

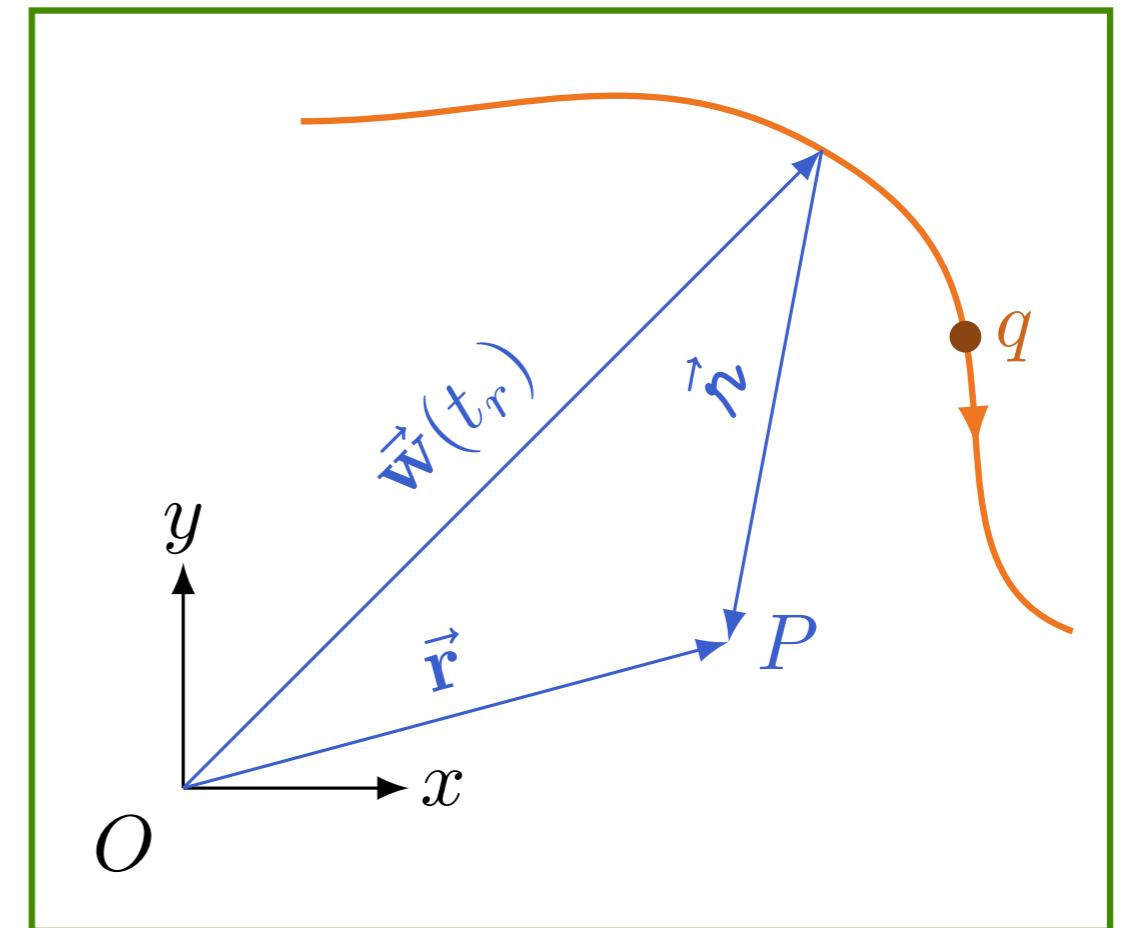
# Eletromagnetismo Avançado

3º ciclo  
Aula de 12 de  
novembro

# Potenciais de Liénard e Wiechert

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\|\vec{r}\| - \vec{r} \cdot \frac{\vec{v}}{c}}$$

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{q\vec{v}}{\|\vec{r}\| - \vec{r} \cdot \frac{\vec{v}}{c}}$$



