Simultaneous Equation Econometrics: The Missing Example

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ABSTRACT: For introductory presentation of issues involving identification and estimation of simultaneous equation systems, a natural vehicle is a model consisting of supply and demand relationships to explain price and quantity variables for a single good. One would accordingly expect to find in introductory econometrics textbooks a supply-demand example featuring actual data in which structural estimation methods yield more satisfactory results than ordinary least squares. In a search of 26 existing textbooks, however, we have found no such example—indeed, no example with actual data in which all parameter estimates are of the proper sign and statistically significant. This absence is documented in the present paper. Its main contribution, however, is the development of a simple but satisfying example, for broiler chickens, based on U.S. annual data over 1960-1999. Some discussion of the historically notable beef example of Tintner (1952) is included.

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1. Introduction

Existing textbooks of econometrics, including several that are excellent in most respects, are marred by a surprising and rather disturbing omission relating to simultaneous equation estimation. Ever since the publication of Haavelmo's (1943, 1944) classic papers on simultaneous equation analysis, a central ingredient of the subject of econometrics has been the identification and estimation of structural relationships in simultaneous equation systems.[1] The main vehicle for introductory presentation of the relevant issues has been, for most of these years, a two-equation system consisting of demand and supply relationships for the joint determination of price and quantity exchanged for a non-durable good.[2] Accordingly, one might expect to find, in most if not all introductory textbooks, a supply-demand example featuring actual data in which structural estimation methods (such as instrumental variables, two-stage least squares, or full-information maximum likelihood) are shown to yield more plausible estimates than ordinary least squares. Also, such an example should, to be satisfactory, feature theoretically appropriate signs on each of the estimated structural parameters with all of the important estimates being significantly different from zero at conventional significance levels.

Examination of 26 of the leading textbooks reveals that most present the simultaneous equations modeling methodology using the two-equation supply and demand system. It seems clear, however, that the authors of these texts have struggled to find a suitable example to illustrate the power of the approach. We think it likely that all these authors would agree that they have not hit upon a fully satisfactory example. In fact, none of them includes an

[1] Hausman (1983, p. 392) suggests that "The simultaneous equation model is perhaps the most remarkable development in econometrics."
[2] Such systems are included in 22 of the 26 textbooks listed below.

example that meets all of the criteria suggested in the preceding paragraph. Instead, most include either no numerical application for the supply-demand example or else one based on hypothetical data created by the writer. A few provide estimates based on actual price-quantity data, but in most cases the results are unsatisfying because crucial parameter estimates are statistically insignificant and/or of the theoretically-incorrect sign—e.g., downward-sloping supply curves.[3]

The purpose of the present paper, accordingly, is to present an example that has the desirable characteristics mentioned above. Specifically, we develop and estimate a simple demand-supply system involving annual U.S. time series data for 1960-1999 for chicken. Our specification of the demand and supply functions attempts to be theoretically sensible, and our instrumental variable estimation yields statistically significant estimates of all structural parameters, each of which is of the appropriate sign and plausible in magnitude. The instrumental variable estimates, moreover, are more satisfactory than ones obtained by application of ordinary least squares to the structural equations.

We begin in Section 2 with a discussion of a few textbook presentations that are both relevant and notable, plus a table indicating the manner in which each of the 26 textbooks examined fail to meet our criterion for the supply-demand example. Then in Section 3 we specify the model to be used in our example and report data sources. Least squares estimates are reported in Section 4 together with a discussion of various alternative specifications. Our structural estimates, obtained via two-stage least squares, are developed in Section 5, after which Section 6 presents a graphical portrayal of our estimated demand and supply relationships. Section 7 provides a brief conclusion.

[3] There is one example, introduced by Tintner (1952), that comes close to being satisfactory but falls slightly short. It will be discussed below.

2. Existing Textbook Treatments

Table 1 reports, for 26 leading textbooks of econometrics, the manner in which each of these fails to have a fully satisfactory quantitative supply-demand example based on actual data. In 12 cases, the book discusses the supply-demand system, but does not provide any quantitative example, while a few—only four, in fact—omit discussion of the supply-demand system entirely. Three of the textbooks, those of Kmenta (1971), Kelejian and Oates (1974), and Hill, Griffiths, and Judge (2000), include quantitative examples that are excellent in terms of the points being made, but utilize hypothetical data constructed by the author(s). Finally, there are seven textbooks that do include estimates involving actual market data, but feature results that fail to be satisfactory in terms of illustrating the basic pedagogic points about simultaneous-equation estimation. For example, Merrill and Fox (1970) and Maddala (1992) each include examples in which the structural equation estimates for the supply function imply negative responses of quantity supplied to price of the product.[4] [5]

The most nearly satisfactory examples come from the earliest textbook examined, that of Tintner (1952). It includes two examples for the U.S. market for meat, both based on annual data for the years 1919-1941. The difference between the two cases is that one (but not the other) is for a just-identified system, with one exogenous variable appearing in each structural equation and excluded from the other. In this case, all of the parameter estimates of the structural system are of the proper sign, but no standard errors are reported to permit

[4] In a footnote on p. 545, Merrill and Fox (1970) report a system with all signs correct, but an insignificant response of quantity supplied to price.

[5] Since this paper was written, an example with actual data has been provided by Schmidt (2004). No indications are reported regarding serial correlation of residuals, however, which is a serious limitation given that monthly time series data is used and the good studied (alcoholic beverages) is one for which habit formation is quite likely. In addition, the inclusion of labor force participation rate and unemployment rate variables in the supply function is problematic (as is acknowledged by the author's footnote on p. 278) and the negative (significant) sign on the unemployment rate is (we believe) difficult to rationalize.

Table 1: Supply-Demand Examples in Existing Textbooks

Author(s)	Date	No S&D Example	No Numer. Example	Hypoth. Data	Incorrect Sign	Non-Significant
Tintner	1952				1	2
Klein	1953	*				
Valavanis	1959		*			
Klein	1962	*				
Johnston	1963	*				
Goldberger	1964				3	3
Christ	1966		*			
Malinvaud	1966		4			
Merrill and Fox	1970				*	*
Theil	1971		*			
Frank	1971		*			
Kmenta	1971			*		5
Beals	1972		7			
Murphy	1973					
Kelejian & Oates	1974			*		
Pindyck & Rubinfeld	1976		*			
Maddala	1977		*			
Wonnacott&Wonnacott	1979		*			
Kennedy	1979	*				
Wallace & Silver	1988					*
Maddala	1992				*	
Gujarati	1995					6
Johnston & DiNardo	1997		*			
Hill, Griffiths, & Judge	2001			*		
Wooldridge	2003		*			
Ashenfelter, et. al.	2003		*			

Notes: 1. Overidentified example on pp. 177-184; see Goldberger, p. 328 .
2. Just identified example on pp. 168-172; no standard errors reported.
3. Tintner's overidentified example.
4. A demand function is estimated, but no supply function.
5. Tintner's just-identified example.
6. Incomplete example; one equation only.
7. Data (whether artificial or actual is unclear) provided only in problem.

determination of the statistical significance of the various estimates. We have obtained the data utilized by Tintner, however, and have determined that neither of the elasticities in the supply function is significantly different from zero at the 0.05 significance level.[6] Accordingly, we have entered this example in our table as yielding insignificant estimates. Tintner's second system differs from the first only by including an additional exogenous variable in the supply function, thereby making the demand function over-identified. Since the supply function is again exactly identified, the indirect least squares, two-stage least squares, and limited-information maximum likelihood estimates are the same. In this case, Tintner reports estimates (but not standard errors) only for the demand function. Goldberger (1964) reports estimates for the supply function for this example, however, and finds the price elasticity to be negative.

