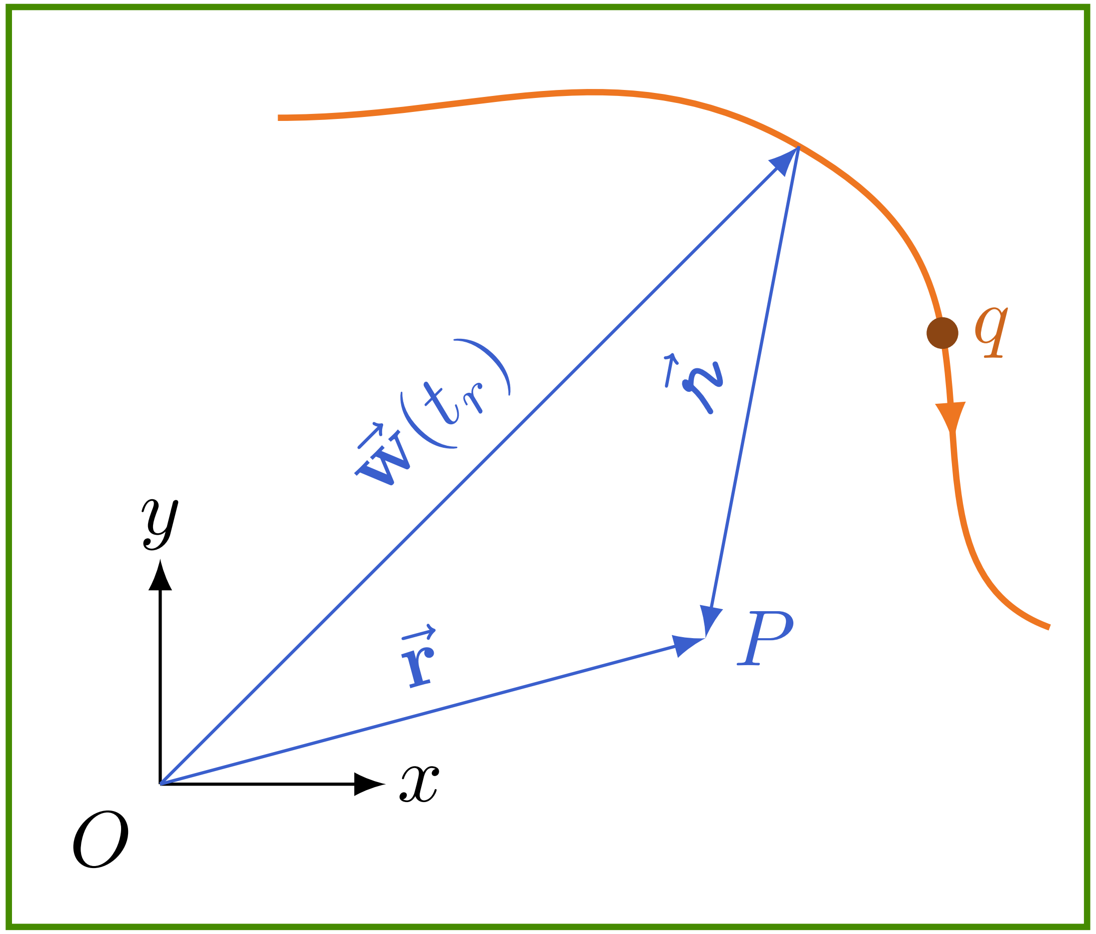
Eletromagnetismo Avançado

3º ciclo
Aula de 12 de novembro

Potenciais de Liénard e Wiechert

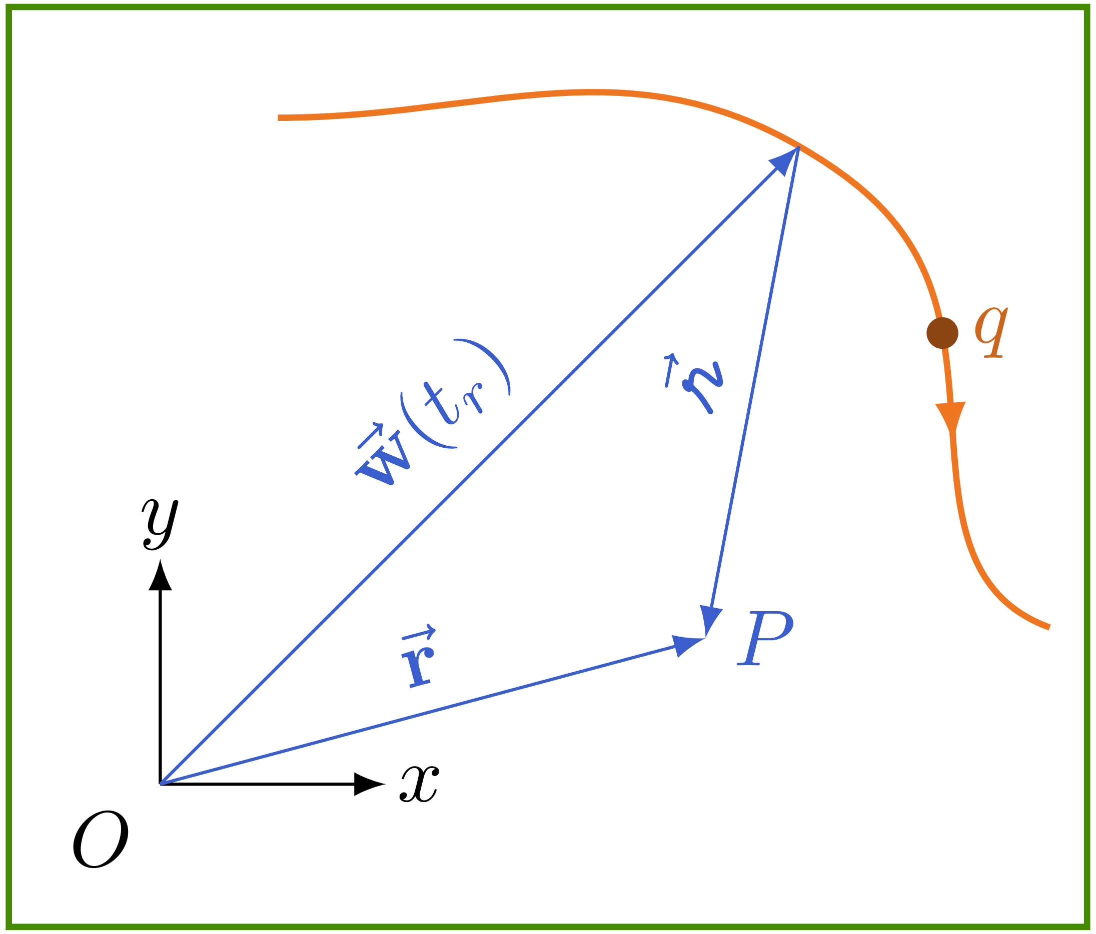
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathcal{R} - \vec{\mathcal{R}} \cdot \frac{\vec{\mathbf{v}}}{c}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}}}{\mathcal{R} - \vec{\mathcal{R}} \cdot \frac{\vec{\mathbf{v}}}{c}}$$



Potenciais de Liénard e Wiechert

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{\vec{\mathscr{r}} \cdot \vec{\mathbf{u}}}$$



$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\vec{\mathbf{v}}}{c^2} V(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{\mathscr{r}} \cdot \vec{\mathbf{u}})^3}[(c^2 - v^2)\vec{\mathbf{u}} + \vec{\mathscr{r}} \times (\vec{\mathbf{u}} \times \vec{\mathbf{a}})]$$

