MONITORIA MACRO IV — 06/12/21 — LISTA 2

Questão 4 EFICIÊNCIA NO MODELO DE ROMER

$$g = \frac{\lambda L\alpha - \rho}{\gamma + \alpha}$$ → Modelo de 3 Setores

Retomando o problema

Bem final: (Competitivo)

$$\max_{LY_t, \{x_{i_t}\}_{i \in [0, A_t]}} LY_t^{1-\alpha} \int_0^{A_t} x_{i_t}^{\alpha} \, di - w_t LY_t - \int_0^{A_t} p_{i_t} x_{i_t} \, di$$

CPO:

$$[x_{i_t}] \quad \alpha LY_t^{1-\alpha} x_{i_t}^{\alpha-1} - p_{i_t} = 0$$

$$[LY_t] \quad (1-\alpha) LY_t^{-\alpha} \int_0^{A_t} x_{i_t}^{\alpha} \, di - w_t = 0$$

Rearranjando:

$$w_t = (1-\alpha) LY_t^{-\alpha} \int_0^{A_t} x_{i_t}^{\alpha} \, di \quad (1)$$ → D Por Trabalho

$$x_{i_t} = \left(\frac{p_{i_t}}{\alpha LY_t^{1-\alpha}} \right)^{\frac{1}{\alpha-1}} \quad (2)$$ → D. Pelo Bem Intermediário

Bem Intermediário (monopólio) → para garantir monopólio, precisa adquirir a patente (perpétuo)

2 estágios < comprar / se compra, produzir / preço

2º Estágio (caso patente seja comprado)

$$\max_{p_{it}, x_{it}} \left\{ p_{it} x_{it} - r_t \underbrace{k_{it}}_{= x_{it}} \; : \; p_{it} = \alpha L y_t^{1-\alpha} x_{it}^{\alpha-1} \right\}$$

$$\max_{x_{it}} \left\{ \alpha L y_t^{1-\alpha} x_{it}^{\alpha} - r_t x_{it} \right\}$$

CPO:
$$\alpha \cdot \alpha \cdot L y_t^{1-\alpha} x_{it}^{\alpha-1} - r_t = 0$$
$$\alpha p_{it} - r_t = 0$$

$$p_{it} = \frac{r_t}{\alpha} \longrightarrow \forall i \longrightarrow \text{tudo } \underline{\text{igual}}$$

Com isso: $$\pi_t = (1-\alpha)\alpha L y_t^{1-\alpha} x_t^{\alpha}$$

Primeiro Estágio (aquisição da patente)

$$P_{At} = V_t = \int_t^{\infty} \exp\left\{ -\int_t^{s} r_\tau \, d\tau \right\} \pi_s \, ds$$

de dado de:

$$P_A = \int_t^{\infty} e^{-\int_t^{s} r \, d\tau} \pi \, ds = \pi \int_t^{\infty} e^{-r(s-t)} ds =$$

$$= \pi \cdot \left. \frac{e^{-r(s-t)}}{-r} \right|_t^{\infty} = \pi \left(\lim_{s \to \infty} \frac{e^{-r(s-t)}}{-r} + \frac{1}{r} \right) =$$

$$= \pi \left(0 + \frac{1}{r} \right) = \frac{\pi}{r}$$

$$\therefore P_A = \frac{\pi}{r} = \frac{\alpha(1-\alpha) L y^{1-\alpha} x^{\alpha}}{r} \quad (3)$$

Setor de Pesquisa

$$\dot{A}_t = \lambda A_t L_{A_t}$$

$$\pi^A_t = p_{A_t} \lambda A_t L_{A_t} - w_t L_{A_t} \xrightarrow[\text{ótimo}]{\text{no}} w_t = p_{A_t} \lambda A_t \quad (4)$$

Consumidores:

$$\max U_0 = \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\gamma} - 1}{1-\gamma} dt \; ; \quad \rho, \gamma > 0 \qquad \text{sa } C_t + \dot{K}_t \leq w_t L + r_t K_t + \pi_t$$

→ Euler: $\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma}$

Crescimento Balanceado:

$$\frac{\dot{A}_t}{A_t} = \lambda L_{A_t} \to L_A \text{ cte} \Rightarrow L_Y \text{ cte}$$

