- Como descrever sistemas Compostos:

↳ Descrever sistemas de partículas (sistemas) que são por si só espaços vetoriais.

⇒ produtos tensoriais: Combina diferentes espaços num espaço composto

$$\mathcal{E} = V \otimes W \quad \left\{ \begin{array}{l} V : \text{dim. } p \\ W : \text{dim. } q \end{array} \right. \Rightarrow V \otimes W : \text{dim. } (p \cdot q)$$

* Vetores: $|v\rangle \in V \;;\; |w\rangle \in W \Rightarrow |v\rangle \otimes |w\rangle \in V \otimes W$

Notação: $|v\rangle|w\rangle$ ou $|vw\rangle$

* operadores: $(\hat{A} \otimes \hat{B})\,|v\rangle|w\rangle = (\hat{A}|v\rangle)(\hat{B}|w\rangle)$

* Norma: se $|\psi\rangle = |v\rangle|w\rangle = |vw\rangle$

$$\Downarrow$$

$$\langle\psi|\psi\rangle = \langle v|v\rangle\langle w|w\rangle$$ norma

→ Representação Matricial do Prod. tensorial

$$\left.\begin{array}{l} \hat{A} \rightarrow A : m \times n \\ \hat{B} \rightarrow B : p \times q \end{array}\right\} \Rightarrow \hat{A} \otimes \hat{B} = AB = \begin{pmatrix} A_{ij}B & \cdots \\ \vdots & \ddots \end{pmatrix}$$

$A \otimes B : mp \times nq$

$$= \begin{pmatrix} (A_{11}B) & A_{12}B & \cdots & A_{1n}B \\ \vdots & \ddots & & \\ \cdots & & & A_{mn}B \end{pmatrix}$$

Exemplos:

- $\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

- $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \left(\begin{array}{cc|cc} aw & ax & bw & bx \\ ay & az & by & bz \\ \hline cw & cx & dw & dx \\ cy & cz & dy & dz \end{array}\right)$

Exemplo: Considere um sistema composto q-bits $|A\rangle, |B\rangle \rightarrow \{|0\rangle, |1\rangle\}$

O estado singleto é uma combinação linear

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right)$$

O operador $\hat{X}_A$ atua no q-bit $|A\rangle$

Qual o resultado da ação de $\hat{X}_A$ no estado singleto?

$$\left.\begin{aligned} X|0\rangle &= |1\rangle \\ X|1\rangle &= |0\rangle \end{aligned}\right\} \Rightarrow \quad X_A|\psi\rangle = \frac{1}{\sqrt{2}}\left(X_A|0\rangle_A|1\rangle_B - X_A|1\rangle_A|0\rangle_B\right)$$

$$X_A|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle|1\rangle - |0\rangle|0\rangle\right)$$

Exemplo Suponha $|\psi\rangle = \dfrac{|0\rangle|0\rangle + |1\rangle|1\rangle}{\sqrt{2}}$

$$(\hat{Z}_A \otimes \hat{X}_B)|\psi\rangle$$

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \qquad Z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \qquad X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$

$\Rightarrow$

$Z \otimes X$

$$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \otimes \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & -1\\0 & 0 & -1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} ; \quad \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$(Z \otimes X) \cdot (|0\rangle|0\rangle) = \begin{pmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & -1\\0 & 0 & -1 & 0\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$\Downarrow$

$$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \rightarrow Z|0\rangle = |0\rangle$$

$$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} \rightarrow X|0\rangle = |1\rangle$$

$\Downarrow$

$$(Z \otimes X)\,(|0\rangle|0\rangle) = (|0\rangle|1\rangle) \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = (|0\rangle|1\rangle)$$

Sugestão: ... desenvolva o restante do exercício p/ praticar ...

