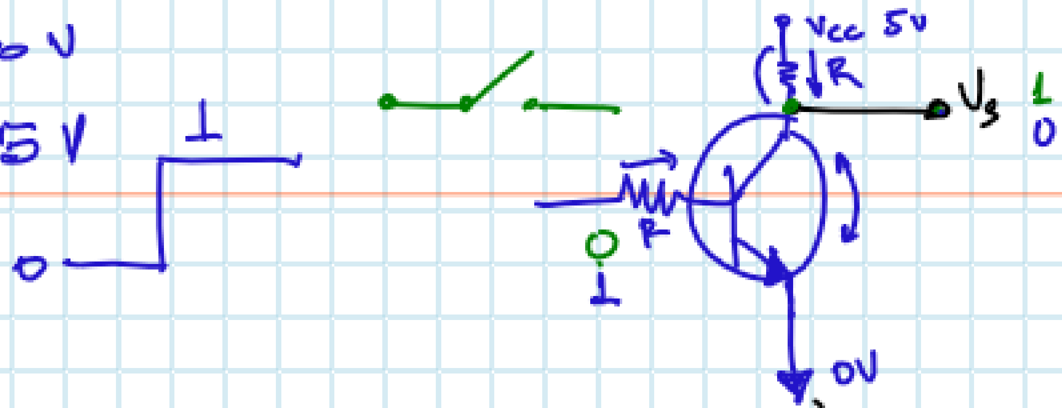
TTL $\begin{cases} |0\rangle : 0\,V \\ |1\rangle : 5\,V \end{cases}$



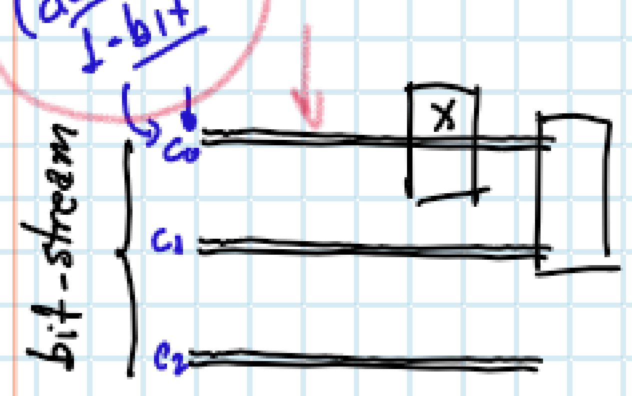
Circuitos Quânticos

- circ. clássicos $\Rightarrow \{ |0\rangle, |1\rangle \Rightarrow (b_j) = \{0,1\} \in \mathbb{R} \}$

