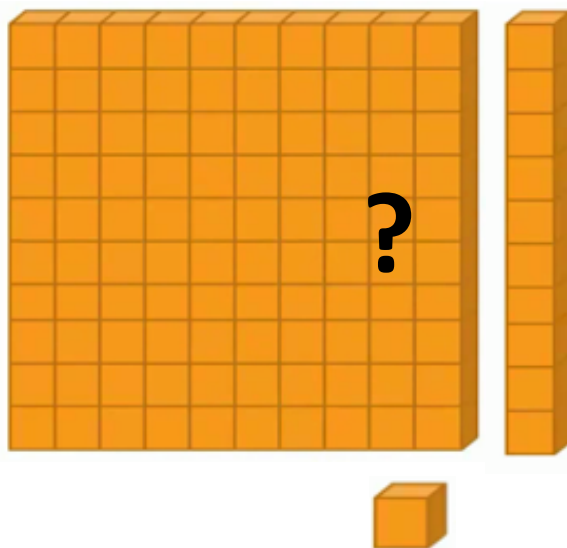
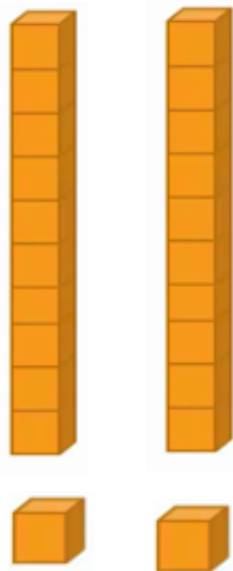
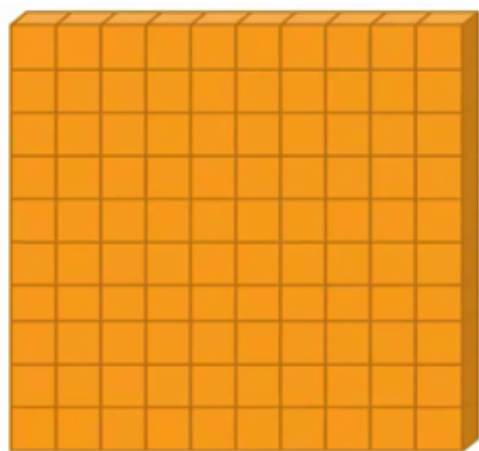
Correspondência 1 a 1 ou, 2 a 1.

Divisão ligada à ação de repartir/distribuir.