- Operador Densidade

$$\boxed{\hat{\rho} = \sum_{i=1}^{n} \underline{p_i} |\psi_i\rangle\langle\psi_i|}$$ prob. $\{0 \dots 1\}$ (estados de Mistura)

$\hat{\rho} = \sum |\psi_i\rangle\langle\psi_i| \rightarrow \hat{\rho} = |\psi\rangle\langle\psi|$ (estados puros)

- **Propriedades do op. Densidade:**

- Hermitiano ✓ $\rightarrow \rho^{\dagger} = \rho$
- Traço unitário: $Tr(\rho) = 1$
- P/ estados puros: $Tr(\rho^2) = 1$; pois $\rho^2 = \rho$
- P/ " Mistura: $Tr(\rho^2) < 1$
- Autovalores satisfaz $0 \leq \lambda_i \leq 1$
- Valor esperado $\boxed{\langle \hat{A} \rangle = Tr(\hat{\rho}\hat{A})}$

$\hat{\rho} = |\psi\rangle\langle\psi|$

$\hat{\rho}^2 = \hat{\rho}\hat{\rho} = |\psi\rangle\underbrace{\langle\psi|\psi\rangle}_{1}\langle\psi| = |\psi\rangle\langle\psi| \dots = \hat{\rho}$

$\hat{\rho} = \sum_{i=1}^{n} p_i |i\rangle\langle i|$

$\hat{\rho} = p_1 |1\rangle\langle 1| + p_2 |2\rangle\langle 2|$

$\hat{\rho}^2 = \hat{\rho}\cdot\hat{\rho} = (p_1 |1\rangle\langle 1| + p_2 |2\rangle\langle 2|)(p_1 |1\rangle\langle 1| + p_2 |2\rangle\langle 2|)$

$= p_1^2 \underbrace{|1\rangle\langle 1|1\rangle\langle 1|}_{|1\rangle\langle 1|} + p_1 p_2 |1\rangle\langle 1|2\rangle\langle 2| + p_2 p_1 |2\rangle\langle 2|1\rangle\langle 1| + p_2^2 |2\rangle\langle 2|$

$\hat{\rho}^2 = p_1^2 |1\rangle\langle 1| + p_2^2 |2\rangle\langle 2| \Rightarrow Tr(\hat{\rho}^2) = p_1^2 + p_2^2 < 1$

$\hat{\rho} = p_1 |1\rangle\langle 1| + p_2 |2\rangle\langle 2| \quad Tr(\rho) = p_1 + p_2 = 1$

$\langle \hat{A} \rangle_\psi = \langle \psi | \hat{A} | \psi \rangle = \langle \psi | \hat{\mathbb{1}} \hat{A} | \psi \rangle$

$= \sum_i \langle \psi | u_n \rangle \langle u_n | \hat{A} | \psi \rangle = \sum_i \langle u_i | \hat{A} | \psi \rangle \langle \psi | u_i \rangle$

$= \sum_i \underbrace{\langle u_i | \hat{A}\hat{\rho} | u_i \rangle}_{(\hat{A}\hat{\rho})_{ii}} = Tr(\hat{A}\hat{\rho})$

Exemplo: $\rho = \begin{pmatrix} 3/4 & \frac{1+i}{4} \\ \frac{1-i}{4} & 1/4 \end{pmatrix}$

a) Puro?

b) $\langle Z \rangle$? ; $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

a) $\rho^2 = \rho\cdot\rho = \begin{pmatrix} 3/4 & (1+i)/4 \\ \frac{(1-i)}{4} & 1/4 \end{pmatrix}\begin{pmatrix} 3/4 & \frac{1+i}{4} \\ \frac{1-i}{4} & 1/4 \end{pmatrix} = \begin{pmatrix} 11/16 & (1+i)/4 \\ \frac{(1-i)}{4} & 3/16 \end{pmatrix}$

