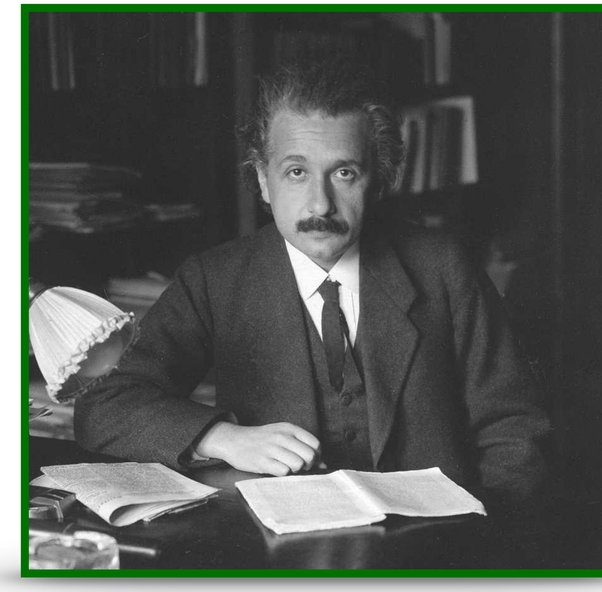
Eletromagnetismo Avançado

4º ciclo
Aula de 15 de dezembro

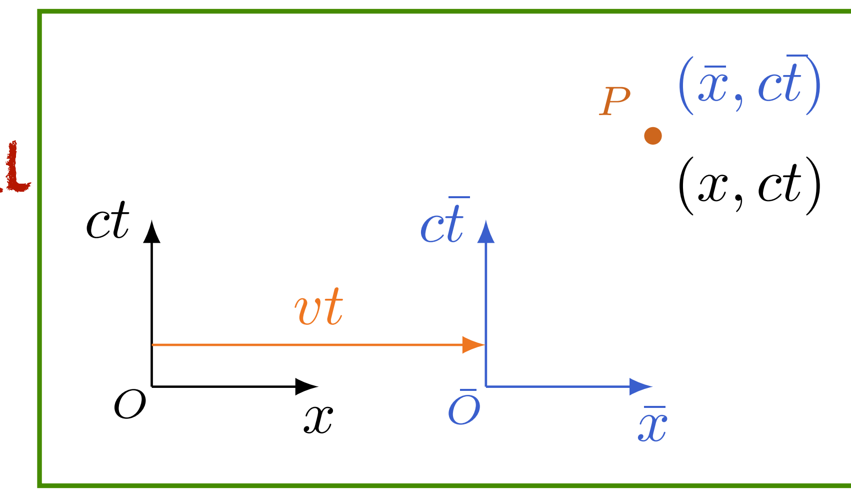
Os princípios da relatividade especial

Referenciais em movimento uniforme



I. Leis independem do referencial

II. Velocidade luz independe do referencial



$$c^2\bar{t}^2 - \bar{x}^2 = c^2t^2 - x^2$$

Quadrivetores

$$\vec{\mathbf{r}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow x^{\mu} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

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$$\vec{\mathbf{r}} = \begin{pmatrix} x & y & z \end{pmatrix} \longrightarrow x_\mu = \begin{pmatrix} -ct & x & y & z \end{pmatrix}$$

Quadrivetores

contravariante

$$\vec{\mathbf{r}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

covariante

$$\vec{\mathbf{r}} = \begin{pmatrix} x & y & z \end{pmatrix} \longrightarrow x_\mu = \begin{pmatrix} -ct & x & y & z \end{pmatrix}$$

$$\vec{\mathbf{r}} \cdot \vec{\mathbf{r}} = \begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x^2 + y^2 + z^2 \longrightarrow x_\mu x^\mu = -c^2t^2 + x^2 + y^2 + z^2$$

Quadrivetores

$$\begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$\bar{x}^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$$

$$\Lambda \equiv \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Quadrivetores

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$$a_{\mu} b^{\mu} = -a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3$$

Intervalo

$\text{Evento A} \equiv x^\mu_A$

$\text{Evento B} \equiv x^\mu_B$

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$$\Delta x^\mu = x^\mu_A - x^\mu_B$$ Deslocamento

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$$\Delta x^\mu \Delta x_\mu = -\Delta x_0^2 + \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2$$

Intervalo

$\text{Evento A} \equiv x_A^\mu$

$\text{Evento B} \equiv x_B^\mu$

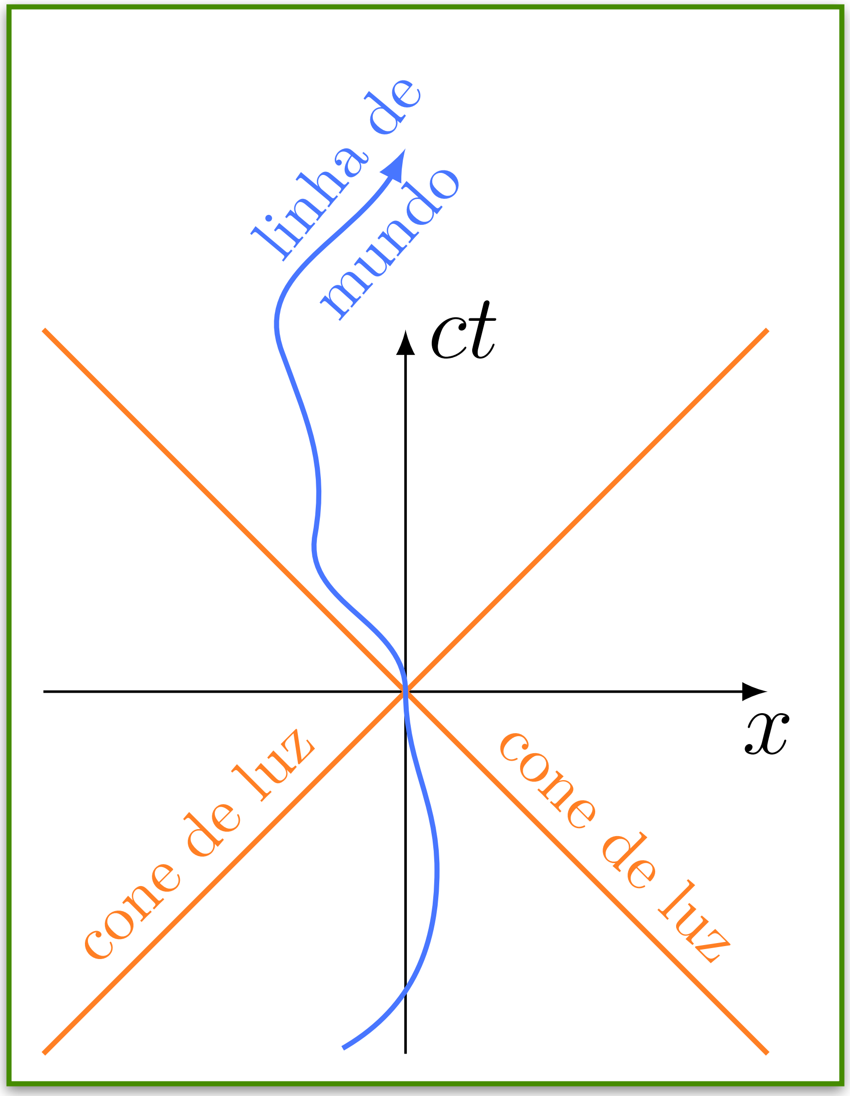
$$\Delta x^\mu = x_A^\mu - x_B^\mu$$ Deslocamento

$$\Delta x^\mu \Delta x_\mu = -\Delta x_0^2 + \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2$$

$$I_{AB} \equiv \Delta x^\mu \Delta x_\mu$$ Intervalo (invariante)

