Eletromagnetismo Avançado

3º ciclo
Aula de 24 de novembro

Radiação de dipolo

$$q(t) = q_0 \cos(\omega t)$$

$$r \gg \lambda \gg d$$

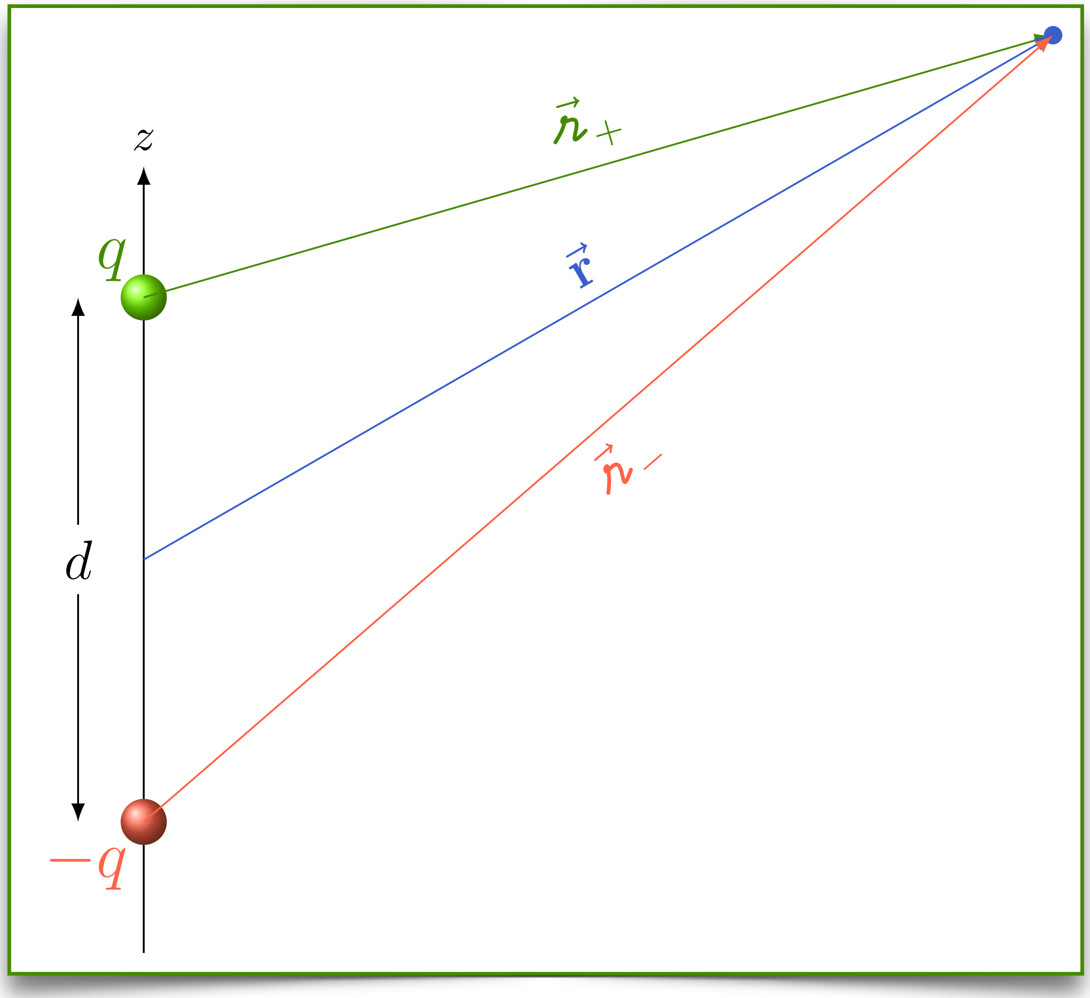
$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin\omega(t - \frac{r}{c})\, \hat{\mathbf{z}}$$

$$\boxed{\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = -\frac{\mu_0 p_0 \omega^2}{4\pi} \frac{\sin\theta}{r} \cos\omega(t - \tfrac{r}{c})\, \hat{\theta}}$$

$$\boxed{\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\omega(t - \tfrac{r}{c})\, \hat{\phi}}$$

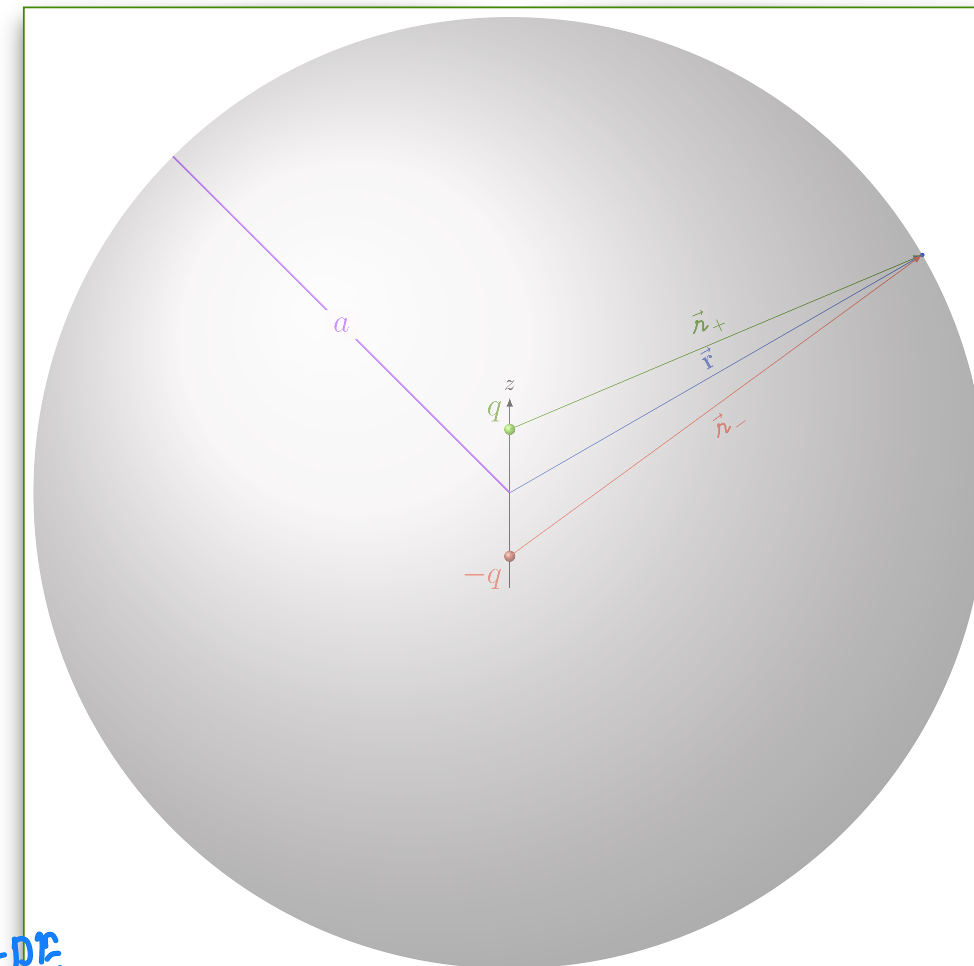
$$\boxed{\langle \vec{\mathbf{S}} \rangle = \frac{\mu_0 p_0^2 \omega^4}{8\pi c} \frac{\sin^2\theta}{4\pi r^2}\, \hat{\mathbf{r}}}$$



$$\langle \vec{\mathbf{S}} \rangle = \frac{\mu_0 p_0^2 \omega^4}{8\pi c} \frac{\sin^2\theta}{4\pi r^2} \hat{\mathbf{r}}$$

Pratique o que aprendeu

$$q(t) = q_0 \cos(\omega t) \qquad\qquad r \gg \lambda \gg d$$



Qual é a resistência radiativa do fio?

$$P = R\frac{q_0^2\omega^2}{2}$$

$$P = \frac{\mu_0 d^2 \omega^2}{6\pi c} \frac{q_0^2 \omega^2}{2}$$

$$\omega = \frac{2\pi}{\lambda} c$$

$$\boxed{R = \frac{2\pi}{3}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(\frac{d}{\lambda}\right)^2}$$

MEDIDA DA PROPORCIONALIDADE ENTRE d E λ

EX.: LUZ E MOLÉCULAS ATMOSFÉRICAS

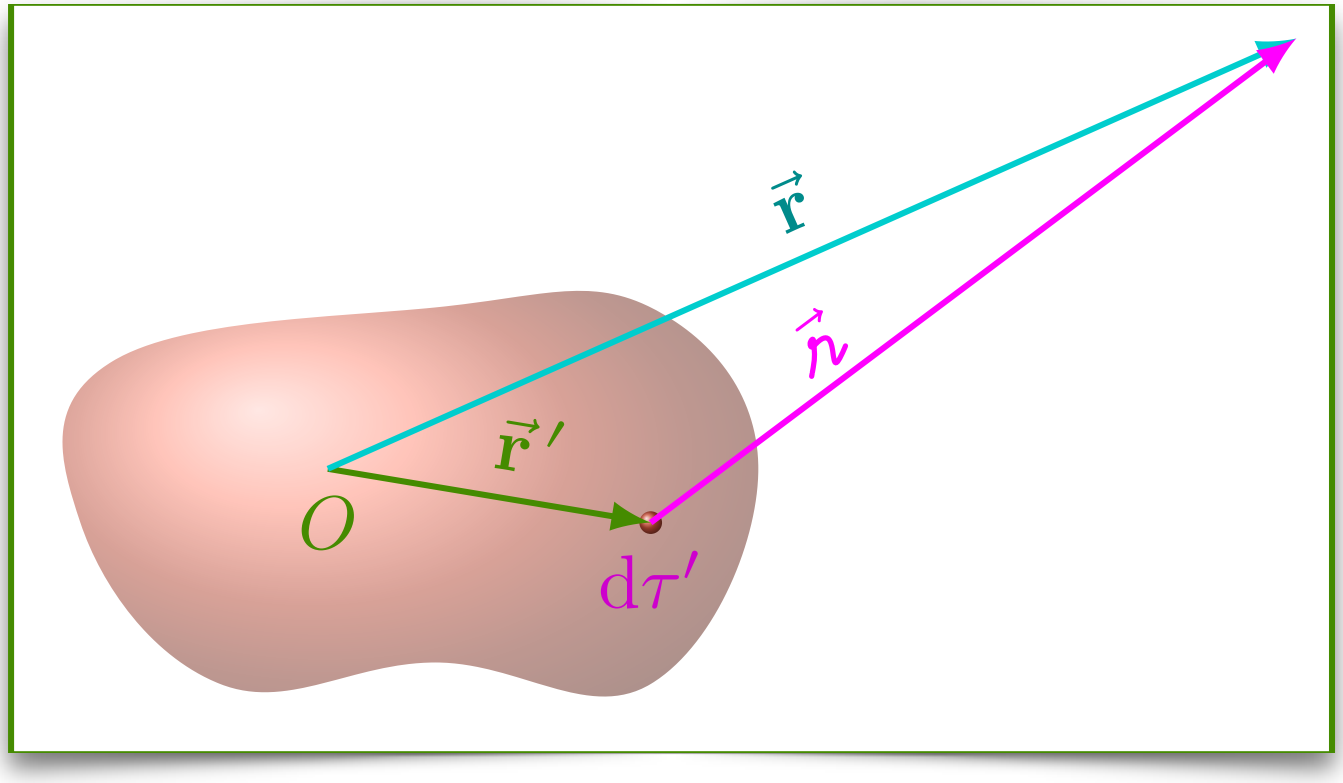
$d \simeq 0{,}5\,nm$

$\lambda_{AZUL} = 400\,nm$ → $\frac{d}{\lambda} = 1{,}25 \times 10^{-3}$

$\lambda_{VERMELHO} = 700\,nm$ → $\frac{d}{\lambda} = 0{,}7 \times 10^{-3}$

Radiação de distribuição de cargas

$$\left.\begin{array}{l}\rho = \rho(\vec{\mathbf{r}}\,', t) \\ \vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t)\end{array}\right\} \text{CONHECIDOS}$$