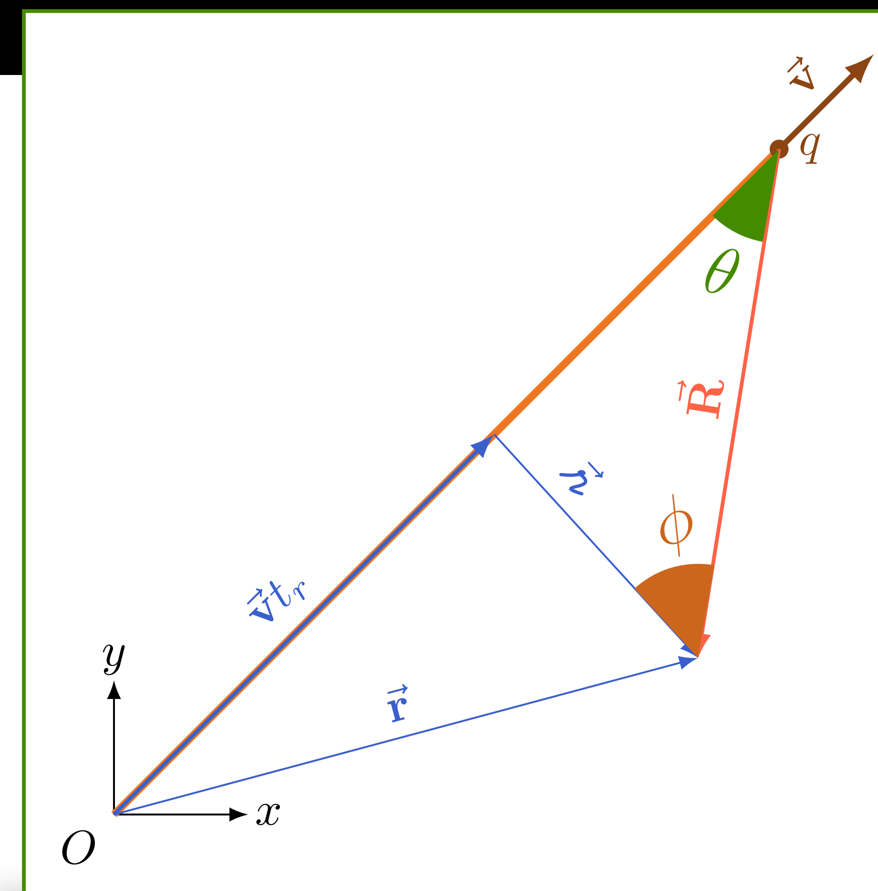
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c}\hat{\mathscr{r}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, t)$$

Pratique o que aprendeu

$$\vec{\mathbf{u}} \equiv c\hat{\mathscr{r}} - \vec{\mathbf{v}}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\vec{\mathscr{r}} \cdot \vec{\mathbf{u}})^3} [(c^2 - v^2)\vec{\mathbf{u}} + \vec{\mathscr{r}} \times (\vec{\mathbf{u}} \times \vec{\mathbf{a}})]$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2\theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$



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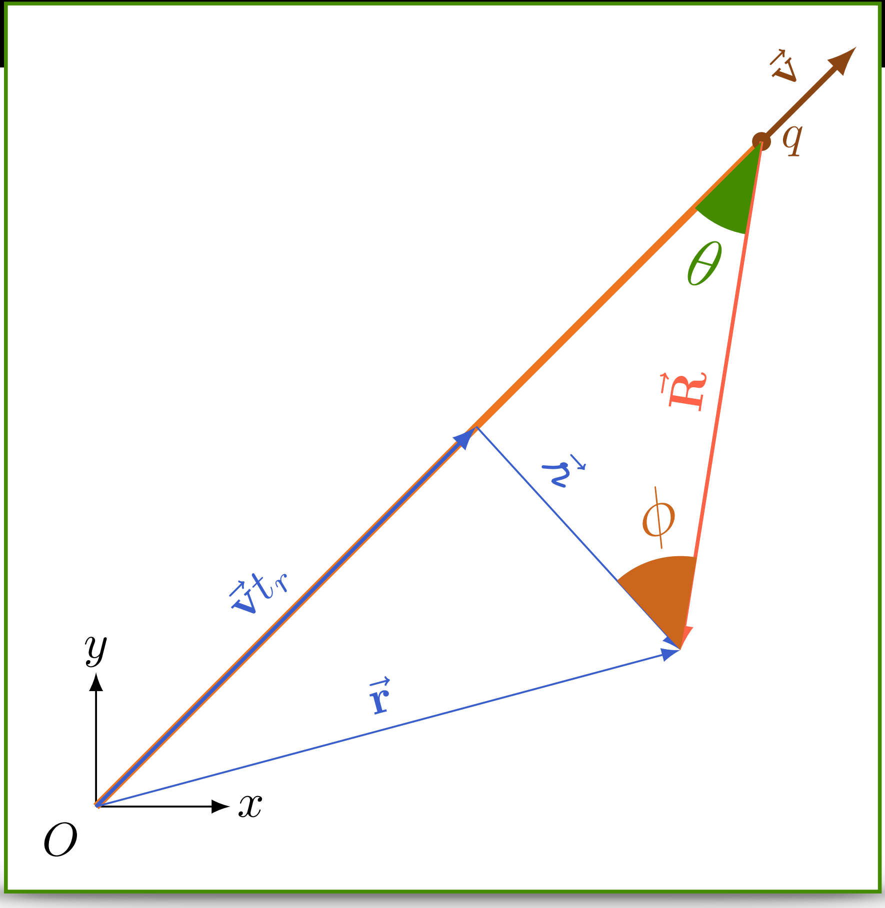

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$$\vec{\mathscr{r}} = \vec{\mathbf{r}} - \vec{\mathbf{v}}t_r$$

↳ SOMAR E SUBTRAIR $\vec{v}t$

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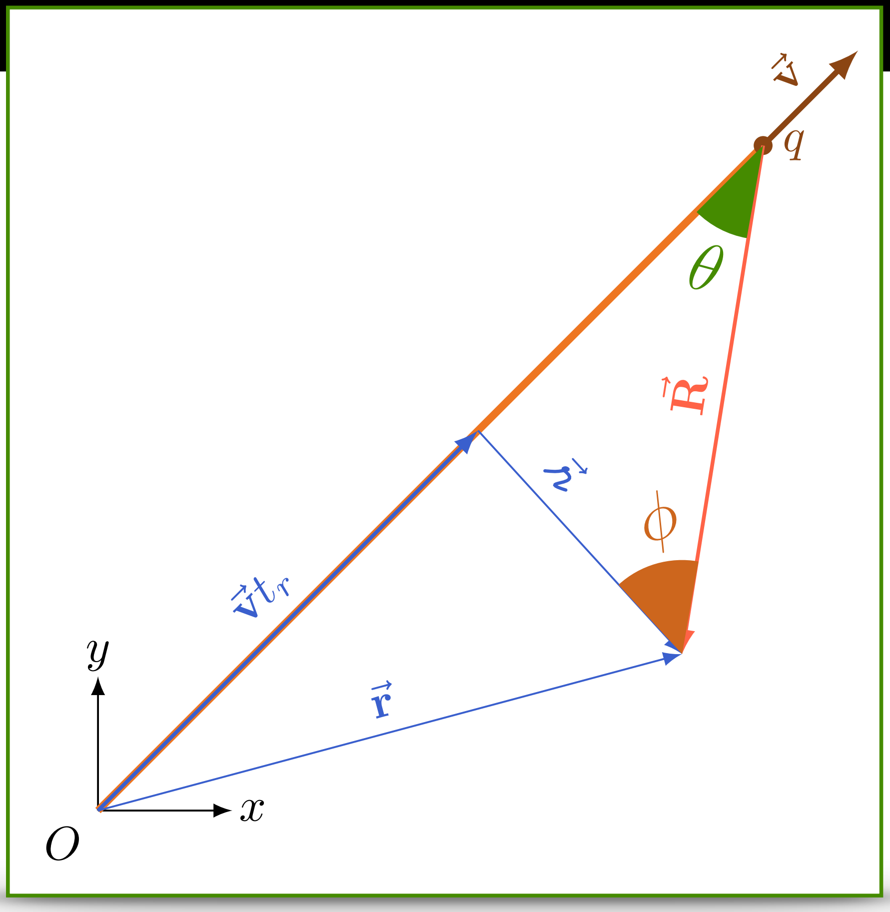