As implied in the foregoing paragraphs, Tintner's early example has been adapted by other authors, including Goldberger (1964) and Kmenta (1971). It would appear that these later discussions are based on the second-moment matrices reported by Tintner, who does not provide raw observations on the relevant variables. Consequently, because of their historical significance, we report in Appendix A the raw data series as they appear in French (1949).

3. Basic Model Specification

For the type of simple supply-demand model with which we are concerned, the jointly determined variables would be market price P and quantity Q. The most basic partial equilibrium supply and demand functions could be written as Q = S(P, W) and Q = D(P, Y) with W denoting the price of important factors of production and Y the income level of

[6] The estimated meat price coefficient is 0.62 with a standard error of 0.39 while a cost variable has a coefficient of −0.42 with standard error of 0.24. Our statement about significance presumes a two-sided test. If we used a one-sided test, the cost variable would be significant but not the price variable. The data come from the 1949 Iowa State College M.S. thesis by French (1949).

potential demanders. The partial derivatives of S and D would be expected to have the following signs: $S_1 > 0$, $S_2 < 0$, $D_1 < 0$, and $D_2 > 0$. The model's quantity variables should be expressed in physical units per time period and prices in real, relative-price terms. Assuming that the relationships can be approximated in constant-elasticity form and presuming analysis with time series data for some economy such as the United States, we then specify the most basic version of the model as follows, with lower-case letters denoting logarithms of the underlying variables:

(1) $\quad q_t = \alpha_0 + \alpha_1 p_t + \alpha_2 w_t + u_t \qquad$ (supply)

(2) $\quad q_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + v_t. \qquad$ (demand)

As suggested above, we presume that $\alpha_1 > 0$, $\alpha_2 < 0$, $\beta_1 < 0$, and $\beta_2 > 0$. In (1) and (2), u_t and v_t are stochastic disturbances representing measurement error, a multitude of individually-unimportant omitted variables, and purely random influences. We assume that $Eu_t = 0$, $Ev_t = 0$, $Eu_t^2 = \sigma_u^2$, and $Ev_t = \sigma_v^2$ for all $t = 1, 2, \ldots, T$. We also assume that w_t and y_t can legitimately be treated as exogenous to the particular market under consideration, so that w_t and y_t will be uncorrelated with values of u_t and v_t for all current and past periods.

The market to be studied is that for the edible meat of young chicken, often termed "broilers," in the United States. A large volume of data pertaining to the production and consumption of chicken is collected and reported by the U.S. Department of Agriculture (USDA). Some price data are generated by the USDA but most of the price series utilized below represent indexes developed by the U.S. Labor Department's Bureau of Labor Statistics (BLS). Per capita income levels for U.S. consumers are generated by the U.S. Commerce Department's Bureau of Economic Analysis (BEA). Our reported supply-demand estimates will be based on annual U.S. time series observations from the post-

World-War-II era, with the exact dates (reported below) determined by data availability.

Our aim is to obtain satisfactory estimates of basic structural equations such as (1) and (2), keeping the specifications as simple as possible. Some possible complexities must, however, be recognized. One is due to the rapid improvements in technology for the production of broilers that have taken place over the postwar era, thereby shifting the supply function. Also, there have been major changes in the price of chicken relative to those for other types of meat, so the price of some substitute goods might be expected to appear in the demand function. In addition, the improvement of transportation facilities has been so rapid that in recent years it has become the case that a significant fraction of U.S. broiler production is exported abroad, primarily to Russia and Hong Kong.

One specific issue that we have been forced to face is the precise definition of our main quantity variable. In terms of consumption, the USDA Poultry Yearbook reports per capita consumption of young chicken on both a ready-to-cook basis and a retail weight basis. Our preferred series, however, comes from the USDA Economic Research Service's electronic "data system," which reports per capita consumption of chicken on the following basis: boneless, trimmed (edible) weight, pounds per capita per year. For this "boneless equivalent" measure we were able to obtain a consistent series for the time span 1909-2001, and its behavior during the 1950's seems to be less affected by changing tastes than that of young chicken, retail weight.[7] From the perspective of quantity supplied, however, it seems preferable to utilize a measure of production, perhaps on an aggregate (rather than per-capita)

[7] Between 1950 and 1960, the per capita consumption of chicken almost tripled on a retail weight basis, while increasing by about 34 percent on a boneless equivalent basis. Our belief is that consumers began to eat primarily the better parts of the chicken, discarding some of those that were often consumed during earlier years. We would therefore expect to find a more stable demand function for consumption expressed in terms of the boneless equivalent basis. We have not been able to find a long consistent series for the ready-to-cook measure.

basis. The way in which we face this difficulty will be detailed in Section 5 below, and the itemization of the precise series used for the various variables will be provided in Appendix B together with the data series themselves.

4. Least Squares Estimates

We begin with exploratory estimation of the structural supply and demand equations, initially using (inconsistent) least squares methods. Consider first the demand function. If we straightforwardly regress q on p and y, as suggested by equation (2), the results for 1950-2001 are as follows:

$$q = -4.860 + \underset{(0.669)}{} \; \underset{(0.068)}{0.871\,y} - \underset{(0.070)}{0.277\,p} \tag{3}$$

(0.669) (0.068) (0.070)

$R^2 = 0.980$ SE = 0.0572 DW = 0.343 T = 52

Here, and in results reported below, the figures in parentheses are standard errors. Also, the R^2 statistic is unadjusted, SE is the estimated standard deviation of the disturbance term, DW is the Durbin-Watson statistic, and T is the number of observations. The results in (3) are encouraging in the sense that the income and price variables have the expected signs and are statistically significant. The DW statistic suggests very strong serial correlation of the disturbances, however, so more work is needed on this relation.[8] One natural variable to add to a demand function is the price of a substitute good, so in (4) we add the (log) real price of beef, denoted pb:

(4) $q = -4.679 + 0.852\,y - 0.264\,p - 0.118\,pb$

(0.675) (0.069) (0.070) (0.084)

$R^2 = 0.981$ SE = 0.0566 DW = 0.443 T = 52

[8] Regarding limitations of the DW statistic, see footnote 11 below.