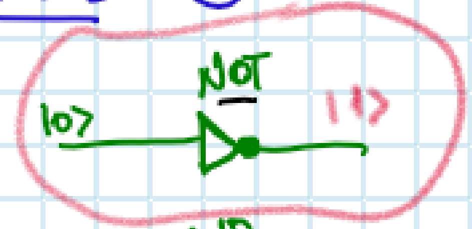
↑ notação p/ estados

(clássico) 1-bit

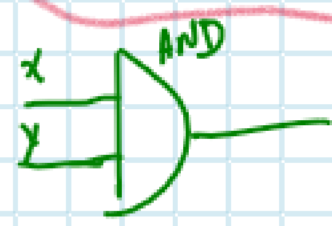
3-bit : $\{ |000\rangle$



Exemplos

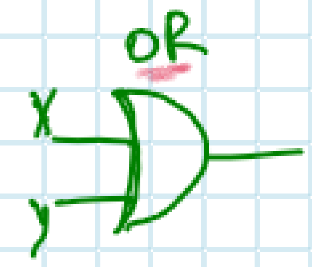


x	(NOT x)
0	1
1	0



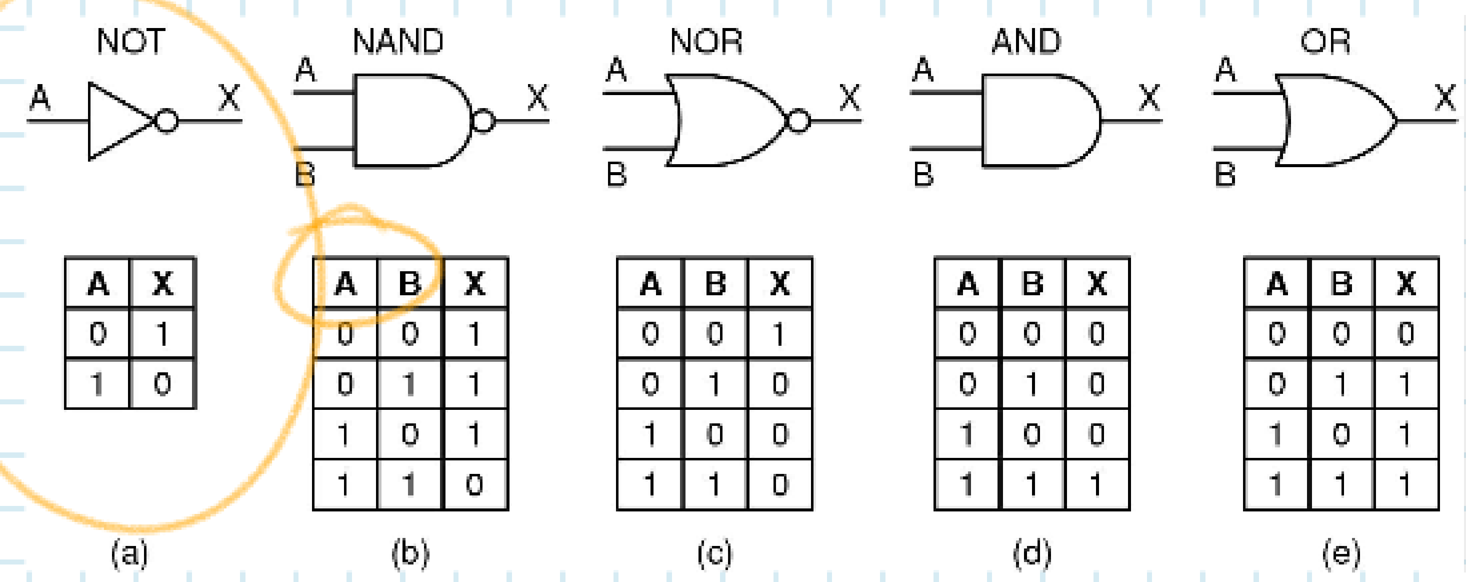
x y	x (AND) y
0 0	0
0 1	0
1 0	0
1 1	1

$\rightarrow x \cdot y =$



$x + y$

x y	
0 0	0
0 1	1
1 0	1
1 1	1



A	X
0	1
1	0

(a)

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

(b)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(c)

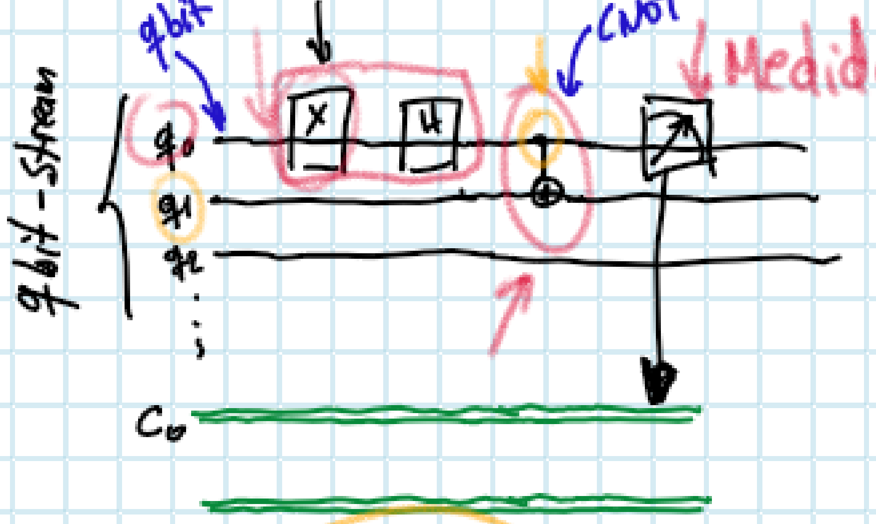
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

(d)

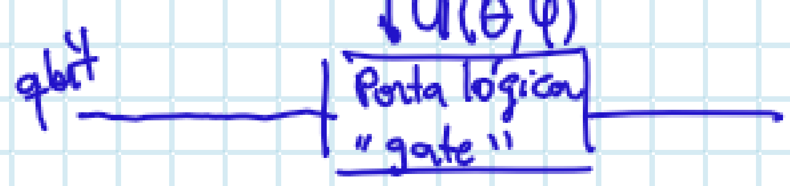
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(e)

- Circ. Quânticos $\Rightarrow \{ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \Rightarrow \alpha, \beta \in \mathbb{C}^2 \}$



op. unitária



$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \sim U(\theta,\varphi) = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$$

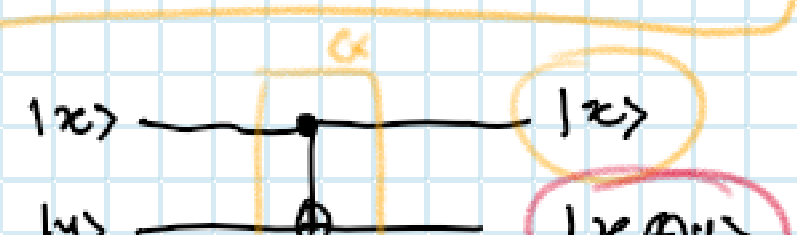
qbits xy	
00	00
01	01
10	11
11	10

$|00\rangle \rightarrow$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \text{“}\mathbb{1} \otimes \sigma_x\text{”} \rightarrow |0\rangle\langle 0| + |0\rangle\langle 0| + |1\rangle\langle 1| + |1\rangle\langle 0|$$

Control - NOT

CX



XOR

Reversível!!

$$\text{CNOT} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$0 \oplus 0 = 0$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