Diagramas de Minkowski

$$I_{AB} \equiv \Delta x^{\mu} \Delta x_{\mu}$$

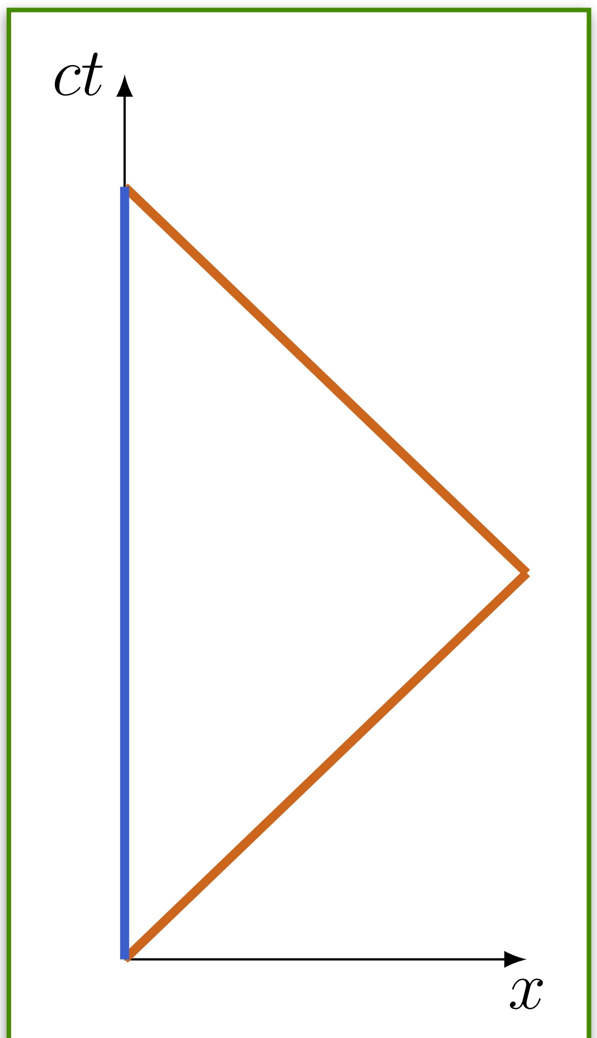


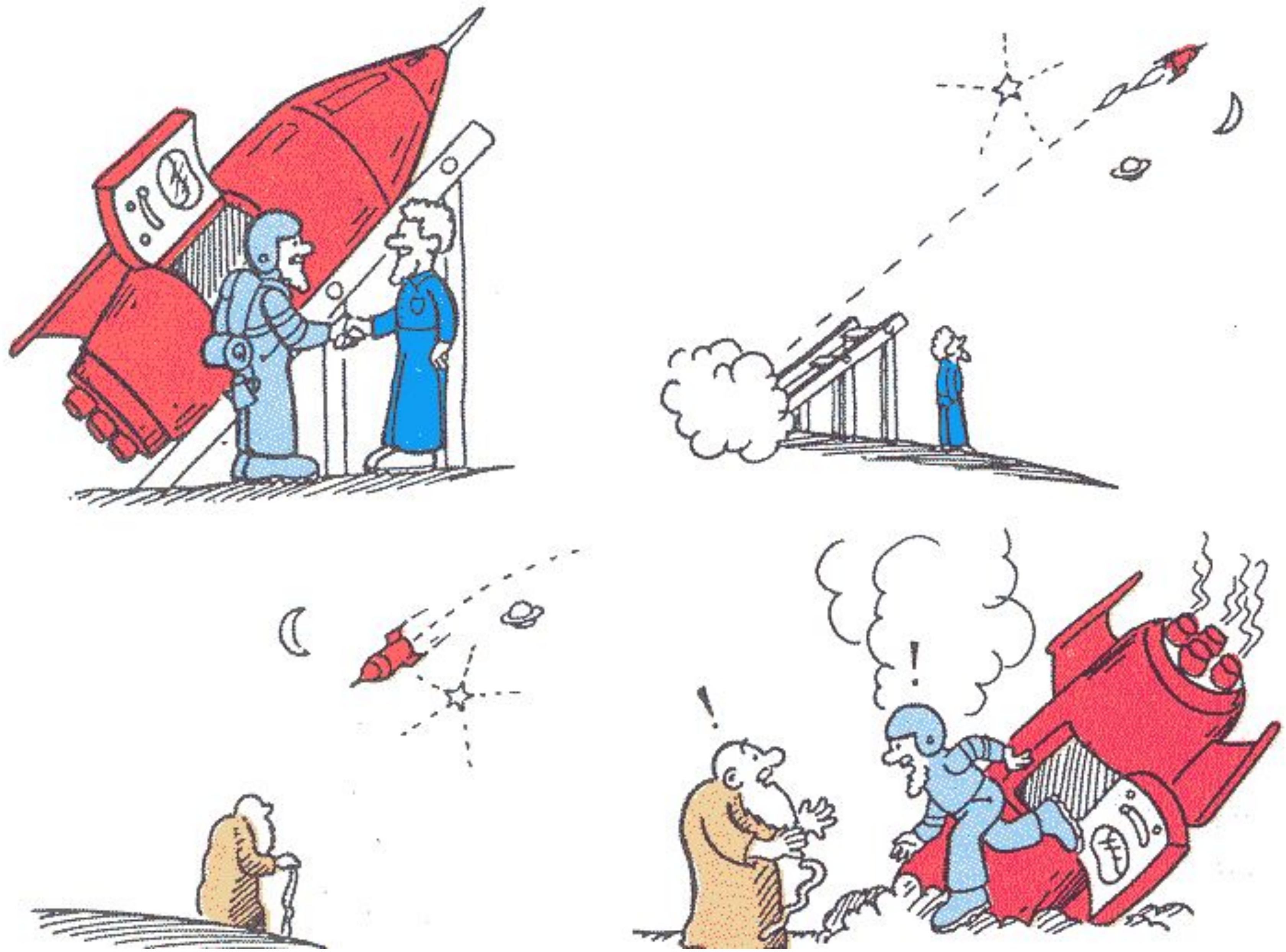
$$\begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Pratique o que aprendeu