$$355 \div 2$$



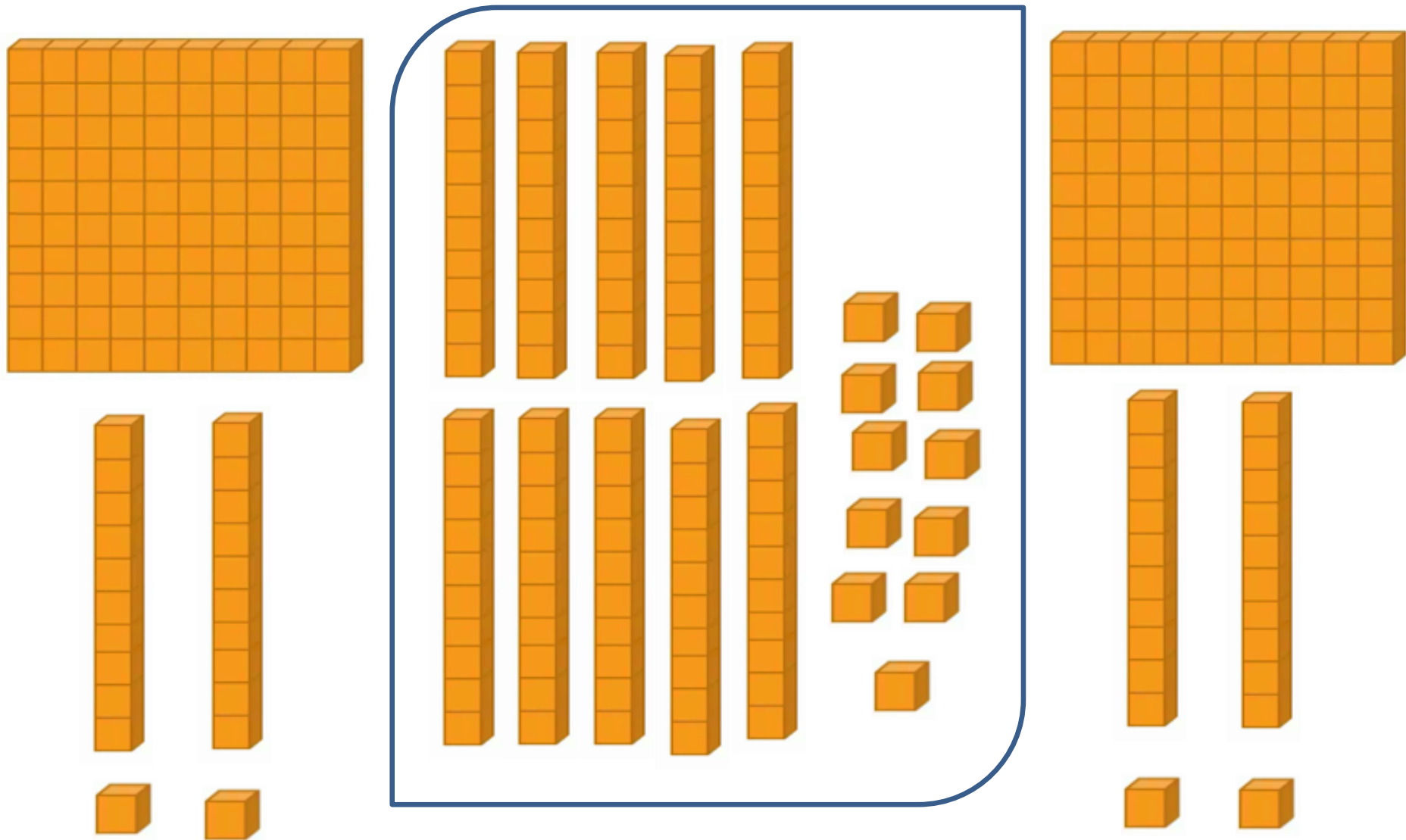
# $355 \div 2$ Divisão - repartir



Cada pessoa ficou com 122 para cada. Mas sobrou 111.