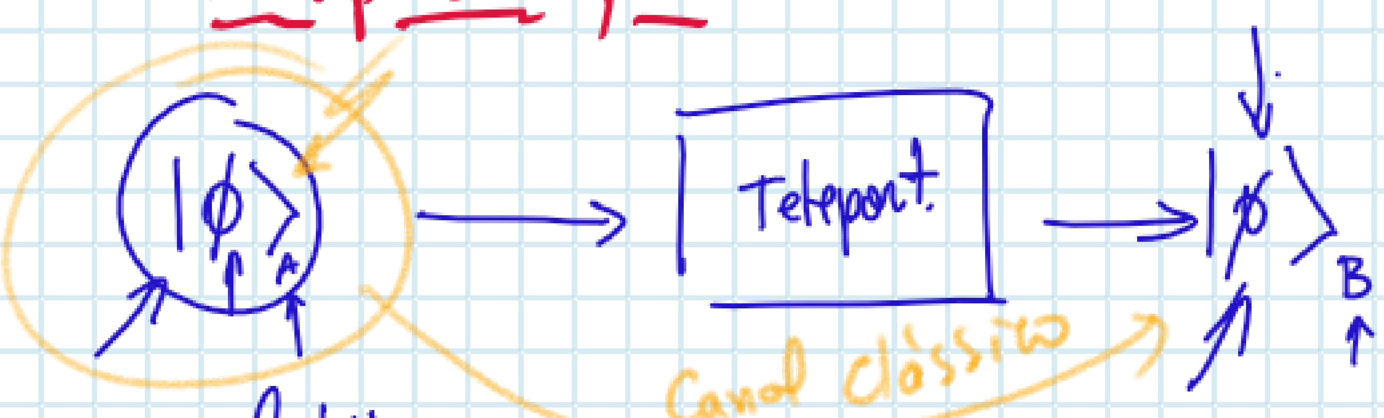
$1 \oplus 1 = 0$

Exemplo: teleportação

Motivação : "No-Cloning theorem"

$\Rightarrow$ Cópia de informação quântica NÃO é permitida

Mas é possível "teleportar" estado quântico p/ qbit remoto



$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

Emaranhamento

Q^do "$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$"

↳ ñ pode ser escrito como um estado produto

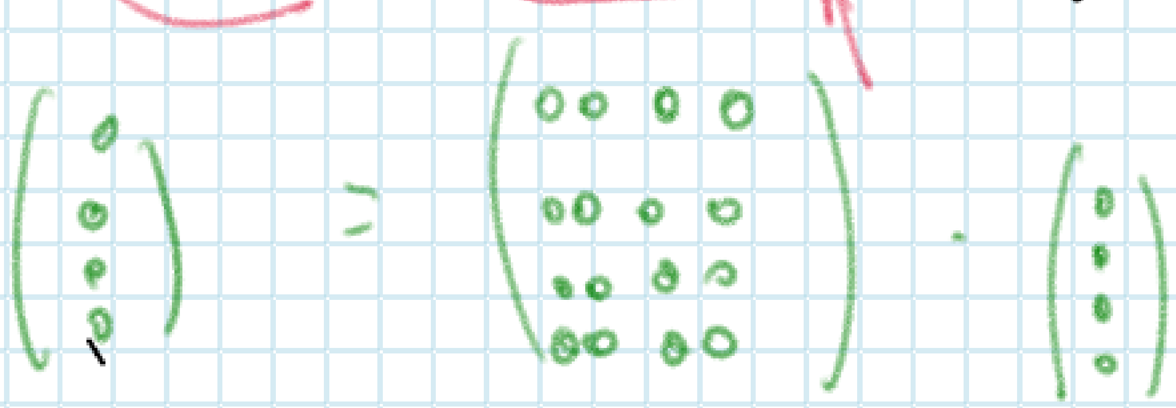
Estados de Bell

: Estados maximamente emaranhados!!

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right); \qquad |\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right); \qquad |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

$$|\beta_{ij}\rangle = \boxed{\mathbb{1} \otimes \sigma_x^{j}\, \sigma_z^{i}}\, |\beta_{00}\rangle$$



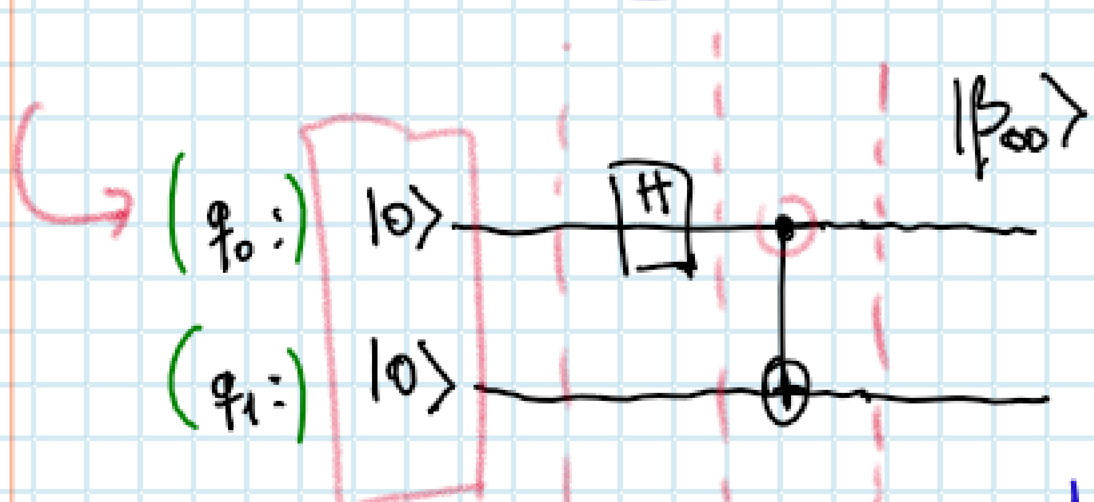
Exemplos

$|\beta_{00}\rangle = \mathbb{1} \times \mathbb{1}$ ($i=0$, $j=0$)

$|\beta_{01}\rangle = \mathbb{1} \otimes \mathbb{1}\, \sigma_x\, |\beta_{00}\rangle$ ($i=0$, $j=1$)

$= \mathbb{1} \otimes \sigma_x\, |\beta_{00}\rangle$

↳ Como criar esses estados?