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\gamma} \Rightarrow \frac{\dot{C}_t}{C_t} = \frac{r - \rho}{\gamma}$$

$$p_A = (1-\alpha)\alpha L_Y^{1-\alpha} x^{\alpha}$$

$$Y_t = L_Y^{1-\alpha} \int_0^{A_t} x_{i_t}^{\alpha} di = L_Y^{1-\alpha} A_t x^{\alpha}$$

$$K_t = \int_0^{A_t} k_{i_t} di = A_t x$$

tudo cresce à mesma taxa de A

(3)

$$Y_t = C_t + \dot{K}_t$$

$$\underbrace{\frac{Y_t}{K_t}}_{\substack{\text{memo tx} \\ = \text{cost} \\ \text{no BGP}}} = \underbrace{\frac{C_t}{K_t}}_{\substack{\text{cost} \\ \text{no BGP}}} + \underbrace{\frac{\dot{K}_t}{K_t}}_{\substack{\text{cost} \\ \text{no BGP}}}$$

$\Rightarrow$ C e K crescono al memo tasso

$$\therefore \quad \frac{\dot{C}_t}{C_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{A}_t}{A_t} = \lambda l_A = g$$

(1) il lavoro ott:

$$w_t = (1-\alpha) L_y^{-\alpha} A_t x^{\alpha}$$

ma

$$w_t = p_{A_t} \lambda A_t = \frac{(1-\alpha)\alpha L_y^{1-\alpha} x^{\alpha} \lambda A_t}{r}$$

$$\therefore \quad (1-\alpha) L_y^{-\alpha} A_t x^{\alpha} = \frac{(1-\alpha)\alpha L_y^{1-\alpha} x^{\alpha} \lambda A_t}{r}$$

$$L_y^{-\alpha} = \frac{\alpha L_y^{1-\alpha} \lambda}{r}$$

$$L_y = \frac{r}{\alpha \lambda} \quad \Rightarrow \quad L_A = L - \frac{r}{\lambda \alpha}$$

$$\therefore \quad g = \lambda l_A = \lambda L - \frac{r}{\alpha}$$

Do Euler: $g = \frac{r - \rho}{\gamma} \Rightarrow r = \gamma g + \rho$

$\therefore \quad g = \lambda L - \frac{(\gamma g + \rho)}{\alpha} \Rightarrow \alpha g = \alpha \lambda L - \gamma g - \rho$

$\Rightarrow (\alpha + \gamma) g = \alpha \lambda L - \rho \Rightarrow \boxed{g = \frac{\alpha \lambda L - \rho}{\gamma + \alpha}}$

A) Estática Comparativa

$\frac{\partial g}{\partial \lambda} = \frac{\alpha L}{\gamma + \alpha} > 0 \Rightarrow$ ↑ λ ↑ progresso técnico

↑ crescimento

$\frac{\partial g}{\partial L} = \frac{\alpha \lambda}{\gamma + \alpha} > 0 \Rightarrow$ ↑ L ↑ L_A ↑ prog. técnico

↑ crescimento

$\frac{\partial g}{\partial \gamma} = \frac{-(\alpha \lambda L - \rho)}{(\gamma + \alpha)^2} < 0 \Rightarrow$ ↑ γ (av. ao risco) ↓ L_A

↓ prog técnico ↓ crescimento

$\frac{\partial g}{\partial \rho} = -\frac{1}{\gamma + \alpha} < 0 \Rightarrow$ ↑ ρ (impaciência) ↓ L_A

↓ prog. técnico ↓ crescimento

B) Planejador Central

$$U_0=\int_0^{\infty} e^{-\rho t}\,\frac{C_t^{1-\gamma}-1}{1-\gamma}\,dt$$

Restrições: $\dot{K}_t = Y_t - C_t = (L_{Y,t})^{\alpha}\int_0^{A_t} x_{it}^{1-\alpha}\,di - C_t \Rightarrow \underbrace{L_{Y_t}^{\alpha} A_t x_t^{1-\alpha}}_{L_{Y_t}^{\alpha} A_t^{\alpha} K_t^{1-\alpha}} - C_t$ (1)

$K_t = \int_0^{A_t} x_t(i)\,di \xrightarrow{\bar{x}} K_t = A_t x_t$ (2)

$\dot{A}_t = \lambda A_t L_{A,t}$ (3)

$L_{A_t} + L_{Y_t} = L$ (4)

K_0, A_0 dados

$x_t(i) = \bar{x}_t$

Queremos máx U_0 s.a (1) – (4)

$$\mathcal{H} = e^{-\rho t}\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}\right) + \mu_t\left[\lambda A_t L_{A_t}\right] + \theta_t\left[L_{Y_t}^{\alpha} A_t^{\alpha} K_t^{1-\alpha} - C_t\right]$$

$$= e^{-\rho t}\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}\right) + \mu_t\left[\lambda A_t (L-L_{Y_t})\right] + \theta_t\left[L_{Y_t}^{\alpha} A_t^{\alpha} K_t^{1-\alpha} - C_t\right]$$

$\mathcal{H}_{C_t}=0 \Rightarrow e^{-\rho t}\cdot C_t^{-\gamma} - \theta_t = 0$ (5)