$$\beta = \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

O paradoxo dos gêmeos





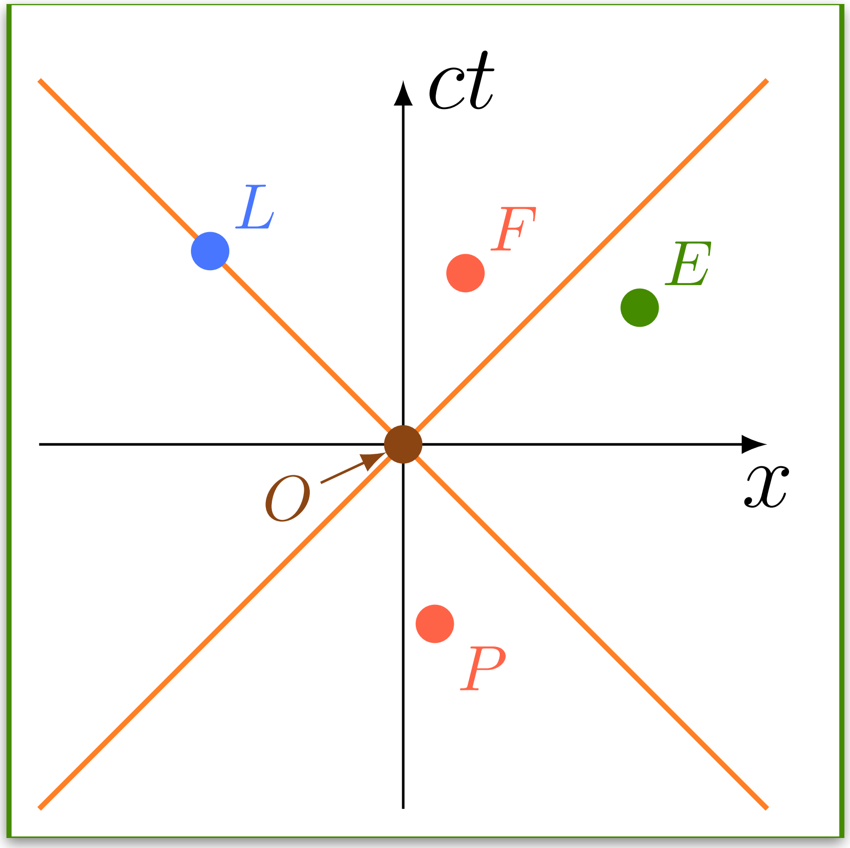
Diagramas de Minkowski

$$I_{AB} \equiv \Delta x^{\mu} \Delta x_{\mu}$$

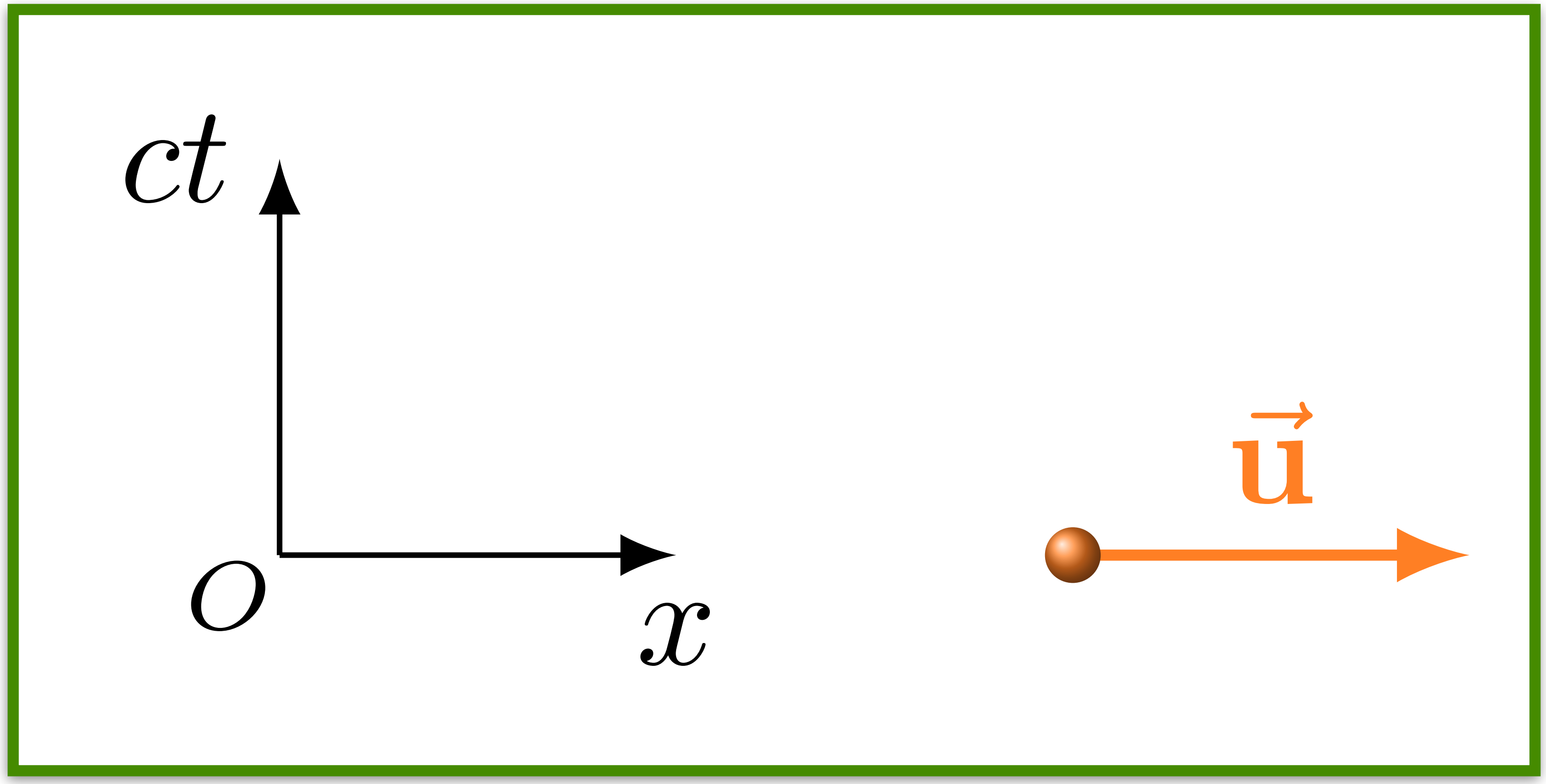
$F, P:$ tipo tempo $(I < 0)$

$E:$ tipo espaço $(I > 0)$

$L:$ tipo luz $(I = 0)$

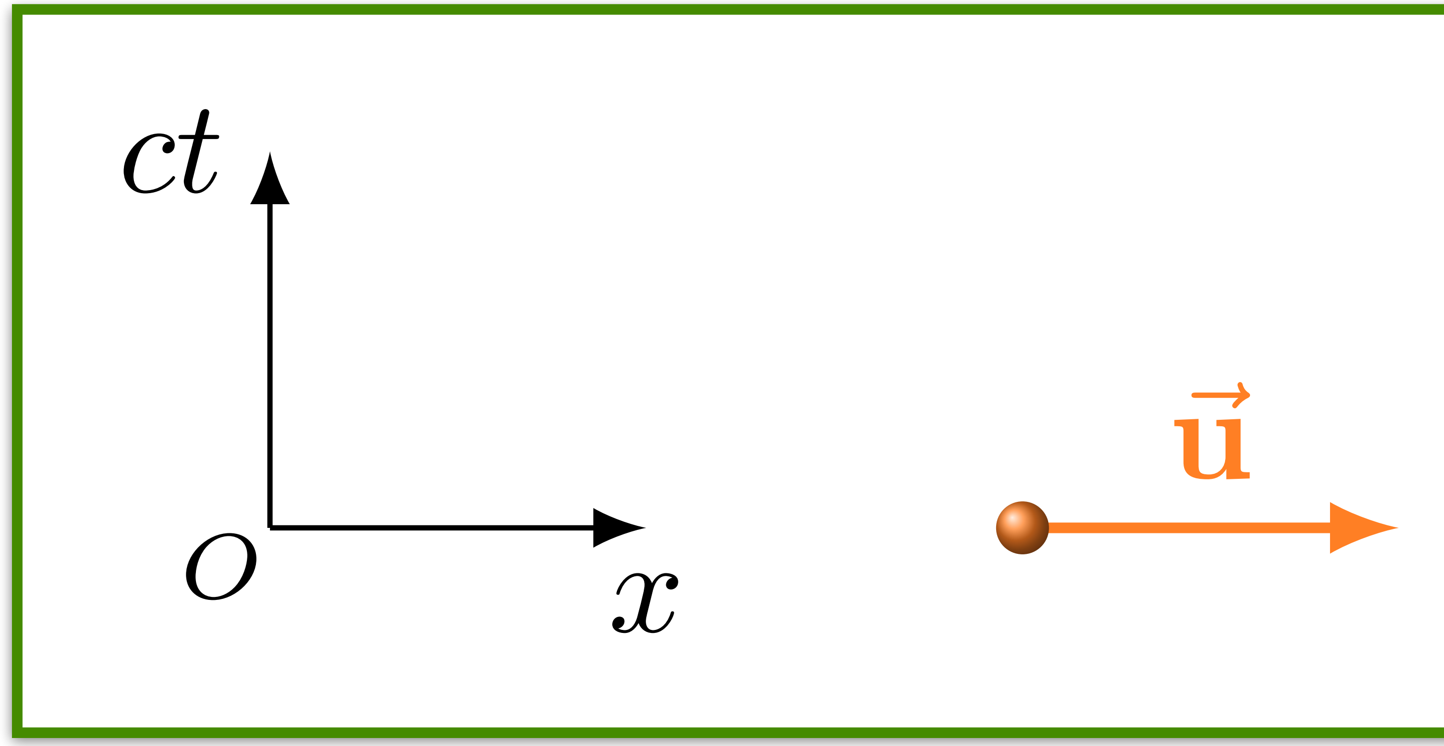


Velocidade própria



Velocidade própria

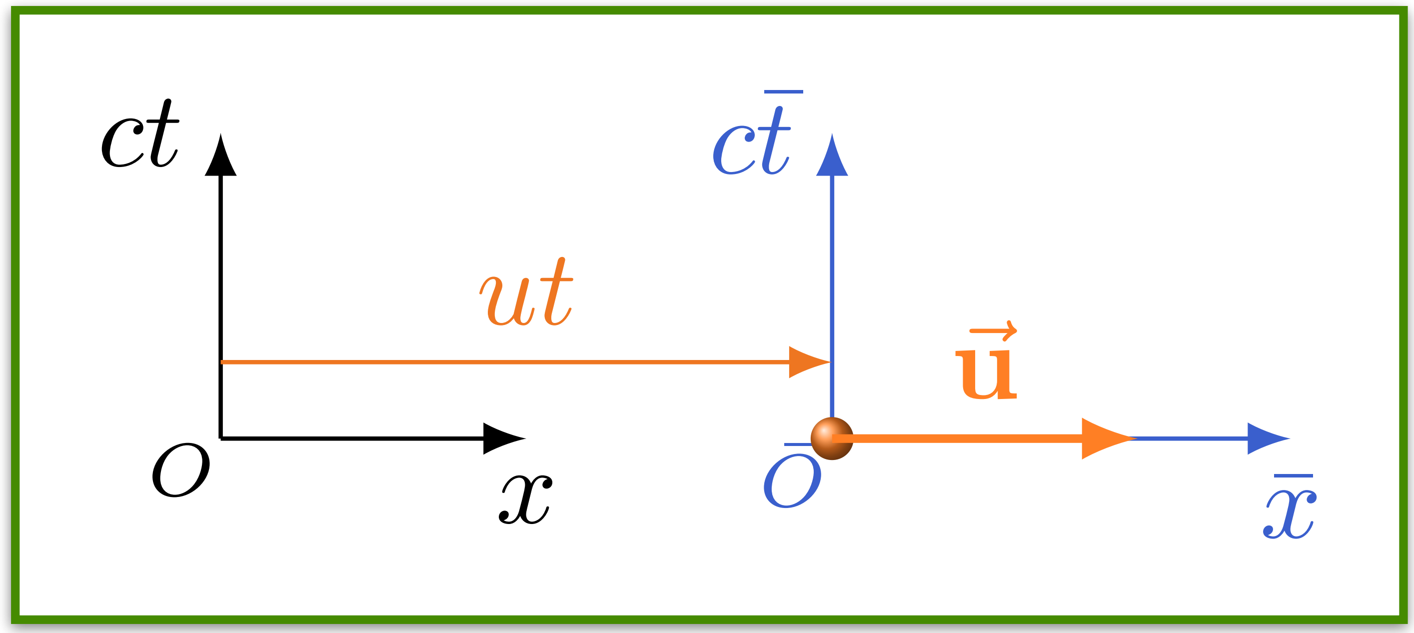
$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$



Velocidade própria

$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$\bar{\eta}^\mu = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



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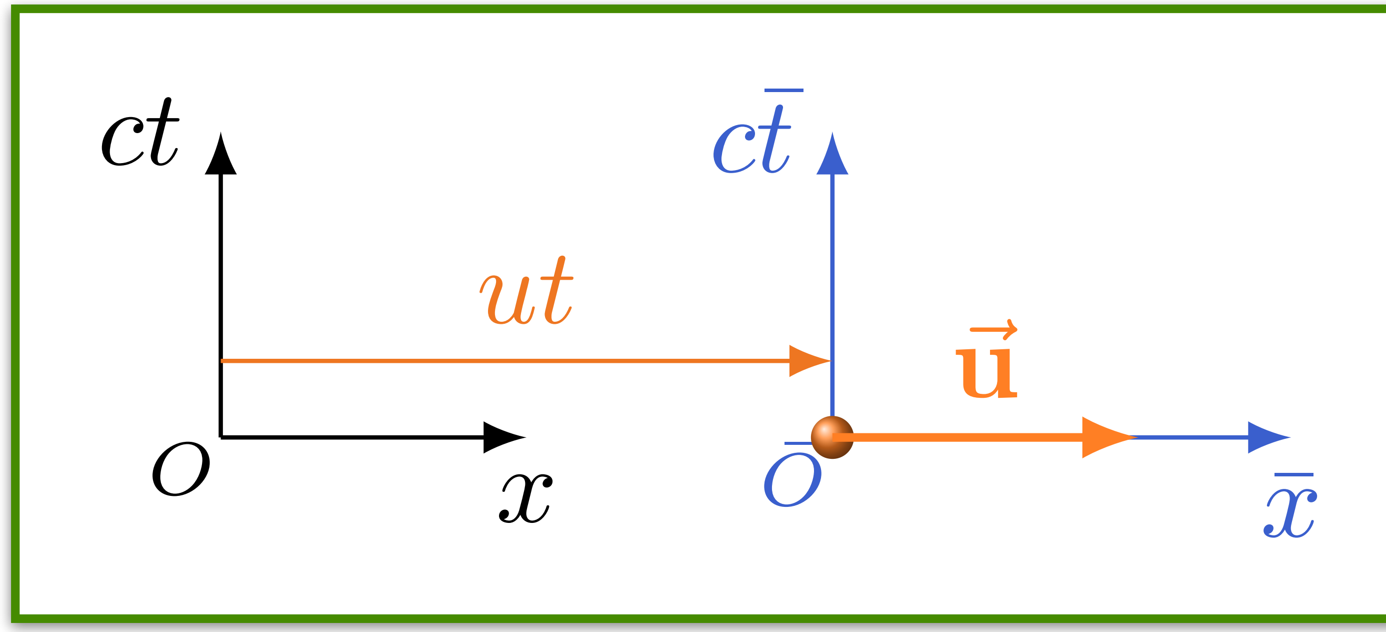
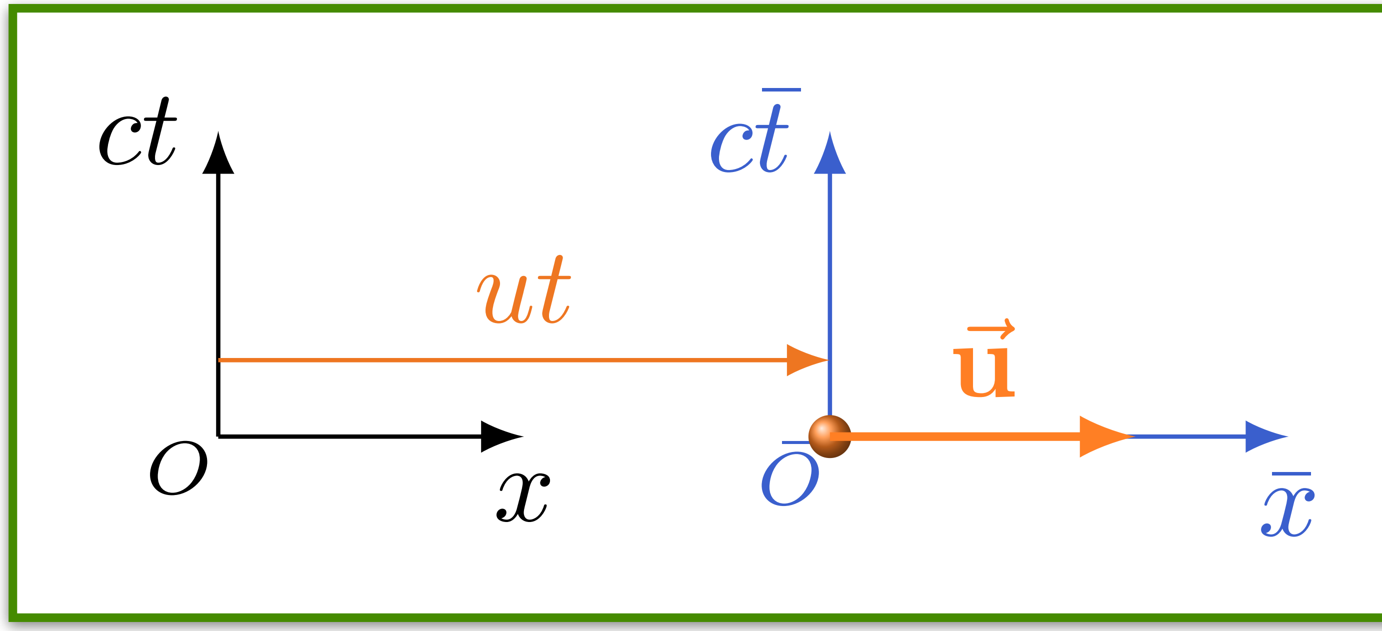
$$\eta^\mu = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



