Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3210 - Mecânica dos Sólidos I

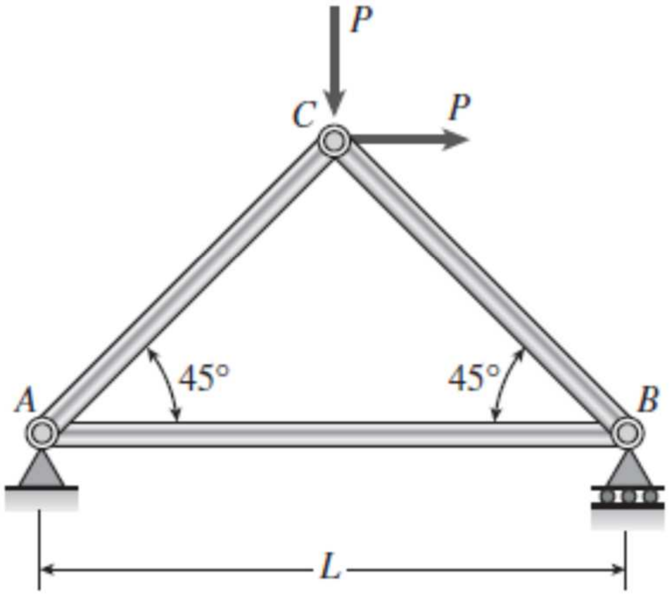
Aula #06

Prof. Dr. Roberto Ramos Jr.

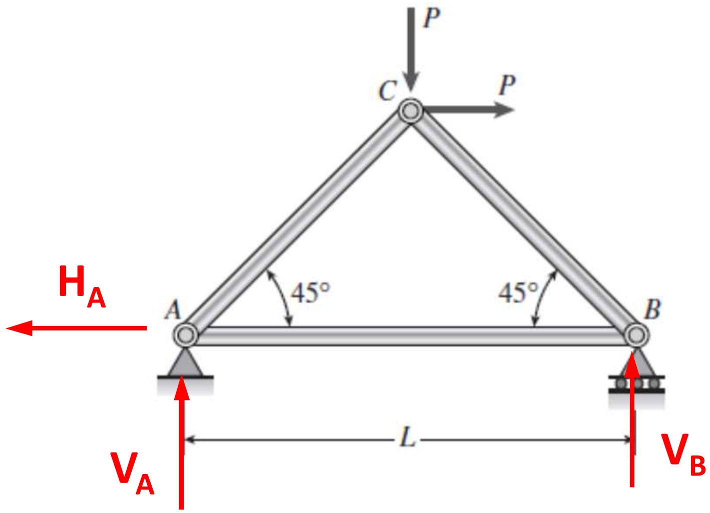
08/04/2022

Problem 2.2-8 The three-bar truss *ABC* shown in the figure has a span $L = 3$ m and is constructed of steel pipes having cross-sectional area $A = 3900$ mm^2 and modulus of elasticity $E = 200$ GPa. Identical loads *P* act both vertically and horizontally at joint *C*, as shown.

(a) If $P = 650$ kN, what is the horizontal displacement of joint *B*?
(b) What is the maximum permissible load value P_{max} if the displacement of joint *B* is limited to 1.5 mm?



Solução:

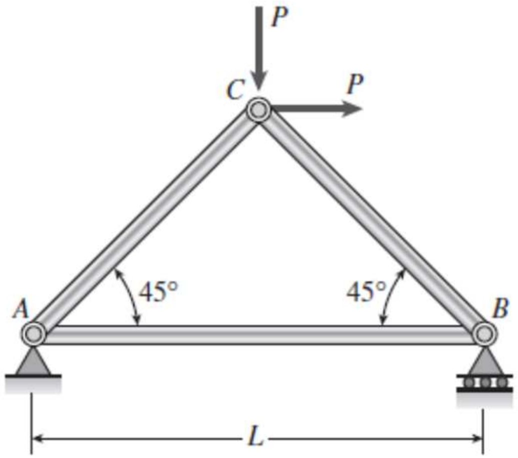


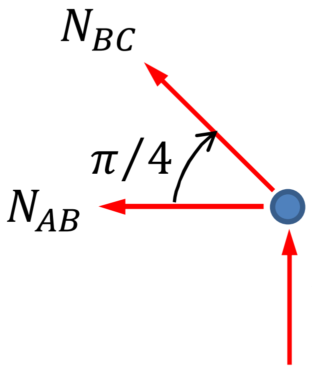
$$\sum F_x = 0 \;\Leftrightarrow\; H_A = P$$