$Tr(\rho^2) = \frac{11}{16} + \frac{3}{16} = \frac{14}{16} = \frac{7}{8} < 1$ Mistura

b) $\langle Z \rangle_\psi = \langle \psi | Z | \psi \rangle$

$\langle Z \rangle = Tr(\rho Z) = Tr\left[\underbrace{\begin{pmatrix} 3/4 & \frac{1+i}{4} \\ \frac{1-i}{4} & 1/4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\begin{pmatrix} 3/4 & -\left(\frac{1+i}{4}\right) \\ \frac{1-i}{4} & -1/4 \end{pmatrix}}\right] = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$$|\psi\rangle = c_1|u_1\rangle + c_2|u_2\rangle + \cdots + c_n|u_n\rangle$$ estado puro

$$\hat{\rho} = |\psi\rangle\langle\psi| = \left(c_1|u_1\rangle + c_2|u_2\rangle + \cdots + c_n|u_n\rangle\right)\left(c_1^*\langle u_1| + \cdots + c_n^*\langle u_n|\right)$$

$$= \underbrace{\sum_{i=1}^{n} |c_i|^2 |u_i\rangle\langle u_i|}_{\text{populações}} + \underbrace{\sum_{i\neq j}^{n} c_j^* c_i |u_i\rangle\langle u_j|}_{\text{Coerências}}$$

população: $\rho_{11} = |c_1|^2$

coerência

$$\begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & & \\ \vdots & & \rho_{33} & \\ \rho_{n1} & \cdots & & \rho_{nn} \end{pmatrix}$$

$$c_j = |c_j| e^{i\theta_j} \qquad z = r e^{i\theta}$$

Coerencia

$$c_i c_j^* = |c_i||c_j| e^{i(\theta_i - \theta_j)}$$

Resumindo

- Estados puros
- Estados de mistura
  - → Parcial
  - → completa ⇒ Exemplos: 2 níveis $\left\{ \begin{array}{l} 50\% \ |0\rangle \\ 50\% \ |1\rangle \end{array} \right.$ (Mistura completa) estatística

$$\frac{1}{n} \leq Tr(\rho^2) \leq 1$$

dimensão do espaço

Mistura completa

invariante (independente da base)

estado puro

$$\hat{\rho} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\rho^2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} (1/2)^2 & 0 \\ 0 & (1/2)^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} \Rightarrow Tr(\rho^2) = \frac{1}{2} = \frac{1}{n}$$

Exemplo: estado puro (superposição)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ ; \ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \psi = \alpha|0\rangle + \beta|1\rangle$$

operador Hadamard

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |-\rangle$$

$$\hat{\rho}_+ = |+\rangle\langle +| = \frac{1}{2}\left(|0\rangle + |1\rangle\right)\left(\langle 0| + \langle 1|\right)$$

$$= \frac{1}{2}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$\rho_+ = \begin{pmatrix} \langle 0|\hat{\rho}_+|0\rangle & \langle 0|\hat{\rho}_+|1\rangle \\ \langle 1|\hat{\rho}_+|0\rangle & \langle 1|\hat{\rho}_+|1\rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\rho_+^2 = \rho_+ \cdot \rho_+ = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \Rightarrow Tr(\rho_+^2) = 1$$

* Verifique

$$\hat{\rho}_- \ ; \ (\rho_-) = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

MATRIZ

$$\hookrightarrow \boxed{\hat{\rho}_M = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|} \quad \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\hat{\rho} = \sum_{i=0}^{1} p_i |\psi_i\rangle\langle\psi_i|$$

probabilidades $\sum_i p_i = 1$

Mistura estatística

$$\hookrightarrow \hat{\rho}_+ = \frac{1}{2}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$\hat{\rho}_- = \frac{1}{2}\left(|0\rangle\langle 0| - |0\rangle\langle 1| - |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

estados puros

Exemplo: Qual a probabilidade de medir $|+\rangle$, $|-\rangle$ ou $\hat{\rho}_M$?
Qual o valor esperado de $\hat{z}$: $\langle z_+\rangle$, $\langle z_-\rangle$, $\langle z_M\rangle$

Pauli

$$\hookrightarrow P(|+\rangle) = P_+ = Tr(\hat{\rho}\hat{\mathbb{P}}_+) \Rightarrow \hat{\mathbb{P}}_+ = |+\rangle\langle +| = \hat{\rho}_+$$