Velocidade própria



$$u = \frac{\mathrm{d}x}{\mathrm{d}t}$$ 🥺

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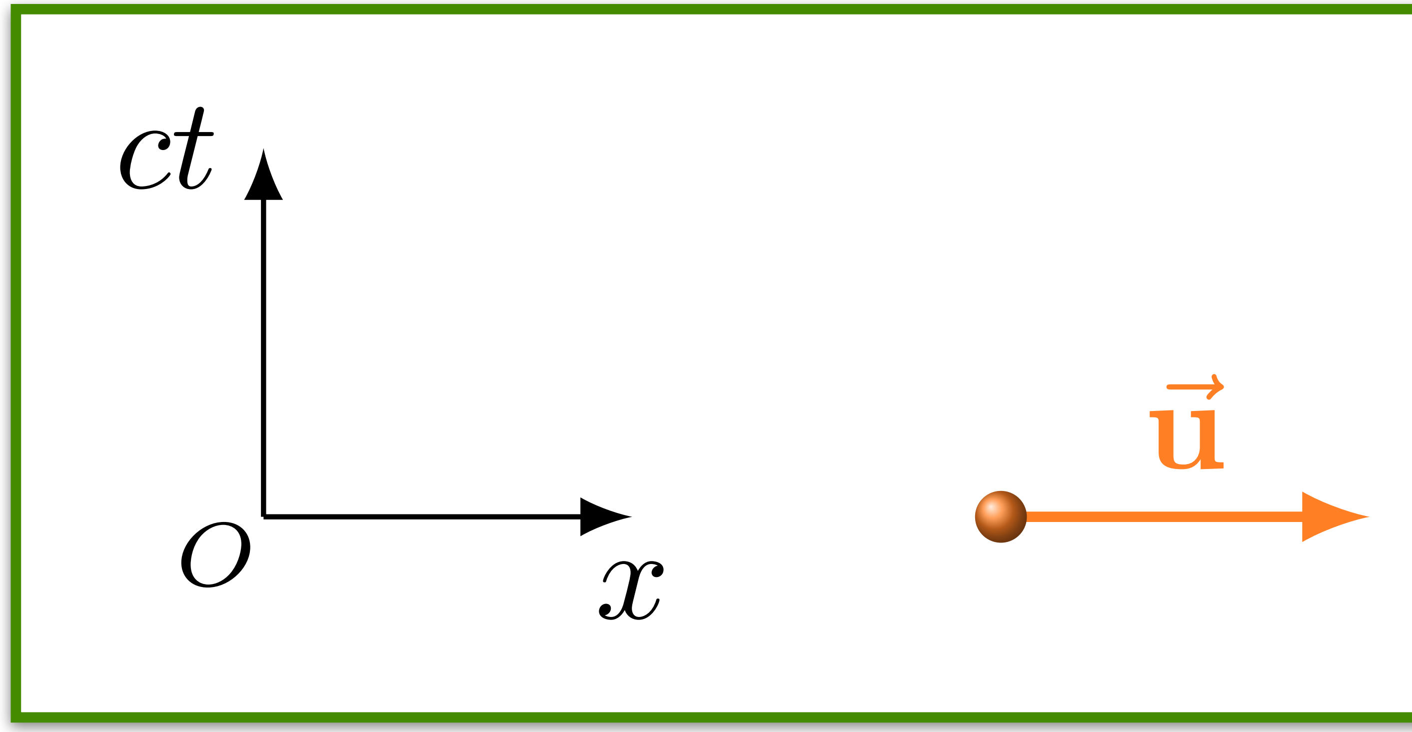
$$\eta^{\mu} = \begin{pmatrix} \gamma c \\ \beta\gamma c \\ 0 \\ 0 \end{pmatrix}$$

Velocidade própria



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$$\eta^{\mu} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

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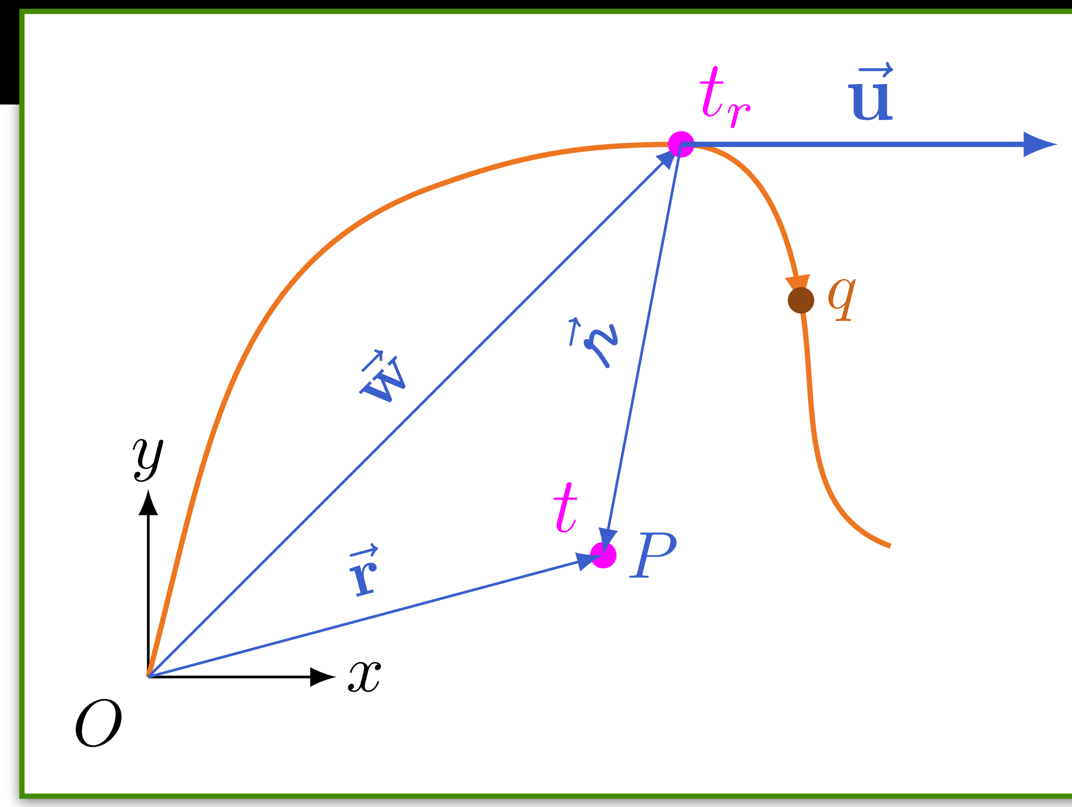
Pratique o que aprendeu