Here pb enters with the wrong sign (for a substitute) and the DW is still unacceptably low, indicating very strong autocorrelation. Thus we specify the disturbance term as following a first-order autoregressive [AR(1)] process and obtain the following:[9]

$$(5)\qquad \underset{(0.188)}{q = 5.939} + \underset{(0.272)}{0.272\, y} - \underset{(0.070)}{0.307\, p} + \underset{(0.084)}{0.247\, pb} + \underset{(0.019)}{0.997\, u(-1)}$$

$R^2 = 0.995 \quad SE = 0.0288 \quad DW = 2.396 \quad T = 51$

Now the price of beef enters significantly and with the correct sign, and the residual autocorrelation is greatly reduced. The value of the estimated AR(1) parameter for the disturbance is so close to 1.0, however, that we are led to impose the value 1.0 and estimate the equation in first-difference form. No constant term is included, since it would represent a time trend in the log-levels regression. Our results are:

$$(6)\qquad \Delta q = \underset{(0.150)}{0.711\, \Delta y} - \underset{(0.058)}{0.374\, \Delta p} + \underset{(0.068)}{0.251\, \Delta pb}$$

$R^2 = 0.331 \quad SE = 0.0294 \quad DW = 2.38 \quad T = 51$

Here the R^2 statistic is much smaller, but pertains to a different dependent variable.[10] The SE statistic indicates more informatively that the equation's explanatory power is almost as high as for (5). All variables have the theoretically appropriate signs and there is no strong indication of autocorrelated disturbances. Consequently, we adopt (6) as a promising demand specification to carry into our simultaneous-equation estimation attempts to be made below.

Turning now to the chicken supply function, we begin with a counterpart to (1) above, with pcor representing the real price of corn, an important input price since corn is the

[9] Use of the AR(1) specification for v_t leads to loss of the observation for 1950.
[10] The implied R^2 for q is 0.994. Analogous values for all demand functions below exceed 0.992.

primary grain used as chicken feed. One difference from the demand function is that supply is formulated in aggregate (not per capita) terms, with q^A the aggregate counterpart to q, i.e., $q^A = q + pop$, with pop representing the log of population. The results of this first attempt are as follows.

$$(7) \qquad q^A = 9.185 - 1.203\, p - 0.338\, pcor$$
$$\qquad (0.029) \quad (0.110) \quad (0.075)$$
$$R^2 = 0.942 \qquad SE = 0.1412 \qquad DW = 0.591 \qquad T = 52$$

These clearly indicate the need for respecification, since the chicken price variable enters very strongly with the wrong sign and residual autocorrelation is strong. There are two additions to the list of regressors that suggest themselves readily. The first is a time trend, to represent technical progress that reduces marginal cost for given input prices. The second is the previous period's value of output, again represented by q^A, to reflect adjustment costs that tend to make one period's output positively related to that of the previous period. Thus we enter the variables time and $q^A(-1)$, with their coefficients expected both to be positive (and the second to lie between 0 and 1). We obtain:

$$(8) \qquad q^A = 2.652 - 0.143\, p - 0.029\, pcor + 0.0099\, time + 0.629\, q^A(-1)$$
$$\qquad (0.605) \quad (0.046) \quad (0.019) \qquad (0.0031) \qquad (0.091)$$
$$R^2 = 0.997 \qquad SE = 0.0305 \qquad DW = 2.054 \qquad T = 51$$

These results are clearly more encouraging, since all variables (except for the price of chicken) have the correct sign and there is no evidence of autocorrelated disturbances.[11] Nevertheless, the existence of a USDA Poultry Yearbook price index specifically representing feed for young chickens suggests that it be used in place of the price of corn, even though observations are available only for 1960-1999.[12] With that one change, the estimated supply function becomes:

(9) $q^A = 2.478 - 0.041\ p - 0.083\ pf + 0.0102\ time + 0.647\ q^A(-1)$

$$\begin{array}{ccccc} (0.698) & (0.052) & (0.032) & (0.0038) & (0.108) \end{array}$$

$R^2 = 0.997$ SE = 0.0252 DW = 1.883 T = 39.

These results are encouraging. All variables but one enter significantly and with the proper sign, the exception being the troublesome price of chicken. Even with that variable there is improvement relative to (8) since its coefficient is now insignificant (its t statistic is smaller than 1). There is no sign of autocorrelated residuals and the equation's explanatory power is good. Consequently, we suggest that relations (6) and (9) should provide a good starting point for our exercise in simultaneous equation estimation of demand and supply functions for broiler chickens in the United States.

5. Simultaneous Equation Estimates

Our first step is to obtain two-stage least squares estimates of equations (6) and (9). Because of the presence of the pf variable, the data sample will be limited to 1960-1999. The first-stage regressors, often termed instruments, include a constant, time, $q^A(-1)$, pf, Δy, Δpb, Δpop, and p(-1), the latter two included because of the identities $\Delta q = q^A - q^A(-1) - \Delta pop$ and $\Delta p = p - p(-1)$. The estimates are:

(10) $\Delta q = 0.843\ \Delta y - 0.404\ \Delta p + 0.279\ \Delta pb$

$$\begin{array}{ccc} (0.143) & (0.086) & (0.093) \end{array}$$

$R^2 = 0.291$ SE = 0.0253 DW = 1.929 T = 40

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[11] Of course the DW statistic is often biased toward 2.0 when a lagged value of the dependent variable is included as a regressor. Accordingly, in all subsequent equations, we have conducted a Breusch-Godfrey LM tests with two lags—and have obtain results indicative of no significant autocorrelation.

$$(11) \quad q^A = 2.371 + 0.105\ p - 0.113\ pf + 0.0123\ time + 0.640\ q^A(-1)$$

$$(0.773) \quad (0.077) \quad (0.037) \qquad (0.0043) \qquad (0.119)$$

$$R^2 = 0.996 \qquad SE = 0.0279 \quad DW = 1.869 \quad T = 40$$

Here the results are almost what we would have hoped for. All of the seven parameter estimates are of the theoretically appropriate sign and six are clearly significant. The coefficient on the price of chicken in the supply function is still the weakest link, but now the sign of the estimate is positive and the t-ratio is 1.36. The equations' SE values remain low and the DW statistics are close to 2.0. All in all, these two equations come close to providing the type of result mentioned in our introduction, namely, a supply-demand example featuring actual data in which structural estimation methods are shown to yield more plausible estimates than ordinary least squares.

