Disciplina 4300357

Oscilações e Ondas

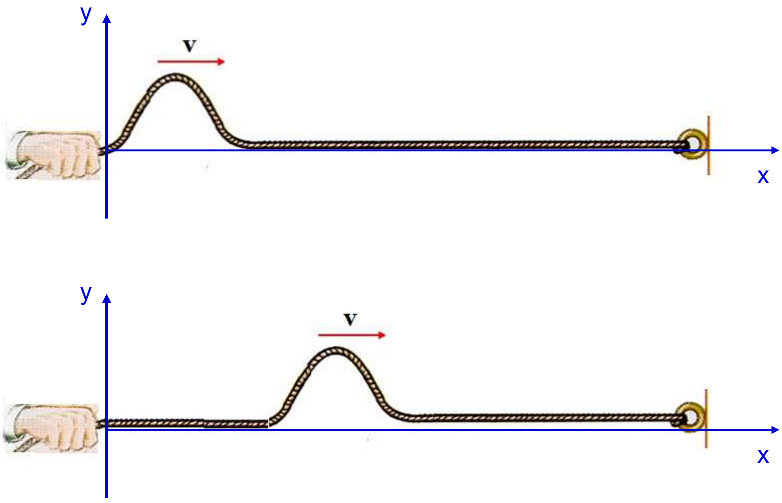
Aula 09

Equação da onda

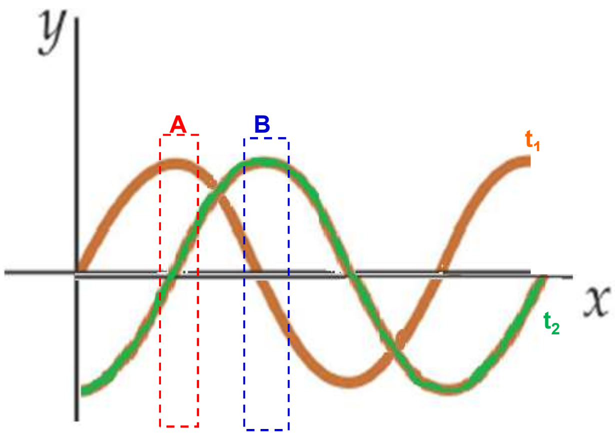
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{y(x,t) = Asen(kx - \omega t)}$$

Energia no Movimento Ondulatório



Energia no Movimento Ondulatório



A
t_1: repouso
t_2: U_{max}

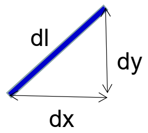
A
$t_1 \Rightarrow t_2$
ganho K

A
$t_1 \Rightarrow t_2$
ganho U

Parcelas de Energia (K e U)

Em cada segmento

$$\mu = \frac{dm}{dx} \qquad dm = \mu dx$$



inclinação dy/dx

$$y(x,t) = Asen(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = -A\omega cos(kx - \omega t)$$

A energia cinética

$$K = \frac{mv^2}{2} \qquad dK = \frac{dmv^2}{2}$$

$$dK = \frac{\mu dx(-A\omega cos(kx - \omega t))^2}{2}$$

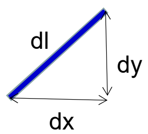
$$dK = \frac{\mu dx A^2 \omega^2 cos^2(kx - \omega t))^2}{2}$$

$$\frac{dK}{dt} = \frac{\mu dx A^2 \omega^2 cos^2(kx - \omega t))^2}{dt \qquad 2}$$

$$\frac{dK}{dt} = \frac{\mu v A^2 \omega^2 cos^2(kx - \omega t))^2}{2}$$

Parcelas de Energia (K e U)

A energia potencial



$$U = W\ (T) = \mathrm{T} \cdot ds$$

$$dU = Tds$$

$$dU = T(dl - dx)$$

$$dU = T(\sqrt{dx^2 + dy^2} - dx)$$

$$dU = T\left(\sqrt{dx^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)} - dx\right)$$

$$dU = Tdx\left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - 1\right)$$

$$dU = Tdx\left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - 1\right)$$

$$dU = Tdx\left(\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2$$

$$dU = Tdx\left(\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

$$dU = Tdx\frac{1}{2}\left(\frac{dy}{dx}\right)^2 \quad = \frac{Tdx}{2}(Akcos(kx-\omega t)^2$$

$$\frac{dU}{dt} = \frac{Tdx}{2dt}(Akcos(kx-\omega t)^2 \quad = \frac{T}{2}vA^2k^2cos^2(kx-\omega t)$$

$$\frac{dU}{dt} = \frac{\mu v^2}{2}vA^2\frac{\omega^2}{v^2}cos^2(kx-\omega t)$$

$$\frac{dU}{dt} = \frac{\mu vA^2\omega^2cos^2(kx-\omega t)}{2}$$

$$v^2 = \frac{T}{\mu} \qquad v = \frac{\omega}{k}$$

$$y(x,t) = Asen(kx-\omega t) \qquad \frac{\partial y}{\partial x} = Akcos(kx-\omega t) \qquad T = \mu v^2 \qquad k = \frac{\omega}{v}$$

$$E = K + U$$

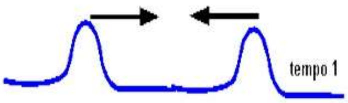
$$\frac{dE}{dt} = \frac{dK}{dt} + \frac{dU}{dt}$$

$$\frac{dK}{dt} = \frac{\mu vA^2\omega^2cos^2(kx-\omega t))^2}{2} \qquad \frac{dU}{dt} = \frac{\mu vA^2\omega^2cos^2(kx-\omega t)}{2}$$

$$\frac{dK}{dt} = \frac{dU}{dt}$$

$$\frac{dE}{dt} = 2\frac{dU}{dt}$$

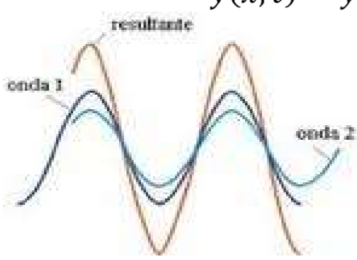
Superposição de Ondas

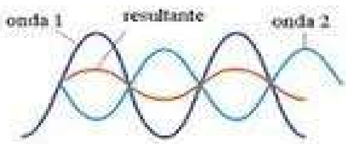


Superposição de Ondas

Quando várias ondas se combinam em um ponto, o deslocamento de cada partícula num dado instante de tempo é simplesmente a soma dos deslocamentos nela causados por cada onda considerada agindo isoladamente das demais.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$





Superposição de Ondas

$$y_1(x,t) = Asen(kx - \omega t) \qquad\qquad y_2(x,t) = Asen(kx - \omega t + \emptyset)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$y(x,t) = Asen(kx - \omega t) \ + Asen(kx - \omega t + \emptyset)$$

$$y(x,t) = A(sen(kx - \omega t) \ + sen(kx - \omega t + \emptyset))$$

$$senB + senC = 2sen\frac{1}{2}(B + C)cos\frac{1}{2}(B - C)$$

$$y(x,t) = 2Asen\frac{1}{2}(kx - \omega t + kx - \omega t + \emptyset)cos\frac{1}{2}(kx - \omega t - kx + \omega t - \emptyset)$$

$$y(x,t) = 2Asen\frac{1}{2}(2kx - 2\omega t + \emptyset)cos\frac{1}{2}(-\emptyset)$$

$$y(x,t) = 2Asen(kx - \omega t + \emptyset/2)cos\ \emptyset/2$$

$$y(x,t) = 2Acos\ \emptyset/2sen(kx - \omega t + \emptyset/2)$$