$$\mathscr{r}_{\mu}\eta^{\mu} = ?$$

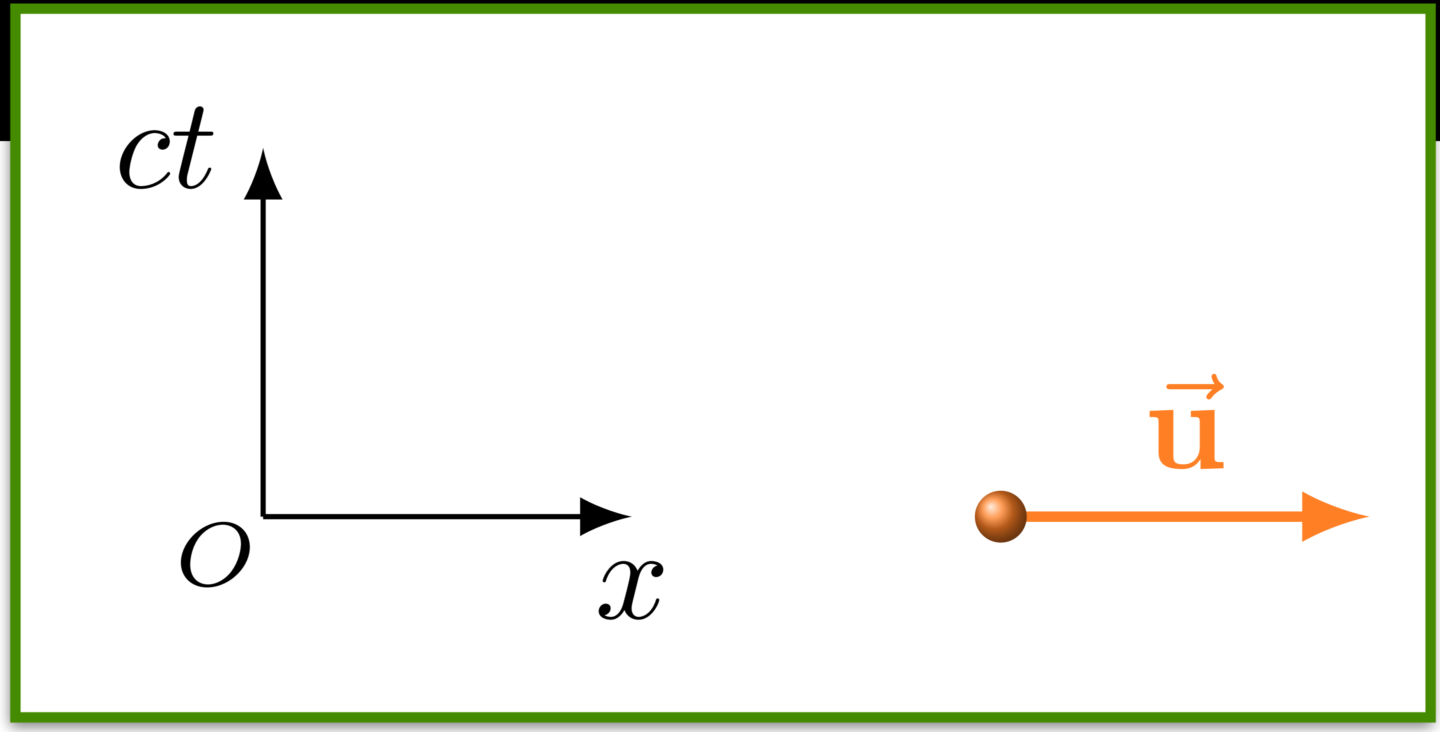
$$\mathscr{r}_{\mu} = \begin{pmatrix} -c(t - t_r) & \mathscr{r}_1 & \mathscr{r}_2 & \mathscr{r}_3 \end{pmatrix}$$

$$c(t - t_r) = \mathscr{r}$$

$$\mathscr{r}_{\mu}\eta^{\mu} = \gamma(c\mathscr{r} - \vec{\mathbf{u}} \cdot \vec{\mathscr{r}})$$

$$\eta^{\mu} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{pmatrix} c \\ u \\ 0 \\ 0 \end{pmatrix}$$

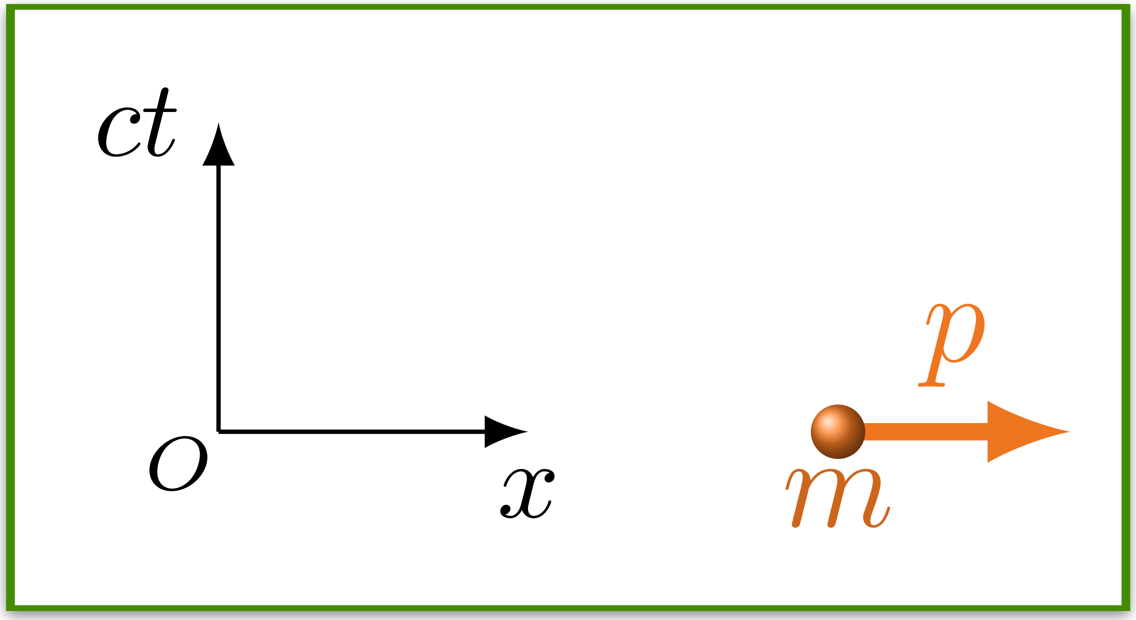
Pratique o que aprendeu



$$x_{\mu}\eta^{\mu} = ?$$

Dinâmica relativística

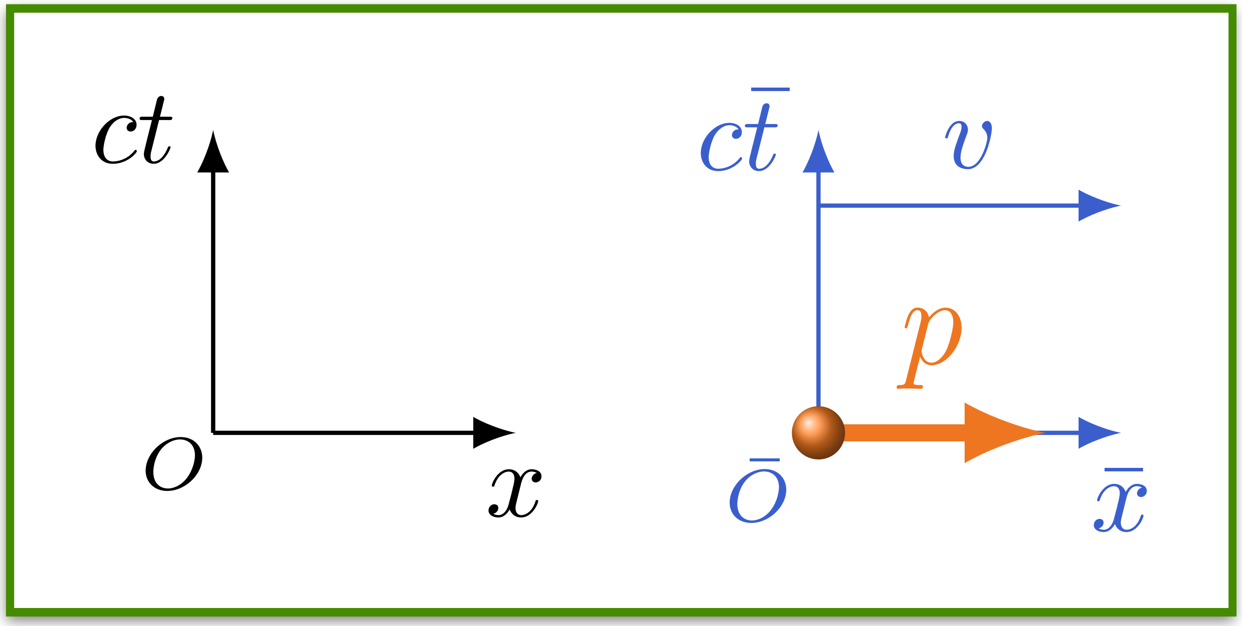
$$p^\mu = \begin{pmatrix} p_0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$



Dinâmica relativística

$$p^\mu = \begin{pmatrix} p_0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{p}^\mu = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} p_0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$



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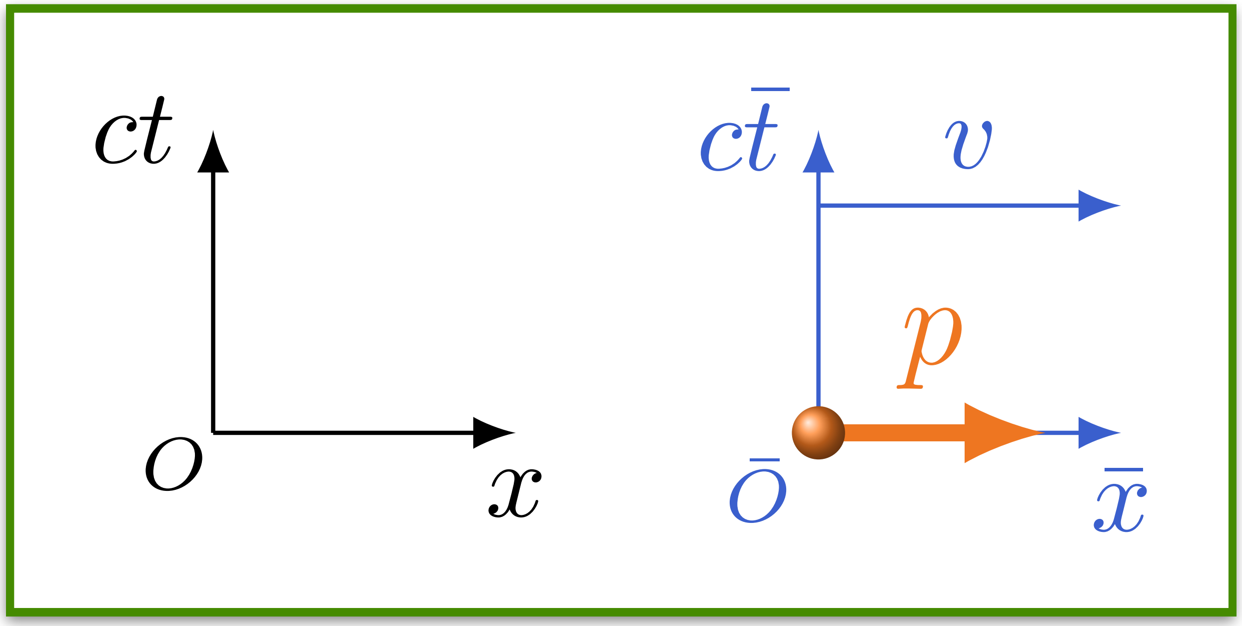

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$$\bar{p}_0 = \gamma p_0 - \beta\gamma mv$$