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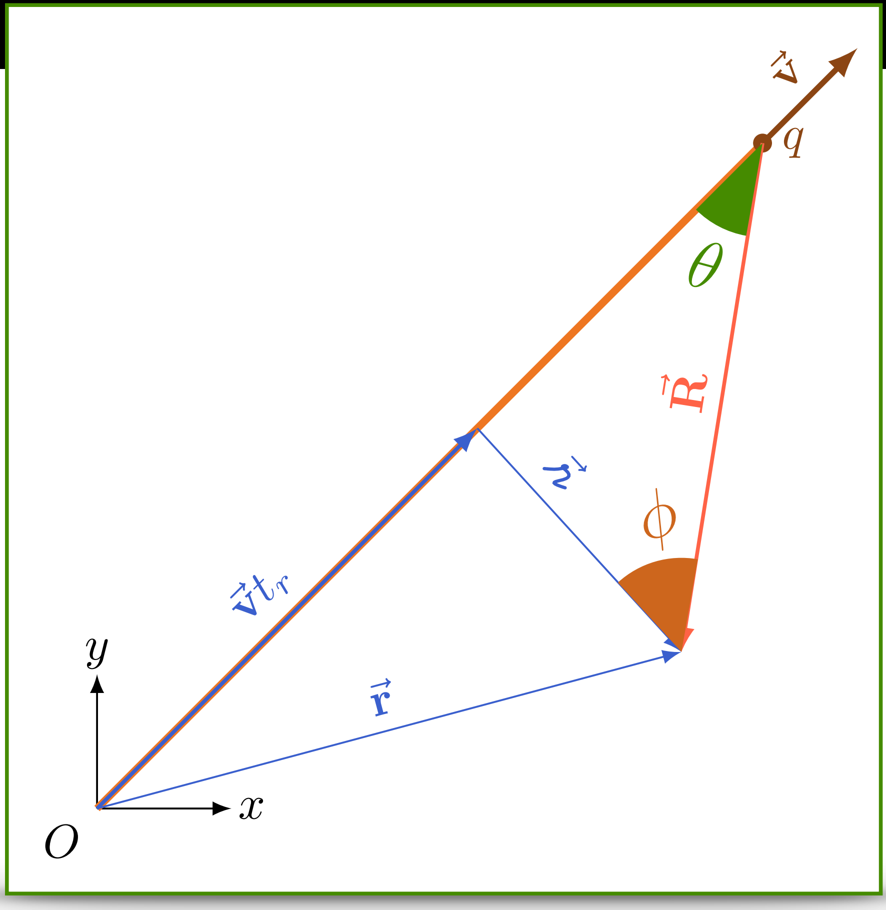

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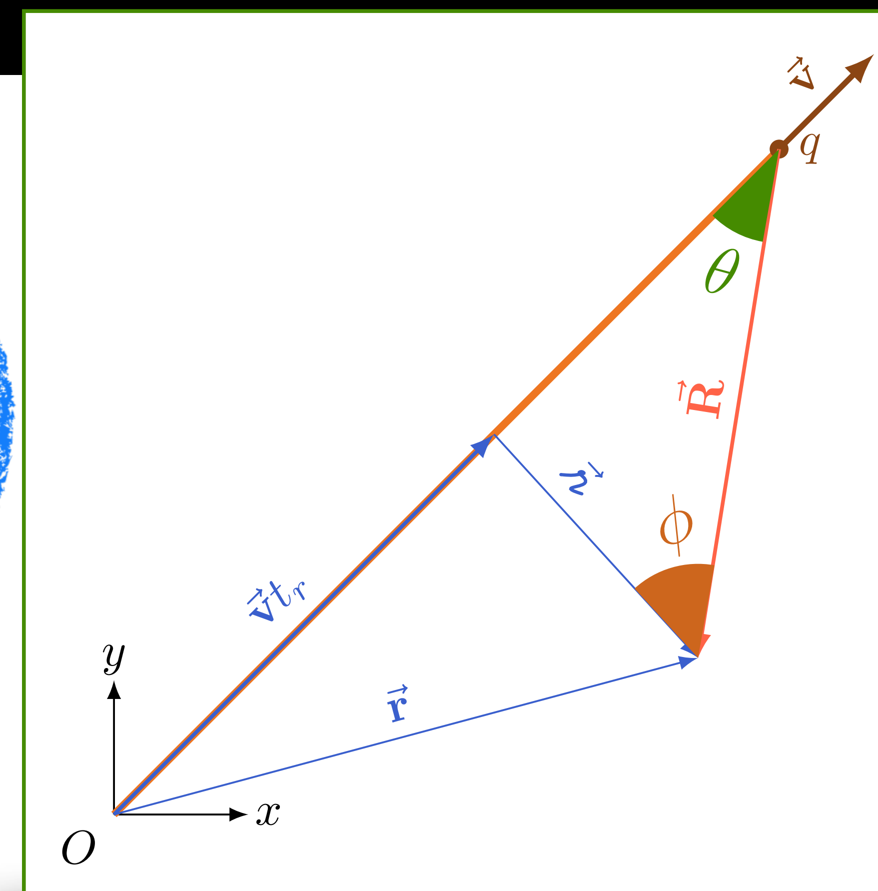

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$\vec{R}$ (E $\vec{R} // \vec{E}$)

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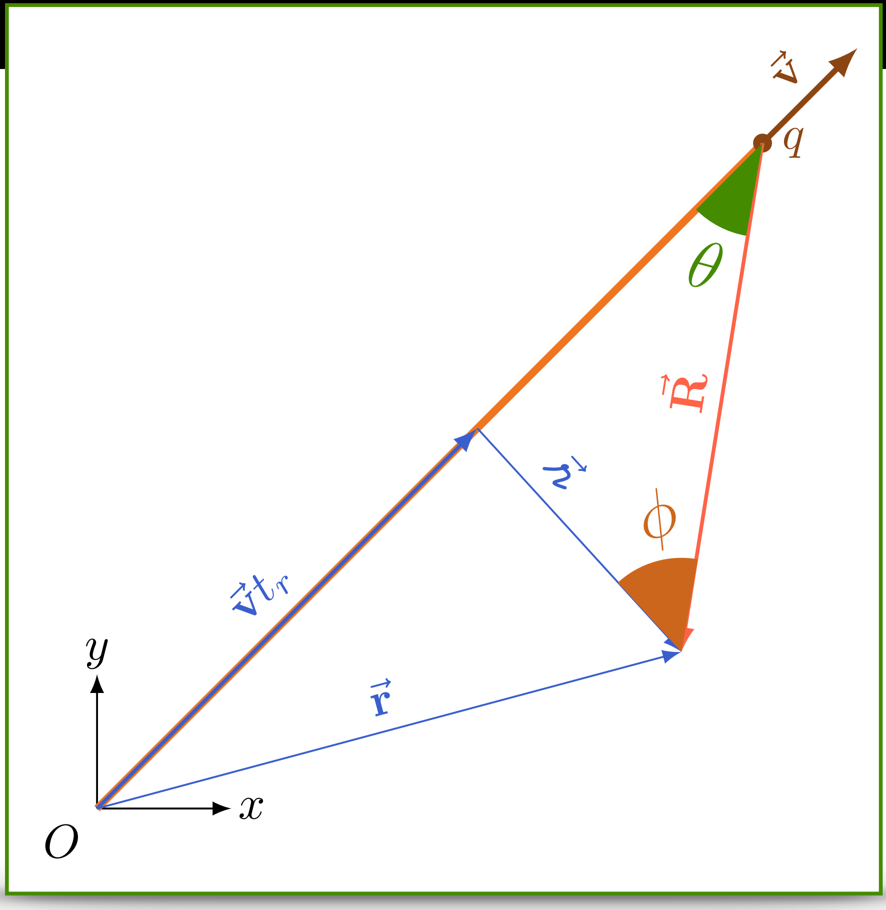