$$\sum M_{pólo\ A} = 0 \;\Leftrightarrow\; V_B L = P\frac{L}{2} + P\frac{L}{2} \;\Leftrightarrow\; V_B = P$$

$$\sum F_y = 0 \;\Leftrightarrow\; V_A + V_B = P \;\Leftrightarrow\; V_A = 0$$

Equilíbrio do nó B:





$$\frac{\sqrt{2}}{2} N_{BC} + P = 0 \quad \Leftrightarrow \quad N_{BC} = -\sqrt{2}P$$

$$N_{AB} + \frac{\sqrt{2}}{2} N_{BC} = 0 \quad \Leftrightarrow \quad N_{AB} = P$$

Logo: $$\delta_B = \frac{N_{AB}L}{EA} = \frac{PL}{EA} = \frac{(6{,}5 \times 10^5)(3000)}{(2 \times 10^5)(3900)} = 2{,}5\ mm$$

b) Se o deslocamento horizontal de B está limitado a 1,5 mm, então a máxima força que pode ser aplicada é:

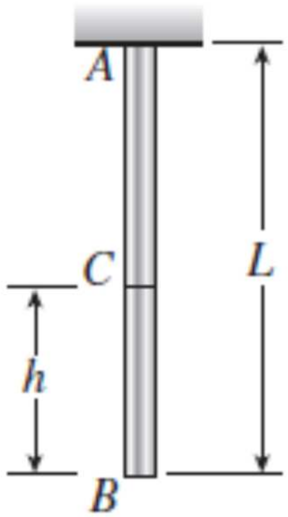
$$P_{máx} = 1{,}5mm \frac{650\ kN}{2{,}5mm} = 390\ kN$$

Problem 2.3-12 A prismatic bar AB of length L, cross-sectional area A, modulus of elasticity E, and weight W hangs vertically under its own weight (see figure).

(a) Derive a formula for the downward displacement δ_C of point C, located at distance h from the lower end of the bar.

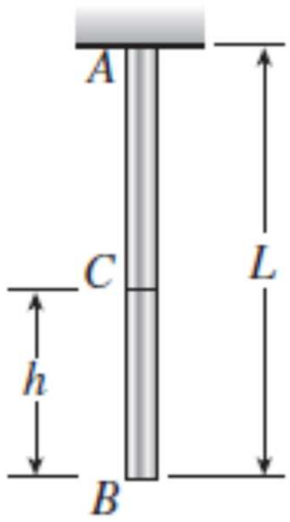
(b) What is the elongation δ_B of the entire bar?

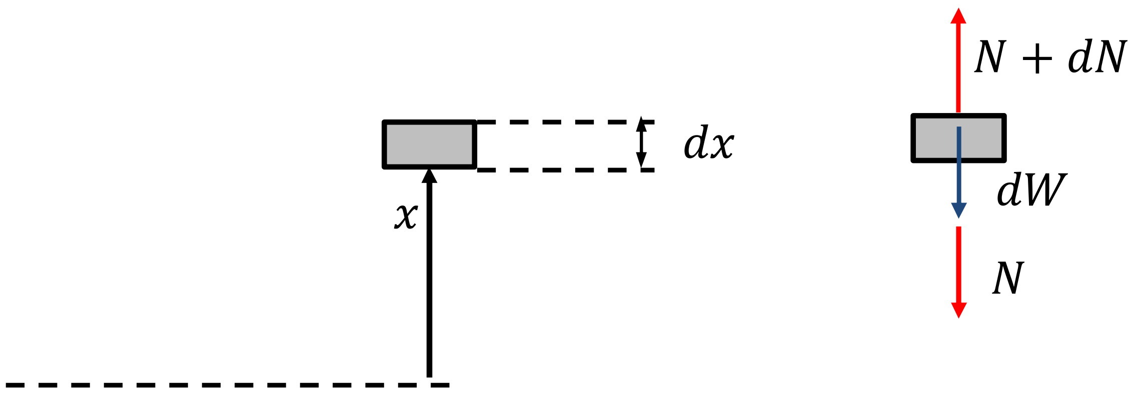
(c) What is the ratio β of the elongation of the upper half of the bar to the elongation of the lower half of the bar?