Consideration of the recalcitrant supply price elasticity has led us, however, to consider a slight extension of the model. The basic problem, we believe, is that the quantity variable used in both relations is the quantity of chicken consumed. That is appropriate for the demand function, but in the supply function the variable should instead reflect quantity produced. Broiler inventory stocks are not so large as to make their neglect implausible, at least with annual data, but in recent years the United States has begun to export a rather substantial fraction of broiler production. In 2001, for example, exports amounted to approximately 17 percent of production.[13] Accordingly, we wish to re-estimate relations (10) and (11) with $qprod^A$, the log of broilers produced, used in place of q^A in the supply function.

As an approximation, we initially take broiler exports to be exogenous and thus use

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[12] The log of the broiler grower feed variable is denoted pf. We know that the slight model specifications to be introduced in the next section will necessitate limitation of the sample period for additional reasons.

the variable expts = $qprod^A - q^A$ as a first-stage regressor. The lagged value $qprod^A(-1)$ is added to that list, while Δpop and $q^A(-1)$ continue to belong as well because of the identity $\Delta q = qprod^A - (qprod^A - q^A) - q^A(-1) - \Delta pop$. The two-stage least squares estimates for 1960-1999 are as follows:

$$(12) \quad \Delta q = \underset{(0.142)}{0.841}\, \Delta y - \underset{(0.086)}{0.397}\, \Delta p + \underset{(0.093)}{0.274}\, \Delta pb$$

$R^2 = 0.299$ SE = 0.0251 DW = 1.920 T = 40

$$(13) \quad qprod^A = \underset{(0.695)}{2.030} + \underset{(0.106)}{0.221}\, p - \underset{(0.052)}{0.146}\, pf + \underset{(0.0063)}{0.0184}\, time + \underset{(0.125)}{0.631}\, qprod^A(-1)$$

$R^2 = 0.996$ SE = 0.0351 DW = 2.011 T = 40

Here the only substantial change from equations (10) and (11) is that the main weakness of the latter has been eliminated: the chicken price variable now enters the supply function with a positive coefficient and a t-ratio in excess of 2.0, indicating statistical significance.

Exports of chicken are not truly exogenous, of course. We suggest, however, that to a great extent the major trends and fluctuations in the quantity of chicken exports over our sample period have been due to improvements in shipping technology and to altering political relationships involving the two main foreign markets for U.S. chicken, Russia and Hong Kong.[14] Nevertheless, we have estimated relationships that differ from (12) and (13) in that the chicken export variable is not included in the list of first stage regressors, but is replaced with U.S. exports of meat (beef, veal, and pork)—a variable that should be more nearly exogenous. The results are so nearly the same as in (12) and (13) that there seems to

[13] Furthermore, the boneless-equivalent measure is not as well suited for production as for consumption.
[14] Over the period 1995 to 1999, the Russian Federation was the largest market for U.S. chicken exports, accounted for more than 30% of the total whereas there were no U.S. chicken exports to the Russion Federation

be no point in taking the space to report them.

6. Some Illustrative Plots

To illustrate our results, we plot supply and demand functions implied by our estimated equations. We begin by deriving the demand function in levels that is implied by our equation in first differences. Neglecting error terms, the latter is:

$\Delta q_t = \alpha_y \Delta y_t - \alpha_p \Delta p_t + \alpha_{pb} \Delta pb_t$.

For any variable, z: $\sum_{s=0}^{s=t} \Delta z_s = z_t - z_0$. Thus, summing both sides of the preceding equation over the interval 0 to t, our demand function for date t can be rewritten as

$q_t - q_0 = \alpha_y(y_t - y_0) - \alpha_p(p_t - p_0) + \alpha_{pb}(pb_t - pb_0)$.

Let $\alpha_0 = q_0 - \alpha_y y_0 - \alpha_p p_0 - \alpha_{pb} pb_0$. Using our estimated coefficients from equation (14) and the values of the variables q, y, p, and pb from 1959, we estimate α_0 to be –4.507.

Solving for p and substituting in the estimated coefficients, we obtain an equation for the demand curve at date t. We choose to plot demand and supply curves in conventional rather than log units. Accordingly, we write the demand curve in terms of Q and P:

(14) $\ln(P) = [\ln(Q) - (-4.507 + 0.841 y_t + 0.2775\, pb_t)]/(-0.397)$.

Here we have deleted the t subscripts on P and Q since the set of (Q, P) pairs that satisfy this equation constitute the date t demand curve. To plot the demand curve for date t, we simply insert the observed values of the exogenous variables y_t and pb_t for date t .

To plot the long-run supply function, we set $qprod^A = qprod^A(-1)$ and solve equation (13) for p_t as follows:

(14) $p_t = [(1 - 0.631) qprod^A_t - (2.030 - 0.146\, pf_t + 0.0184\, t]/0.221$.

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until the early 1990's. Hong Kong was the second largest market for chicken exports in the 1995 to 1999 period, accounting for 20% of U.S. chicken exports.

In order to plot the supply and demand functions using common variables, we rewrite the preceding equation in terms of per capita domestic supply using the identity: $QPROD^{A}_{t} = Q_tN_t + X_t$ where N_t denotes (unlogged) population and X_t denotes observed exports at date t. The long-run supply curve for date t with price as a function of quantity per capita is then the set of (Q, P) pairs that satisfy:

(15) $\ln(P) = [(1-.631)\ln(N_t{*}Q+X_t) - (2.030 \; - 0.146\, pf_t + 0.0184\, t]/0.221.$

To obtain the short-run supply curve for a given date, we set $qprod^{A}(-1)$ equal to the value of $qprod^{A}$ that equates demand and long run supply—the intersection of the two curves in equations (14) and (15). Let $qprod_t^{A*}$ denote this long-run equilibrium value. Then the short-run supply curve for date t is the set of (Q, P) pairs that satisfy:

(16) $\ln(P) = [\ln(N_t{*}Q+X_t) - (2.030 \; - 0.146\, pf_t + 0.0184\, t + 0.631\, qprod_t^{A*})]/0.221.$

Using equations (14) through (16), we plot in Figure 1 the demand curve and both short- and long-run supply curves for 1995. As the reader will see, this plot nicely conforms to the usual textbook depiction of the demand curve and short- and long-run supply curves. To illustrate the shifting of demand and supply curves over time that results from changes in the exogenous variables, we plot in Figure 2 the demand and long-run supply curves for 1960 and 1995.[15] The outward shift of demand from 1960 to 1995 is due to an increase in real per capita income of roughly 225% over this period. The price of the substitute good, beef, decreased by roughly 25% over this period. While this decline in price of the substitute offset a portion of the growth in demand for chicken, this effect is relatively modest compared to the effect of growing per capita income. The outward shift in the supply curve is a result of a fall in the price of the primary input (chicken feed) by roughly 50%, and to a substantial

[15] In the interest of clarity of the diagram, we omit the short-run supply functions.