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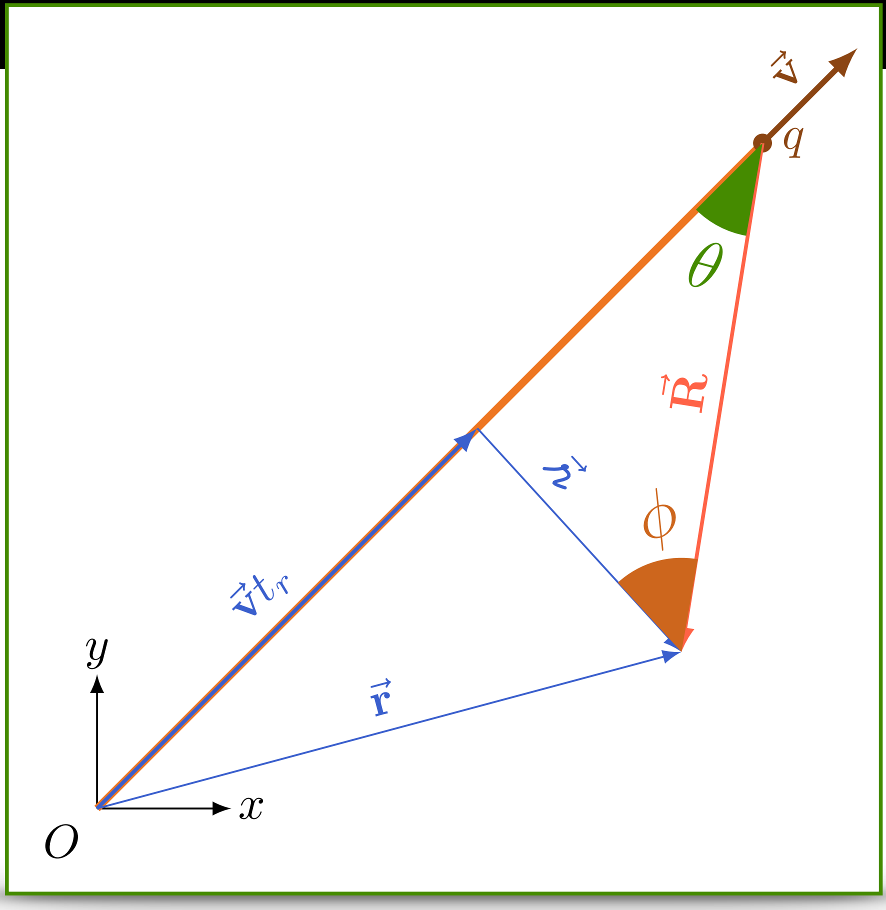

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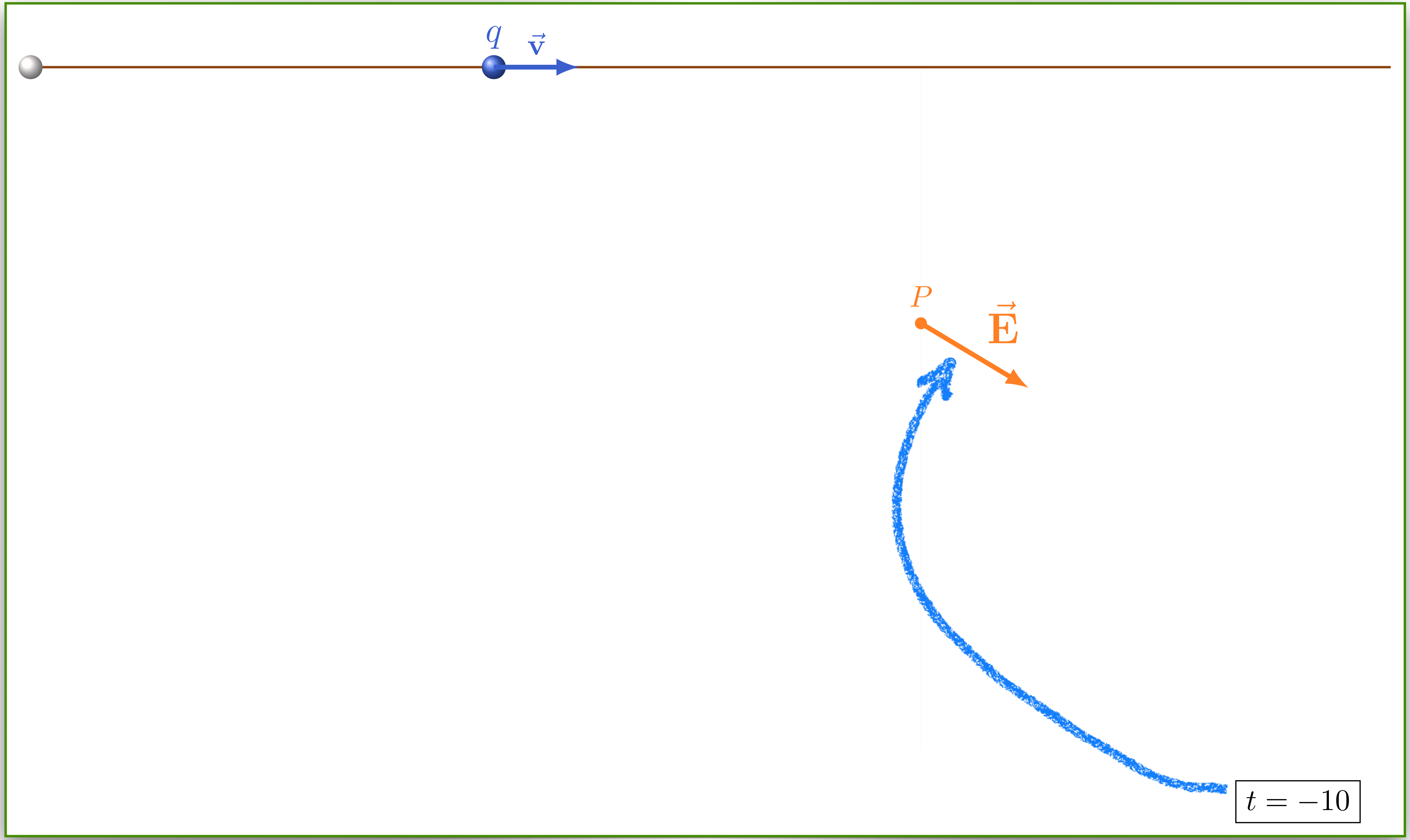

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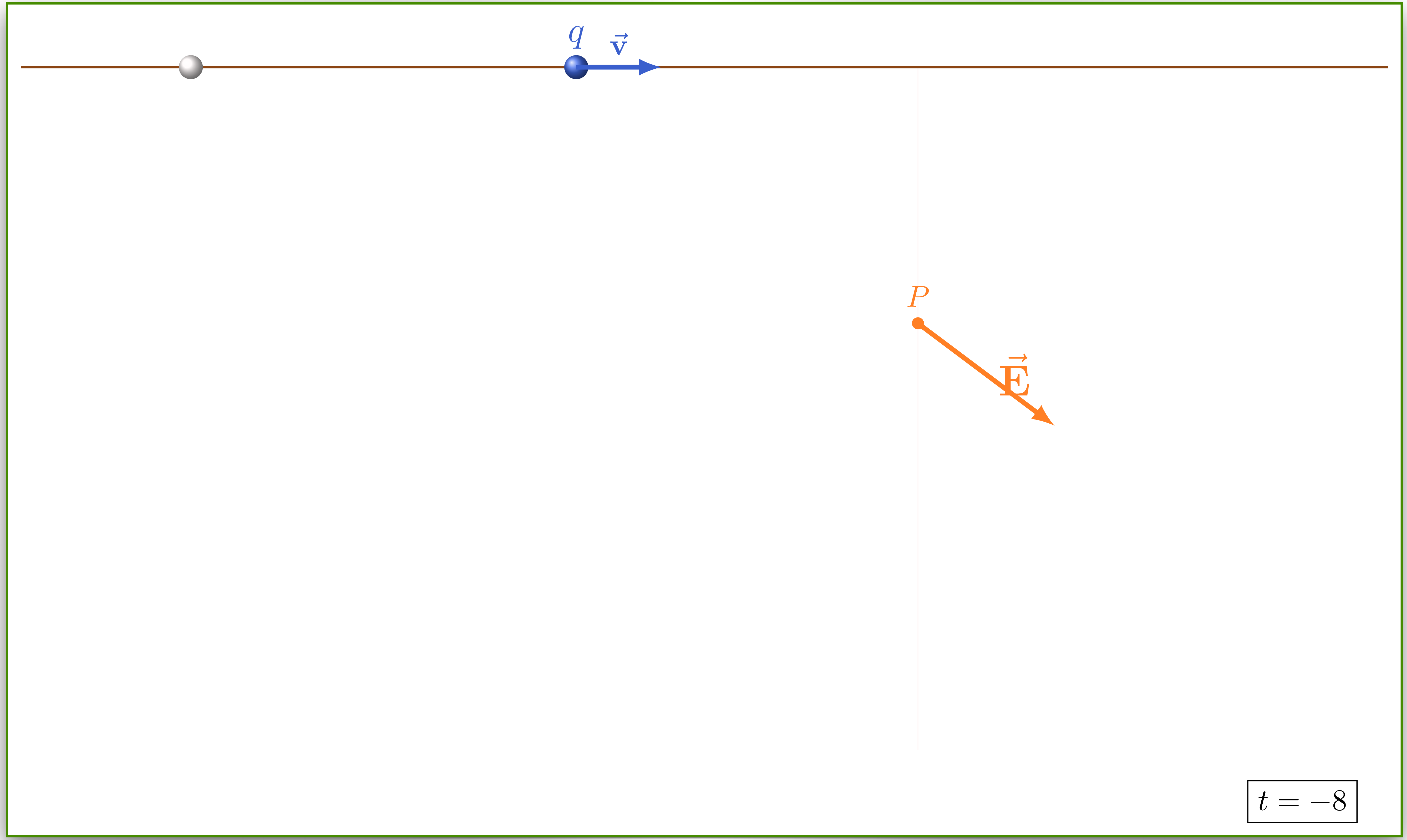