# Potenciais de Liénard e Wiechert

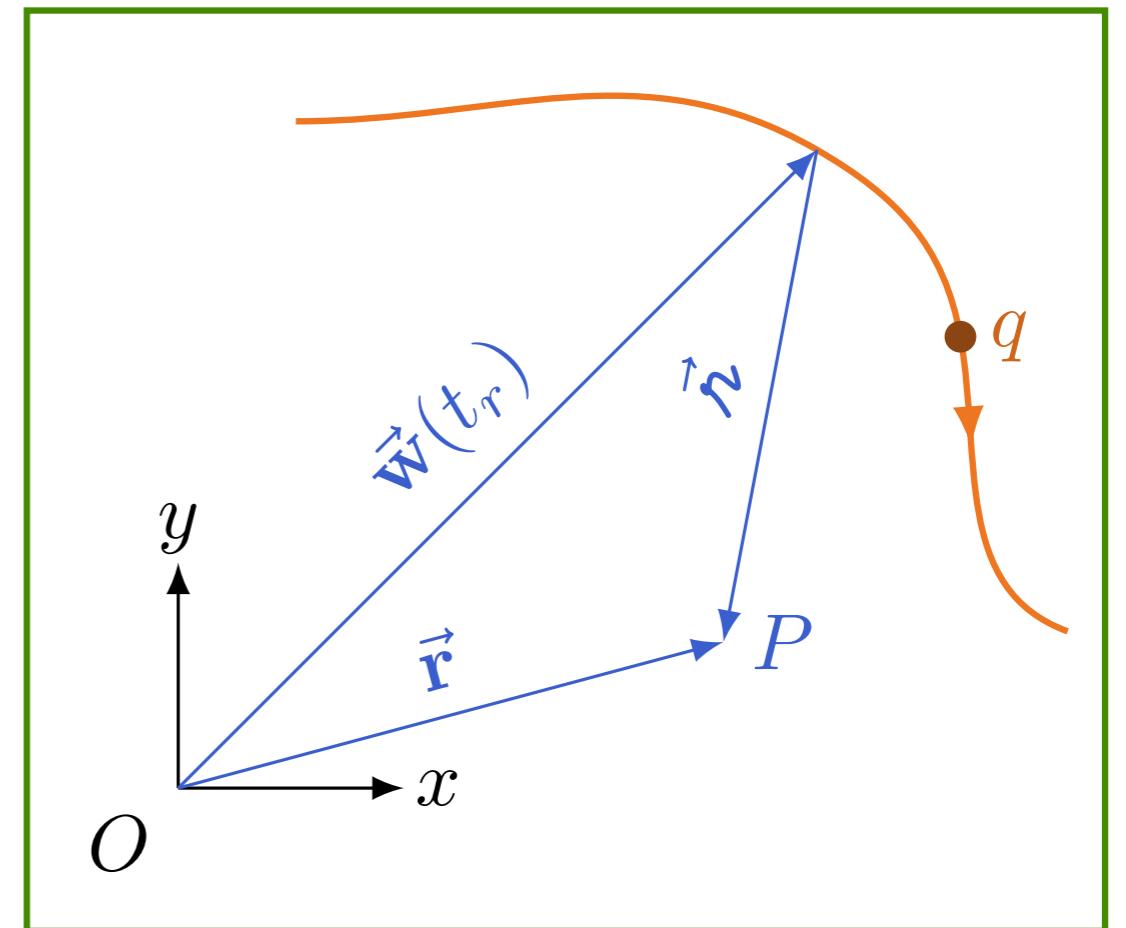
$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{\vec{\boldsymbol{\lambda}} \cdot \vec{\mathbf{u}}}$$

$$\vec{\mathbf{A}}(\vec{r}, t) = \frac{\vec{\mathbf{v}}}{c^2} V(\vec{r}, t)$$

$$\vec{\mathbf{E}}(\vec{r}, t) = -\vec{\nabla}V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\vec{\mathbf{E}}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\vec{\boldsymbol{\lambda}}}{(\vec{\boldsymbol{\lambda}} \cdot \vec{\mathbf{u}})^3} [(c^2 - v^2)\vec{\mathbf{u}} + \vec{\boldsymbol{\lambda}} \times (\vec{\mathbf{u}} \times \vec{\mathbf{a}})]$$

$$\vec{\mathbf{B}}(\vec{r}, t) = \frac{1}{c} \hat{\vec{\boldsymbol{\lambda}}} \times \vec{\mathbf{E}}(\vec{r}, t)$$