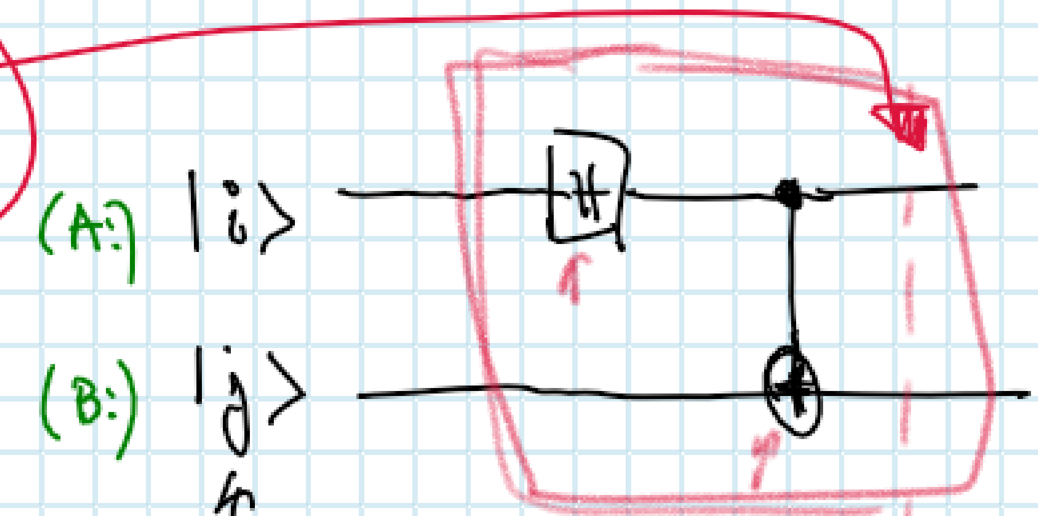
$|00\rangle$

$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes |0\rangle$

$\frac{1}{\sqrt{2}}(|00\rangle+|10\rangle)$

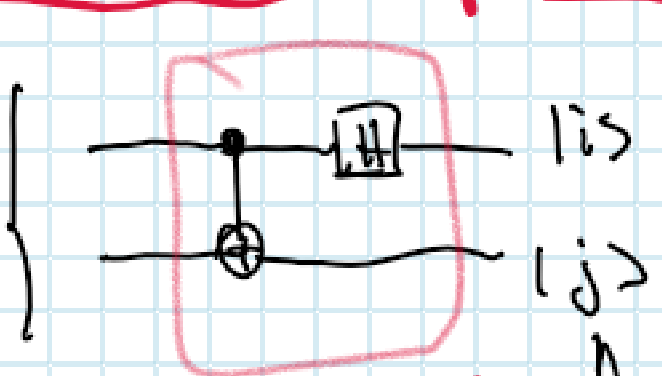
$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$

$\Rightarrow$ $|\beta_{ij}\rangle_{AB}$



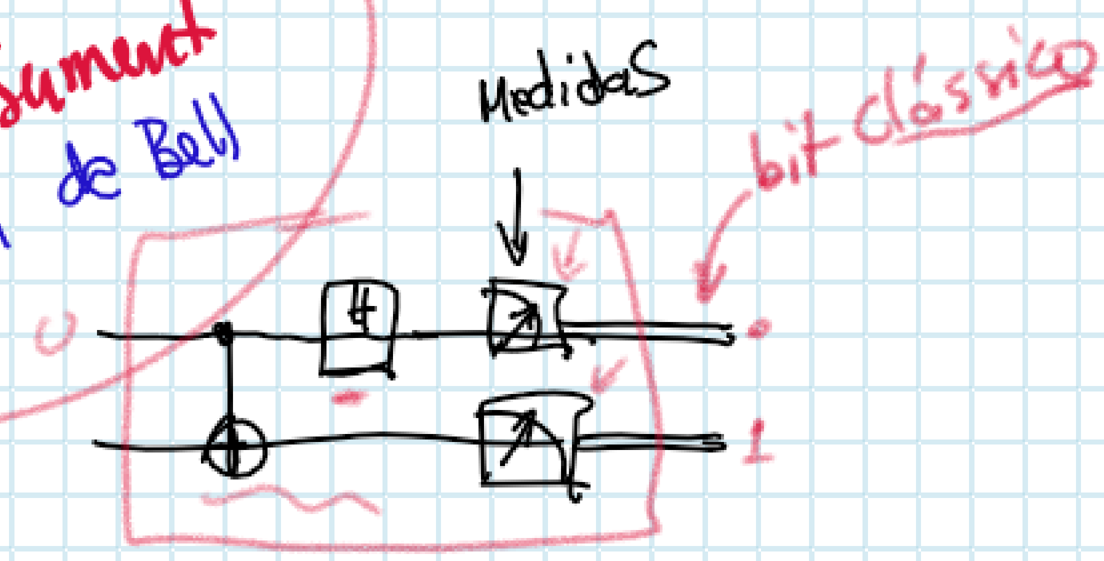
INICIAL	"FINAL"
$\|00\rangle$	$\|\beta_{00}\rangle$
$\|01\rangle$	$\|\beta_{01}\rangle$
$\|10\rangle$	$\|\beta_{10}\rangle$
$\|11\rangle$	$\|\beta_{11}\rangle$

↳ Invertendo a operação



"Bell Measument"