Solução:

Equilíbrio de um segmento infinitesimal de comprimento dx da barra:

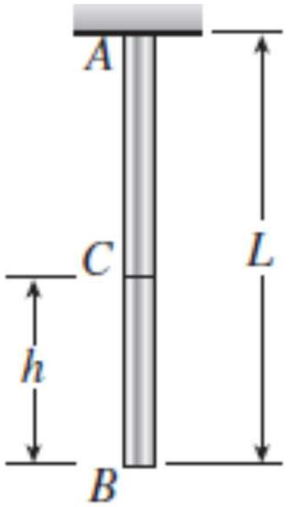




$$N + dN = N + dW \quad \Leftrightarrow \quad dN = dW = \frac{W}{L} dx$$

Logo: $$N(x) = N(0) + \int_0^x \left(\frac{W}{L}\right) dx = \frac{W}{L} x$$

a) O deslocamento vertical do ponto C corresponde ao alongamento do trecho AC da barra. Assim:



$$\delta_C = \int_h^L \frac{N(x)}{EA} dx = \int_h^L \frac{Wx}{LEA} dx = \frac{W}{LEA} \frac{x^2}{2}\bigg|_h^L$$

$$\delta_C = \frac{W}{LEA} \frac{(L^2 - h^2)}{2}$$

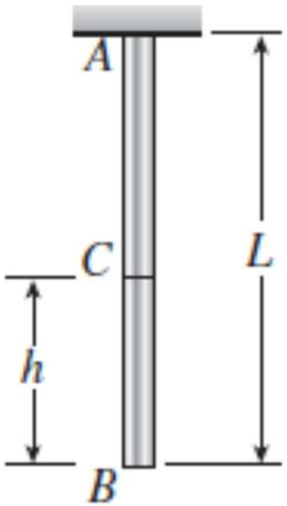
Obs:

1) Verifique sempre se a equação obtida é dimensionalmente correta!
2) Verifique se a equação atende a requisitos óbvios (ex: qual deve ser o deslocamento esperado no ponto A?)

b) O alongamento total da barra corresponde ao deslocamento vertical do ponto B que pode ser calculado tomando $h = 0$. Logo:

$$\delta_B = \frac{WL}{2EA}$$

c) Pelo enunciado a razão β é dada por: $\beta = \dfrac{\Delta L_{AC}}{\Delta L_{CB}}$ (...para $h = L/2$)



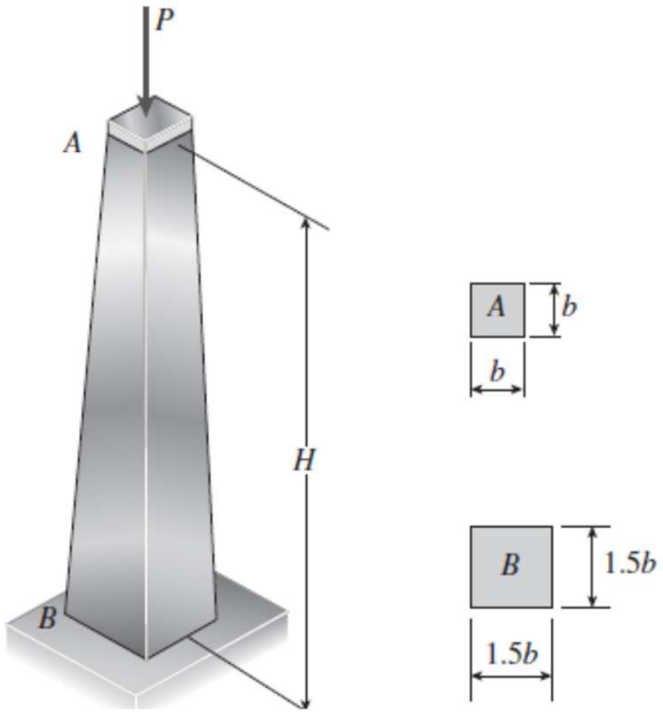
$$\Delta L_{AC} = \frac{W}{LEA}\frac{(L^2 - (L/2)^2)}{2} = \frac{3}{8}\frac{WL}{EA}$$