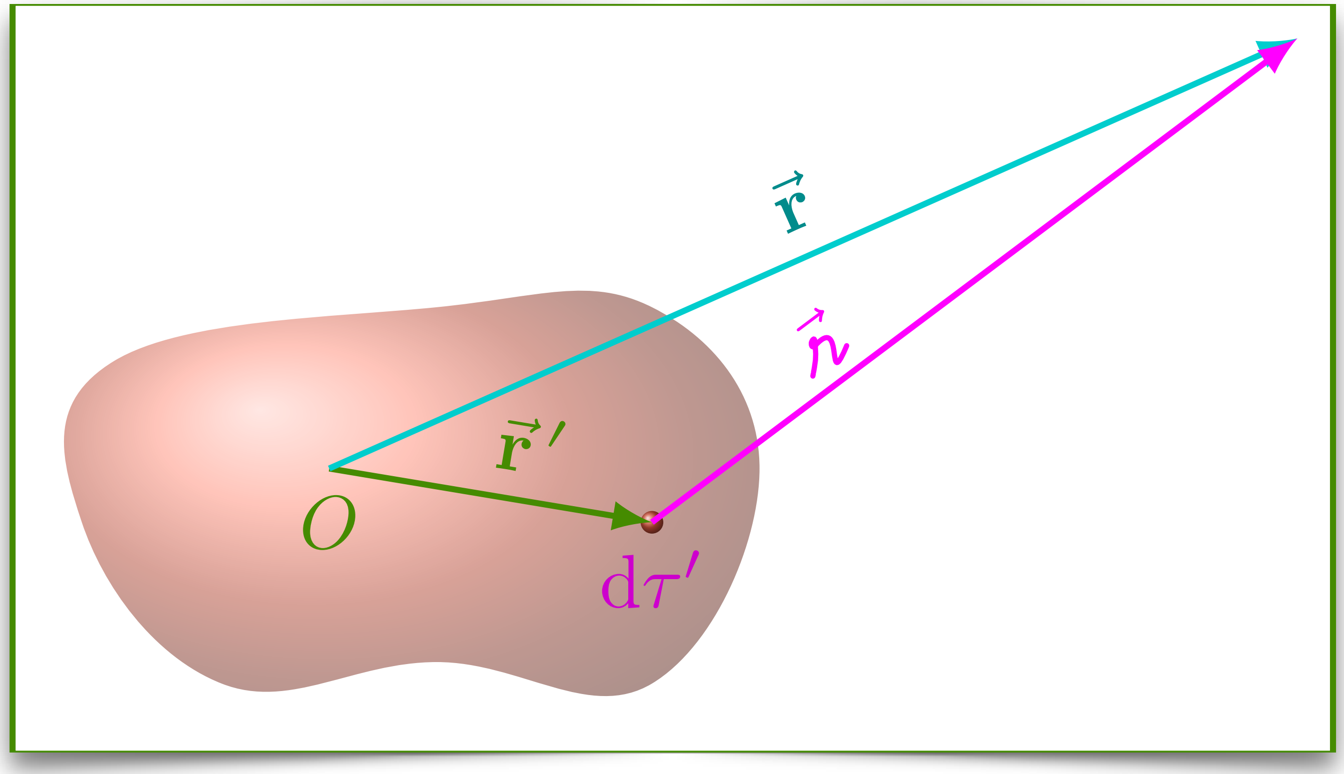
Radiação de distribuição de cargas

$\rho = \rho(\vec{\mathbf{r}}\,', t)$

$\vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t)$

RADIAÇÃO EMITIDA

$r' \ll \lambda \ll r$



Radiação de distribuição de cargas

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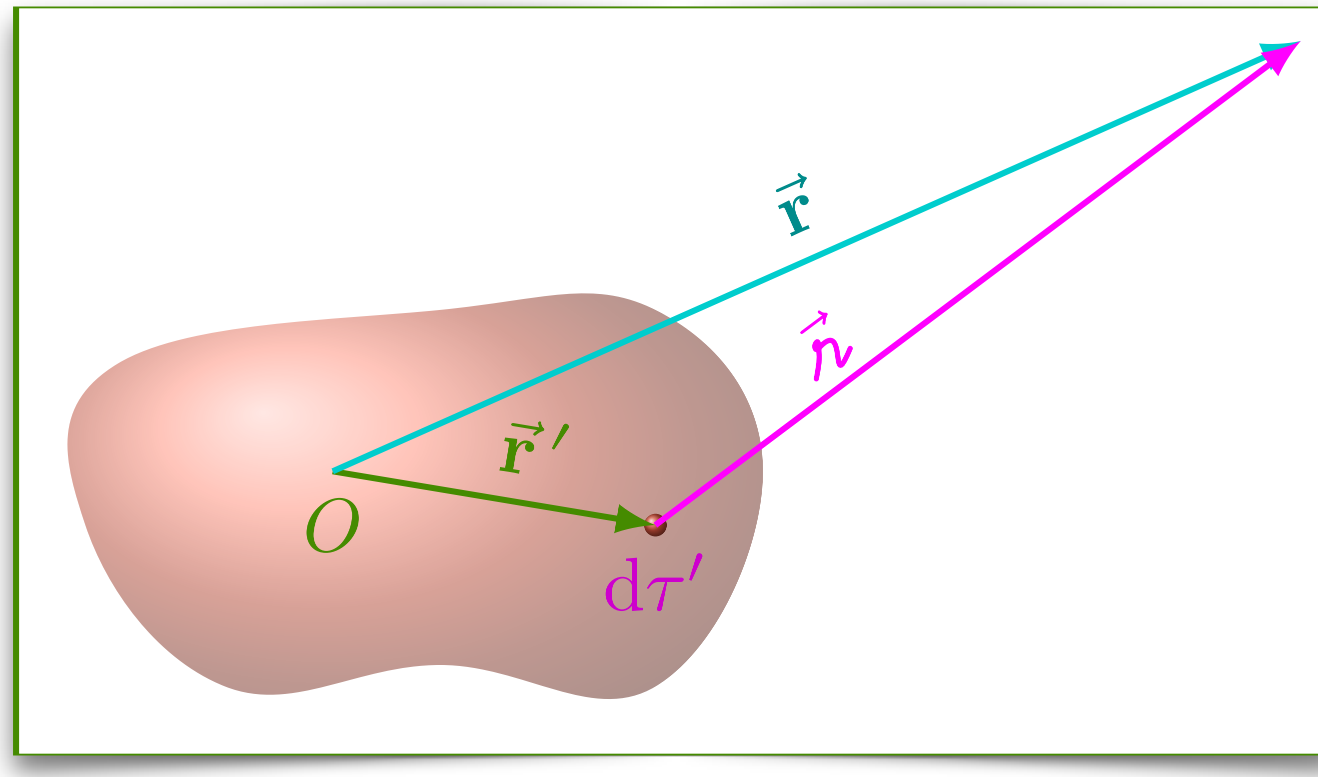


$$r' \ll \lambda \ll r$$

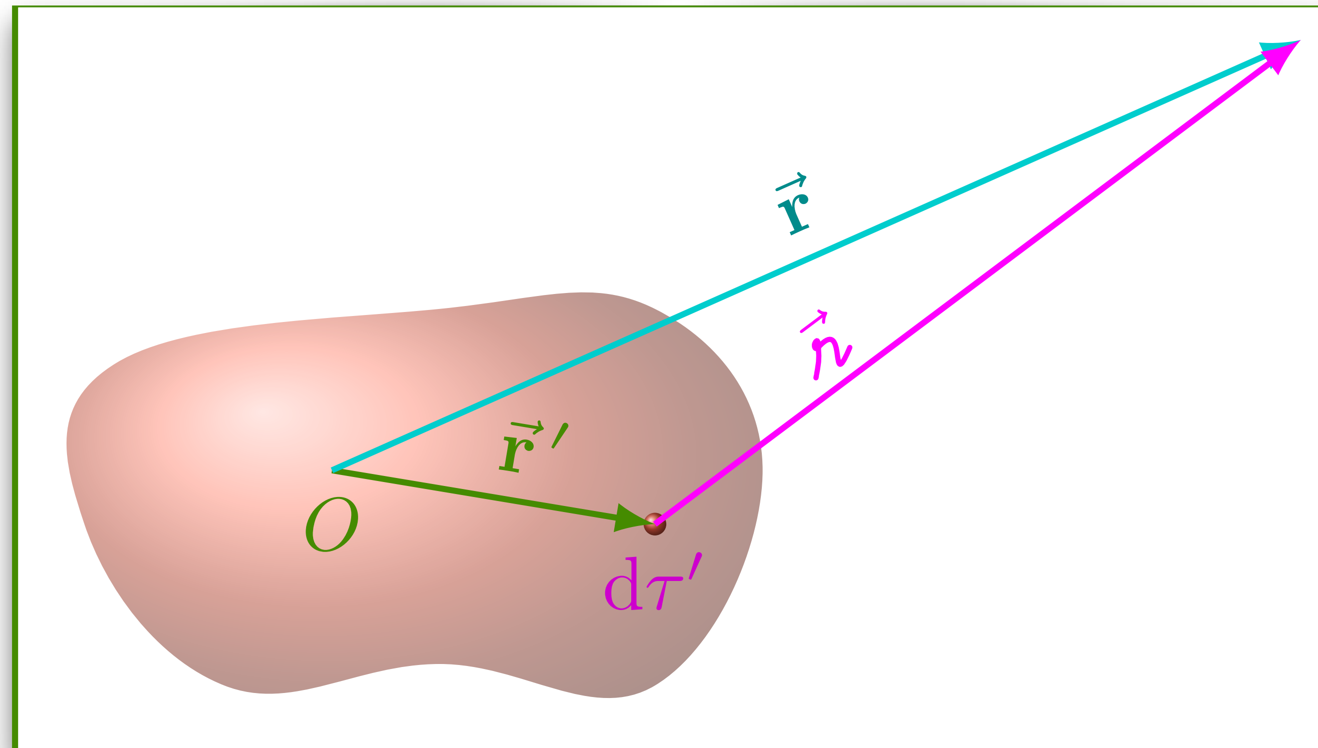
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}}\, \mathrm{d}\tau'$$

$$t - \frac{\mathscr{r}}{c}$$

Radiação de distribuição de cargas

$$\rho = \rho(\vec{\mathbf{r}}\,', t)$$

$$\vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t)$$



$$r' \ll \lambda \ll r$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$

$\mathscr{r} = r$ É APROXIMAÇÃO MAIS SIMPLES, MAS É RUIM

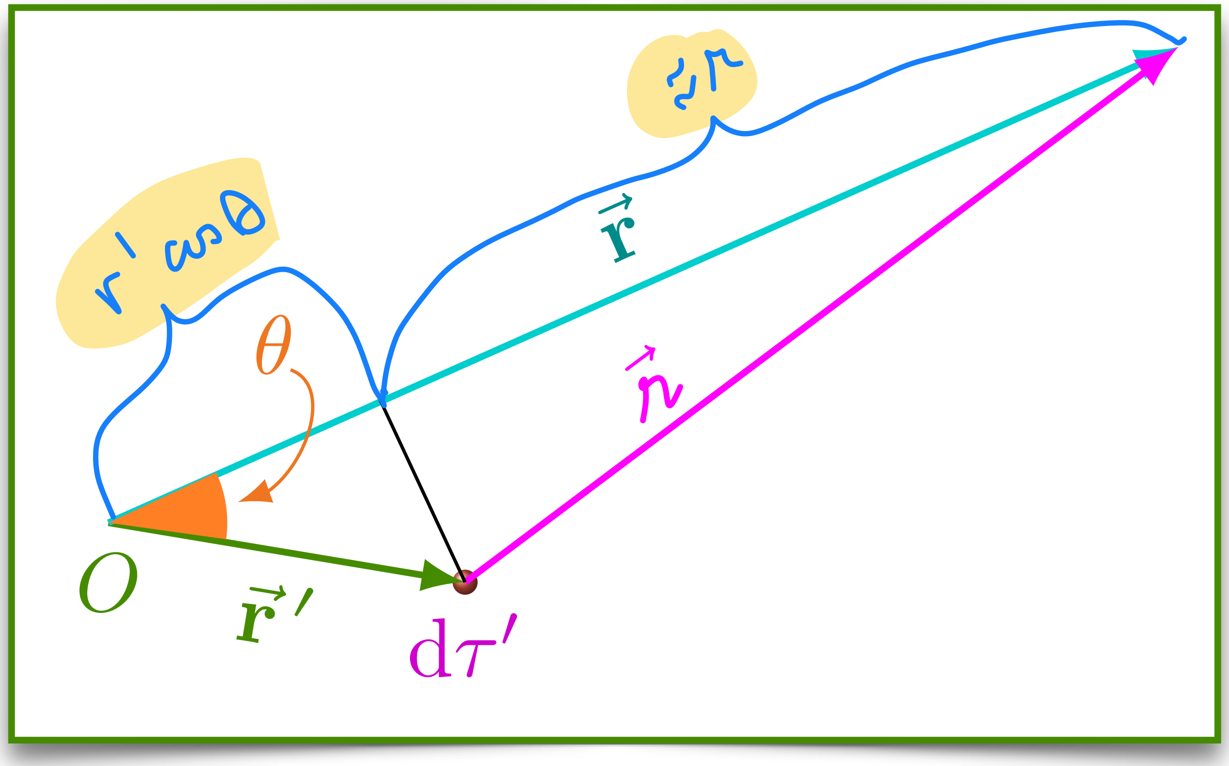
$\cdot\ \mathscr{r} = r \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$, ONDE $Q = \int \rho \, d\tau'$ É A CARGA NO SISTEMA.

$\Rightarrow$ NÃO HAVERIA RADIAÇÃO

Radiação de distribuição de cargas

$$\rho = \rho(\vec{\mathbf{r}}\,', t)$$

$$\vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t)$$



$$r' \ll \lambda \ll r$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{𝓇}\, \mathrm{d}\tau'$$

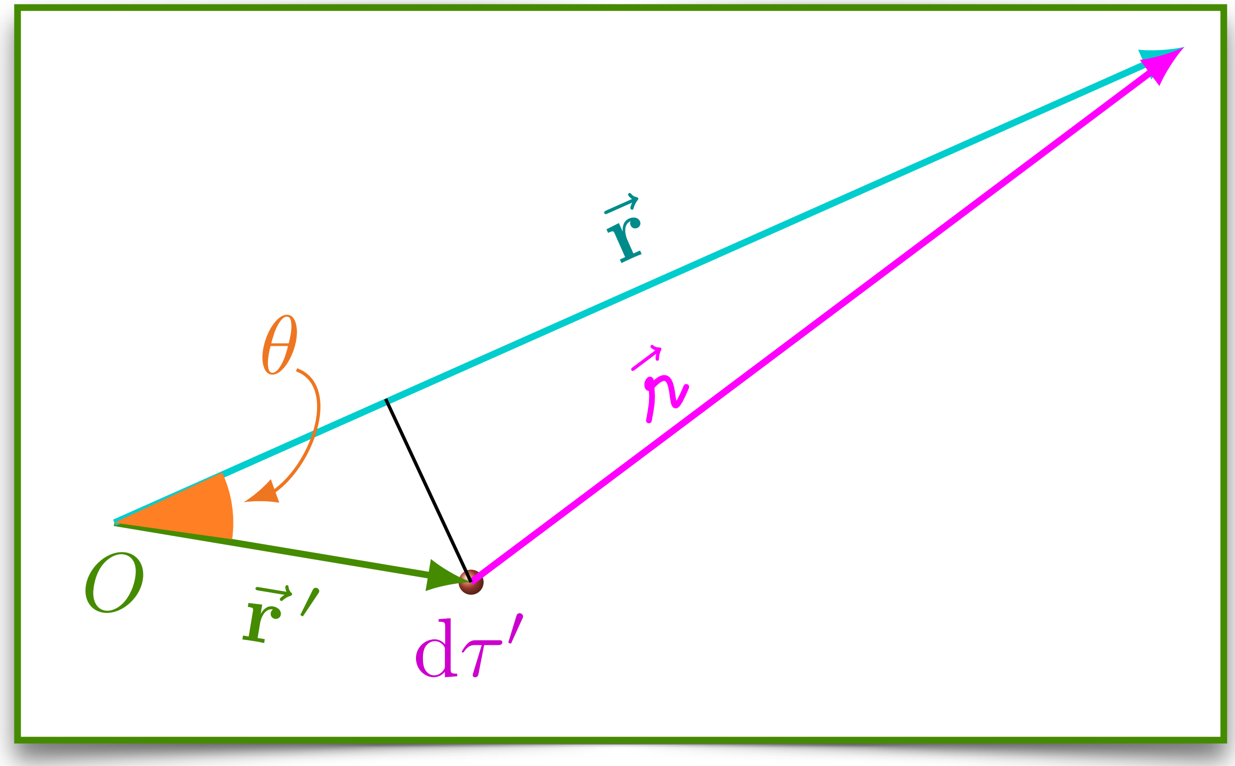
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{𝓇}{c})}{𝓇}\, \mathrm{d}\tau'$$