$\hat{\mathbb{P}}_+ = |+\rangle\langle +|$ $\quad \langle\hat{\mathbb{P}}_+\rangle$

$$\hookrightarrow (\hat{\rho}_+\hat{\mathbb{P}}_+) = \hat{\rho}_+^2 = \hat{\rho}_+\hat{\rho}_+ \Rightarrow Tr\left[\left(\frac{1}{2}\right)^2\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\1&1\end{pmatrix}\right] = \frac{1}{4}4 = 1$$

$$Tr(\hat{\rho}_M\hat{\mathbb{P}}_+) = Tr\left(\frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}1&1\\1&1\end{pmatrix}\right) = \frac{1}{4}Tr\begin{pmatrix}1&1\\1&1\end{pmatrix} = 1/2$$

$$P_0 = Tr(\rho\hat{\mathbb{P}}_0) = Tr(\rho_+ |0\rangle\langle 0|)$$

$$\Rightarrow P_+(\hat{\rho}_M\hat{\mathbb{P}}_+) = Tr\left[\left(\frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}\right)\left(\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\right)\right] = \frac{1}{4}Tr\left[\begin{pmatrix}1&1\\1&1\end{pmatrix}\right] = \frac{2}{4} = \frac{1}{2}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$+ \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|+\rangle + |-\rangle = \frac{2}{\sqrt{2}}|0\rangle \rightarrow |0\rangle = \frac{\sqrt{2}}{2}(|+\rangle + |-\rangle)$$

Sugestão

$$\langle z_+\rangle = Tr(\hat{\rho}_+\hat{z})$$
$$\langle z_-\rangle = Tr(\hat{\rho}_-\hat{z})$$
$$\langle z_M\rangle = Tr(\hat{\rho}_M\hat{z})$$

$$(\hat{z}) = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

Porque $\hat{z}$?

Matriz de Pauli $\hat{z}$, $\hat{\sigma}_z$

$$\langle\hat{z}_+\rangle = \langle +|\hat{z}|+\rangle = Tr(\hat{\rho}_+\hat{z}) = Tr(\hat{z}\hat{\rho}_+)$$

$$\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1&-1\\1&-1\end{pmatrix}$$

$$\langle z_+\rangle = 0$$
$$\langle z_-\rangle = 0$$
$$\langle z_M\rangle = Tr(\hat{\rho}_M\hat{z}) = 0$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$Tr(AB) = Tr(BA)$$

$A: m\times n$; $B: n\times m$

$A\cdot B = (m\times n)(n\times m): m\times m$

$BA = (n\times m)(m\times n): n\times n$

$$Tr(AB) = \sum_{i=1}^{n}\sum_{j=1}^{m} A_{ij}B_{ji} = \sum_{i,j} B_{ji}A_{ij} = Tr(BA)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
$$+ \quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|+\rangle\langle +| = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1| \cdots)$$

$$|0\rangle = (|+\rangle + |-\rangle)$$

Exemplo: spin 1/2 (sistema 2 níveis)

$$\hat{S}_i = \frac{\hbar}{2}\hat{\sigma}_i \Rightarrow \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z \; ; \; \hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x \; \ldots$$