$$\Delta L_{CB} = \delta_B - \Delta L_{AC} = \frac{WL}{2EA} - \frac{3}{8}\frac{WL}{EA} = \frac{1}{8}\frac{WL}{EA}$$

Logo: $\beta = 3$

Problem 2.3-14 A post AB supporting equipment in a laboratory is tapered uniformly throughout its height H (see figure). The cross sections of the post are square, with dimensions $b \times b$ at the top and $1.5b \times 1.5b$ at the base.

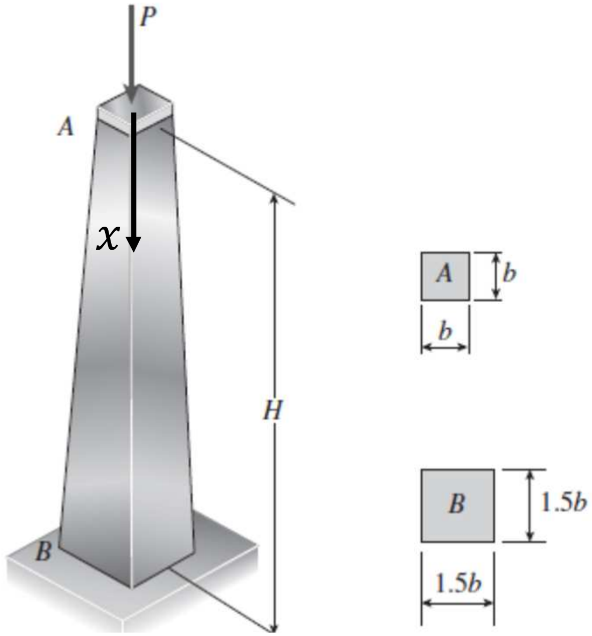
Derive a formula for the shortening δ of the post due to the compressive load P acting at the top. (Assume that the angle of taper is small and disregard the weight of the post itself.)



Solução:

Temos um problema de compressão não uniforme (por conta da variação das seções transversais). O encurtamento total da barra é dado por:

$$\delta = \int_0^H \frac{N(x)}{EA(x)} dx$$



Onde:

$$N(x) = -P$$

$$A(x) = (b(x))^2 = \left(b\left(1 + \frac{x}{2H}\right)\right)^2$$

Chamando: $\eta = 1 + \dfrac{x}{2H}$ Teremos: $d\eta = \dfrac{dx}{2H} \Leftrightarrow dx = 2H d\eta$

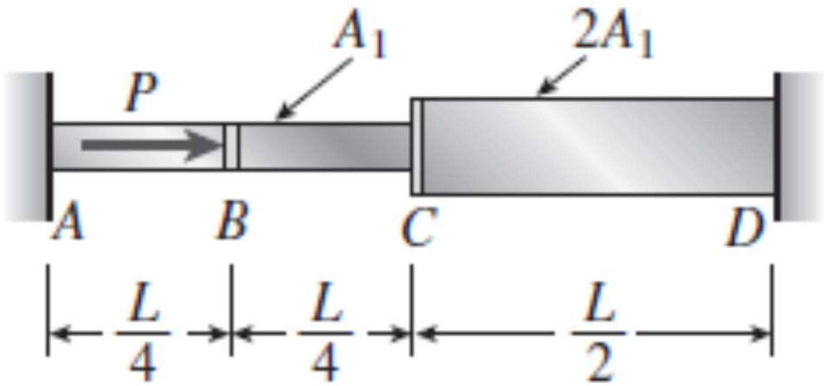
Extremos de integração:
$$\begin{cases} x = 0 \Rightarrow \eta = 1 \\ x = H \Rightarrow \eta = 1{,}5 \end{cases}$$