$r \cong 𝓇 + r' \cos\theta$

Radiação de distribuição de cargas

$$\rho = \rho(\vec{\mathbf{r}}\,', t)$$

$$\vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t)$$



$$r' \ll \lambda \ll r$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}}\, \mathrm{d}\tau'$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}}\, \mathrm{d}\tau'$$

$$\mathscr{r} \approx r - r' \cos\theta$$

Radiação de distribuição de cargas

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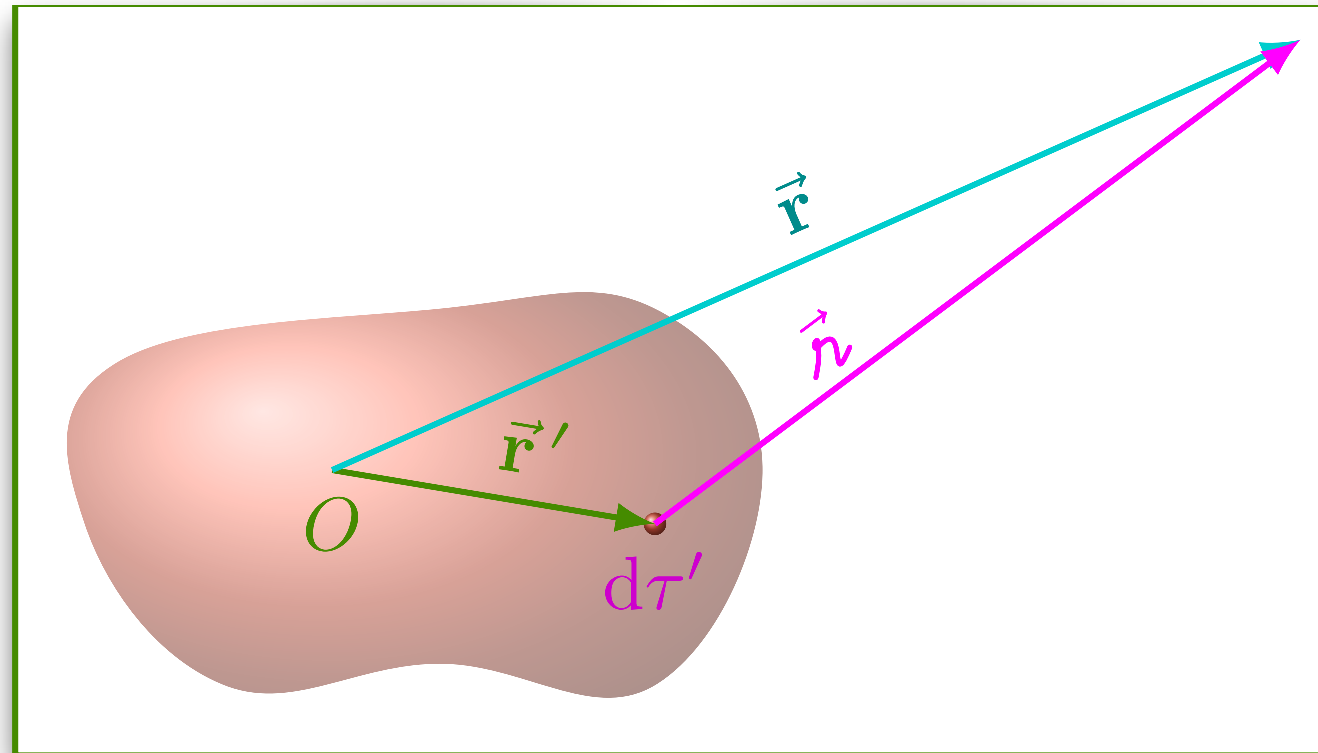


$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}}\, d\tau'$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}}\, d\tau'$$

$$\mathscr{r} \approx r - r' \cos\theta$$

$$\frac{1}{1-x} = 1 + x \quad (\text{TAYLOR, PARA } f(x) = \frac{1}{1-x})$$

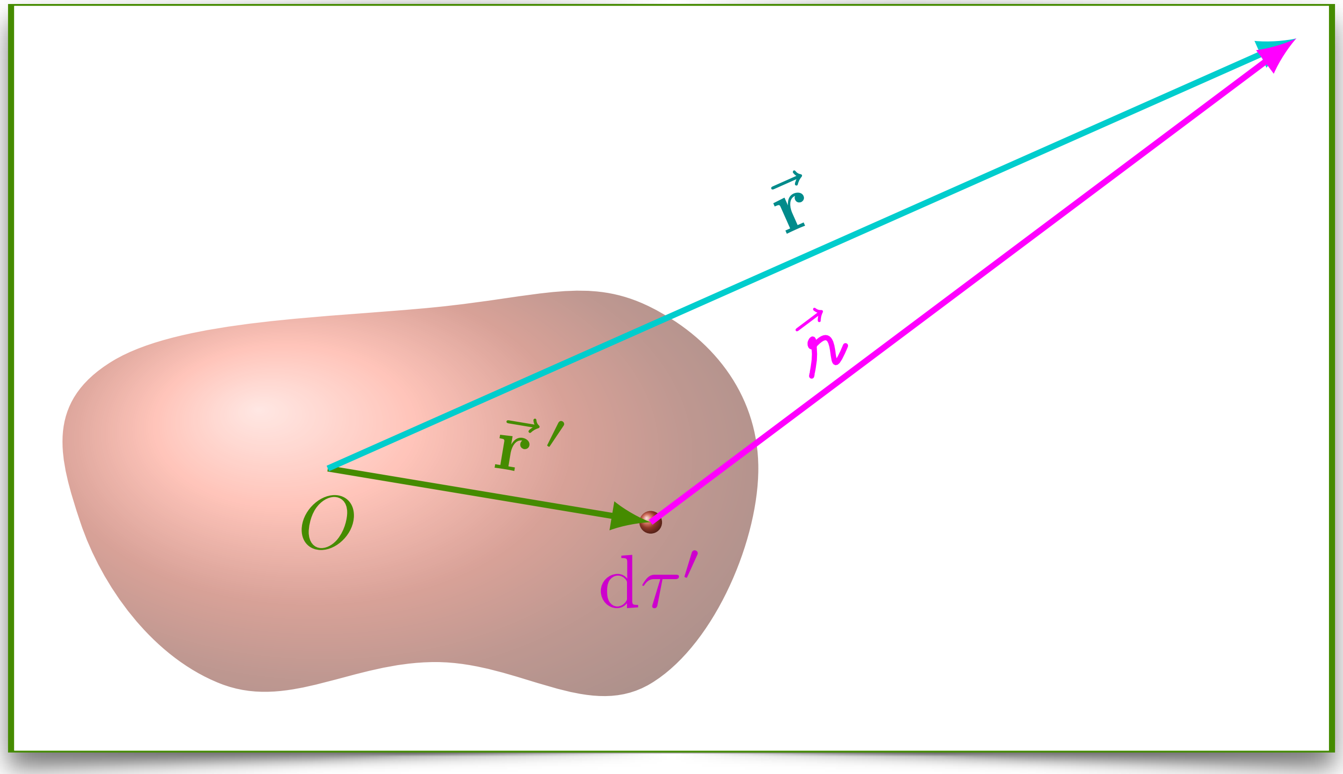
$$\mathscr{r} = r(1 - \frac{r'}{r} \cos\theta)$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r} \frac{1}{1 - \frac{r'}{r}\cos\theta} \simeq \frac{1}{r}\left(1 + \frac{r'}{r}\cos\theta\right)$$

Radiação de distribuição de cargas

$$\rho = \rho(\vec{\mathbf{r}}', t)$$

$$\vec{\mathbf{J}} = \vec{\mathbf{J}}(\vec{\mathbf{r}}', t)$$



$$r' \ll \lambda \ll r$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$\mathscr{r} \approx r - r' \cos\theta$$

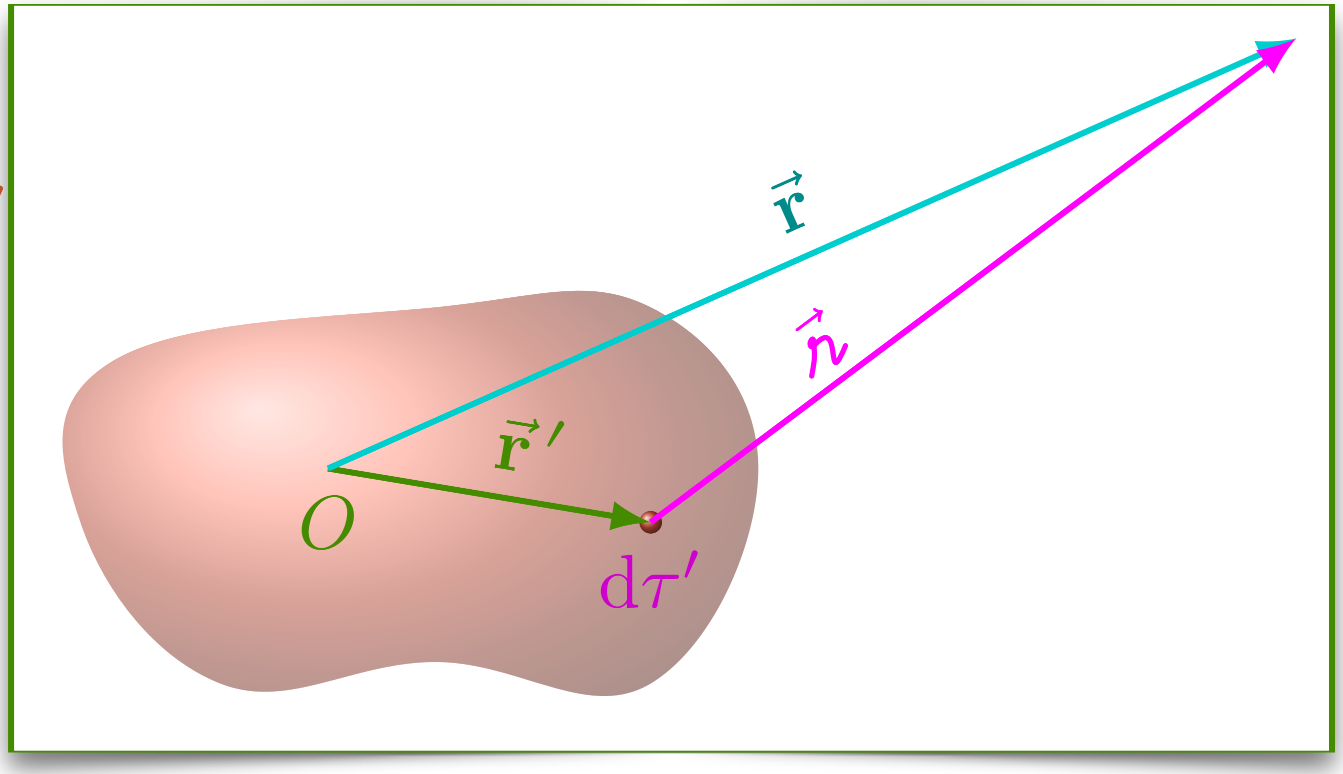
$$\mathscr{r} = r(1 - \frac{r'}{r} \cos\theta)$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1 + \frac{r'}{r} \cos\theta)$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}}\, \mathrm{d}\tau'$$



$$\mathscr{r} \approx r - r' \cos\theta$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1 + \frac{r'}{r} \cos\theta)$$