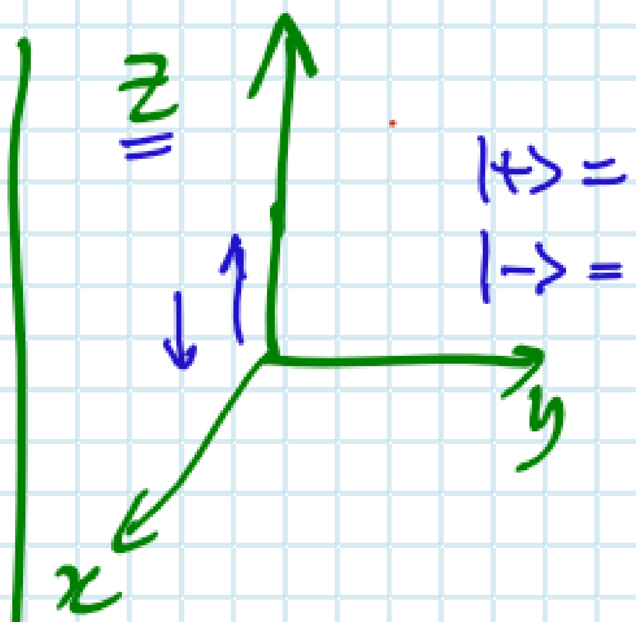
$\hat{\sigma}_i$: Matriz de Pauli

$i = \{1,2,3\} \to \{x,y,z\}$

$$\sigma_1 = \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \hat{x}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \Rightarrow \hat{y}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \hat{z}$$



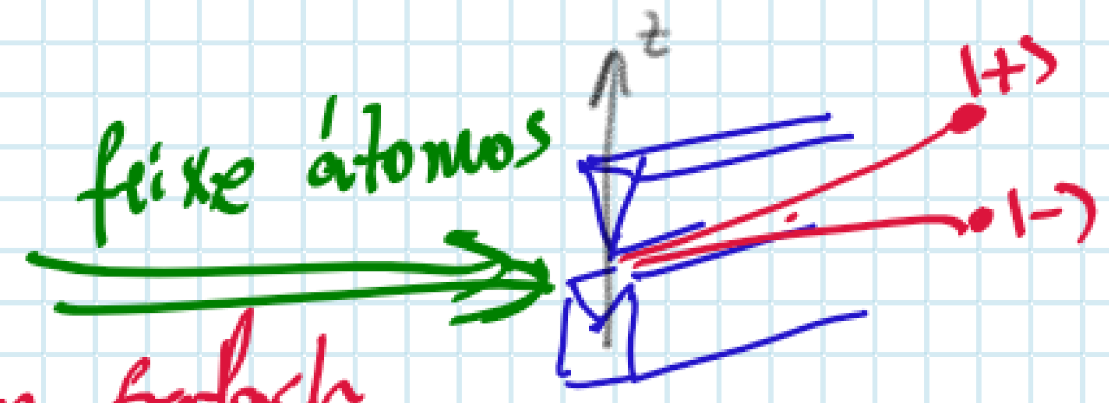
$|+\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|-\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\vec{S}$ ↑ ↓

$$\begin{cases} \hat{S}_z |+\rangle = \frac{\hbar}{2}|+\rangle \Rightarrow \hat{S}_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle \\ \hat{S}_z |-\rangle = -\frac{\hbar}{2}|-\rangle \quad \hat{S}_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle \end{cases}$$

$s = 1/2$



Stern-Gerlach

$\vec{\nabla}B$

$\vec{S} \to \vec{\mu}$

$U_B = -\vec{\mu}\cdot\vec{B}$

$\vec{F}_M = -\nabla(U_B)$

$= \vec{\mu}\cdot\vec{\nabla}B$

$$\hat{S}_x |+_x\rangle = \frac{\hbar}{2}|+_x\rangle$$
$$\hat{S}_x |-_x\rangle = -\frac{\hbar}{2}|+_x\rangle \Rightarrow \hat{S}_j |\pm_j\rangle = \pm\frac{\hbar}{2}|\pm_j\rangle$$

CUIDADO!!

$$\Rightarrow \begin{cases} \hat{S}_x |+_z\rangle = ? \\ \hat{S}_y |+\rangle = ? \end{cases}$$

$$[\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z$$
$$[\hat{S}_y, \hat{S}_z] = i\hbar \hat{S}_x$$
$$[\hat{S}_z, \hat{S}_x] = i\hbar \hat{S}_y$$

$$|s, m\rangle = |1/2, m_z\rangle \begin{cases} |1/2, +1/2\rangle = |+\rangle = |\uparrow\rangle \\ |1/2, -1/2\rangle = |-\rangle = |\downarrow\rangle \end{cases}$$