Logo: $\delta = -\displaystyle\int_0^H \frac{P}{E\left(b\left(1+\frac{x}{2H}\right)\right)^2} dx = -\int_1^{3/2} \frac{2HP}{Eb^2\eta^2} d\eta = \frac{2PH}{Eb^2}\left(\frac{1}{\eta}\right)\Bigg|_1^{3/2}$

$$\delta = \frac{2PH}{Eb^2}\left(\frac{2}{3} - 1\right) = -\frac{2PH}{3Eb^2}$$

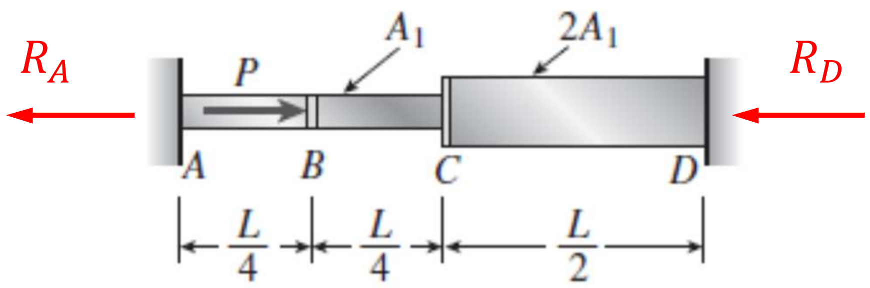
Problem 2.4-7 The axially loaded bar *ABCD* shown in the figure is held between rigid supports. The bar has cross-sectional area A_1 from *A* to *C* and $2A_1$ from *C* to *D*.

(a) Derive formulas for the reactions R_A and R_D at the ends of the bar.
(b) Determine the displacements δ_B and δ_C at points *B* and *C*, respectively.
(c) Draw a diagram in which the abscissa is the distance from the left-hand support to any point in the bar and the ordinate is the horizontal displacement δ at that point.



Solução:

1. D.C.L. e classificação da estrutura (isostática ou hiperestática)

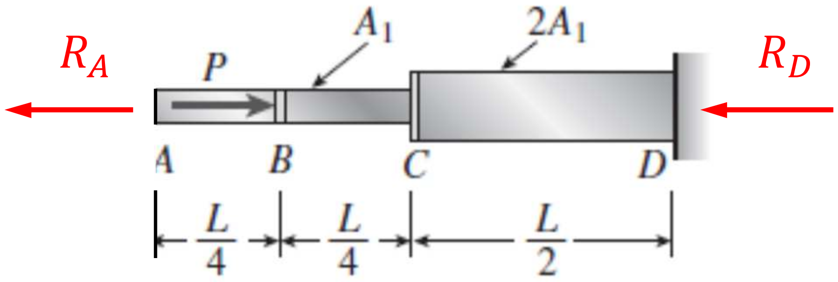


Equilíbrio de forças na direção horizontal: $R_A + R_D = P$

Grau de hiperestaticidade estrutural:

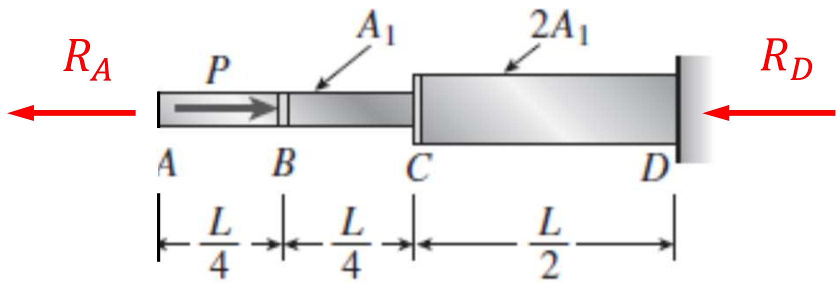
$$g = 2 - 1 = 1$$

2. Escolha da E.I.F. (e consequentemente da incógnita hiperestática):



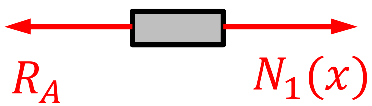
3. Determinação do deslocamento do ponto A (que equivale ao cálculo do alongamento de toda a barra):

$$\delta_A = \delta = \sum_{i=1}^{3} \delta_i = \sum_{i=1}^{3} \frac{N_i L_i}{E A_i}$$