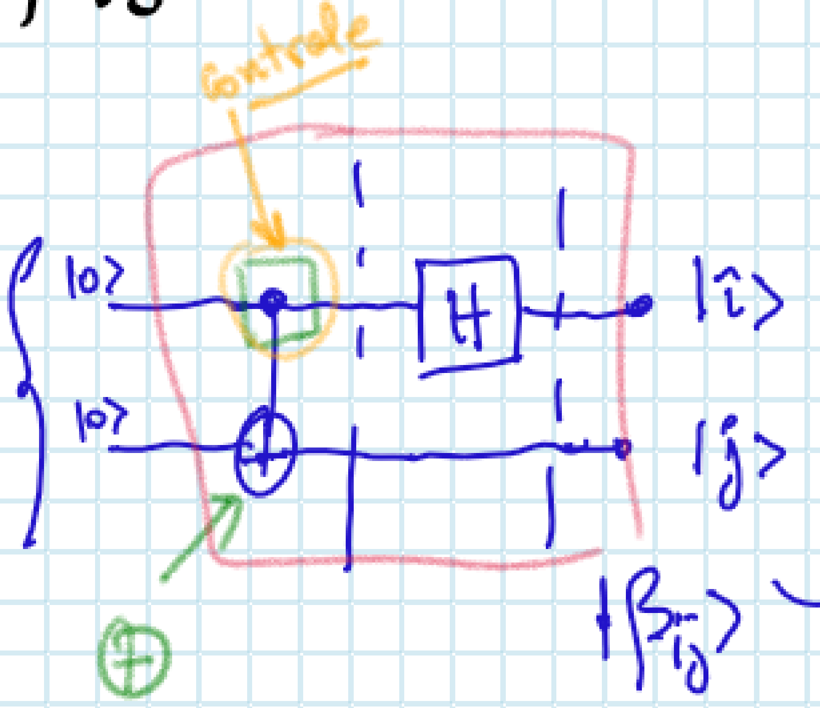
Medida de Bell

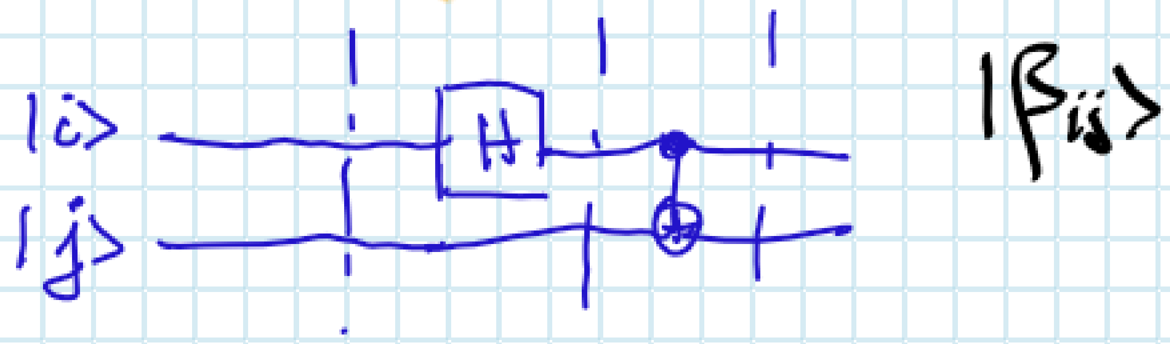


Base de Bell

$$\begin{cases} |\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right); & |\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right); \\ |\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right); & |\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right); \end{cases}$$

$$|\beta_{ij}\rangle = \left(\mathbb{1} \otimes \sigma_x^{j} \sigma_z^{i}\right) |\beta_{00}\rangle$$





$|00\rangle \rightsquigarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$|01\rangle \rightsquigarrow (|01\rangle + |10\rangle)/\sqrt{2}$