Figure 1

DEMAND AND SUPPLY CURVES, 1995

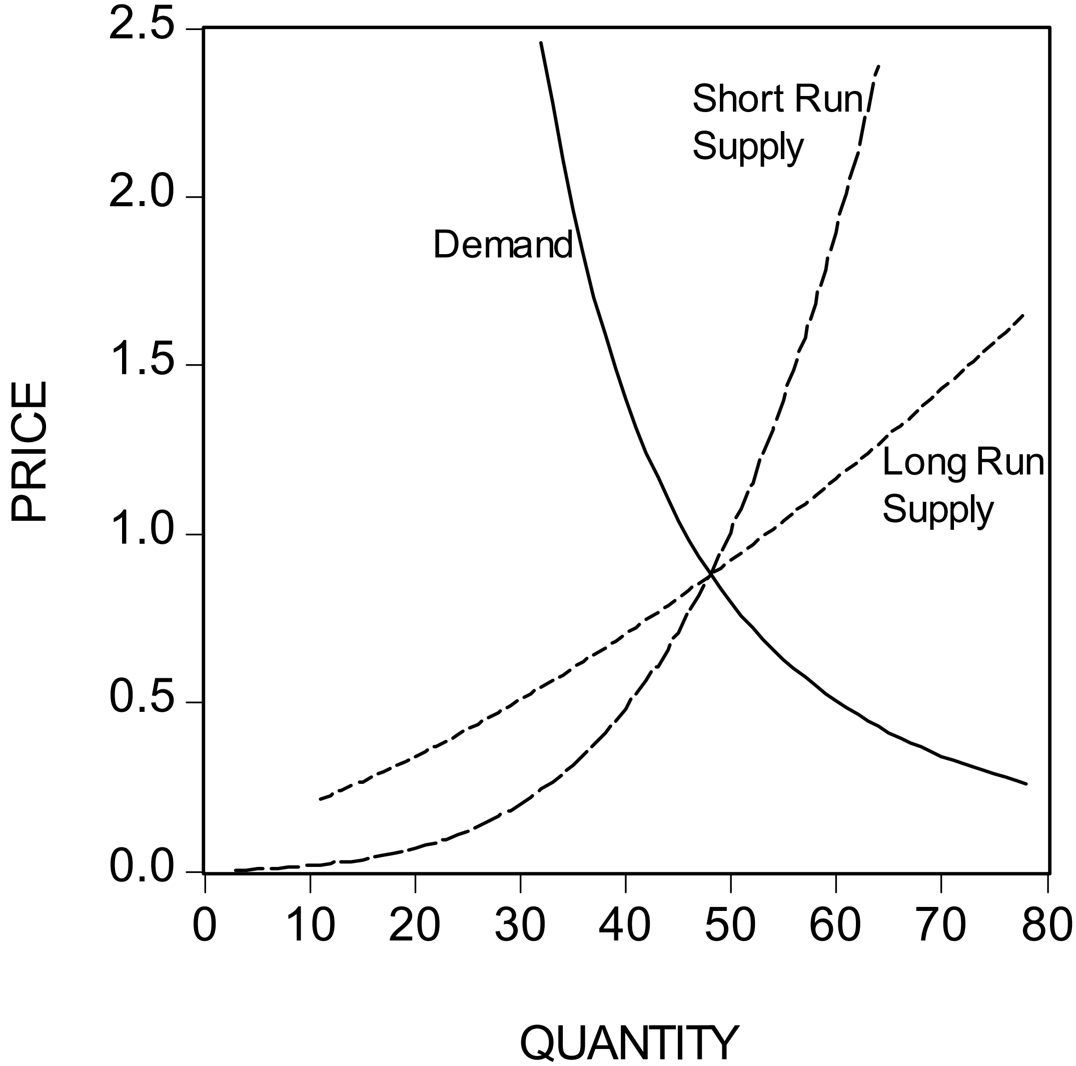
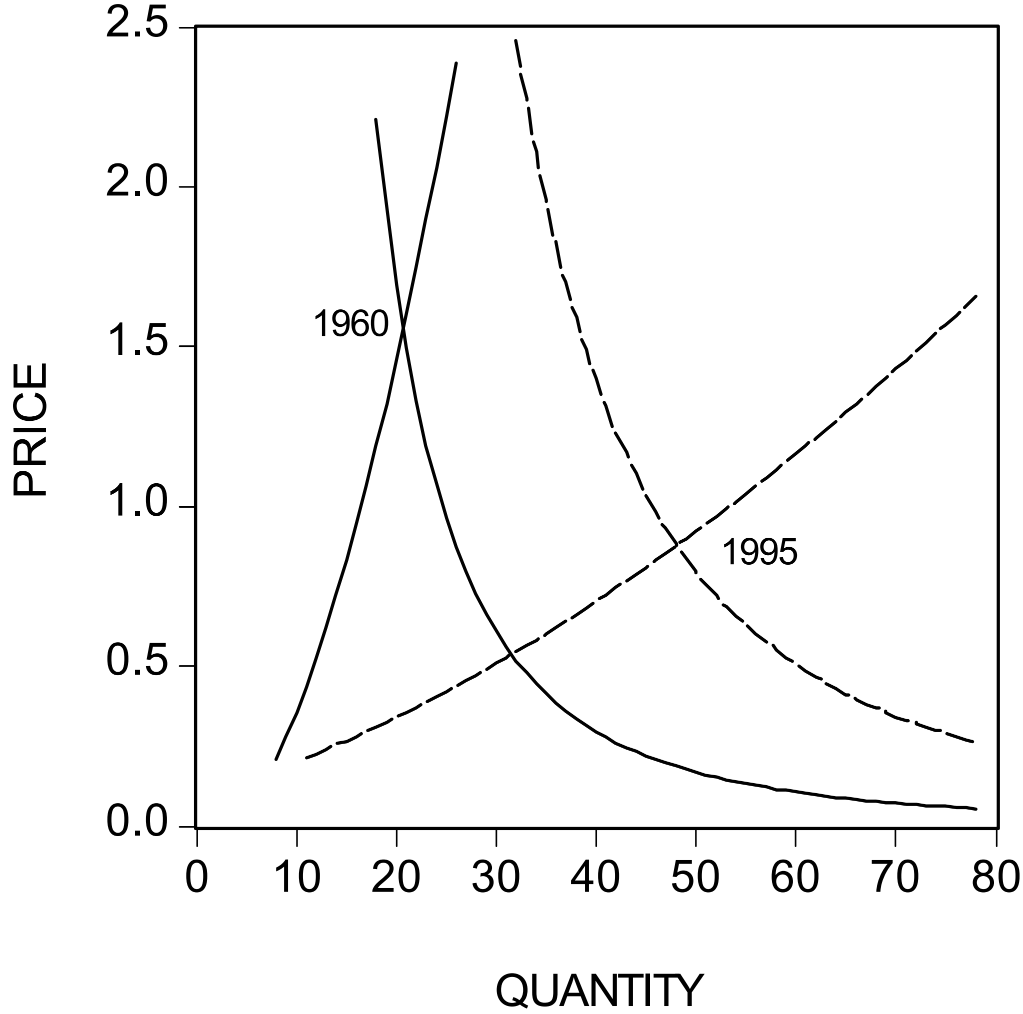