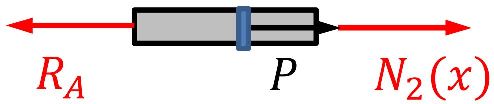
Esforços internos em cada segmento:

Segmento 1 (AB):



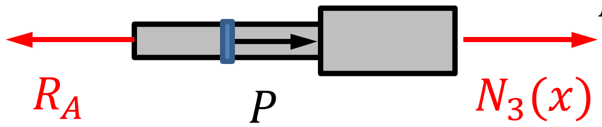
$$N_1(x) = R_A \qquad \left(0 \leq x < \frac{L}{4}\right)$$

Segmento 2 (BC):



$$N_2(x) = R_A - P \qquad \left(\frac{L}{4} < x \leq \frac{L}{2}\right)$$

Segmento 3 (CD):



$$N_3(x) = R_A - P \qquad \left(\frac{L}{2} < x \leq L\right)$$

$$\delta_A = \sum_{i=1}^{3} \frac{N_i L_i}{E A_i} = \frac{R_A (L/4)}{E A_1} + \frac{(R_A - P)(L/4)}{E A_1} + \frac{(R_A - P)(L/2)}{E(2A_1)}$$

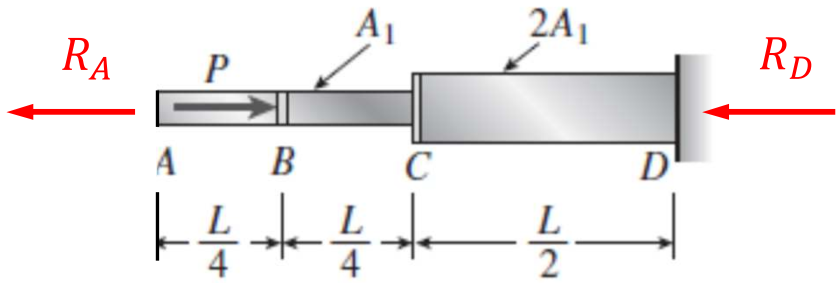
$$\delta_A = \frac{R_A L}{E A_1}\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) - \frac{PL}{E A_1}\left(\frac{1}{4} + \frac{1}{4}\right) = \frac{3}{4}\frac{R_A L}{E A_1} - \frac{1}{2}\frac{PL}{E A_1}$$

4. Compatibilização de deslocamentos:

$$\delta_A = \frac{3}{4}\frac{R_A L}{E A_1} - \frac{1}{2}\frac{PL}{E A_1} = 0 \iff R_A = \frac{2}{3}P$$

Como: $R_A + R_D = P$ Teremos: $R_D = \frac{1}{3}P$

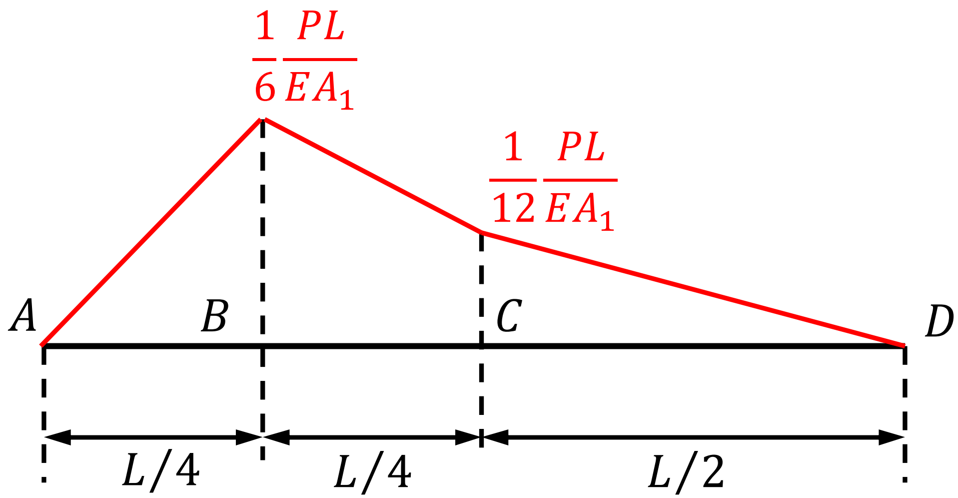
b) Deslocamentos em B e em C:



$$\delta_B = \delta_1 = \frac{R_A(L/4)}{EA_1} = \frac{2P}{3}\left(\frac{L}{4EA_1}\right) = \frac{1}{6}\frac{PL}{EA_1}$$

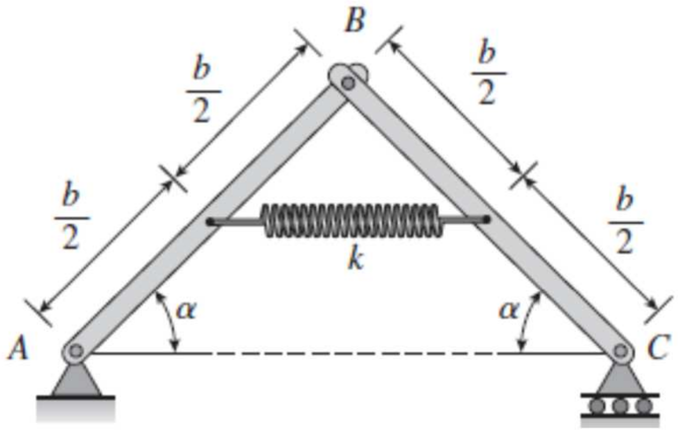
$$\delta_C = \delta_1 + \delta_2 = \frac{R_A(L/4)}{EA_1} + \frac{(R_A - P)(L/4)}{EA_1} = \frac{1}{6}\frac{PL}{EA_1} - \frac{1}{12}\frac{PL}{EA_1} = \frac{1}{12}\frac{PL}{EA_1}$$

c) Diagrama de deslocamentos:



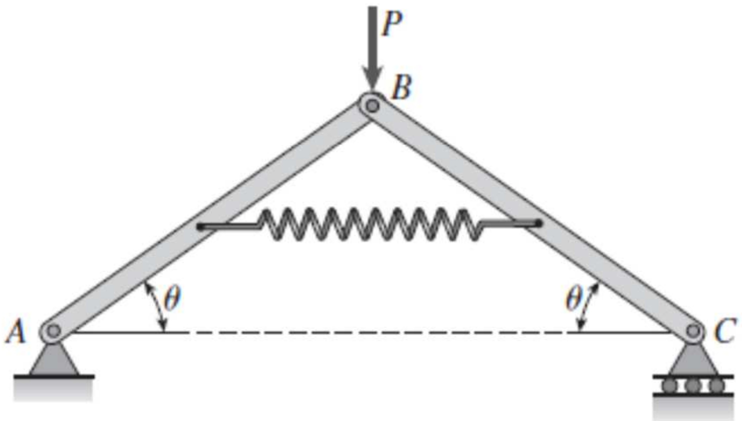
Homework:

Problem 2.2-13 A framework ABC consists of two rigid bars AB and BC, each having length b (see the first part of the figure). The bars have pin connections at A, B, and C and are joined by a spring of stiffness k. The spring is attached at the midpoints of the bars. The framework has a pin support at A and a roller support at C, and the bars are at an angle α to the hoizontal.



When a vertical load P is applied at joint B (see the second part of the figure) the roller support C moves to the right, the spring is stretched, and the angle of the bars decreases from α to the angle θ.