$\vdots \quad \rightsquigarrow |\beta_{ij}\rangle$

$\oplus$

$0+0=0$
$0+1=1$
$1+0=1$
$1+1=0$

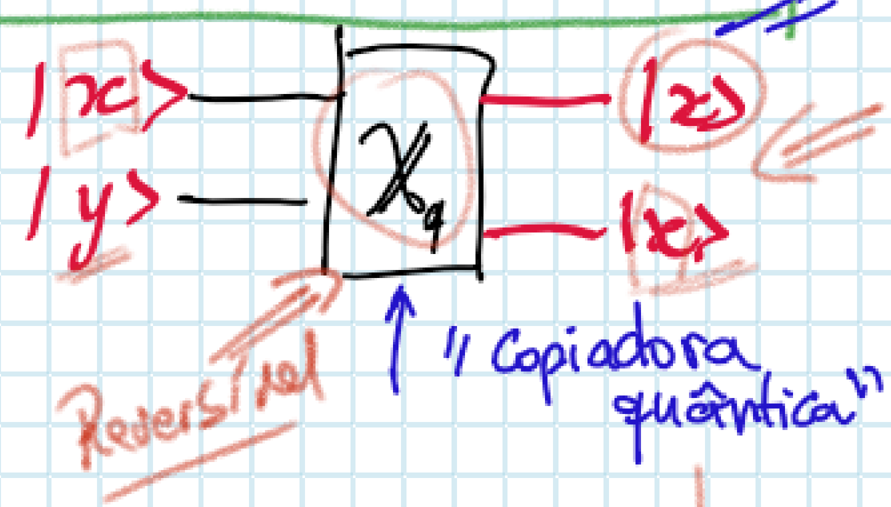
Ex.1 Protocolo de teletransporte

Alice ← ~~Bob~~ Beto
A B

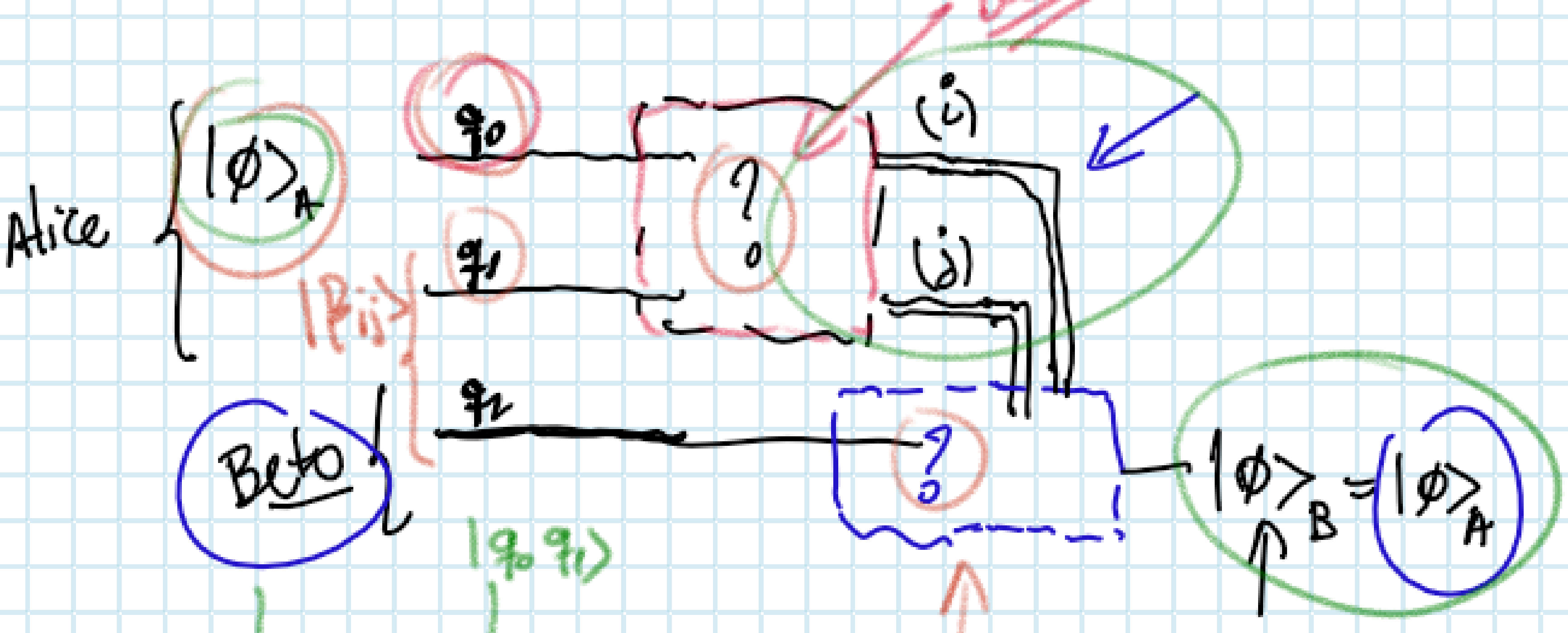
* objetivo: Alice que enviar seu estado $|\phi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$ p/ Beto.
→ Ambos devem compartilhar qbits emaranhados

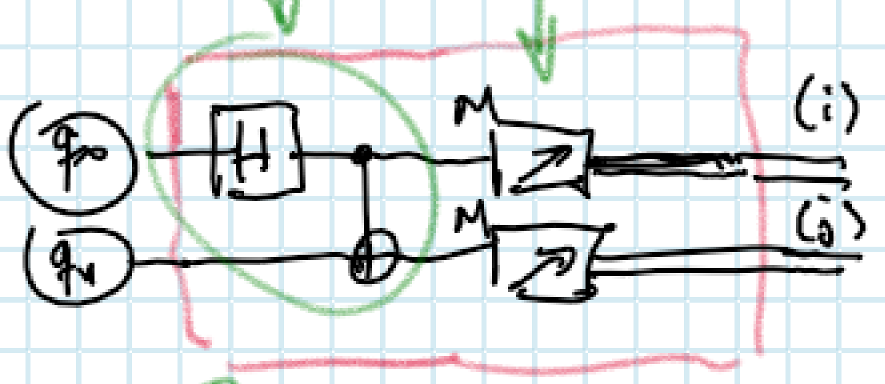
No "cloning" theorem
⇝ Não é possível copiar um qbit

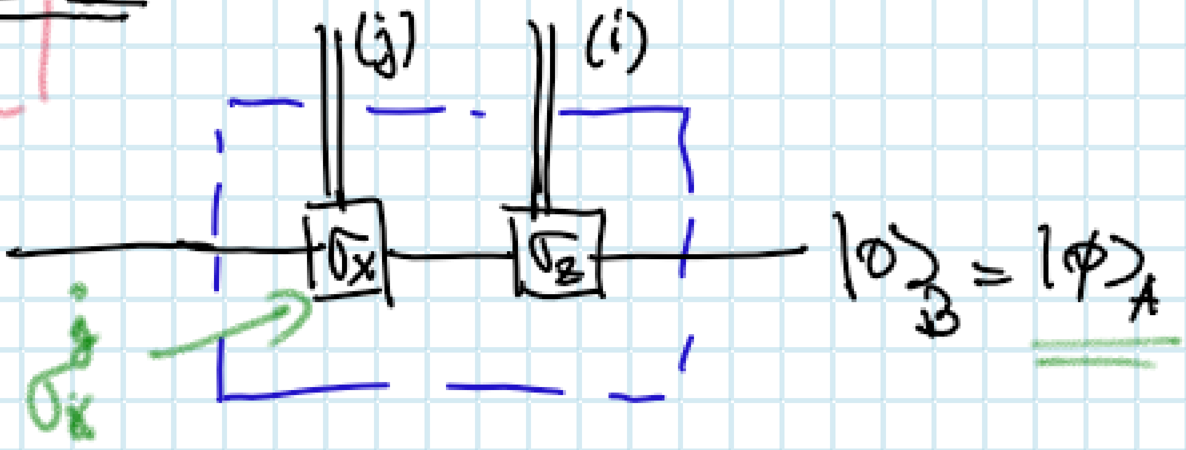
Compartilham $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$



