Postulado 1

Exemplo: sistema 2 níveis – qubit

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$\langle\psi|\psi\rangle = 1 \Rightarrow |\alpha|^2 + |\beta|^2 = 1$

$\alpha = \langle 0|\psi\rangle$, $\beta = \langle 1|\psi\rangle$

$\langle 0|1\rangle = \langle 1|0\rangle = 0$ $\qquad$ $\langle 1|1\rangle = \langle 0|0\rangle = 1$

Postulado 2:

Exemplo: $\hat{A} = \sum_i |u_i\rangle\langle u_j|$

NOT

$X \Rightarrow$ $\underline{X|0\rangle = |1\rangle}$; $\underline{X|1\rangle = |0\rangle}$

$\sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\rightsquigarrow$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$\Longrightarrow$

Exemplo 2:

$$\hat{Y}: \quad \hat{Y}|0\rangle = -i|1\rangle, \quad \hat{Y}|1\rangle = i|0\rangle \Rightarrow Y = \begin{pmatrix} \langle 0|\hat{Y}|0\rangle & \langle 0|\hat{Y}|1\rangle \\ \langle 1|\hat{Y}|0\rangle & \langle 1|\hat{Y}|1\rangle \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \hat{Z} \Rightarrow Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Sugestões

$$\hat{Z}|0\rangle = ?$$

$$\hat{Z}|1\rangle = ?$$

Exemplo 3:

Hadamard

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Rightarrow \text{Ação ?}$$

$|\psi\rangle \rightsquigarrow \langle\psi|\psi\rangle = 1$

$$\overbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}^{\hat{H}} \overbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}^{|0\rangle} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \hat{H}|0\rangle = \frac{1}{\sqrt{2}}\underbrace{(|0\rangle + |1\rangle)}_{|\psi\rangle}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \hat{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$\Rightarrow \hat{H} =$ produto externos

$= |0\rangle\langle 0| + \ldots$

$$\hat{H} = |0\rangle\langle 0| - |1\rangle\langle 1| + |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$\hat{H} = (\hat{X} + \hat{Z})/\sqrt{2}$$

hermitiano ? $\quad S^{\dagger} \stackrel{?}{=} S$

$\Rightarrow$ Desafio: $\quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \rightsquigarrow S^{\dagger} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$

$\neq$

Postulado 3

Exemplo

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar}{2}\hat{X} = \frac{\hbar}{2}\hat{\sigma}_x$$

$$X \to \lambda = \pm 1$$

$$\pm\frac{\hbar}{2} \rightsquigarrow [\hbar] \rightsquigarrow [\hbar] = \text{J.s}$$

ação

$\hbar\omega$

$$\hat{S}_y = \frac{\hbar}{2}\hat{Y} = \frac{\hbar}{2}\hat{\sigma}_y \quad ; \quad \hat{S}_z = \frac{\hbar}{2}\hat{\sigma}_z$$

Postulado 4

Exemplo:

$\hookrightarrow |\psi\rangle = \frac{\sqrt{2}}{3}|u_1\rangle + \frac{\sqrt{3}}{3}|u_2\rangle + \frac{2}{3}|u_3\rangle$

base ortonormal

$\{|u_i\rangle\}$

$\hookrightarrow \langle u_i|u_j\rangle = \delta_{ij}$

Soma das prob.

(1) $\hat{H}|u_n\rangle = n \cdot E|u_n\rangle \Rightarrow \hat{H}|u_2\rangle = 2E|u_2\rangle$

$\sum_i P_i = 1$

$\left(\frac{2}{9}+\frac{3}{9}+\frac{4}{9}=\frac{9}{9}=1\right)$

- probabilidade de medir E : $P_E = 2/9$
- " " " $2E$: $P_{2E} = 3/9 = 1/3$
- " " " $3E$: $P_{3E} = 4/9$

$\hookrightarrow$ Desafio $\langle\hat{H}\rangle = \langle\psi|\hat{H}|\psi\rangle = |\alpha_1|^2$

$|\psi\rangle = \alpha_1|u_1\rangle + \alpha_2|u_2\rangle + \alpha_3|u_3\rangle$

$\langle\hat{H}\rangle = |\alpha_1|^2 \cdot E + |\alpha_2|^2 \cdot 2E + |\alpha_3|^2 \cdot 3E$

$\hookrightarrow \langle\hat{H}\rangle = \left(\frac{2}{9}\right) \cdot E + \left(\frac{3}{9}\right) 2E + \frac{4}{9} \cdot 3E = \frac{20}{9}E$

CASO degenerado

(2) $\hat{H}|u_2\rangle = E|u_2\rangle$

$\left.\begin{array}{l} P_m \\ \uparrow \\ P_{\lambda_m} = P_E \end{array}\right) \Rightarrow P_E = |\alpha_1|^2 + |\alpha_2|^2$

$= \frac{2}{9} + \frac{1}{3} = \boxed{\frac{5}{9}}$

$\hookrightarrow \langle H\rangle = \frac{17}{9}E$