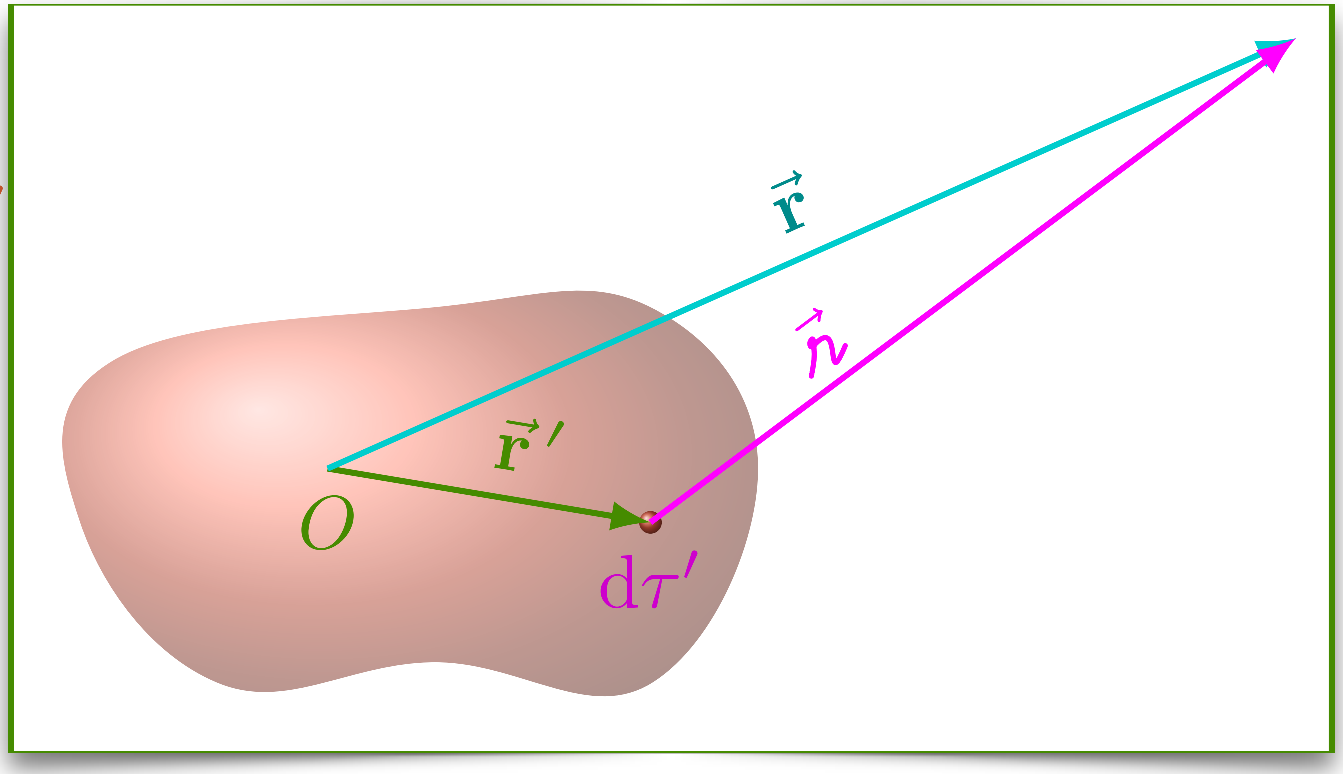
$\hat{r} \cdot \hat{r}' = \cos\theta$

$\hat{r} \cdot \vec{r}\,' = r' \cos\theta$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}}\, \mathrm{d}\tau'$$



$$\mathscr{r} \approx r - r' \cos\theta$$

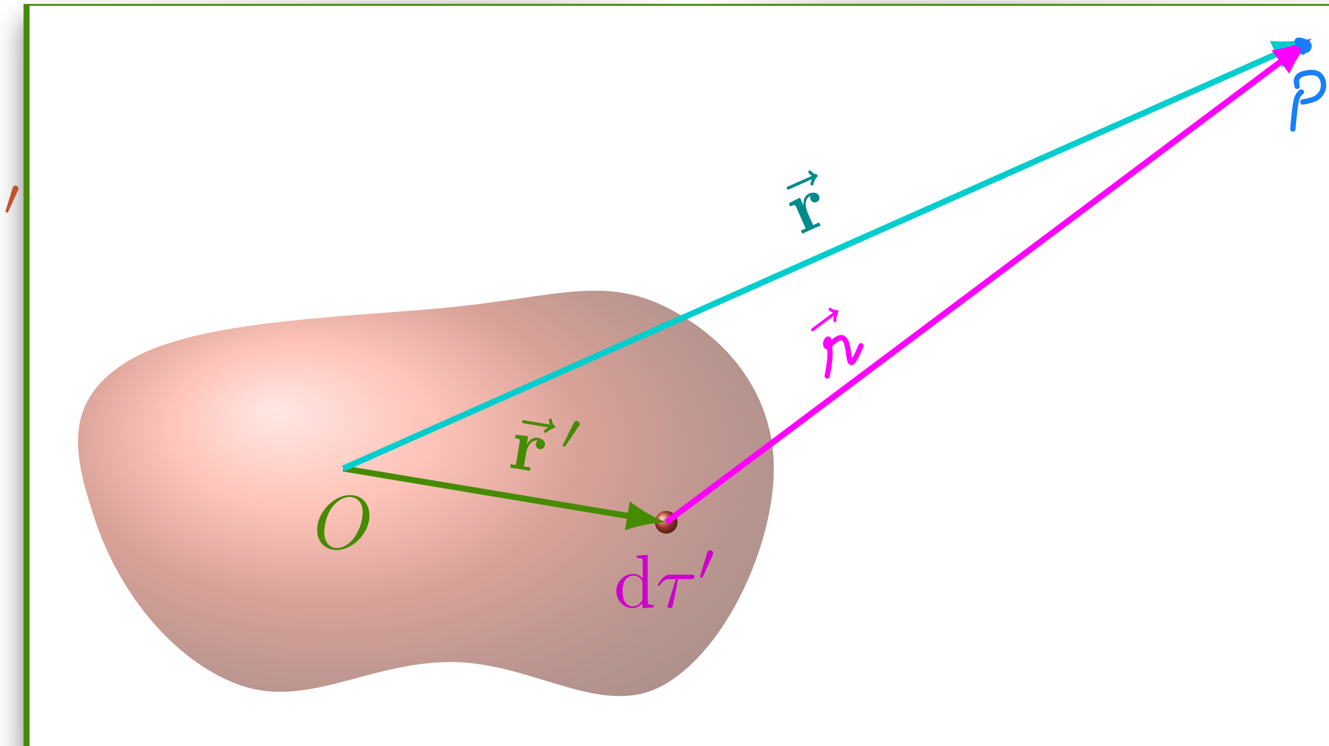
$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1 + \frac{r'}{r} \cos\theta)$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\frac{1}{r} \int \rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})\, \mathrm{d}\tau' + \frac{\hat{\mathbf{r}}}{r^2} \cdot \int \vec{\mathbf{r}}\,' \rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})\, \mathrm{d}\tau' \right)$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$



$$\mathscr{r} \approx r - r' \cos\theta$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left(1 + \frac{r'}{r}\cos\theta\right)$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{r}\int \rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})\, \mathrm{d}\tau' + \frac{\hat{\mathbf{r}}}{r^2} \cdot \int \vec{\mathbf{r}}\,'\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})\, \mathrm{d}\tau'\right)$$

$$\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c}) = \rho(\vec{\mathbf{r}}\,', t_0 + \frac{r'}{c}\cos\theta)$$

$$t - \frac{\mathscr{r}}{c} \cong t - \frac{(r - r'\cos\theta)}{c} = \underbrace{t - \frac{r}{c}}_{t_0} + \frac{r'}{c}\cos\theta = t_0 + \frac{r'}{c}\cos\theta$$

$t_0 \rightarrow$ t - TEMPO P/ LUZ IR DE O A P

Radiação de distribuição de cargas

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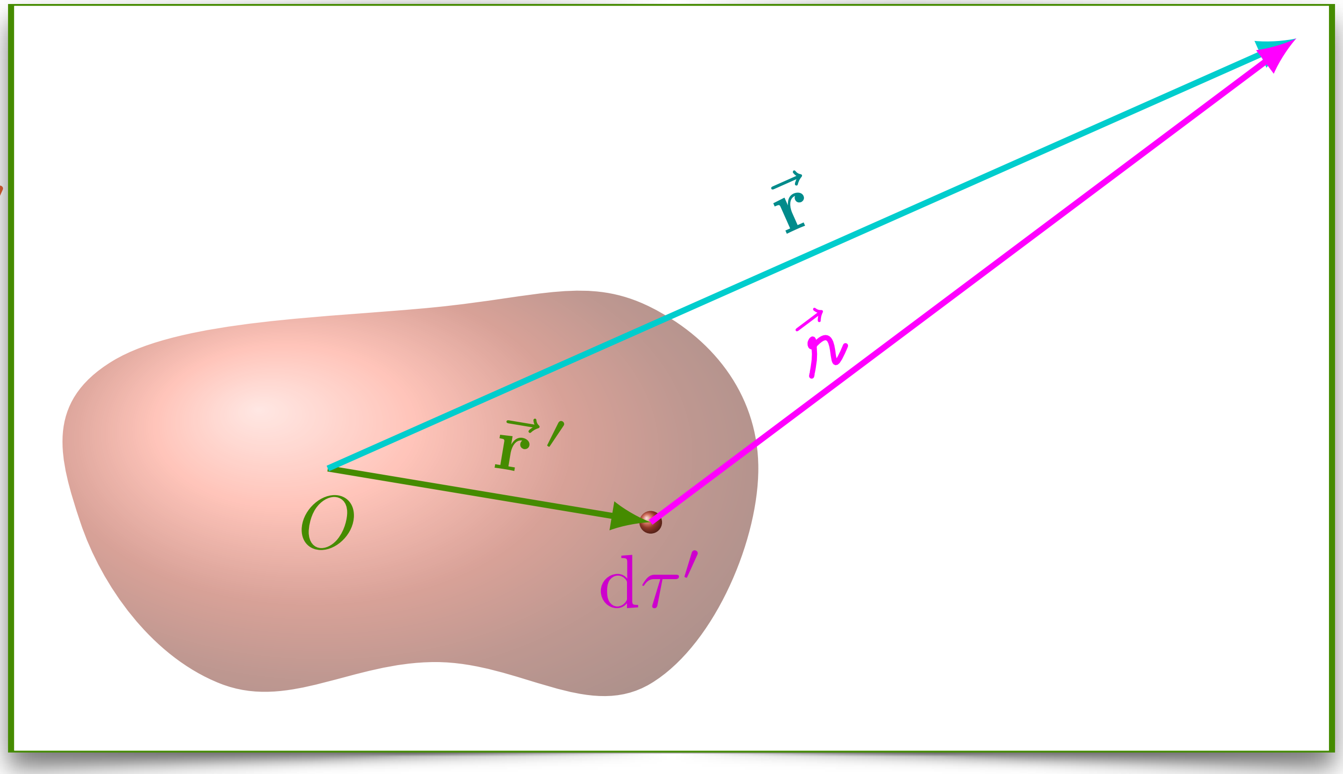

$$\mathscr{r} \approx r - r'\cos\theta$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left(1+\frac{r'}{r}\cos\theta\right)$$

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$$\rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c}) = \rho(\vec{\mathbf{r}}\,',t_0+\frac{r'}{c}\cos\theta)$$

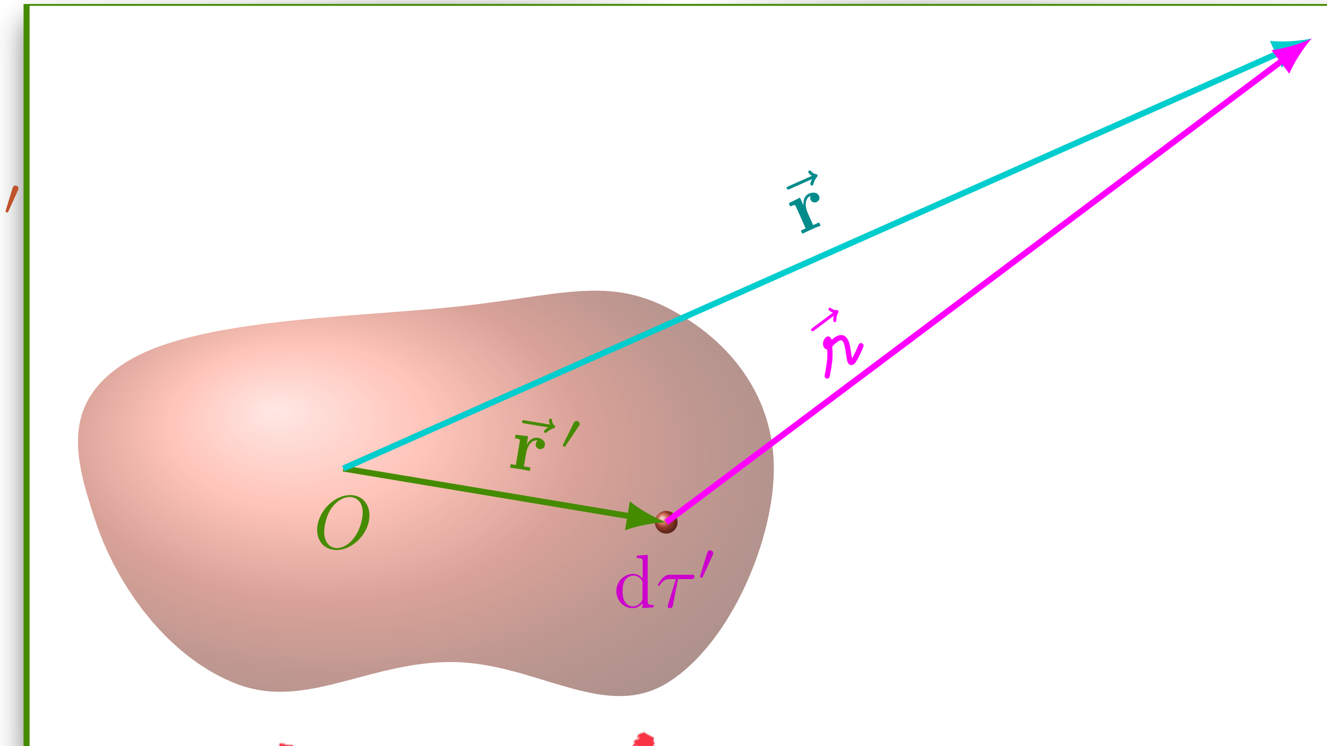
$$\rho(\vec{\mathbf{r}}\,',t_0+\frac{r'}{c}\cos\theta) = \rho(\vec{\mathbf{r}}\,',t_0) + \frac{r'}{c}\cos\theta\frac{\partial\rho}{\partial t}(\vec{\mathbf{r}}\,',t_0)$$

TAYLOR

Radiação de distribuição de cargas

$$r' \ll \lambda \ll r$$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,\mathrm{d}\tau'$$



$$\mathscr{r} \approx r - r'\cos\theta$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1+\frac{r'}{r}\cos\theta)$$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{r}\int \rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})\,\mathrm{d}\tau' + \frac{\hat{\mathbf{r}}}{r^2}\cdot\cancel{\int \vec{\mathbf{r}}\,'\rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})\,\mathrm{d}\tau'}\right)$$