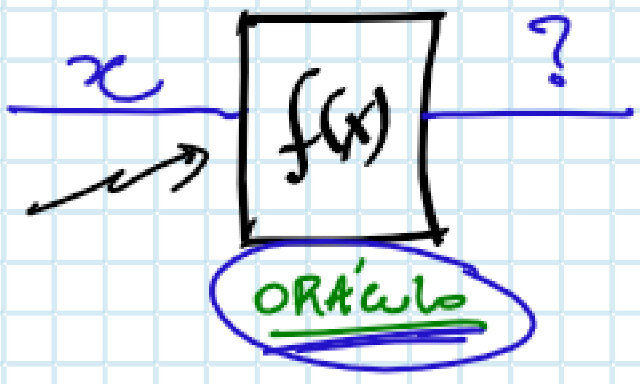
$|x\rangle|y\rangle \longrightarrow |x\rangle|x\rangle$







⊛ Problema de Deutsch-Jozsa:



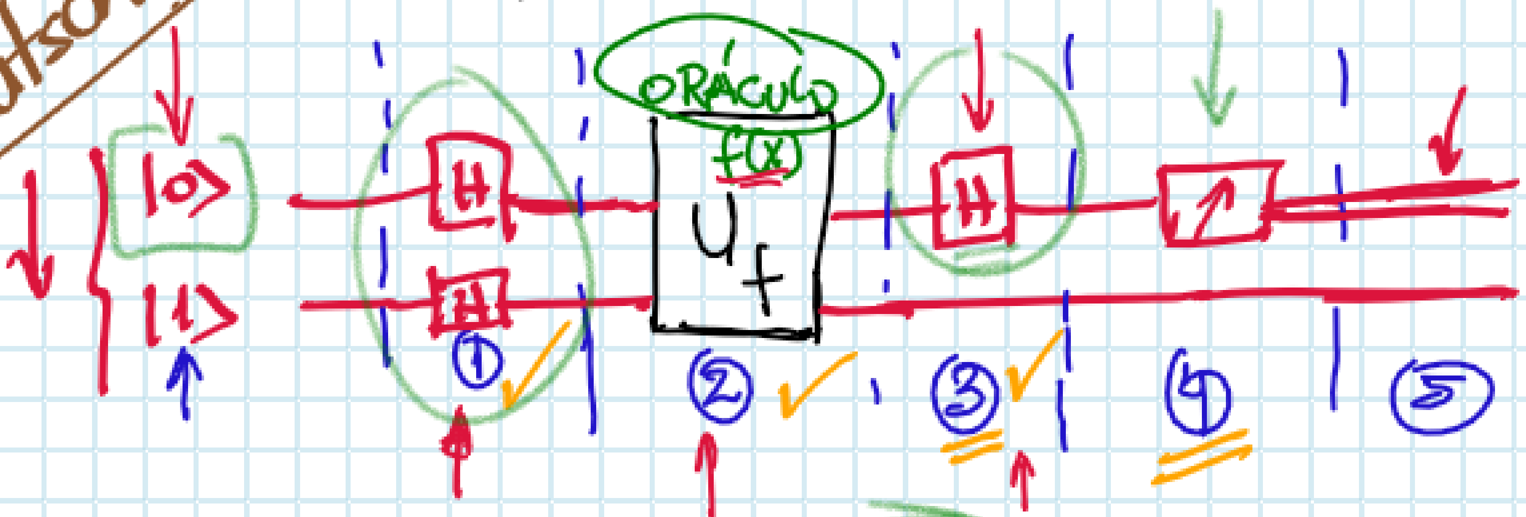
Problema: $f(x): \{0,1\} \to \{0,1\}$

$\boxed{f(0) = f(1)\ ?}$ → *função é constante?

* n - qbits
* 2^n possibilidades

Soma Módulo 2 → $\begin{cases} 0 \oplus 0 = 0 \\ 0 \oplus 1 = 1 \\ 1 \oplus 0 = 1 \\ 1 \oplus 1 = 0 \end{cases}$

* Deutsch
1-qbit



$|01\rangle = |0\rangle|1\rangle$

Início: $|\psi\rangle = |01\rangle = |0\rangle \otimes |1\rangle = |0\rangle|1\rangle$

①: $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$

$= \frac{1}{2}\left[|0\rangle(|0\rangle - |1\rangle) + |1\rangle(|0\rangle - |1\rangle)\right]$

↑ qbit de controle

②: $|\psi\rangle = \frac{1}{2}\Big[|0\rangle\big(|f(0)\oplus 0\rangle - |f(0)+1\rangle\big) +$
$|1\rangle\big(|f(1)\oplus 0\rangle - |f(1)+1\rangle\big)\Big]$

$= \frac{1}{2}\left[(-1)^{f(0)}|0\rangle(|0\rangle - |1\rangle) + (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)\right]$