Figure 2

DEMAND AND LONG RUN SUPPLY CURVES
1960 AND 1995



productivity increase in chicken production. The latter is captured by the coefficient of 0.0183 on the time variable in the supply function. As is evident from Figure 2, the outward shift in supply was more rapid than the outward shift in demand, leading to a substantial fall in the real price of chicken over the 35-year time interval.[16]

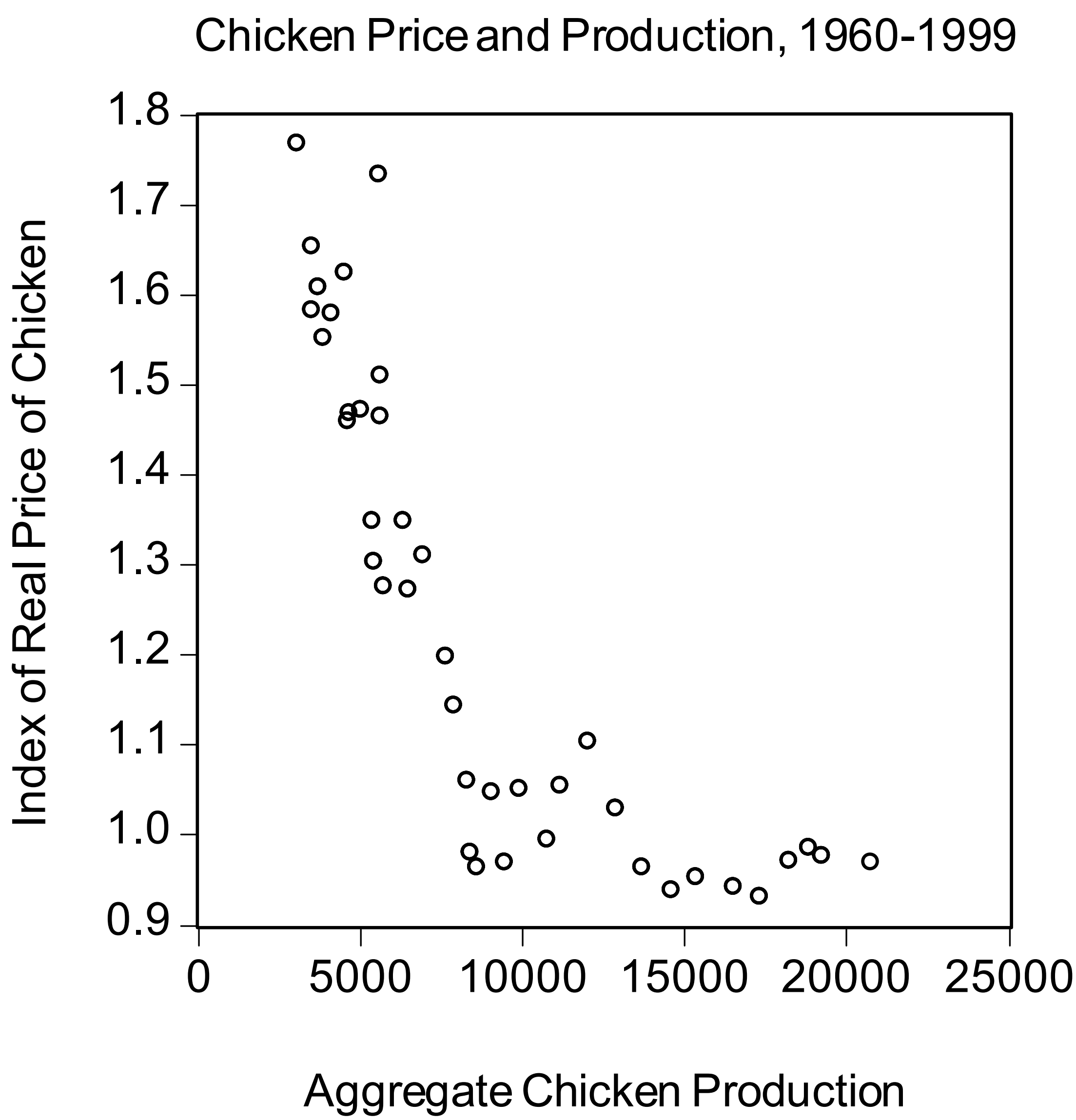
7. Conclusion

The model in equations (12) and (13) meets the objectives we set forth at the outset. The estimated demand coefficients imply an own-price elasticity of –0.40, an income elasticity of 0.84, and a cross-price elasticity with respect to the substitute good (beef) of 0.274. These are of the expected algebraic signs and strike us as being quite reasonable in magnitude. The short-run own-price elasticity of supply is 0.22, and the short-run elasticity of supply with respect to the price of the primary input (feed) is –0.15. The corresponding long-run elasticities are 0.60 and –0.40, respectively. Again, these are of the expected algebraic signs and seem to be quite plausible in magnitude. The estimates also imply a substantial rate of growth of productivity in chicken production. In particular, the short-run supply curves exhibits a shift of 1.84% in each one-year interval, holding constant previous-year production. The long-run supply curve shifts outward by about 5% per year. This rapid rate of productivity growth largely accounts for the falling real price of chicken in the face of the prodigious increase in demand observed over our sample period—an increase of 275% in consumption per capita coupled with a 50% increase in population. In addition to being of the correct signs and reasonable magnitudes, all of our coefficient estimates are statistically significant at the conventional 5% level.

[16] While we have plotted our demand and supply curves in per capita terms, it is of interest to note that population increased by almost 50% from 1960 to 1995. Thus the total physical quantities produced and consumed increased by a correspondingly greater proportion than the per capita values shown in our plots.

Our results, particularly for the supply equation, also illustrate the payoff from estimating the equations as a simultaneous system. The single-equation coefficient estimates for the supply equation (9) yield a supply price elasticity that is of the wrong algebraic sign and statistically insignificant. By contrast, the supply equation (13) estimated with two-stage least squares is of the correct sign and statistically significant. This accomplishment of systems estimation is all the more striking when considered in light of the plot of price versus quantity produced (Figure 3), which exhibits a pronounced inverse relationship between the two. Despite that strong negative relationship, the systems approach produces an estimated supply function in which quantity produced is an increasing and statistically significant function of price.