$\frac{1}{r^2}$ → NÃO CONTRIBUI PARA RADIAÇÃO

$$\rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c}) = \rho(\vec{\mathbf{r}}\,',t_0+\frac{r'}{c}\cos\theta)$$

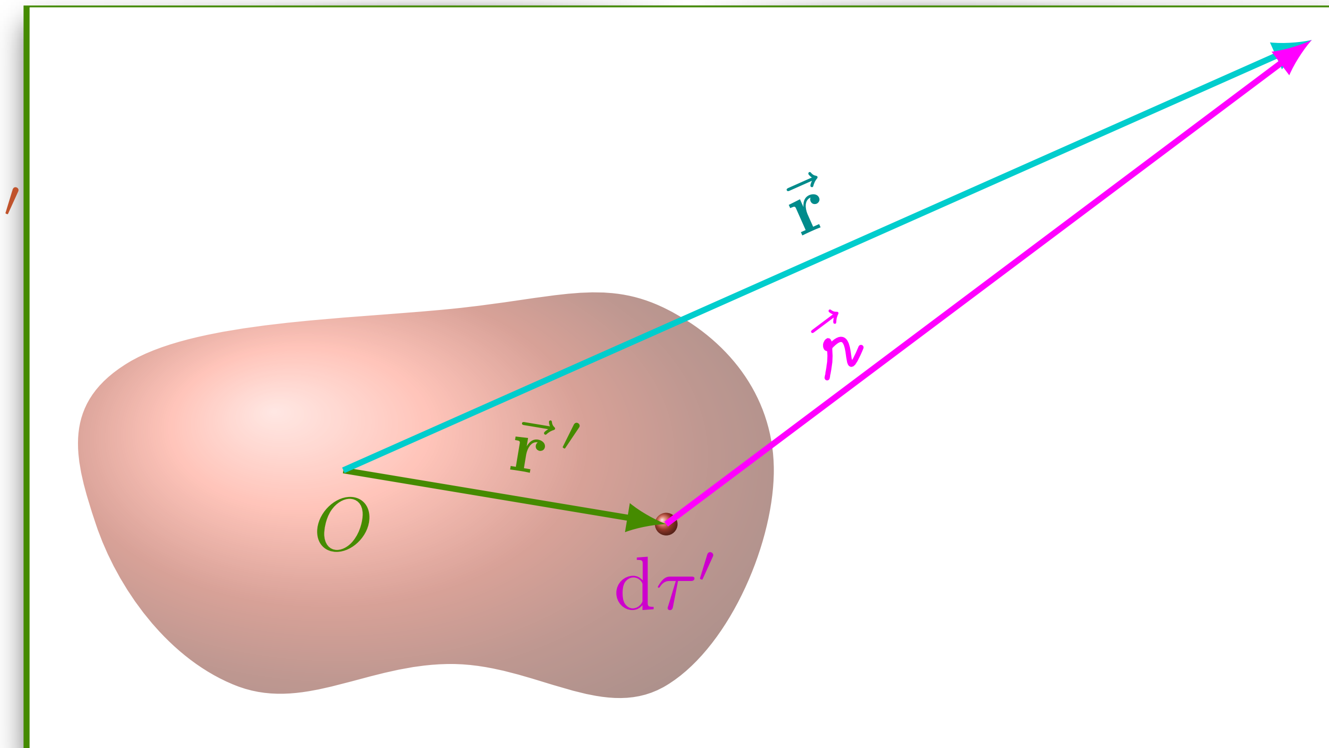
$$\rho(\vec{\mathbf{r}}\,',t_0+\frac{r'}{c}\cos\theta) = \rho(\vec{\mathbf{r}}\,',t_0) + \frac{r'}{c}\cos\theta\,\frac{\partial\rho}{\partial t}(\vec{\mathbf{r}}\,',t_0)$$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{r}\int \rho(\vec{\mathbf{r}}\,',t_0)\,\mathrm{d}\tau' + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\int \vec{\mathbf{r}}\,'\rho(\vec{\mathbf{r}}\,',t=t_0)\,\mathrm{d}\tau'\right)$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}}\, \mathrm{d}\tau'$$



$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\frac{1}{r} \underbrace{\int \rho(\vec{\mathbf{r}}\,', t_0)\, \mathrm{d}\tau'}_{\substack{Q \\ \text{(CARGA)}}} + \frac{\hat{\mathbf{r}}}{rc} \cdot \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\int \vec{\mathbf{r}}\,' \rho(\vec{\mathbf{r}}\,', t = t_0)\, \mathrm{d}\tau'}_{\substack{\vec{p} \\ \text{(MOMENTO DE DIPOLO)}}} \right)$$

Radiação de distribuição de cargas

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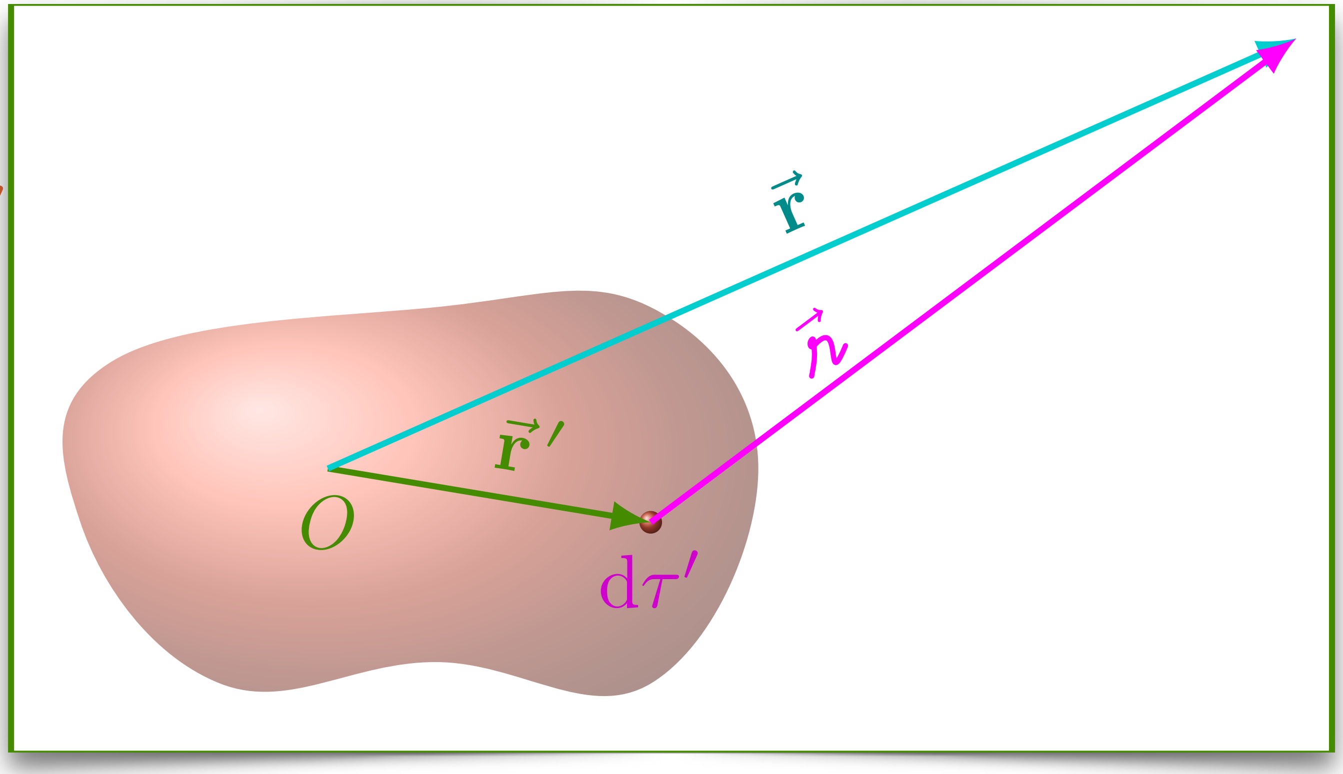

$$t_0 = t - \frac{r}{c}$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\frac{1}{r} \int \rho(\vec{\mathbf{r}}\,', t_0) \, \mathrm{d}\tau' + \frac{\hat{\mathbf{r}}}{rc} \cdot \frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{r}}\,' \rho(\vec{\mathbf{r}}\,', t = t_0) \, \mathrm{d}\tau' \right)$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc} \cdot \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0} \right)$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

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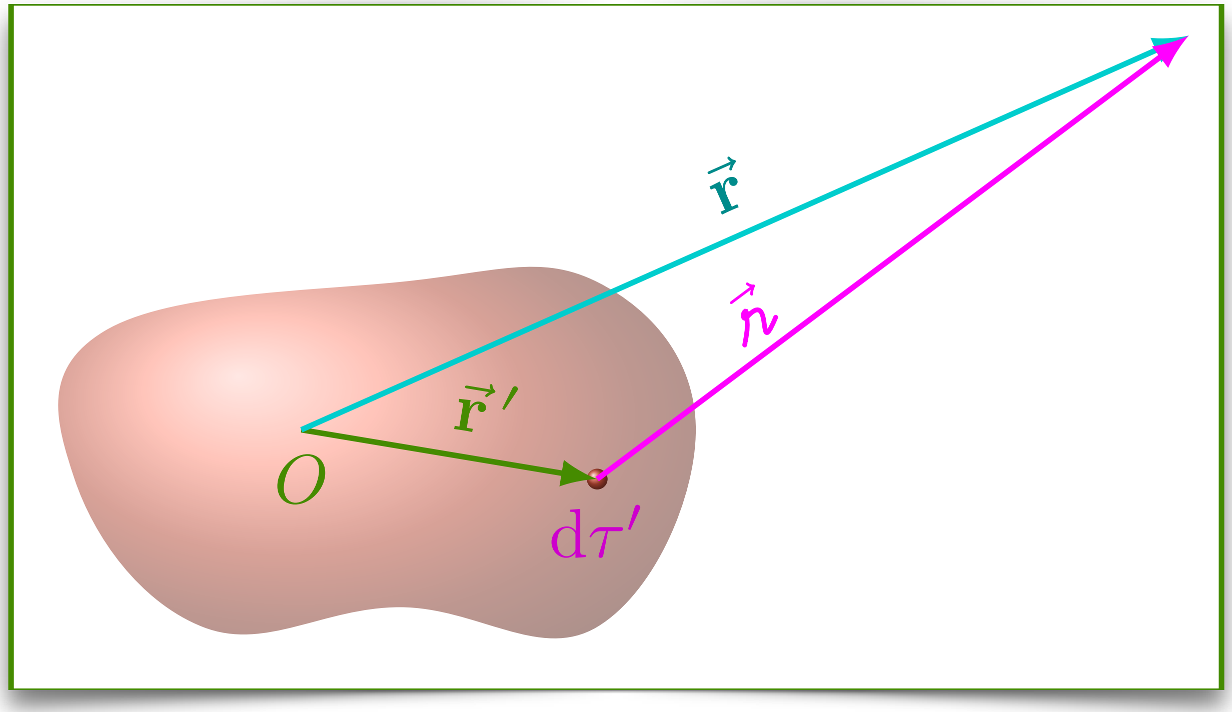
$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\bigg|_{t_0}\right)$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,\mathrm{d}\tau'$$



Radiação de distribuição de cargas

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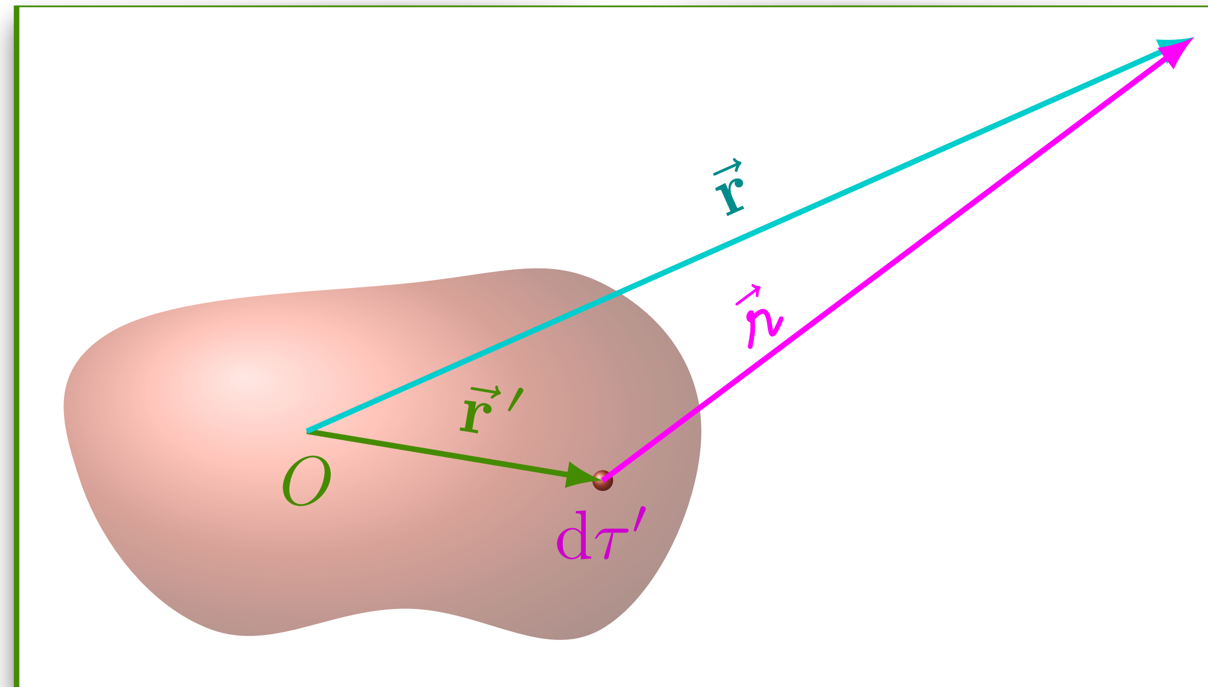
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc} \cdot \left.\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\right|_{t_0} \right)$$