$s = 1/2 \Rightarrow m = \{-1/2, +1/2\}$

• Resumo dos formalismos da M.Q.:

Teoria Quântica: (diferentes formulações)

- funções de onda
- vetores de estado
- operador densidade

$$i\hbar\frac{\partial \Psi(r,t)}{\partial t} = \mathcal{H}\,\Psi(r,t) \leftarrow \langle r|\Psi\rangle$$

$$\begin{cases} i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{\mathcal{H}}|\Psi\rangle \\ |\Psi(t)\rangle = \hat{U}(t,t_0)|\Psi(t_0)\rangle \end{cases}$$

$\Rightarrow |\Psi\rangle \Rightarrow \hat{A}(t) \Rightarrow$ Heisenberg

$$\boxed{i\hbar\frac{\partial}{\partial t}\hat{\rho} = [\hat{H}, \hat{\rho}]}$$

Análogo ao vetor de estado $|\Psi\rangle \Rightarrow$ $\hat{\rho} = |\Psi\rangle\langle\Psi|$

O que interessa é:

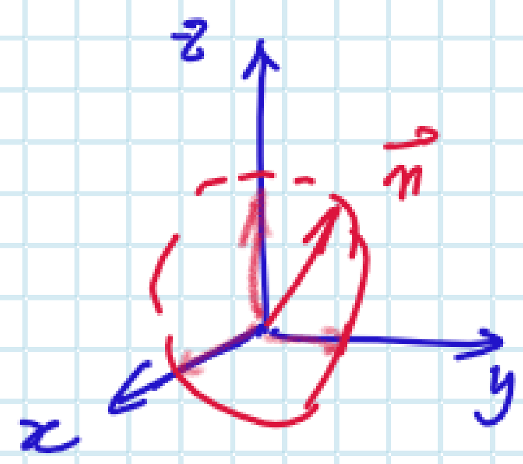
$\langle A\rangle$: Valores esperados (médios) de medidas de observáveis

$\hookrightarrow \langle\Psi|A|\Psi\rangle$; $\int \Psi^* A \Psi\, d\vec{r}$; $Tr(\hat{A}\hat{\rho})$ ...

P_m: probabilidades de medidas/resultados

$\hookrightarrow \langle\phi_m|\Psi\rangle$; $\int \phi_m^* \Psi\, d\vec{r}$; $Tr(\hat{P}_{\phi_m}\hat{\rho})$

• Vetor de Bloch (Felix Bloch)



Sistemas 2 níveis, podemos escrever

$$\hat\rho = \frac{\hat{\mathbb{1}} + \vec n \cdot \vec\sigma}{2}$$

Vetor de Bloch

$\vec n = (n_x, n_y, n_z)$

$$\begin{cases} n_x = \mathrm{Tr}(\hat\rho\,\hat\sigma_x) \\ n_y = \mathrm{Tr}(\hat\rho\,\hat\sigma_y) \\ n_z = \mathrm{Tr}(\hat\rho\,\hat\sigma_z) \end{cases}$$

componentes do vetor Bloch $\vec n$

$\vec\sigma = (\sigma_x, \sigma_y, \sigma_z)$

$$\|\vec n\| \le 1 \begin{cases} = 1 & \text{(puro)} \\ < 1 & \text{(Mistura)} \end{cases}$$

$\vec n \cdot \vec\sigma = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z$

$n_i \in \mathbb{R}$

$$\hat\rho = \frac{1}{2}\left(\hat{\mathbb{1}} + n_x\hat\sigma_x + n_y\hat\sigma_y + n_z\hat\sigma_z\right) \longleftarrow \begin{cases} \mathrm{Tr}(\hat\rho) = 1 \\ ; \\ \mathrm{Tr}(\hat\rho^2) \le 1 \end{cases}$$