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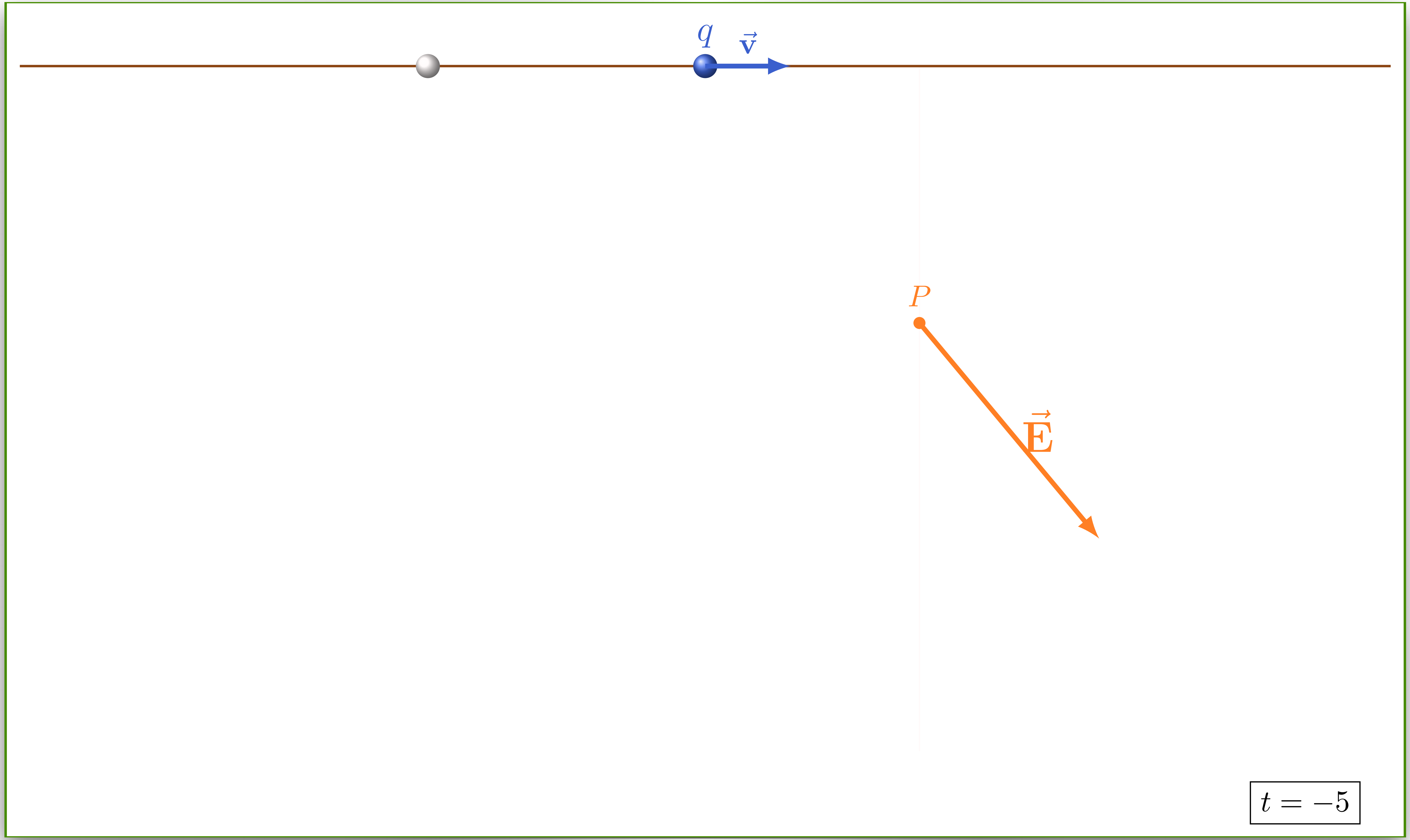

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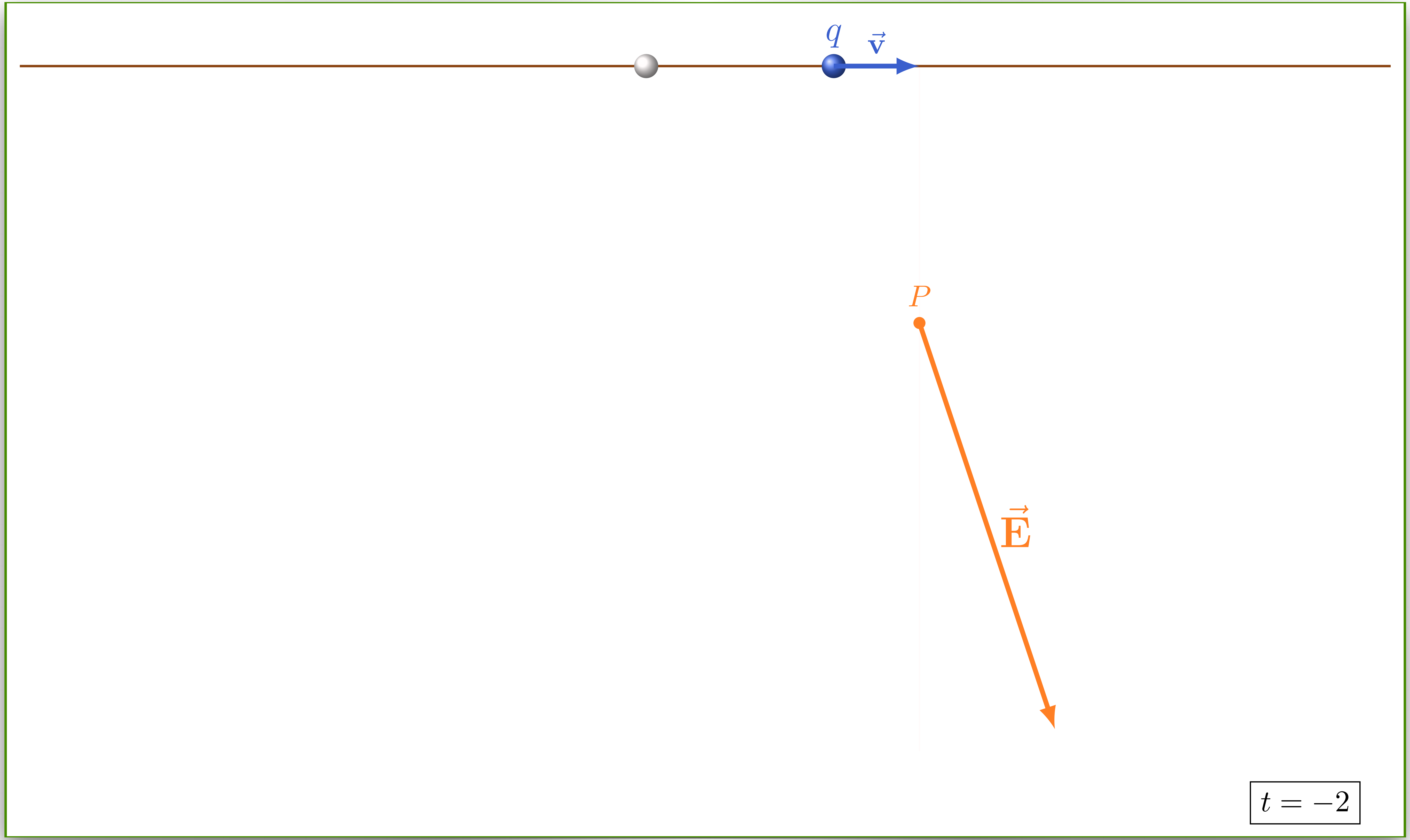