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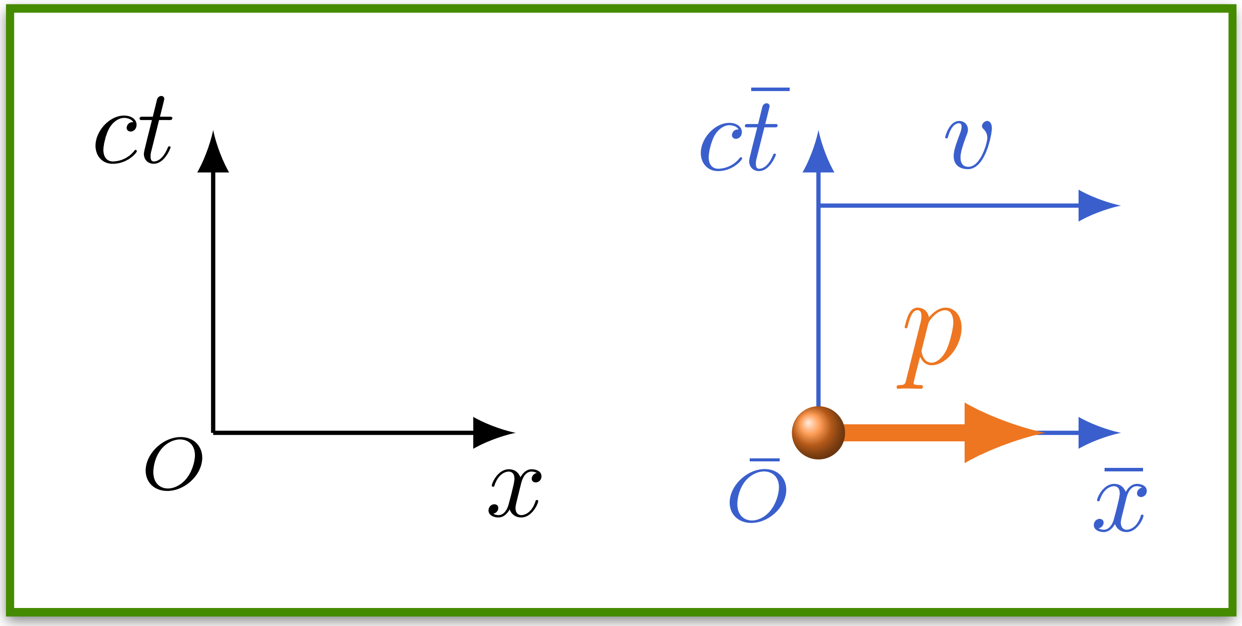

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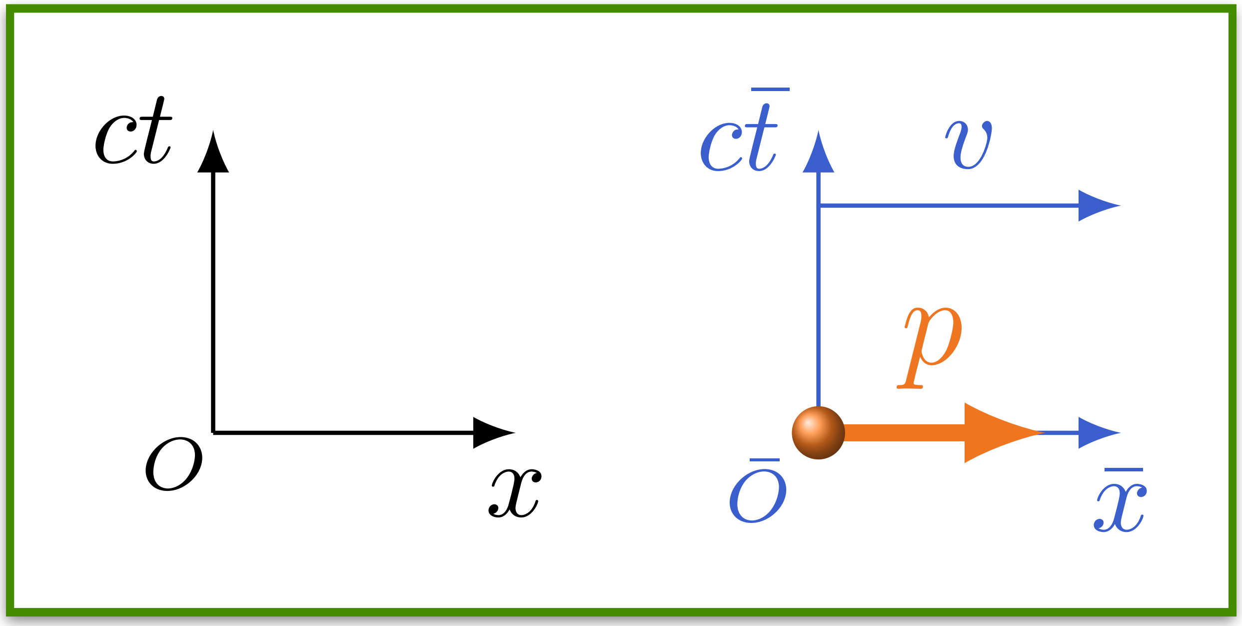
$$\bar{p}_0 = \gamma p_0 - \beta\gamma mv \qquad \Rightarrow \qquad \bar{p}_0 = \gamma(p_0 - mc\beta^2)$$

Dinâmica relativística

$$p^\mu = \begin{pmatrix} p_0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$



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$$\bar{p}_0 = \gamma p_0 - \beta\gamma mv \qquad \Rightarrow \qquad \bar{p}_0 = \gamma(p_0 - mc\beta^2)$$

$$0 = -\beta\gamma p_0 + \gamma mv$$

Dinâmica relativística

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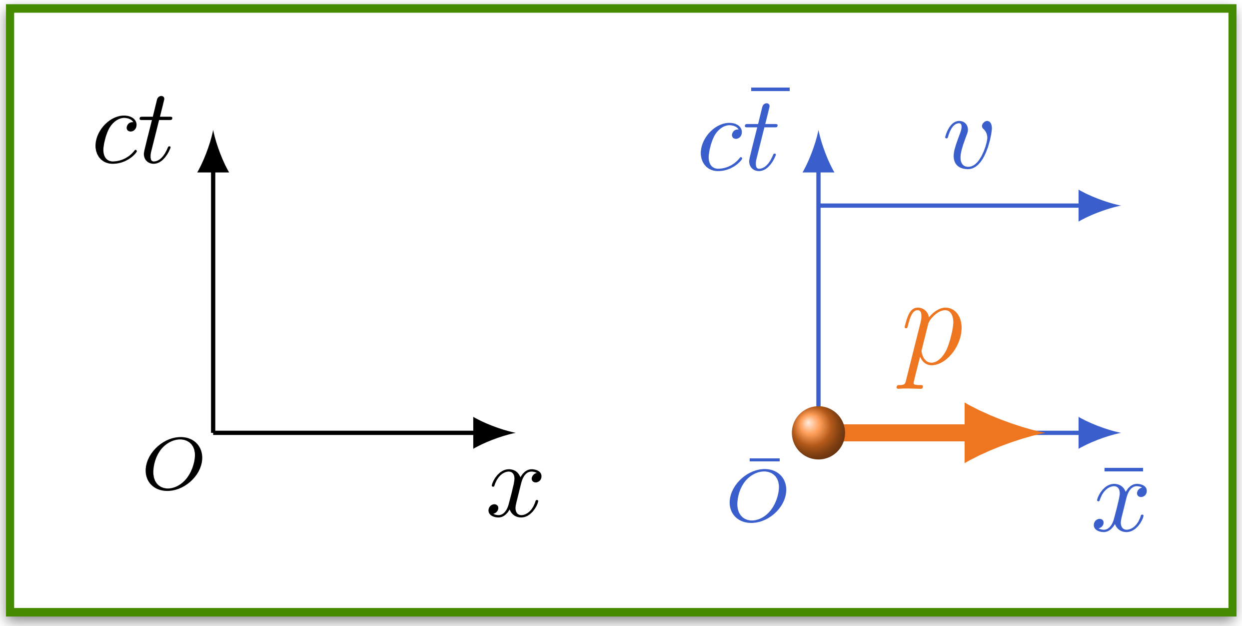

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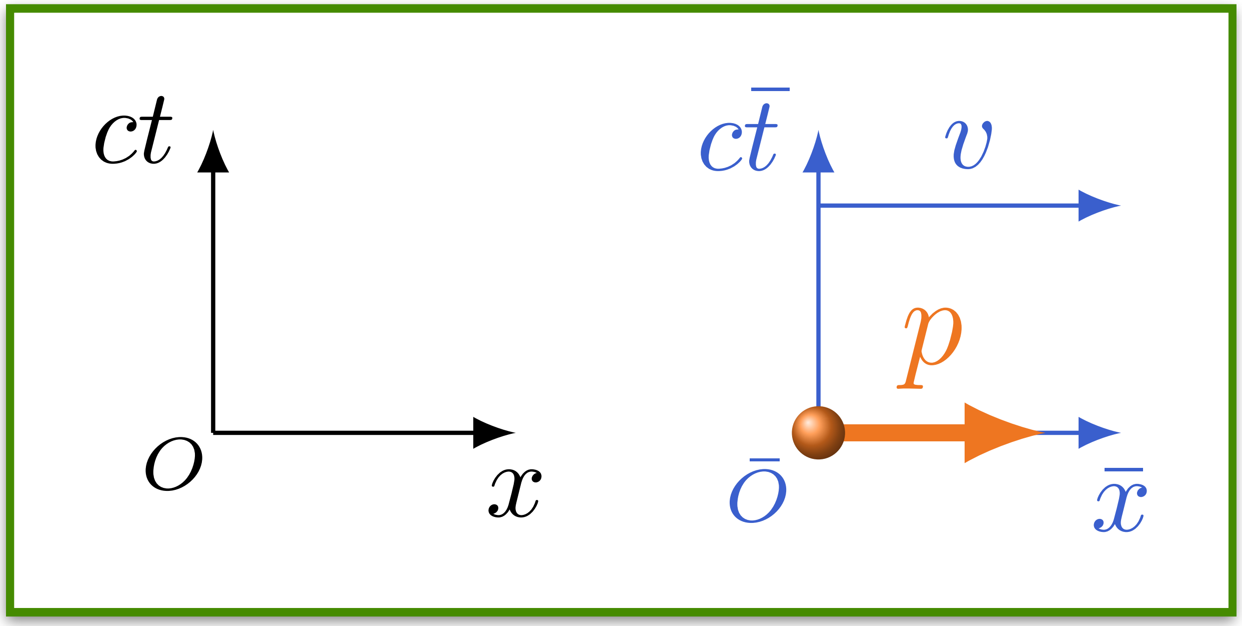
$$\bar{p}_0 = \gamma p_0 - \beta\gamma mv \quad \Rightarrow \quad \bar{p}_0 = \gamma(p_0 - mc\beta^2)$$