$\mathcal{H}_{L_{Y_t}}=0 \Rightarrow -\mu_t\lambda A_t + \theta_t\,\alpha\, L_{Y_t}^{\alpha-1} A_t^{\alpha} K_t^{1-\alpha} = 0$ (6)

$\mathcal{H}_{A_t} = -\dot{\mu}_t \Rightarrow \lambda\mu_t(L-L_{Y_t}) + \theta_t\,\alpha\, L_{Y_t}^{\alpha} A_t^{\alpha-1} K_t^{1-\alpha} = -\dot{\mu}_t$ (7)

$\mathcal{H}_{K_t} = -\dot{\theta}_t \Rightarrow (1-\alpha)\theta_t L_{Y_t}^{\alpha} A_t^{\alpha} K_t^{-\alpha} = -\dot{\theta}_t$ (8)

+ CTs

De (5): $\theta_t = e^{-\rho t} \, c_t^{-\gamma}$

$$\ln \theta_t = -\rho t - \gamma \ln c_t$$

$$\frac{\dot{\theta}_t}{\theta_t} = -\rho - \gamma \frac{\dot{c}_t}{c_t} \qquad (9)$$

De (6): $\mu_t \lambda A_t = \theta_t \alpha LY_t^{\alpha-1} A_t^{\alpha} K_t^{1-\alpha}$

$$\lambda \mu_t = \theta_t \alpha LY_t^{\alpha-1} A_t^{\alpha-1} K_t^{1-\alpha} \qquad (10)$$

De (7): $-\dot{\mu}_t = \lambda \mu_t (L - LY_t) + \alpha \theta_t LY_t^{\alpha} A_t^{\alpha-1} K_t^{1-\alpha} \qquad (11)$

De (8): $-\frac{\dot{\theta}_t}{\theta_t} = (1-\alpha) LY_t^{\alpha} A_t^{\alpha} K_t^{-\alpha} \qquad (12)$

c) G^* $\longrightarrow$ BGP ~~DO~~ PC.

De (9): $\frac{\dot{c}_t}{c_t} = -\frac{1}{\gamma}\left(\frac{\dot{\theta}_t}{\theta_t} + \rho\right) \qquad (13)$

(10) em (11)

$$-\dot{\mu}_t = \lambda \mu_t (L - LY_t) + \underbrace{\theta_t \alpha LY_t^{\alpha} A_t^{\alpha-1} K_t^{1-\alpha}}_{\lambda \mu_t LY_t}$$

$$-\dot{\mu}_t = \lambda \mu_t (L - LY_t) + \lambda \mu_t LY_t$$

$$-\dot{\mu}_t = \lambda \mu_t L$$

$$-\frac{\dot{\mu}_t}{\mu_t} = \lambda L \quad (14)$$

Faço: $\frac{d}{dt} \ln(10)$:

$$\frac{\dot{\mu}_t}{\mu_t} = \frac{\dot{\theta}_t}{\theta_t} + (\alpha-1)\frac{\dot{A}_t}{A_t} + (1-\alpha)\frac{\dot{K}_t}{K_t}$$

(Lembra-se de que L_t é cte)

$$= \frac{\dot{\theta}_t}{\theta_t} + (\alpha-1)g^* + (1-\alpha)g^* = \frac{\dot{\theta}_t}{\theta_t} \quad (15)$$

De (15) e (14): $\frac{\dot{\theta}_t}{\theta_t} = -\lambda L \quad (16)$

(16) em (13): $\frac{\dot{c}_t}{c_t} = g^* = -\frac{1}{\gamma}(-\lambda L + \rho)$

$$\Rightarrow \boxed{g^* = \frac{1}{\gamma}(\lambda L - \rho)}$$

D) $g^* > g$

$$g^* = \frac{\lambda L - \rho}{\gamma}$$

$$g = \frac{\alpha\lambda L - \rho}{\gamma + \alpha}$$

$$g^* - g = \frac{(\gamma+\alpha)(\lambda L - \rho) - \gamma(\alpha\lambda L - \rho)}{\gamma(\gamma+\alpha)}$$

$$g^* - g = \frac{\gamma\lambda L - \cancel{\rho\gamma} + \alpha\lambda L - \rho\alpha - \gamma\alpha\lambda L + \cancel{\gamma\rho}}{\gamma(\gamma+\alpha)}$$

$$= \frac{\alpha(\lambda L - \rho) + \lambda L\gamma(1-\alpha)}{\gamma(\gamma+\alpha)} > 0$$

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→ O planejador central "anula" a ineficiência gerada pelo monopólio no setor de bens intermediários

Questão 5 → Ver Acemoglu p. 465 seção 14.1.4