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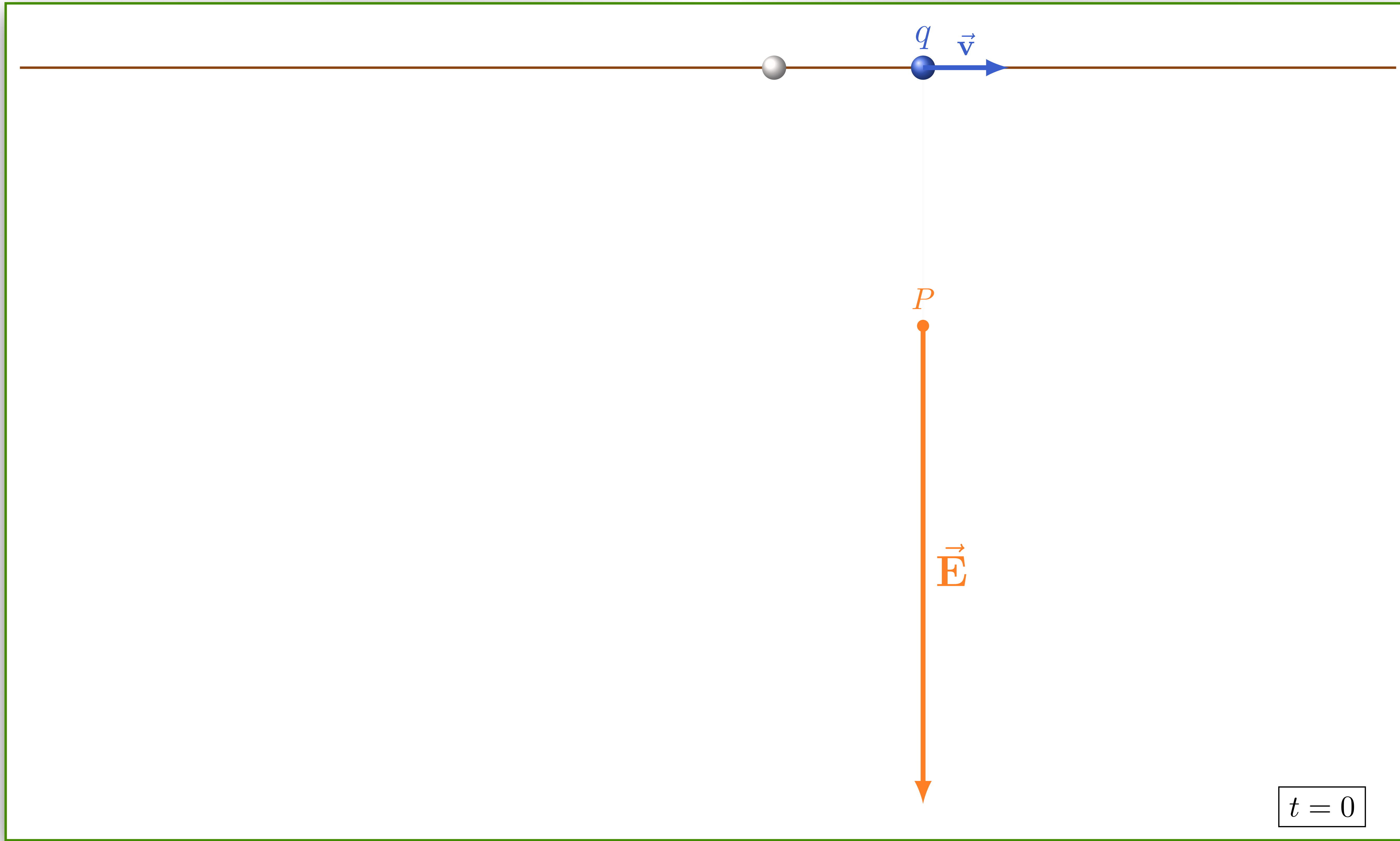

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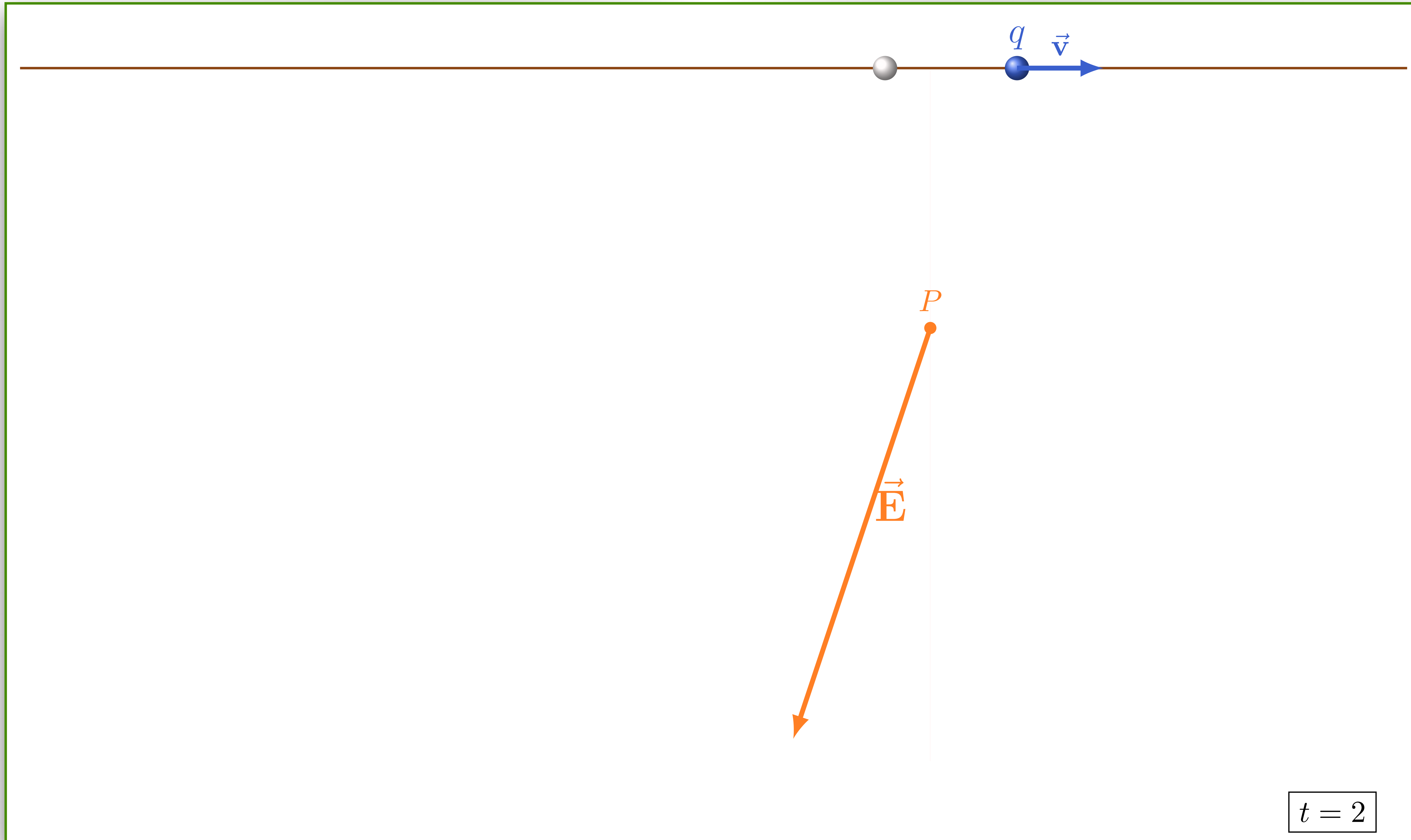