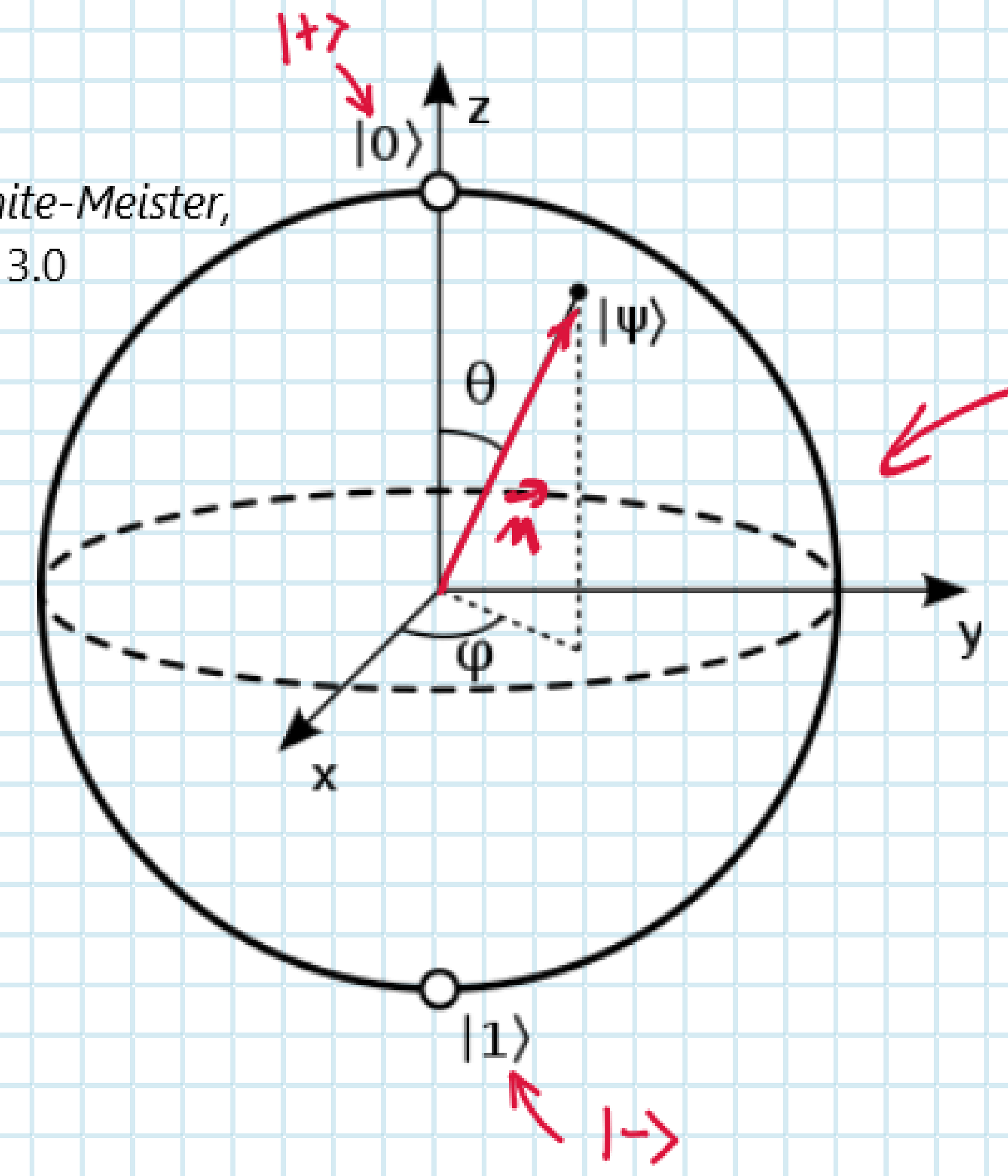
$\rho \rightarrow |\psi\rangle$

$\mathrm{Tr}(\hat{\mathbb{1}}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 2$

$\rho = \sum p_i |\psi_i\rangle$

$$\boxed{\frac{1}{2}\mathbb{1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}}$$

Fig. by Smite-Meister, CC BY-SA 3.0



esfera de Bloch