$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t - \frac{\mathscr{r}}{c})}{\underset{\longrightarrow\ \approx r}{\mathscr{r}}} \, \mathrm{d}\tau'$$

(pois $\frac{1}{\mathscr{r}} = \frac{1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$)

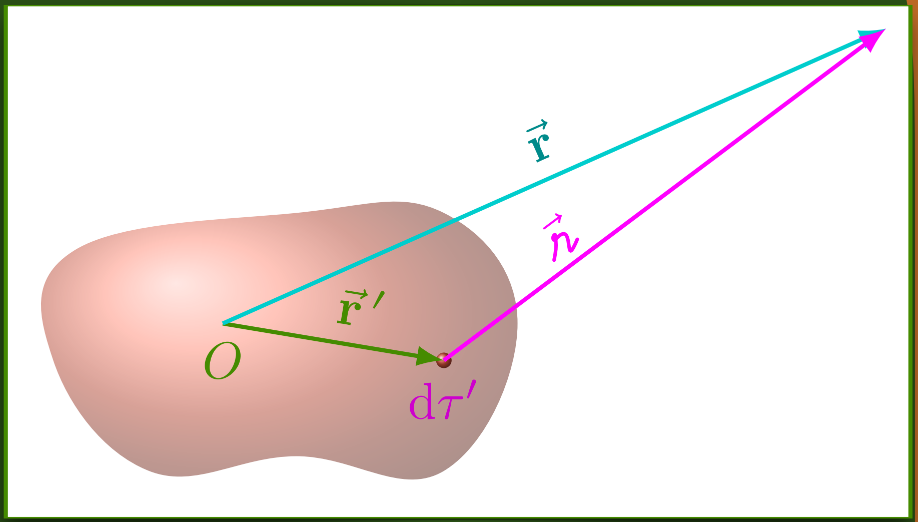
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) \approx \frac{\mu_0}{4\pi r} \int \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t - \frac{r}{c}) \, \mathrm{d}\tau'$$

$$\int \vec{\boldsymbol{\nabla}} \cdot \left(x\vec{\mathbf{J}}\right) \mathrm{d}\tau = \int x\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{J}} \mathrm{d}\tau + \int J_x \mathrm{d}\tau$$

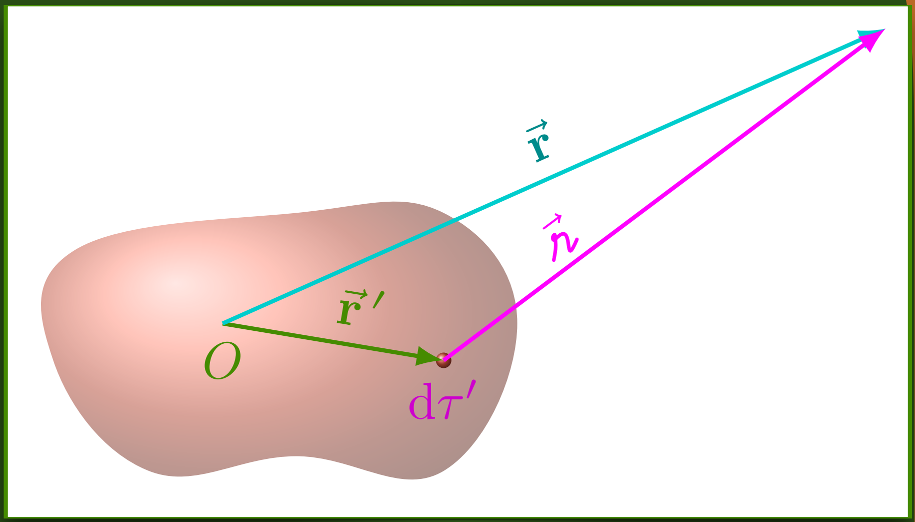
$\Big\downarrow$ GAUSS

$$\int x \underbrace{\vec{J} \cdot \hat{n}}_{} \, dA$$

0 PORQUE
CORRENTE
NEM ENTRA
NEM SAI
DO SISTEMA



$$\int \vec{\boldsymbol{\nabla}}\cdot\left(x\vec{\mathbf{J}}\right)\mathrm{d}\tau = \int x\underbrace{\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{J}}}_{-\frac{\partial\rho}{\partial t}}\mathrm{d}\tau + \int J_x\mathrm{d}\tau$$



$$0 = -\int x\frac{\partial\rho}{\partial t}\mathrm{d}\tau + \int J_x\mathrm{d}\tau$$

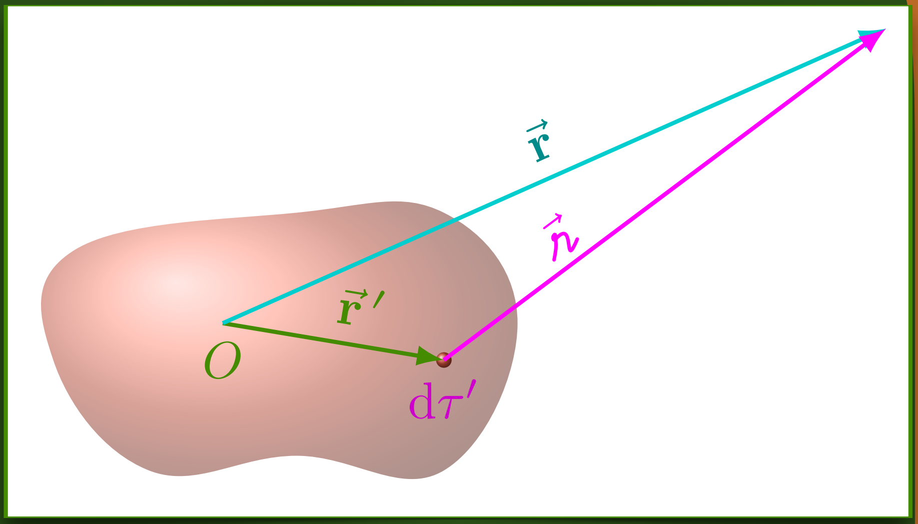
$$\Rightarrow \int J_x\, d\tau = \int x\frac{\partial\rho}{\partial t}\, d\tau$$

$$\int J_y\, d\tau = \int y\frac{\partial\rho}{\partial t}\, d\tau$$

$$\int J_z\, d\tau = \int z\frac{\partial\rho}{\partial t}\, d\tau$$

$$\int \vec{J}\, d\tau = \int \vec{r}\,\frac{\partial\rho}{\partial t}\, d\tau$$

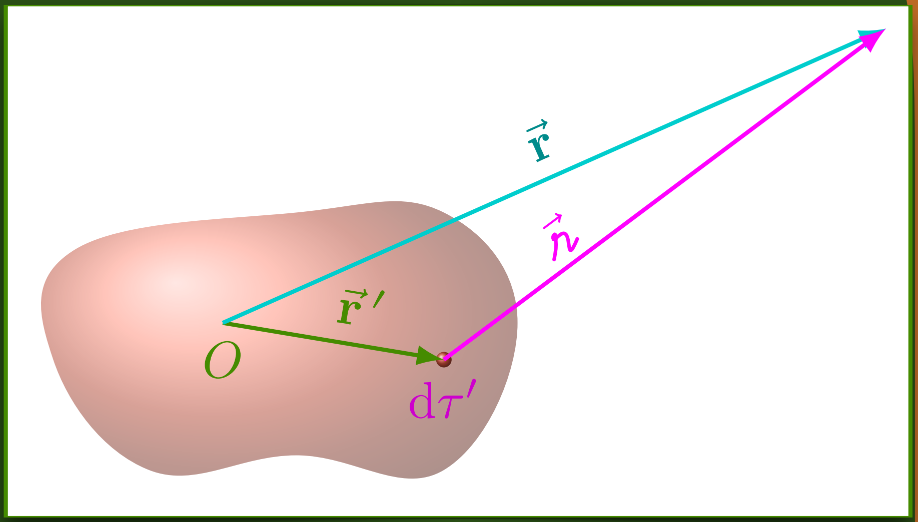
$$\int \vec{\boldsymbol{\nabla}} \cdot \left(x\vec{\mathbf{J}}\right) \mathrm{d}\tau = \int x \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{J}} \mathrm{d}\tau + \int J_x \mathrm{d}\tau$$



$$0 = -\int x \frac{\partial \rho}{\partial t} \mathrm{d}\tau + \int J_x \mathrm{d}\tau$$

$$\int \vec{\mathbf{J}}\, \mathrm{d}\tau = \int \vec{\mathbf{r}} \frac{\partial \rho}{\partial t} \mathrm{d}\tau = \frac{d}{dt} \int \vec{r}\, \rho \, d\tau$$

$$\int \vec{\boldsymbol{\nabla}} \cdot \left(x\vec{\mathbf{J}}\right) \mathrm{d}\tau = \int x \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{J}} \mathrm{d}\tau + \int J_x \mathrm{d}\tau$$



$$0 = -\int x \frac{\partial \rho}{\partial t} \mathrm{d}\tau + \int J_x \mathrm{d}\tau$$

$$\int \vec{\mathbf{J}}\, \mathrm{d}\tau = \int \vec{\mathbf{r}} \frac{\partial \rho}{\partial t} \mathrm{d}\tau$$

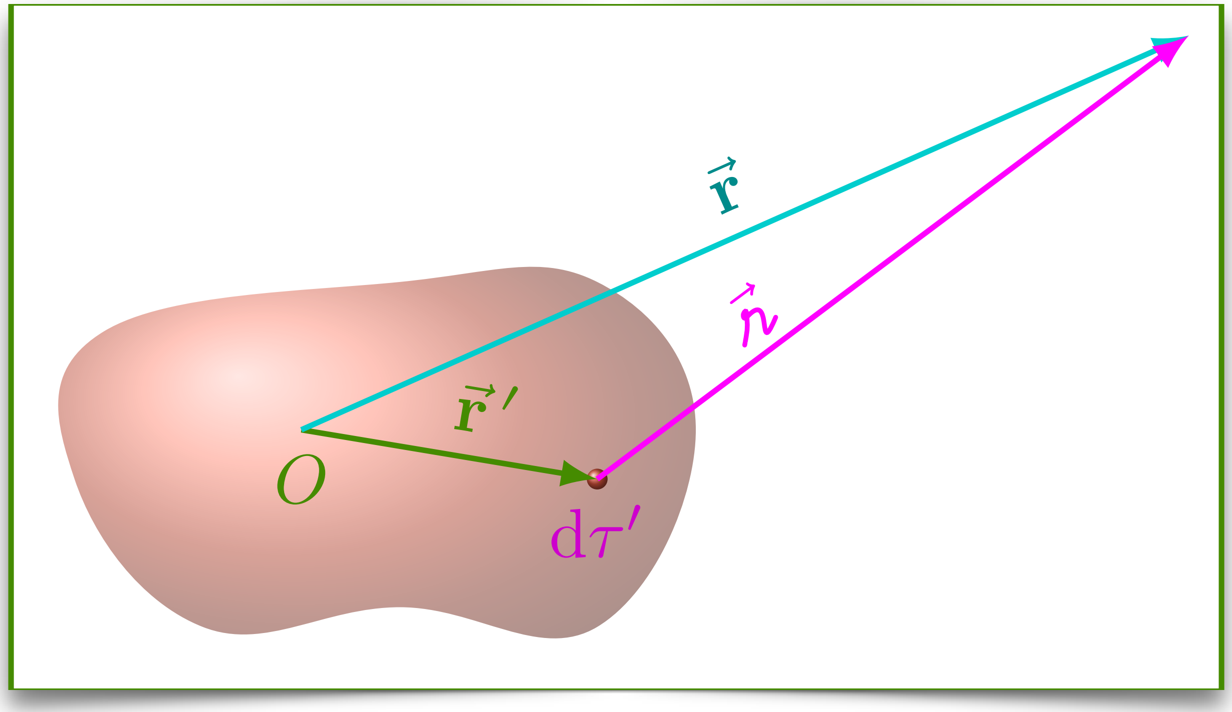
$$\int \vec{\mathbf{J}}\, \mathrm{d}\tau = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\Big(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}\Big)$$



$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,',t-\frac{\mathscr{r}}{c})}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$\int \vec{\mathbf{J}}\,\mathrm{d}\tau = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) \approx \frac{\mu_0}{4\pi r}\int \vec{\mathbf{J}}(\vec{\mathbf{r}}\,',t-\frac{r}{c})\,\mathrm{d}\tau'$$