$$0 = -\beta\gamma p_0 + \gamma mv \quad \Rightarrow \quad p_0 = mc$$

Dinâmica relativística

$$p^\mu = \begin{pmatrix} p_0 \\ mv \\ 0 \\ 0 \end{pmatrix}$$



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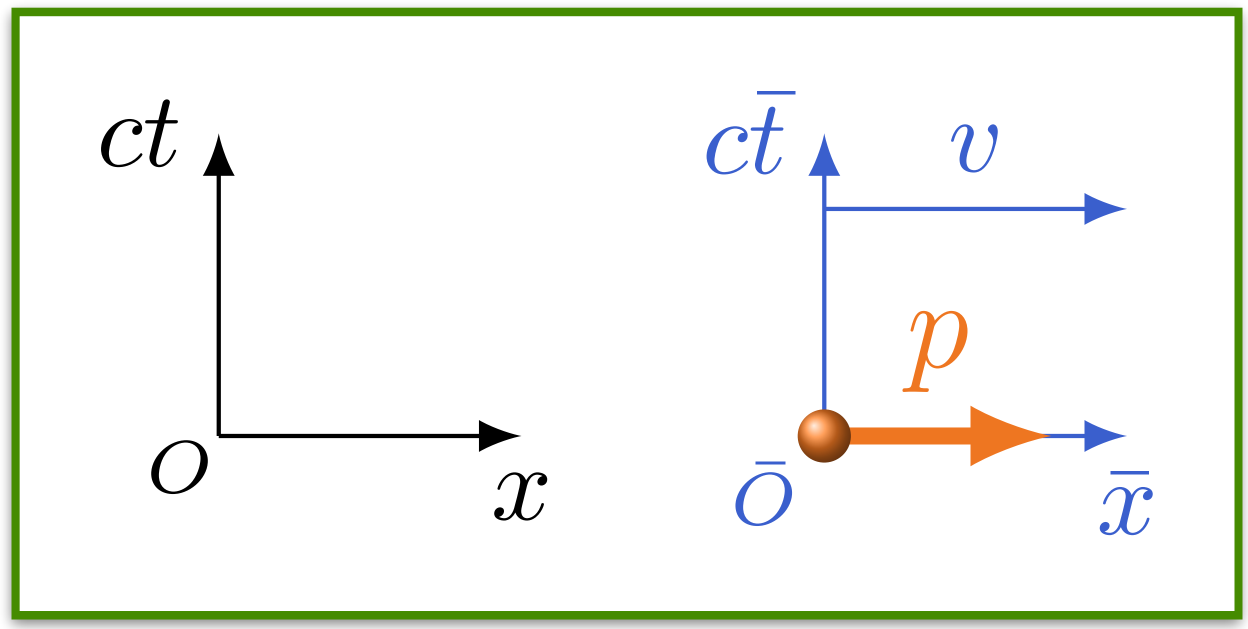
Dinâmica relativística



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$$\bar{p}_0 = \gamma p_0 - \beta\gamma m v \quad \Rightarrow \quad \bar{p}_0 = \gamma(p_0 - mc\beta^2)$$

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$$p_0 = mc$$

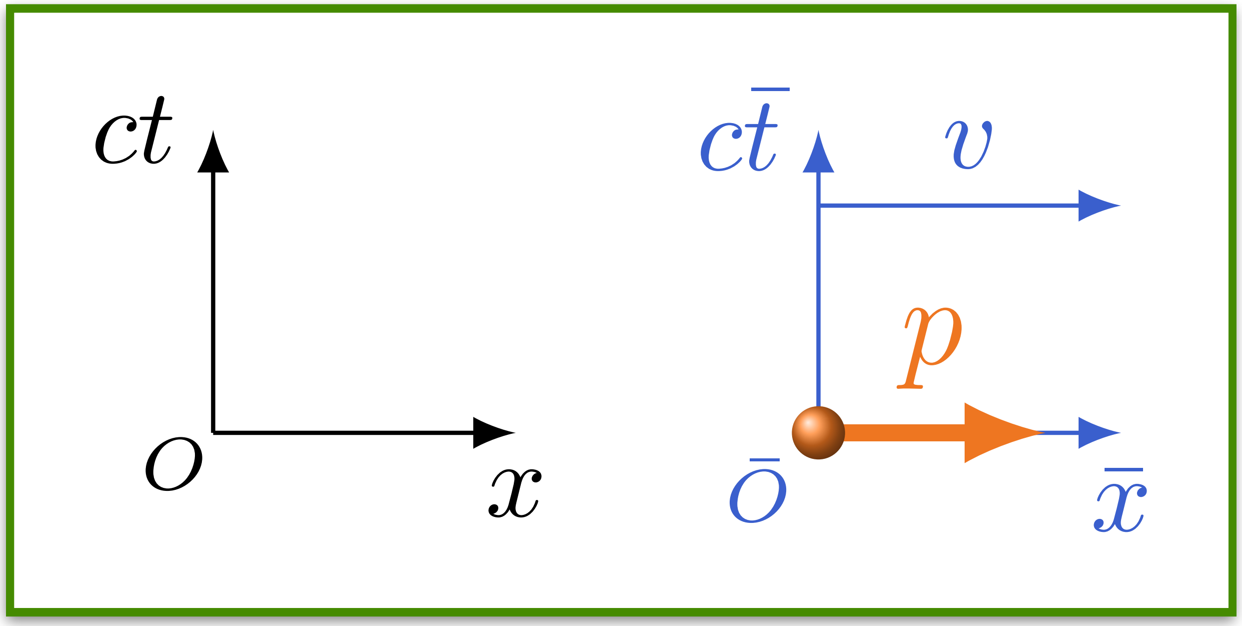
$$p_0 = \gamma \bar{p}_0$$

$$\bar{p}_0 = m_0 c$$

$$m = m_0 \gamma$$

Dinâmica relativística

$$p^\mu = \begin{pmatrix} mc \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}$$