Figure 3


Chicken Price and Production, 1960-1999
Index of Real Price of Chicken
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0
5000
10000
15000
20000
25000
Aggregate Chicken Production

Appendix A

Data from French (1949)

Table 1. The Basic Data of the Statistical Analysis of the Demand for Meat

Year	(a) Y1t Per Capita Consumption of Meat Total (pounds)	(b) Y2t Price of Meat (1935-39 = 100)	(c) Y3t Price of Other Food (1935-39 = 100)	(d) Y4t Price of Non-Food (1935-39 = 100)	(e) Y5t Per Capita Disposable Income (Dollars)	Z1t Time	(f) Z3t Prices Paid to Farmers for Meat Lagged One Year (1935-39 = 100)	(g) Z4 Prices Paid to Farmers For Food Lagged one Year (1935-39 = 100)	(h) Z5 Wage Rates In Industry Average Hourly Earnings (1935-39 = 100)	(i) Z6 Per Capita Investment Expenditures (dollars)	(j) Z7 Cost of Processing Meat (1935-39 = 100)	(k) Z8 Cost of Agricult Producti (1935-39
1919	171.5	101.1	120.3	87.2	506.	1	159.3	188.4	64.4	123.8	107.8	119.3
1920	167.0	88.0	117.8	88.4	454.	2	141.0	175.1	64.7	143.3	98.3	111.2
1921	164.5	84.1	100.5	97.8	398.	3	101.8	147.3	67.4	127.7	88.8	94.0
1922	169.3	85.3	100.2	99.1	449.	4	70.6	98.5	67.4	119.7	80.7	99.5
1923	179.4	83.0	101.7	98.7	502.	5	80.3	113.2	71.6	121.9	82.5	102.0
1924	179.2	83.8	100.5	99.4	499.	6	74.7	122.8	74.9	122.2	78.6	107.0
1925	172.6	88.8	106.0	97.0	507.	7	77.3	126.0	72.9	125.4	80.2	103.9
1926	170.5	93.2	108.7	95.8	514.	8	94.1	119.4	73.9	126.4	77.7	104.5
1927	168.6	93.5	106.7	96.8	519.	9	97.4	108.1	74.3	124.0	80.2	106.2
1928	164.7	100.4	106.7	96.7	530.	10	95.9	110.3	76.8	122.6	80.8	111.8
1929	163.0	103.8	108.2	96.0	553.	11	106.6	120.4	77.3	24.8	81.0	110.9
1930	162.1	99.7	105.5	97.2	502.	12	110.1	113.7	77.3	19.8	81.4	103.2
1931	160.2	93.0	95.6	102.2	467.	13	95.3	100.3	79.3	13.3	77.9	90.4
1932	161.2	81.3	88.6	105.7	392.	14	72.1	79.9	76.4	- 11.5	73.5	81.8
1933	165.8	74.6	91.0	103.5	390.	15	56.1	65.4	80.0	- 10.4	76.6	82.6
1934	163.5	82.4	97.9	101.0	426.	16	55.6	81.5	93.0	- 1.8	99.2	86.2
1935	146.7	101.8	102.3	98.8	465.	17	61.7	102.0	93.8	14.4	102.6	90.9
1936	160.2	99.8	102.2	98.0	521.	18	99.7	109.0	93.8	28.4	94.6	98.3
1937	156.8	103.0	102.5	98.7	537.	19	100.4	110.7	101.7	29.5	107.8	104.1
1938	156.8	88.2	97.0	101.5	500.	20	108.4	101.2	104.0	7.6	100.1	99.4
1939	165.4	97.2	95.8	102.1	539.	21	96.2	90.0	106.6	20.7	94.8	106.8
1940	174.7	95.6	96.4	101.8	572.	22	95.0	89.3	110.4	27.9	92.2	113.0
1941	178.7	102.2	100.3	99.8	656.	23	93.4	97.4	115.9	69.9	96.5	124.0
Sum	3822.4	2128.8	2352.4	2263.2	11398.0	276.0	2143.0	2569.9	1917.8	1489.6	2033.7	2351.0
Mean	166.1913	92.3391	102.2783	98.4000	495.5652	12.0000	93.1739	111.7348	83.3826	64.7652	88.4217	102.2174

Footnotes: See page 27a.

Appendix B: Data

YEAR	Q	Y	PCHICK	PBEEF	PCOR
1950.000	14.30000	7863.000	69.50000	31.20000	59.80000
1951.000	15.10000	7953.000	72.90000	36.50000	72.10000
1952.000	15.30000	8071.000	73.10000	36.20000	71.30000
1953.000	15.20000	8319.000	71.30000	28.50000	62.70000
1954.000	15.80000	8276.000	64.40000	27.40000	63.40000
1955.000	14.70000	8675.000	67.00000	27.10000	56.10000
1956.000	16.80000	8930.000	58.80000	26.70000	57.70000
1957.000	17.60000	8988.000	57.30000	28.70000	51.60000
1958.000	19.30000	8922.000	56.60000	33.40000	50.10000
1959.000	19.80000	9167.000	51.60000	34.40000	48.60000
1960.000	19.20000	9210.000	52.40000	33.50000	46.00000
1961.000	20.60000	9361.000	47.40000	33.00000	45.10000
1962.000	20.60000	9666.000	50.00000	34.20000	44.80000
1963.000	21.10000	9886.000	49.30000	33.80000	49.80000
1964.000	21.30000	10456.00	48.20000	32.80000	49.90000
1965.000	22.90000	10965.00	49.80000	34.40000	51.80000
1966.000	24.50000	11417.00	52.70000	36.20000	54.50000
1967.000	25.10000	11776.00	48.80000	36.40000	51.70000
1968.000	25.20000	12196.00	51.20000	37.90000	45.50000
1969.000	26.30000	12451.00	54.10000	41.70000	49.30000
1970.000	27.40000	12823.00	52.40000	43.50000	54.50000
1971.000	27.40000	13218.00	52.90000	45.50000	55.70000
1972.000	28.30000	13692.00	53.40000	49.70000	52.10000
1973.000	27.10000	14496.00	77.10000	59.60000	89.00000
1974.000	27.00000	14268.00	72.30000	61.30000	128.2000
1975.000	26.40000	14393.00	81.40000	61.90000	115.2000
1976.000	28.50000	14873.00	76.90000	59.90000	107.6000
1977.000	29.00000	15256.00	77.30000	59.50000	88.00000
1978.000	30.40000	15845.00	85.60000	73.10000	92.00000
1979.000	32.80000	16120.00	87.20000	93.10000	104.5000
1980.000	32.70000	16063.00	94.40000	98.40000	119.2000
1981.000	33.70000	16265.00	96.50000	99.20000	125.9000
1982.000	33.90000	16328.00	94.80000	100.6000	100.0000
1983.000	34.00000	16673.00	96.30000	99.10000	128.4000
1984.000	35.30000	17799.00	109.0000	100.3000	129.7000
1985.000	36.40000	18229.00	104.5000	98.20000	105.9000
1986.000	37.20000	18641.00	115.4000	98.80000	83.50000
1987.000	39.40000	18870.00	113.3000	106.3000	67.70000
1988.000	39.60000	19522.00	125.1000	112.1000	97.10000
1989.000	40.90000	19833.00	137.1000	119.3000	102.4000
1990.000	42.40000	20058.00	134.9000	128.8000	100.9000
1991.000	44.10000	19873.00	131.7000	132.4000	97.00000
1992.000	46.50000	20220.00	131.9000	132.3000	96.00000
1993.000	48.20000	20235.00	138.0000	137.1000	92.90000
1994.000	48.80000	20507.00	140.1000	136.0000	100.1000
1995.000	48.20000	20798.00	142.2000	134.9000	109.0000
1996.000	48.80000	21072.00	152.6000	134.5000	158.5000
1997.000	49.50000	21470.00	158.5000	136.8000	110.1000
1998.000	49.80000	22359.00	159.6000	136.5000	91.70000
1999.000	52.90000	22678.00	161.8000	139.2000	78.20000
2000.000	53.20000	23501.00	162.9000	148.1000	76.40000
2001.000	53.90000	23692.00	168.0000	160.5000	78.80000