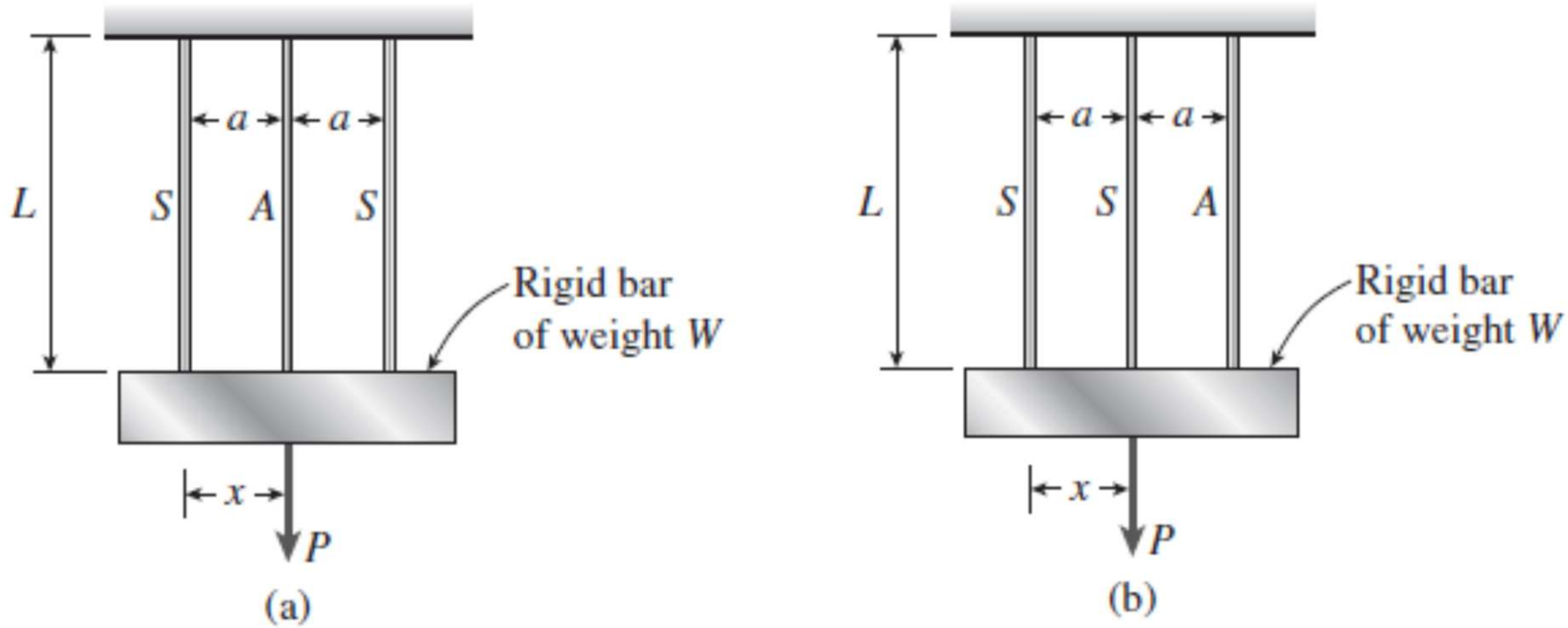
Determine the angle θ and the increase δ in the distance between points A and C. (Use the following data; $b = 8.0$ in., $k = 16$ lb/in., $\alpha = 45°$, and $P = 10$ lb.)



Homework:

Problem 2.4-12 A rigid bar of weight $W = 800$ N hangs from three equally spaced vertical wires (length $L = 150$ mm, spacing $a = 50$ mm): two of steel and one of aluminum. The wires also support a load P acting on the bar. The diameter of the steel wires is $d_s = 2$ mm, and the diameter of the aluminum wire is $d_a = 4$ mm. Assume $E_s = 210$ GPa and $E_a = 70$ GPa.

(a) What load P_{allow} can be supported *at the midpoint of the bar* ($x = a$) if the allowable stress in the steel wires is 220 MPa and in the aluminum wire is 80 MPa? (See figure part a.)
(b) What is P_{allow} if the load is positioned at $x = a/2$? (See figure part a.)
(c) Repeat (b) above if the second and third wires are *switched* as shown in figure part b.



L
a
a
S
A
S
Rigid bar
of weight W
x
P
(a)
L
a
a
S
S
A
Rigid bar
of weight W
x
P
(b)