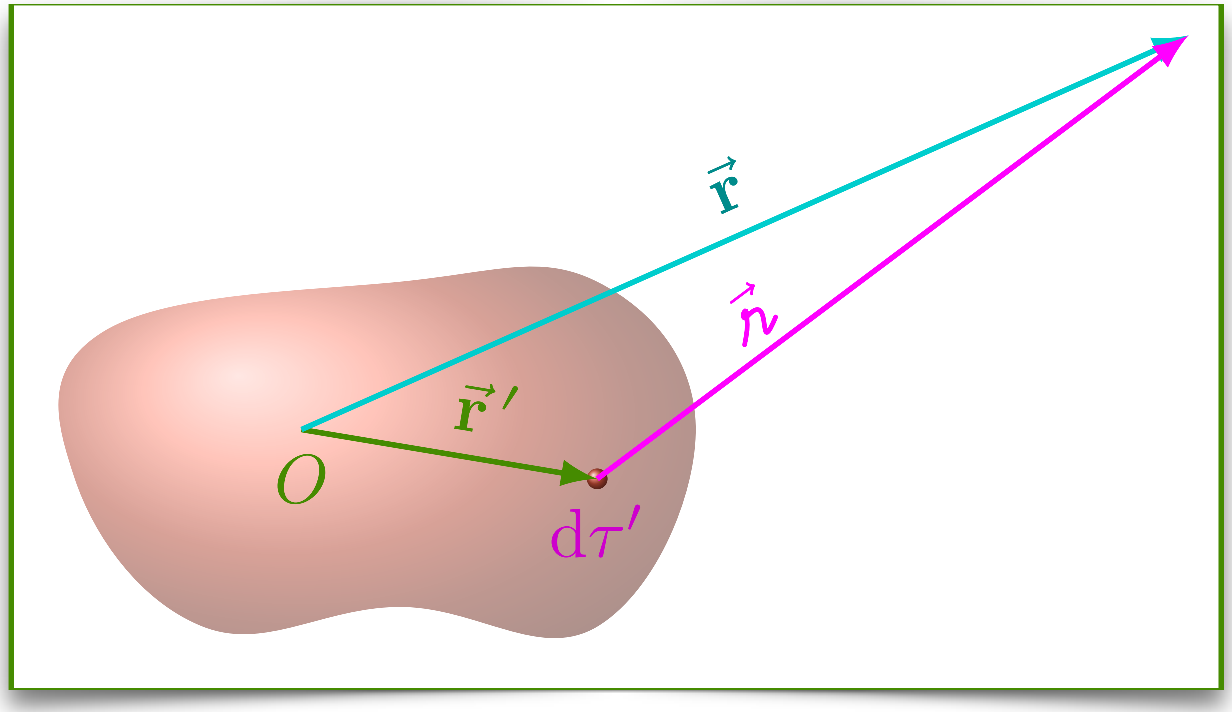
$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) \approx \frac{\mu_0}{4\pi r}\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\Big(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}\Big)$$

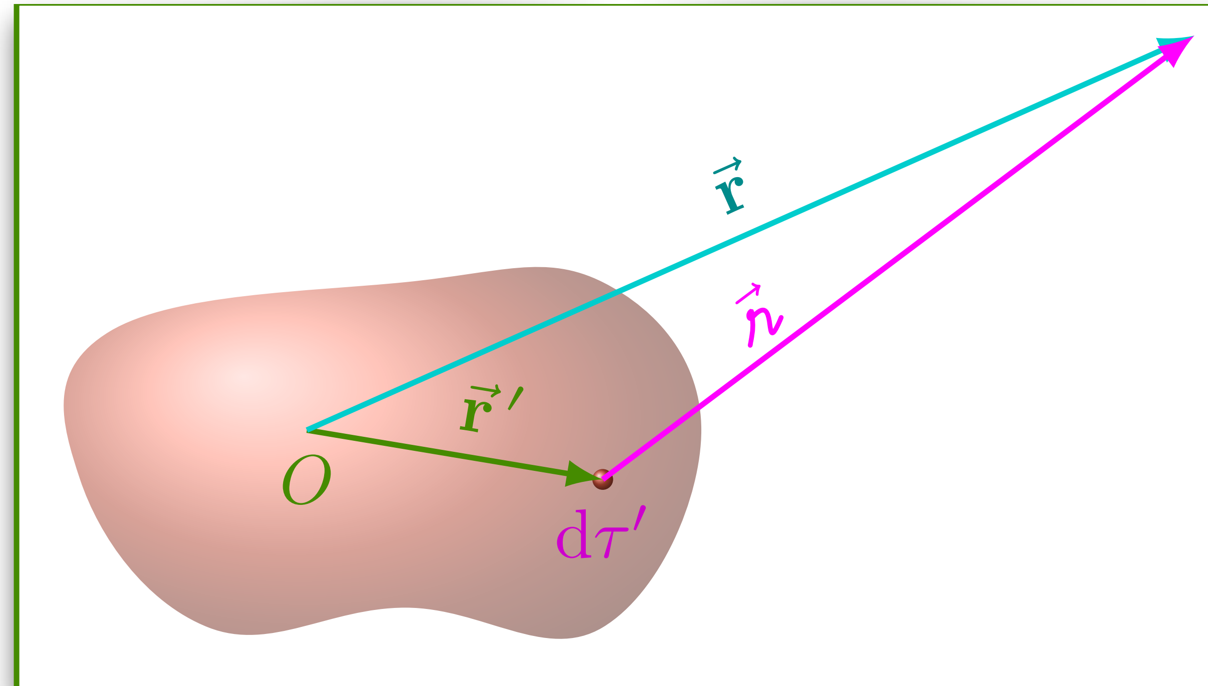
$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi r}\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}$$



Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}\right)$$



$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi r}\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}$$

$$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{rc}\cdot\vec{\nabla}\left(\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}\right)$$

$\frac{d\vec{p}}{dt}$ É FUNÇÃO DE $t_0 = t - \frac{r}{c}$

$$\vec{\nabla} V = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\,\vec{\nabla} t_0$$

$$\vec{\nabla}\left(\frac{d\vec{p}}{dt}\right)_{t_0} = \left(\frac{d^2\vec{p}}{dt^2}\right)_{t_0}\vec{\nabla} t_0$$

$\hat{r}\,\frac{\partial}{\partial r}(r) = \hat{r}$

ESFÉRICAS

$$\vec{\nabla} V = -\frac{1}{4\pi\epsilon_0 c^2}\frac{\hat{\mathbf{r}}}{r}\cdot\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\,\hat{\mathbf{r}}$$

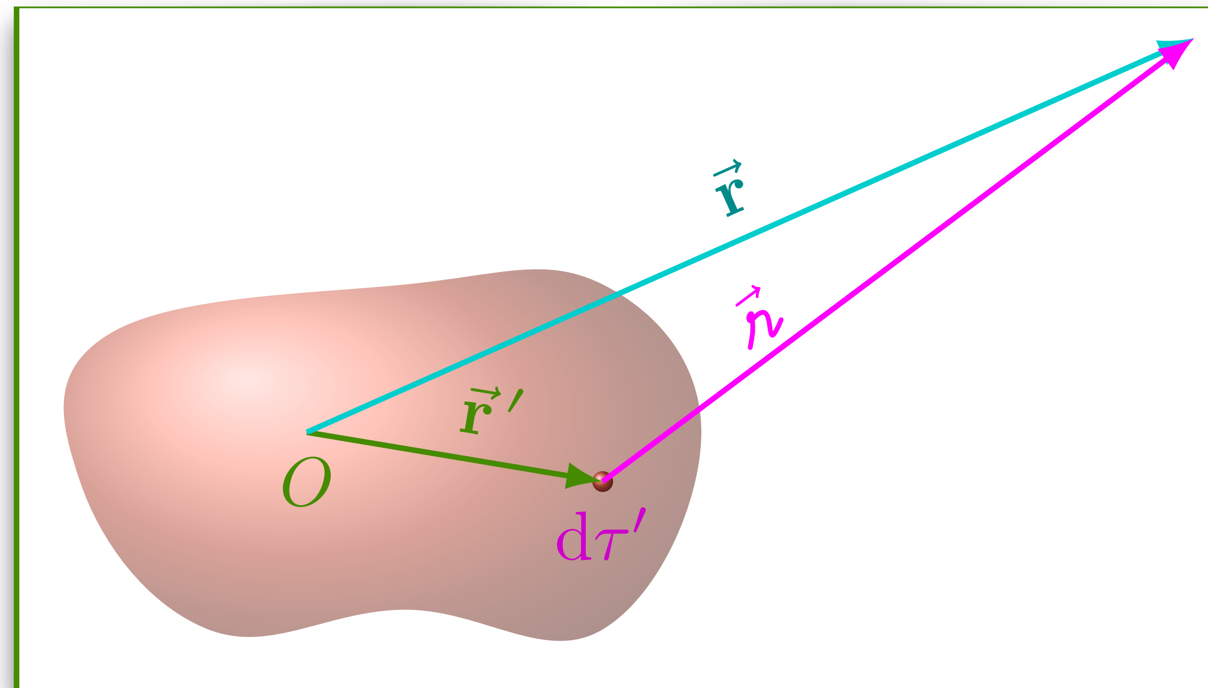
$$\vec{\nabla} t_0 = \vec{\nabla}\left(t - \frac{r}{c}\right) = -\frac{1}{c}\vec{\nabla} r$$

$$\vec{\nabla} t_0 = -\frac{1}{c}\hat{r}$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{\hat{\mathbf{r}}}{rc}\cdot\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}\right)$$



$$\boxed{\vec{\boldsymbol{\nabla}} V = -\frac{1}{4\pi\epsilon_0 c^2}\frac{\hat{\mathbf{r}}}{r}\cdot\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\,\hat{\mathbf{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi r}\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\Big|_{t_0}$$

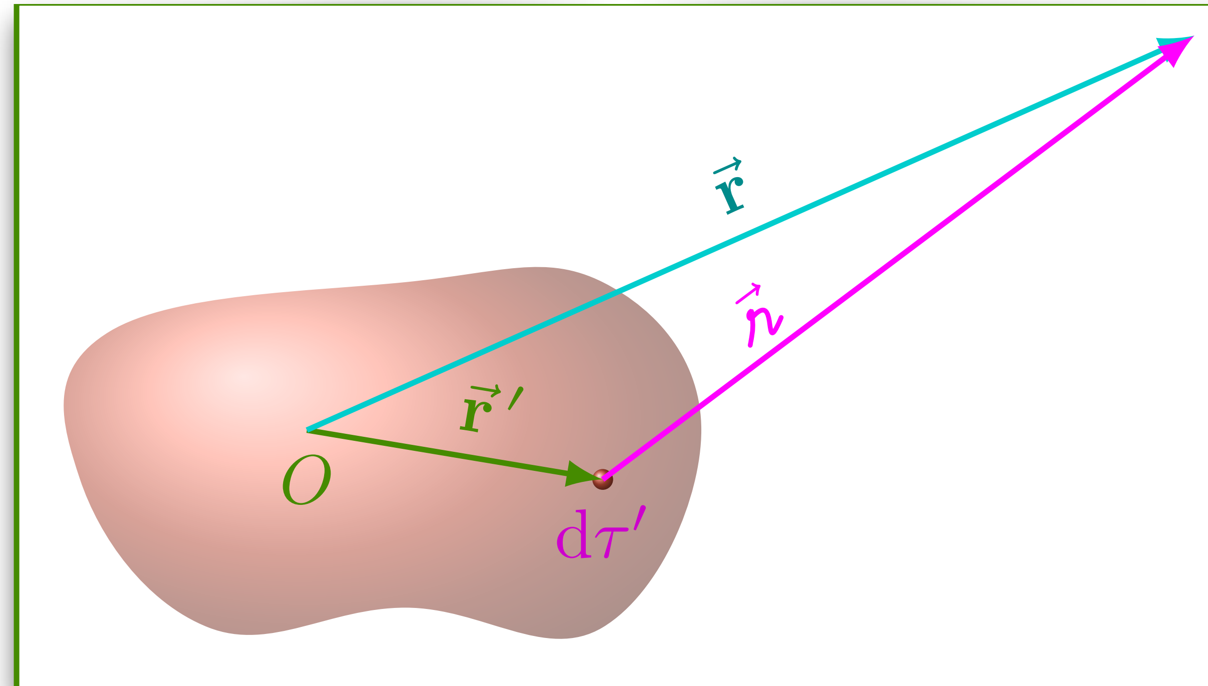
$$\frac{\partial\vec{\mathbf{A}}}{\partial t} = \frac{\mu_0}{4\pi r}\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}$$

$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi}\left(\frac{\hat{\mathbf{r}}}{r}\cdot\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\,\hat{\mathbf{r}} - \frac{1}{r}\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi r}\,\vec{\nabla}\times\left(\frac{d\vec{p}}{dt}\right)_{t_0} = -\frac{\mu_0}{4\pi r c}\left(\vec{r}\times\left(\frac{d^2 \vec{p}}{dt^2}\right)_{t_0}\right)$$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$



$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi r}\left(\left(\hat{\mathbf{r}} \cdot \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)\hat{\mathbf{r}} - \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi r c}\left(\hat{\mathbf{r}} \times \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{E} = -\frac{\mu_0}{4\pi r}\underbrace{\left(\frac{d^2 \vec{p}}{dt^2} - \left(\hat{r}\cdot\frac{d^2\vec{p}}{dt^2}\right)\hat{r}\right)}$$

$\frac{d^2 \vec{p}}{dt^2}$ - SUA PROJEÇÃO NA DIREÇÃO $\hat{r}$

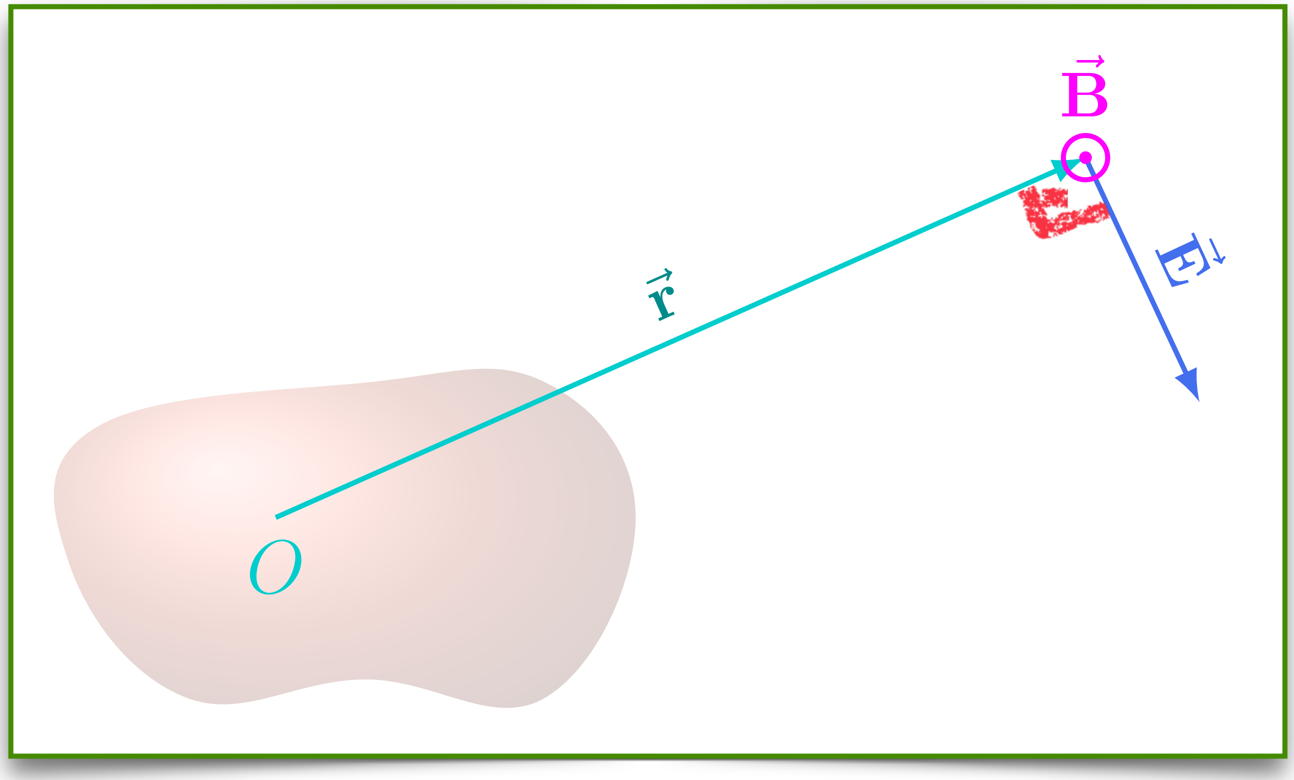
PARTE DE $\frac{d^2\vec{p}}{dt^2}$ $\perp$ a $\hat{r}$