PF	CPI	QPROD[A]	POP	MEATEX	TIME
NA	24.10000	2628500.	151.6840	NA	41.00000
NA	26.00000	2843000.	154.2870	NA	42.00000
NA	26.50000	2851200.	156.9540	NA	43.00000
NA	26.70000	2953900.	159.5650	NA	44.00000
NA	26.90000	3099700.	162.3910	NA	45.00000
NA	26.80000	2958100.	165.2750	NA	46.00000
NA	27.20000	3492200.	168.2210	NA	47.00000
NA	28.10000	3647100.	171.2740	NA	48.00000
NA	28.90000	4144800.	174.1410	NA	49.00000
NA	29.10000	4331118.	177.0730	NA	50.00000
51.53361	29.60000	4333602.	180.6710	50.00000	51.00000
51.86824	29.90000	4944130.	183.6910	49.00000	52.00000
52.09133	30.20000	4997189.	186.5380	46.00000	53.00000
50.97588	30.60000	5269019.	189.2420	80.00000	54.00000
50.75279	31.00000	5443769.	191.8890	78.00000	55.00000
50.97588	31.50000	5871560.	194.3030	49.00000	56.00000
52.48173	32.40000	6437127.	196.5600	44.00000	57.00000
51.86824	33.40000	6552305.	198.7120	45.00000	58.00000
49.52580	34.80000	6653319.	200.7060	59.00000	59.00000
50.36239	36.70000	7174882.	202.6770	87.00000	60.00000
53.15100	38.80000	7686589.	205.0520	49.00000	61.00000
54.54531	40.50000	7723561.	207.6610	57.00000	62.00000
54.82417	41.80000	8146839.	209.8960	76.00000	63.00000
84.66235	44.40000	7961659.	211.9090	119.0000	64.00000
94.03210	49.30000	8034339.	213.8540	76.00000	65.00000
91.13194	53.80000	8019673.	215.9730	120.0000	66.00000
93.86478	56.90000	9012071.	218.0350	184.0000	67.00000
95.25909	60.60000	9279454.	220.2390	180.0000	68.00000
94.42250	65.20000	9902015.	222.5850	204.0000	69.00000
105.5212	72.60000	10926345	225.0550	210.0000	70.00000
115.3371	82.40000	11251965	227.7260	194.0000	71.00000
126.6589	90.90000	11868104	229.9660	239.0000	72.00000
117.0103	96.50000	11995693	232.1880	212.0000	73.00000
124.3165	99.60000	12325516	234.3070	224.0000	74.00000
130.0610	103.9000	12920828	236.3480	226.0000	75.00000
109.7599	107.6000	13519558	238.4660	209.0000	76.00000
104.4615	109.6000	14180145	240.6510	278.0000	77.00000
103.1788	113.6000	15413103	242.8040	326.0000	78.00000
148.3543	118.3000	16006986	245.0210	401.0000	79.00000
143.0559	124.0000	17227111	247.3420	583.0000	80.00000
130.9534	130.7000	18429897	249.9730	564.0000	81.00000
126.6589	136.2000	19591105	253.3360	667.0000	82.00000
125.3761	140.3000	20903765	256.6770	786.0000	83.00000
131.3995	144.5000	22014911	260.0370	780.0000	84.00000
136.4748	148.2000	23666035	263.2260	980.0000	85.00000
138.4826	152.4000	24827130	266.3640	1183.000	86.00000
174.3442	156.9000	26123767	269.4850	1291.000	87.00000
157.7798	160.5000	27041394	272.7560	1443.000	88.00000
128.9456	163.0000	27612361	275.9550	1543.000	89.00000
102.8999	166.6000	29741381	279.1440	1674.000	90.00000
NA	172.2000	30495171	282.4890	1703.000	91.00000
NA	177.1000	NA	286.3620	1737.000	92.00000

Description of variables:

Q: per-capita consumption of chicken, pounds, boneless equivalent (USDA data system)

Y: per-capita real disposable income, chain-linked prices, 1996 = 100 (BEA)

PCHICK: CPI index for whole fresh chicken, 1982-1984 = 100 (Bureau of Labor Statistics)

PBEEF: CPI index for beef, 1982-84 = 100 (Bureau of Labor Statistics)

PCOR: PPI index for corn, 1982 = 100 (Bureau of Labor Statistics)

PF: nominal price index for broiler feed, scaled to imply 1982-84 = 100 (USDA, Poultry Yearbook, 2000)

CPI: Consumer price index, 1982-84 = 100 (Bureau of Labor Statistics)

$QPROD^{A}$: Aggregate production of young chicken, pounds (USDA, Poultry Yearbook, 2000)

POP: U.S. population on July 1, resident plus armed forces, millions (Bureau of Labor Statistics)

MEATEX: exports of beef, veal, and pork, pounds (USDA)

TIME: as used in regressions (TIME = 0 for 1909, TIME = 1 for 1910, …)

PC = PCHICK/CPI

PB = PBEEF/CPI

Note: Capitalized variables are not logarithms.

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