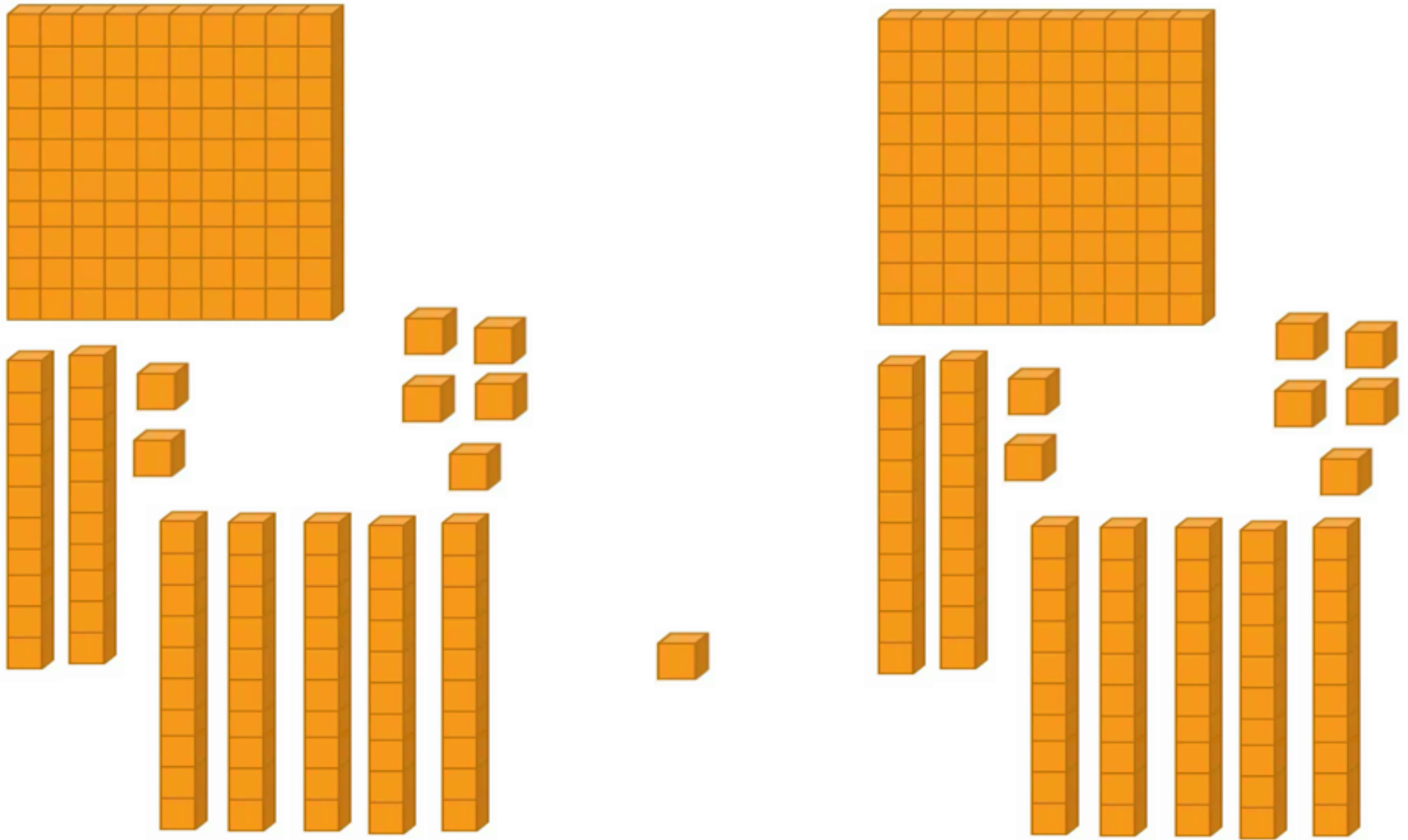


# 355 ÷ 2 Divisão - repartir



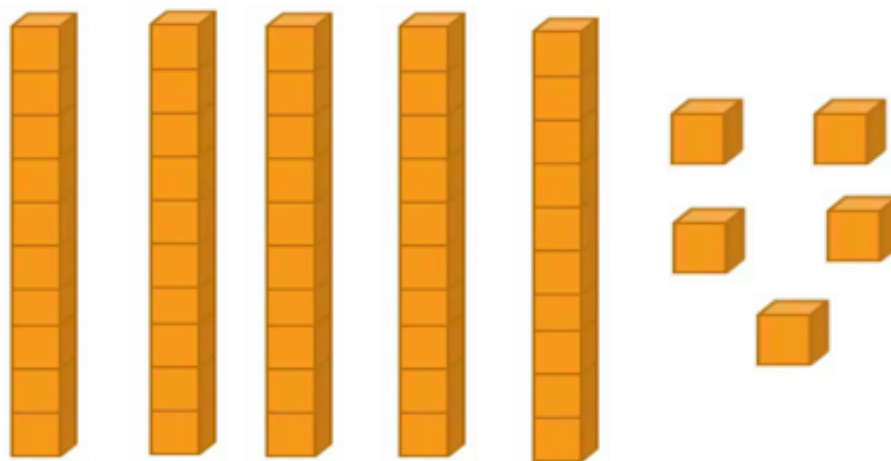
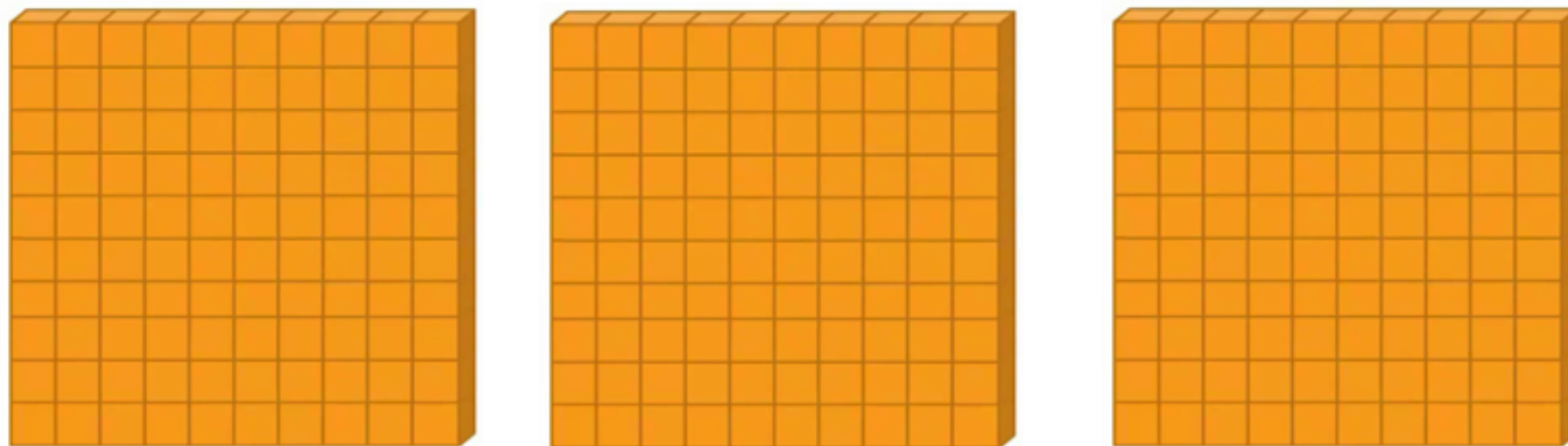
$$\begin{aligned} 1C + 1D + 1U &= \\ 10D + 10U + 1U & \end{aligned}$$

# $355 \div 2$ Divisão - repartir



Cada pessoa ficou com 177 para cada. E sobrou 1.

# 355 ÷ 2 Divisão como repartir



Handwritten annotations for the division process:

- Vertical lines labeled C, D, U, and d (for decimal) in blue, green, and purple.
- Red arrows and numbers indicating the movement of digits: 10 from C to D, 10 from D to U, and 10 from U to d.
- A red checkmark next to the C column.

$$\begin{array}{r} 2 \overline{) 177,5} \\ \underline{12} \phantom{0} \\ 57 \phantom{0} \\ \underline{50} \phantom{0} \\ 77 \phantom{0} \\ \underline{74} \phantom{0} \\ 35 \\ \underline{35} \\ 0 \end{array}$$