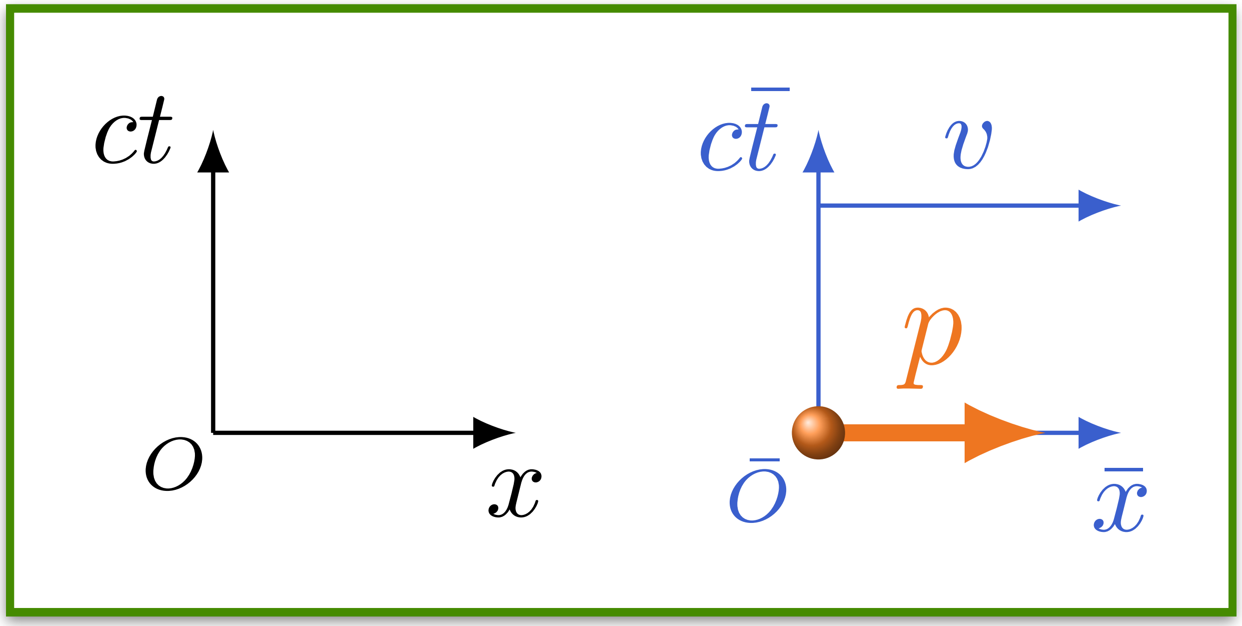
$$\bar{p}_0 = m_0 c$$

$$m = m_0 \gamma$$

Dinâmica relativística

$$m = m_0\gamma$$

$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$

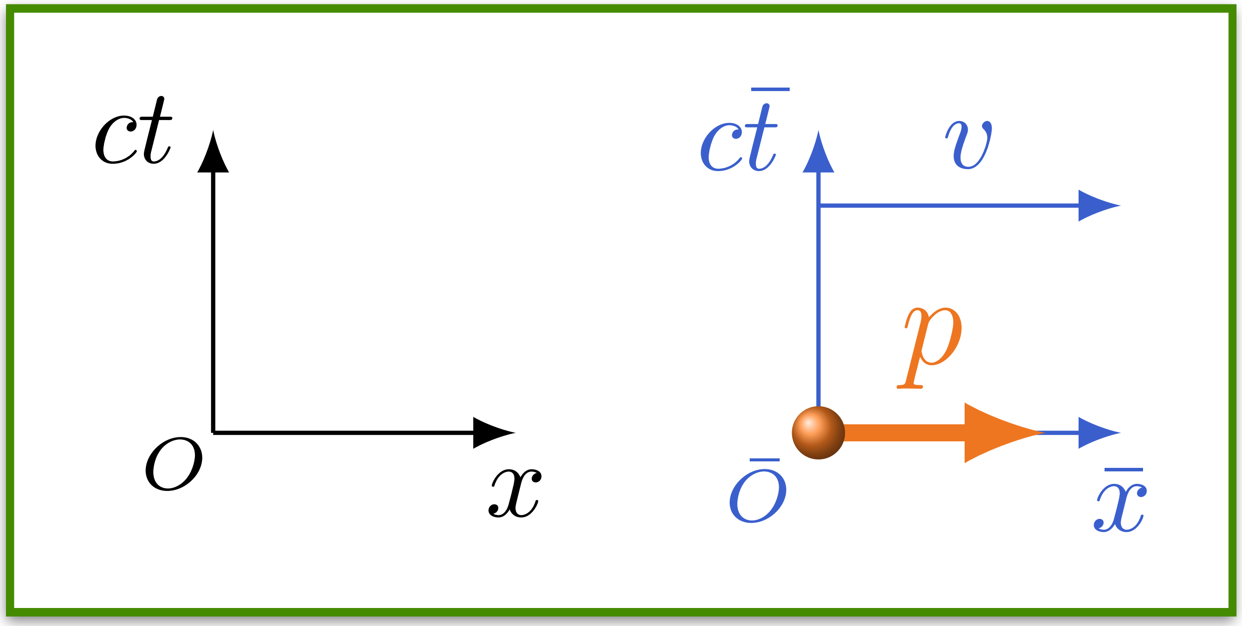


Dinâmica relativística

$$m = m_0 \gamma$$

$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$

$$m = m_0(1 + \frac{v^2}{2c^2}) + \mathcal{O}(v^4/c^4)$$



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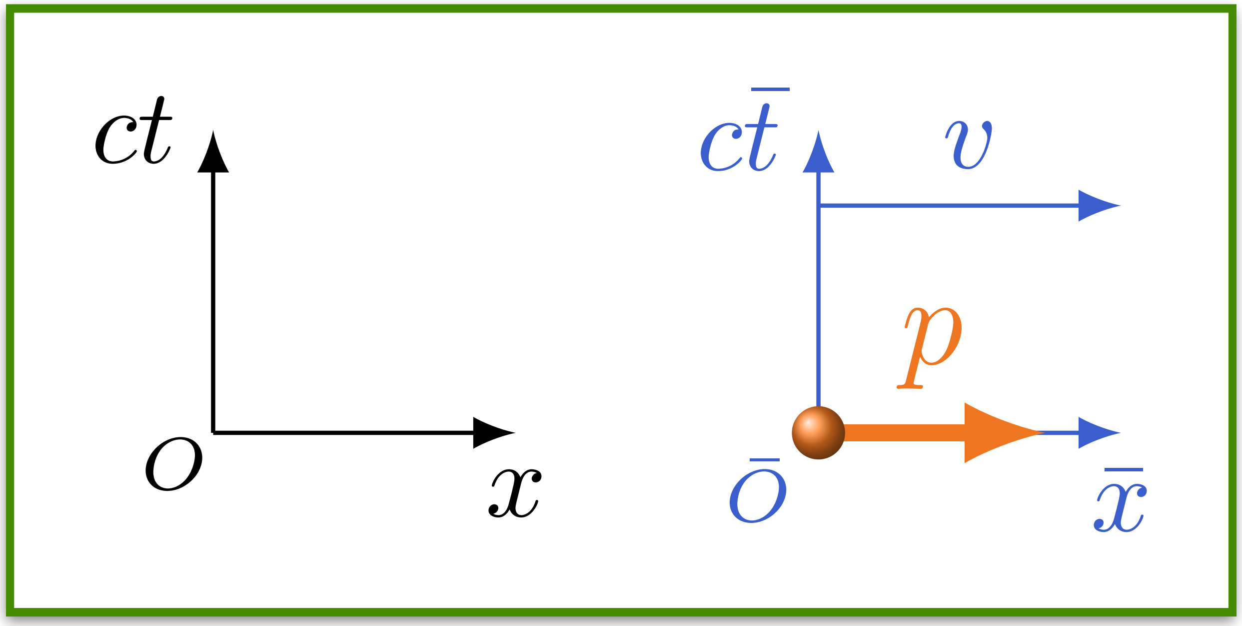

$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$

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$$mc^2 = m_0c^2 + m_0\frac{v^2}{2}$$

Dinâmica relativística

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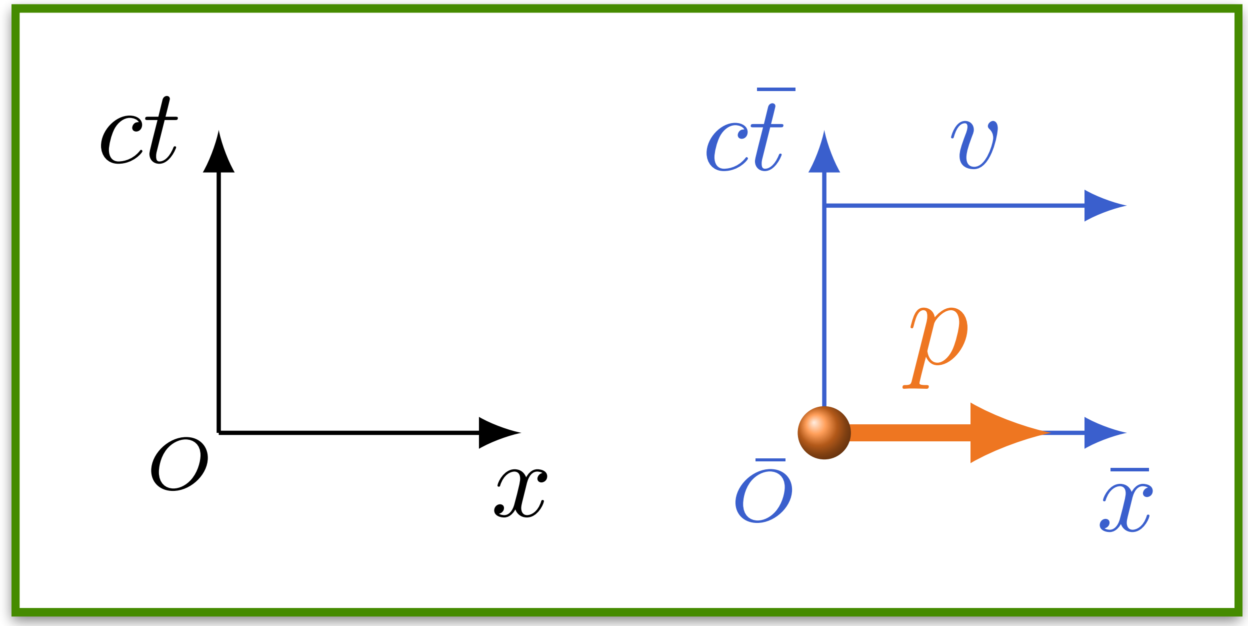

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$$mc^2 = m_0 c^2 + m_0 \frac{v^2}{2}$$

$$p^\mu = \begin{pmatrix} E/c \\ p^1 \\ p^2 \\ p^3 \end{pmatrix}$$

Dinâmica relativística

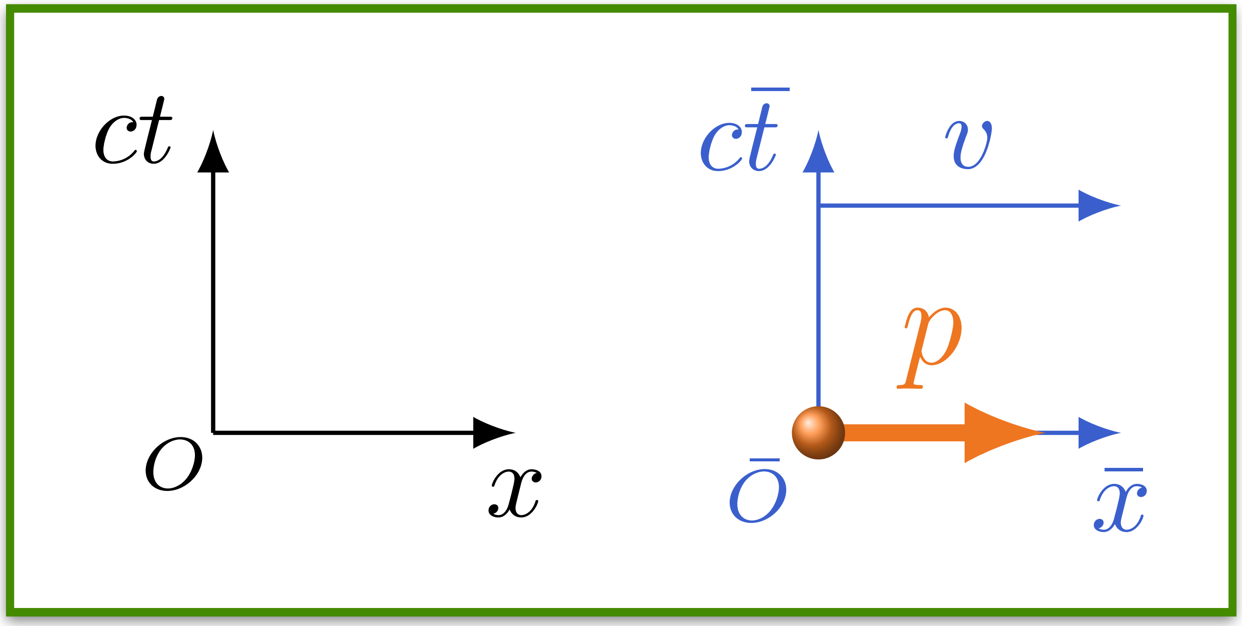
$$m = m_0\gamma$$

$$m = \frac{m_0}{(1-\beta^2)^{1/2}}$$

$$m = m_0\left(1+\frac{v^2}{2c^2}\right) + \mathcal{O}\left(v^4/c^4\right)$$

$$mc^2 = m_0c^2 + m_0\frac{v^2}{2}$$

$$p^\mu = \begin{pmatrix} E/c \\ p^1 \\ p^2 \\ p^3 \end{pmatrix} \Rightarrow \quad E^2 = m_0c^2 + p^2c^2$$



Dinâmica relativística

$$E = \pm\sqrt{m_0c^2 + p^2c^2}$$