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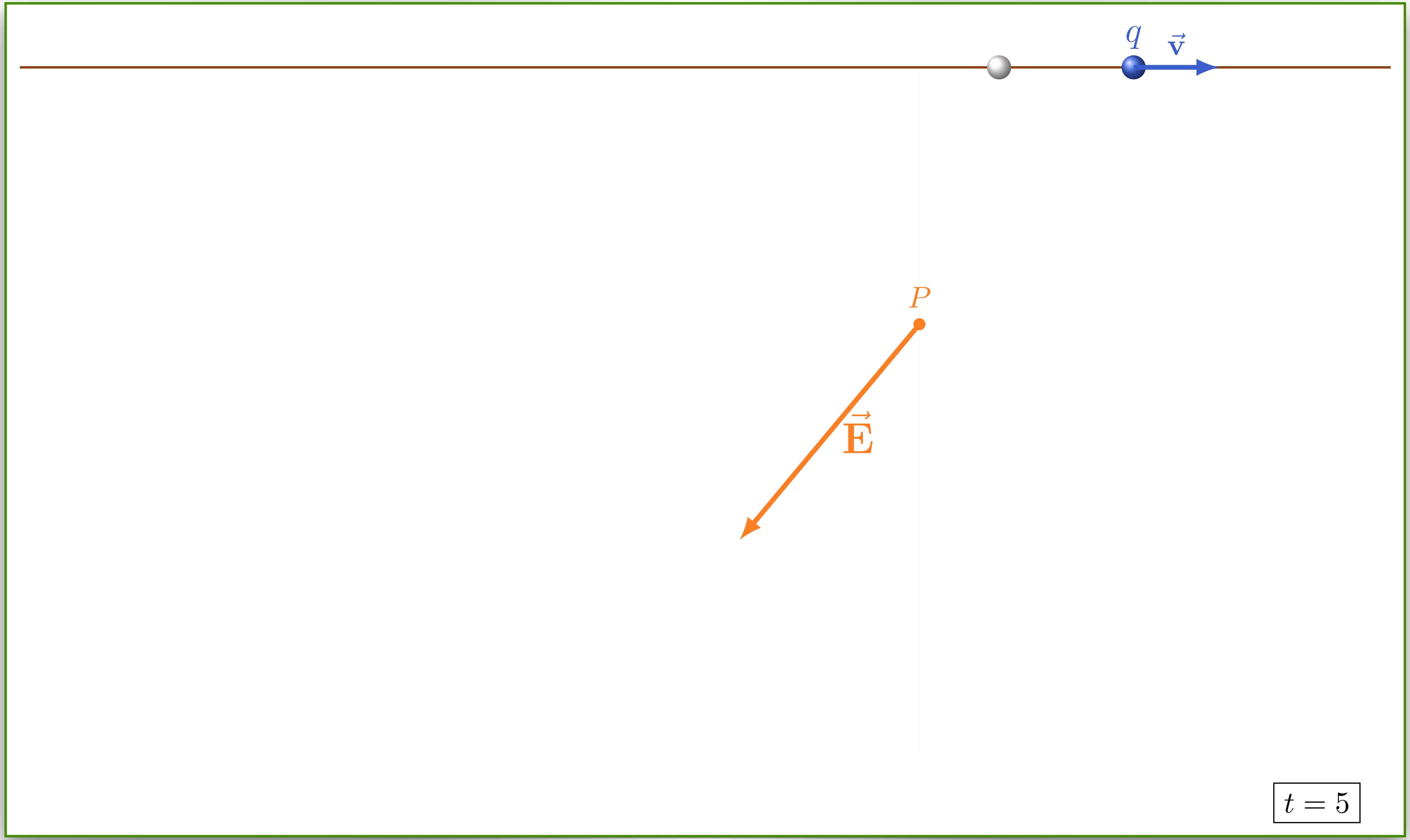

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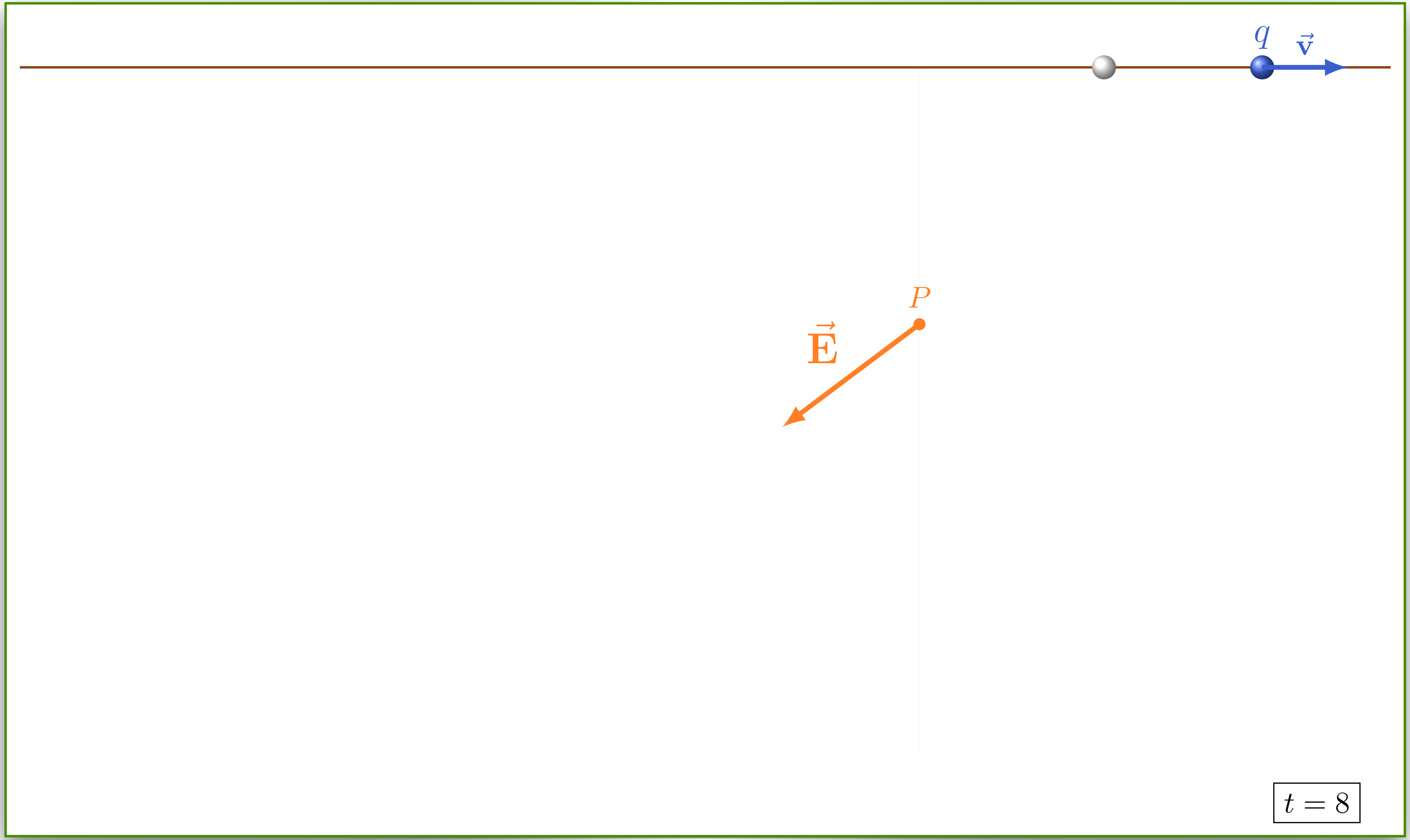