$= \frac{1}{2}(-1)^{f(0)}\Big[\underbrace{|0\rangle + (-1)^{\boxed{f(0)\oplus f(1)}}|1\rangle}_{1^{\circ}\text{ qbit}}\Big]\underbrace{(|0\rangle - |1\rangle)}_{2^{\circ}\text{ qbit}}$

③ $|\psi\rangle = \frac{1}{2}\left(1 + (-1)^{f(0)\oplus f(1)}\right)|0\rangle$
$+ \frac{1}{2}\left(1 - (-1)^{f(0)\oplus f(1)}\right)|1\rangle$

} estado definido* por $f(0)\oplus f(1)$

④ Se Medida for 0 : $\boxed{f(0) = f(1)}$
1 : $\boxed{f(0) \neq f(1)}$

Note que:

$\boxed{f(0)\oplus f(1)} = \begin{cases} 0 & \text{iguais} \\ 1 & \text{diferent} \end{cases}$

* Ao invés de calcular 2 vezes f(x), pode determinar a resposta através de $f(0)\oplus f(1)$.

p/ $f(0) = 0$

• $|f(0)\oplus 0\rangle - |f(0)\oplus 1\rangle =$
$(0\oplus 0 = 0)$ $(0\oplus 1 = 1)$
$(|0\rangle - |1\rangle)$

p/ $f(0) = 1$

• $|f(0)\oplus 0\rangle - |f(1)\oplus 1\rangle =$
$(1\oplus 0 = 1)$ $(1\oplus 1 = 0)$
$(|1\rangle - |0\rangle)$

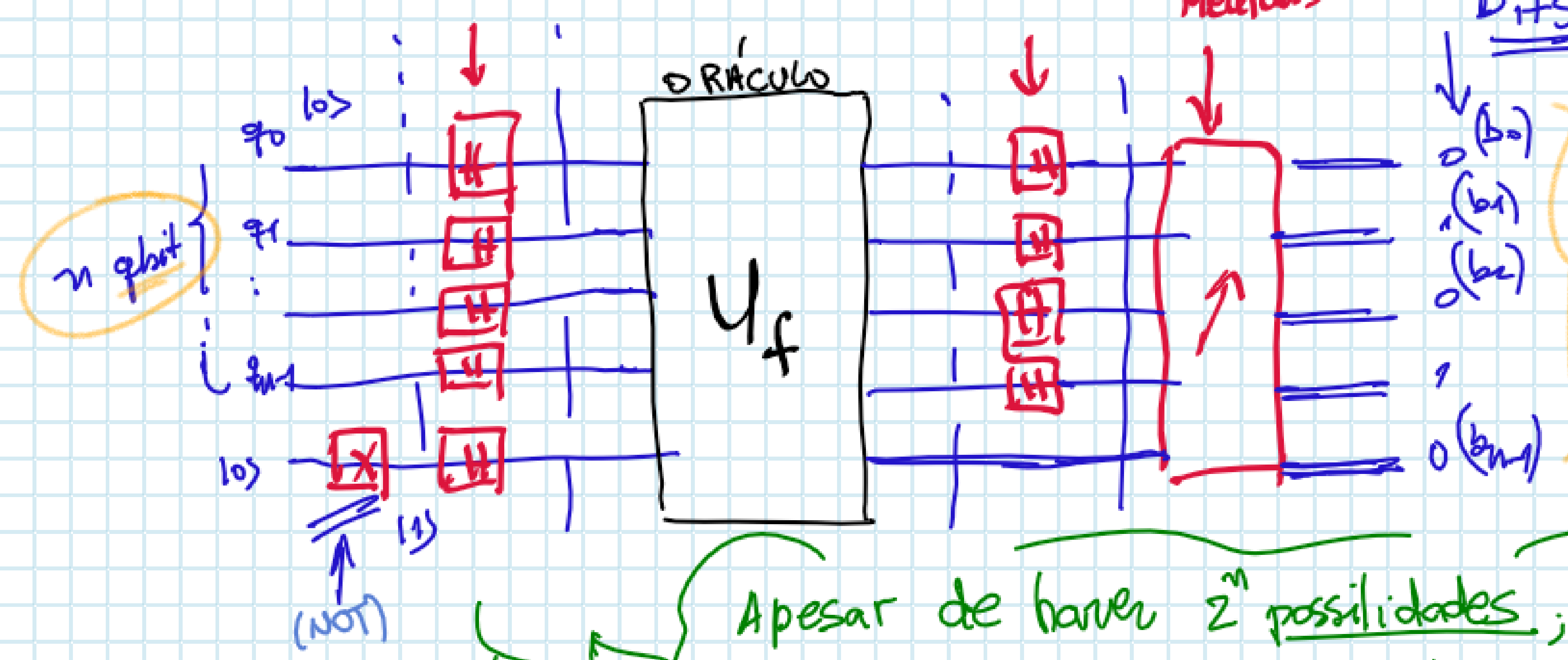
⋮

p/ $f(1) = 1$

$|f(1)+0\rangle - |f(1)+1\rangle =$
$1+0=1$ $1+1$
$(|1\rangle - |0\rangle)$

$n > 1$

* Deutsch-Jozsa: p/ $\boxed{n}$-qbit



bits: $b_0\ b_1\ b_2 \ldots$

$(0\,0\,0\,0)^n$

n bits clássicos

Se: 0000...0 ⇒ $f(x)$ constante!!
∀ outra resposta ⇒ $f(x) \neq$ constante

Apesar de haver 2^n possilidades; basta APENAS 1 pergunta p/ o "oráculo" p/ determinar a resposta!!

⇒ VANTAGEM exponencial !!

↳ **Paralelismo quântico**