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi r}\left(\left(\hat{\mathbf{r}} \cdot \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)\hat{\mathbf{r}} - \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi r c}\left(\hat{\mathbf{r}} \times \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

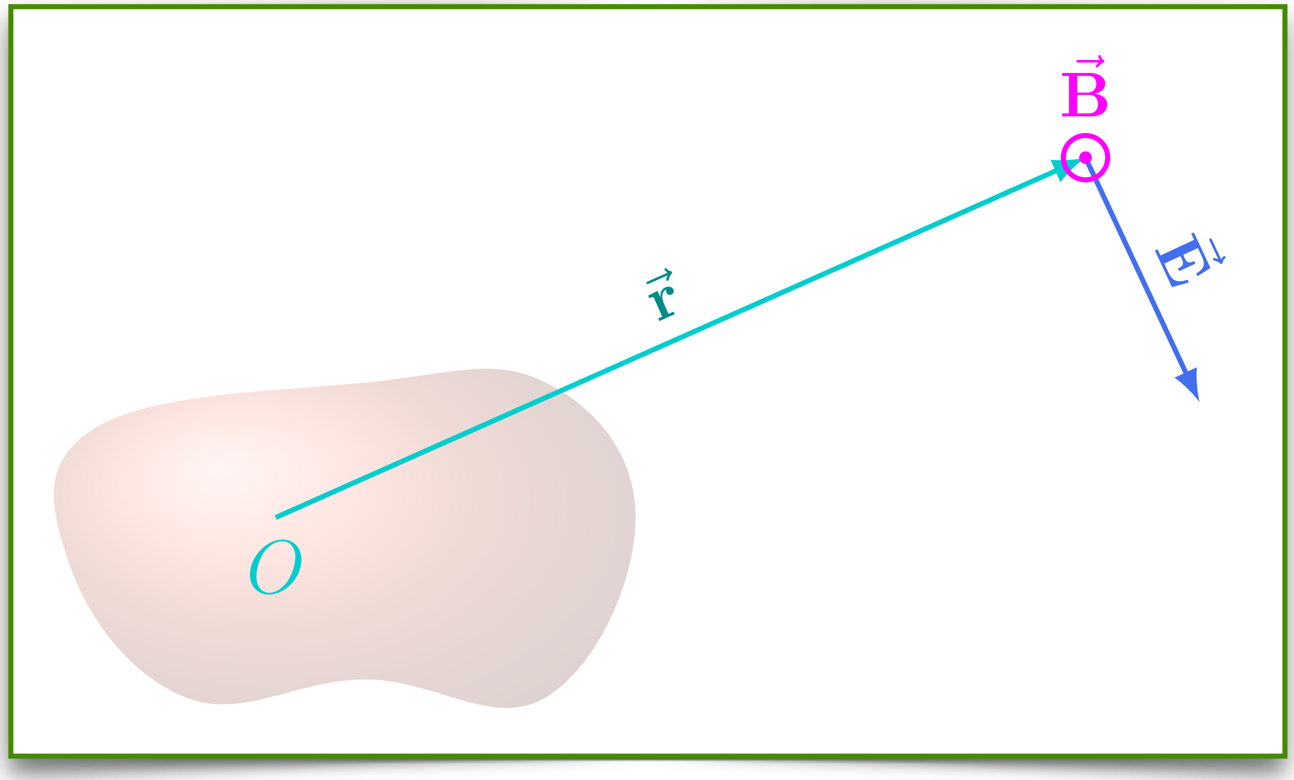


Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi r}\left(\left(\hat{\mathbf{r}} \cdot \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)\hat{\mathbf{r}} - \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi r c}\left(\hat{\mathbf{r}} \times \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$



$$\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2} \parallel \hat{\mathbf{z}} \quad \Rightarrow \quad \left\{ \begin{aligned} \vec{\mathbf{E}} &= \frac{\mu_0}{4\pi r}\left|\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right| \sin\theta\,\hat{\theta} \\ \vec{\mathbf{B}} &= \frac{\mu_0}{4\pi r c}\left|\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right| \sin\theta\,\hat{\phi} \end{aligned} \right\}$$

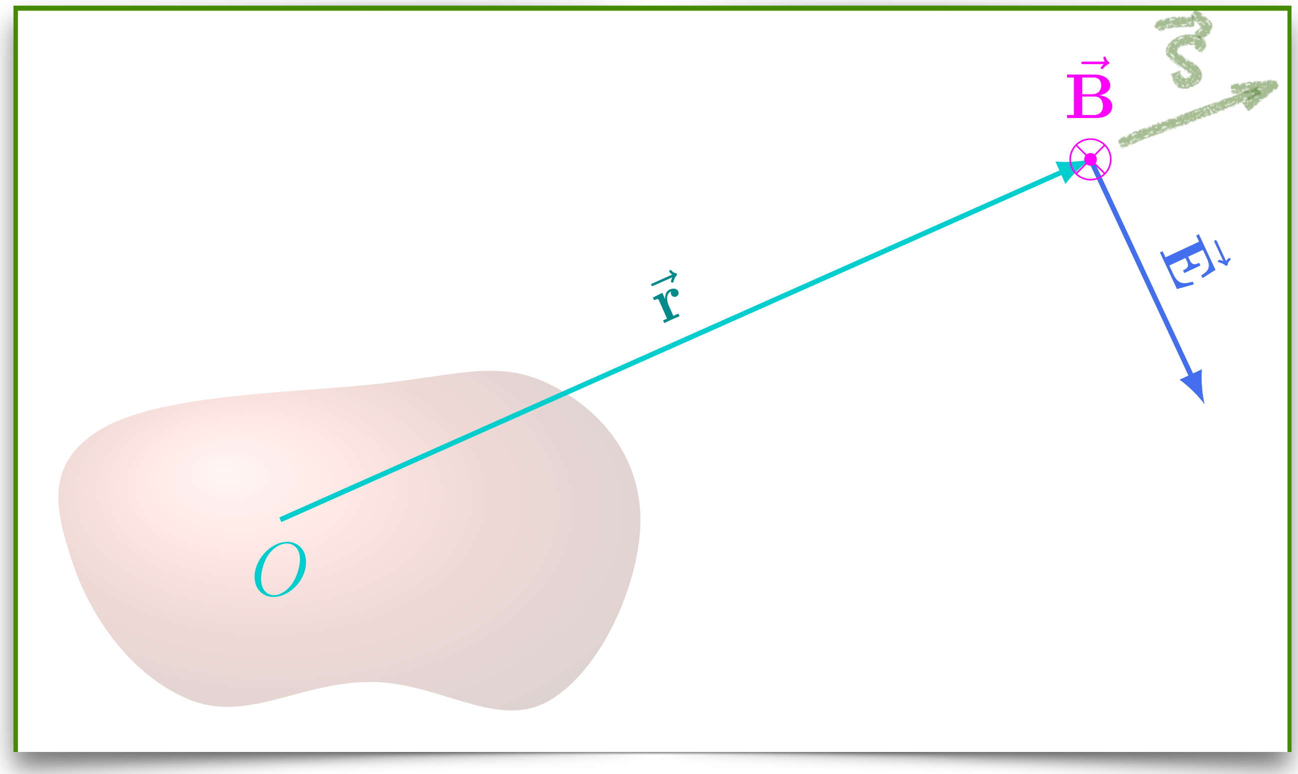
COMO RADIAÇÃO DE DIPOLO OSCILANTE

Radiação de distribuição de cargas

$r' \ll \lambda \ll r$

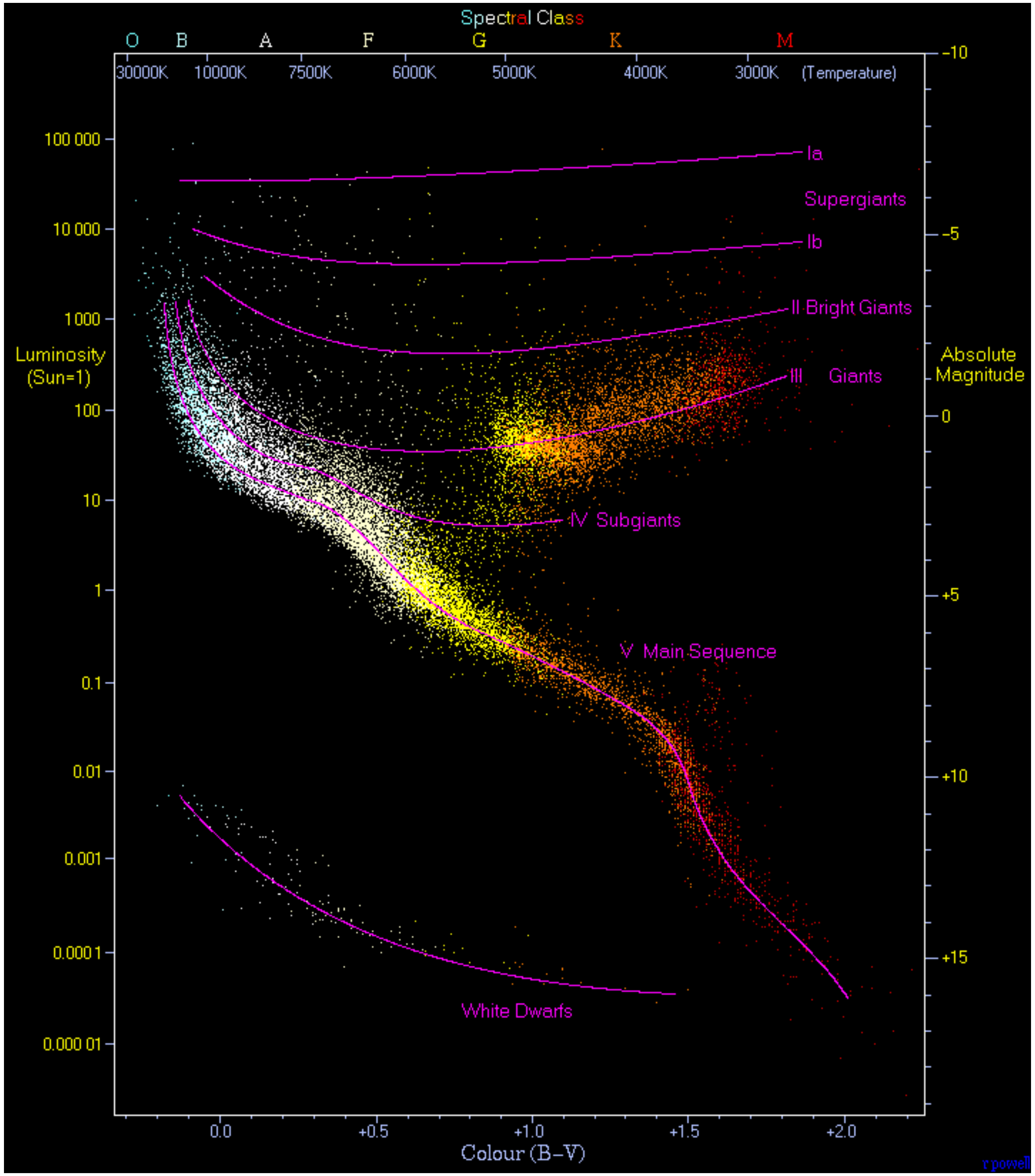
$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi r}\left(\left(\hat{\mathbf{r}}\cdot\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)\hat{\mathbf{r}} - \frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$

$$\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi r c}\left(\hat{\mathbf{r}}\times\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)$$



$$\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2} \parallel \hat{\mathbf{z}} \quad \Rightarrow \quad \begin{cases} \vec{\mathbf{E}} & = \frac{\mu_0}{4\pi r}\left|\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right|\sin\theta\,\hat{\theta} \\ \vec{\mathbf{B}} & = \frac{\mu_0}{4\pi r c}\left|\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right|\sin\theta\,\hat{\phi} \end{cases}$$

$$\vec{\mathbf{S}} = \frac{\mu_0}{(4\pi r)^2 c}\left(\frac{\mathrm{d}^2\vec{\mathbf{p}}}{\mathrm{d}t^2}\right)^2 \sin^2\theta\,\hat{\mathbf{r}}$$


Spectral Class
O
B
A
F
G
K
M
30000K
10000K
7500K
6000K
5000K
4000K
3000K
(Temperature)
Luminosity (Sun=1)
100 000
10 000
1000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
-10
-5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B-V)
r.powell