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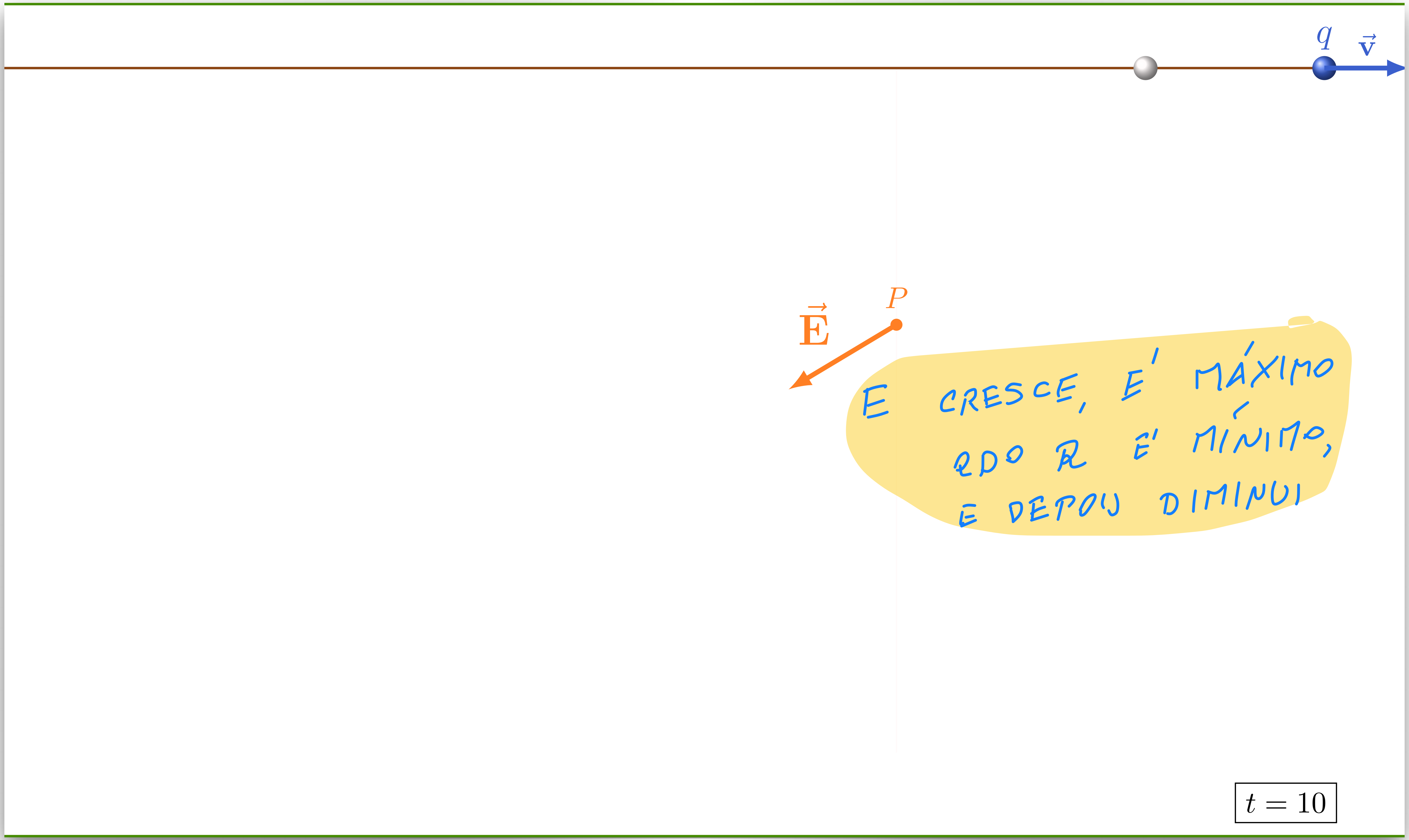

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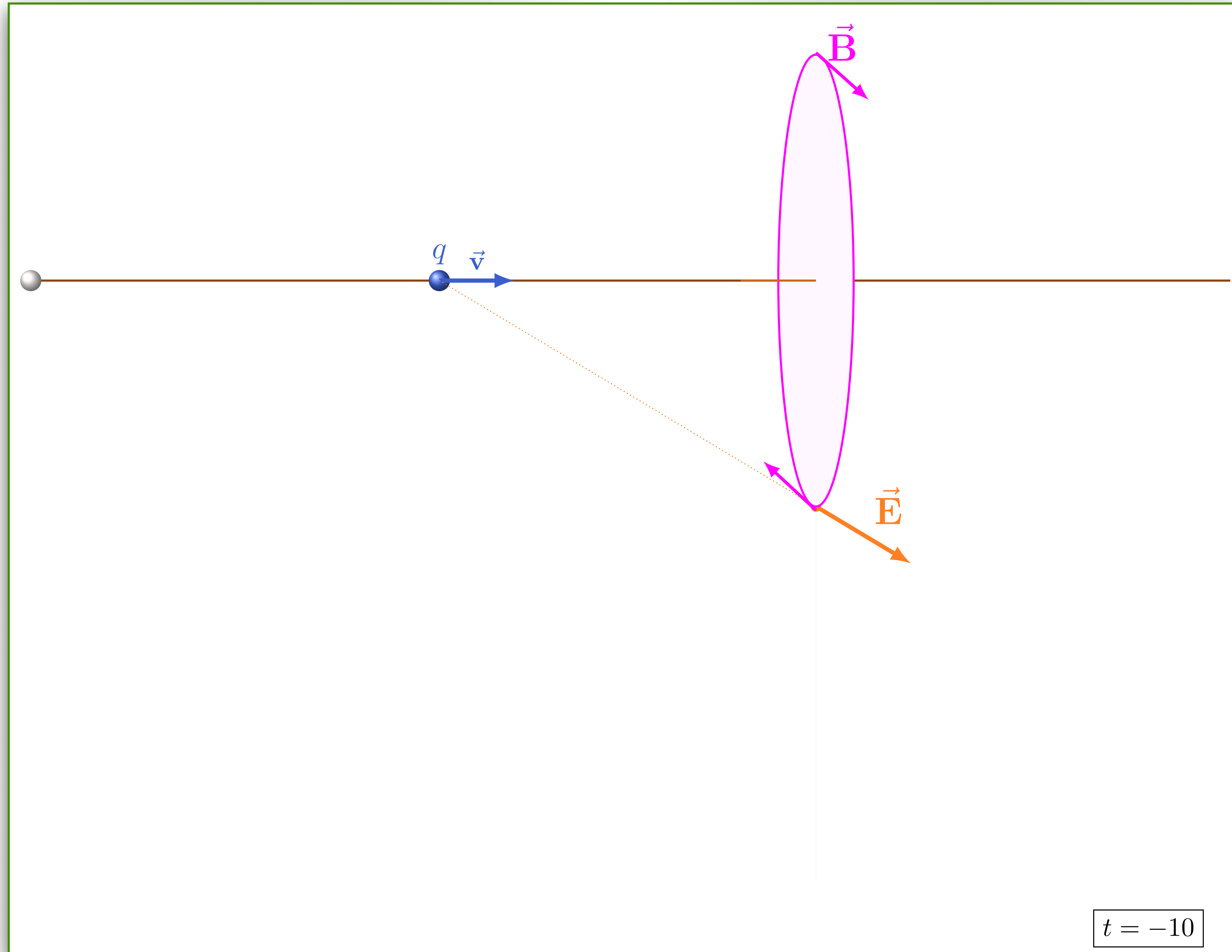

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