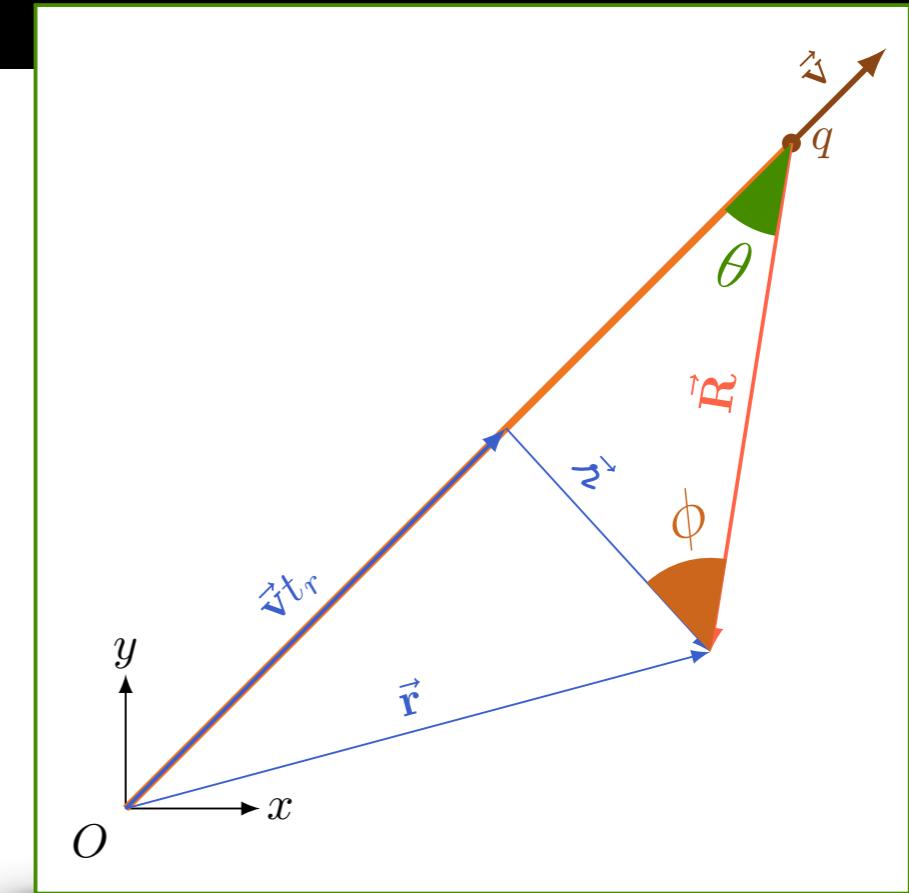


# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\cancel{n}}{(\cancel{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \cancel{n} \times (\vec{u} \times \vec{a})]$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$



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$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\hat{n}}{(\hat{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \hat{n} \times (\vec{u} \times \vec{a})]$$

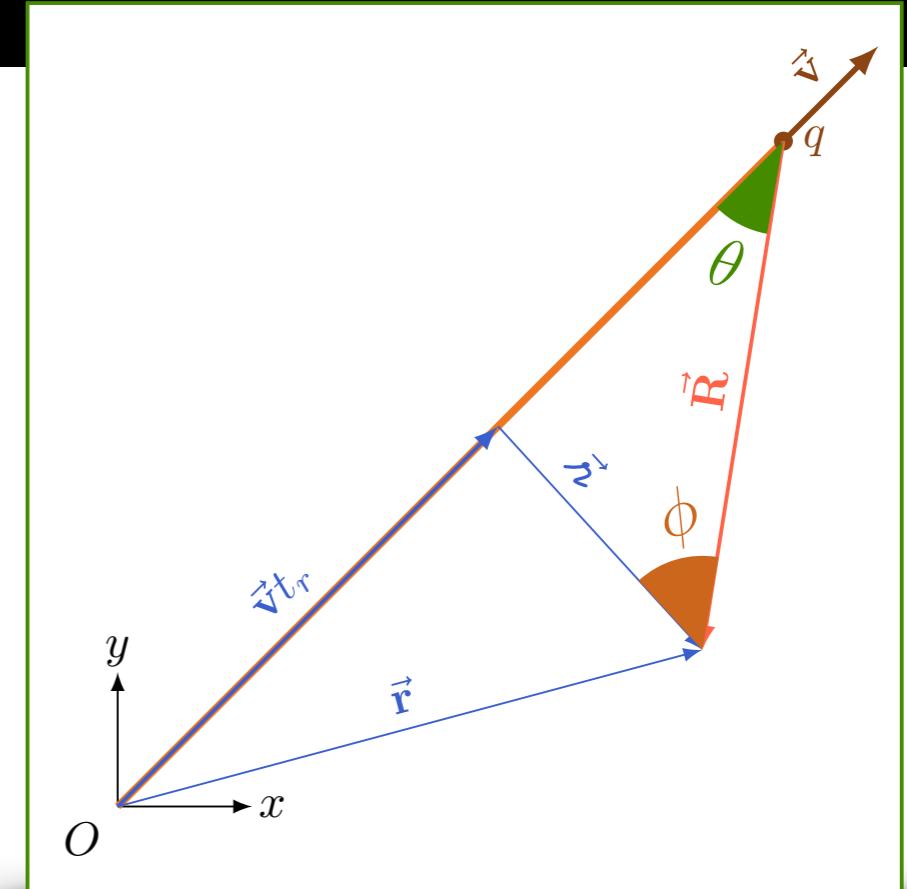
$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

$$\vec{B}(\vec{r}, t) = \frac{1}{n_c} (\hat{n} \times \vec{E})$$

$$\hat{n} = \vec{r} - \vec{v}t_r$$

$\hookrightarrow$  SOMAR E SUBTRAIR  $\vec{v}t$



# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\hat{n}}{(\vec{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{n} \times (\vec{u} \times \vec{a})]$$

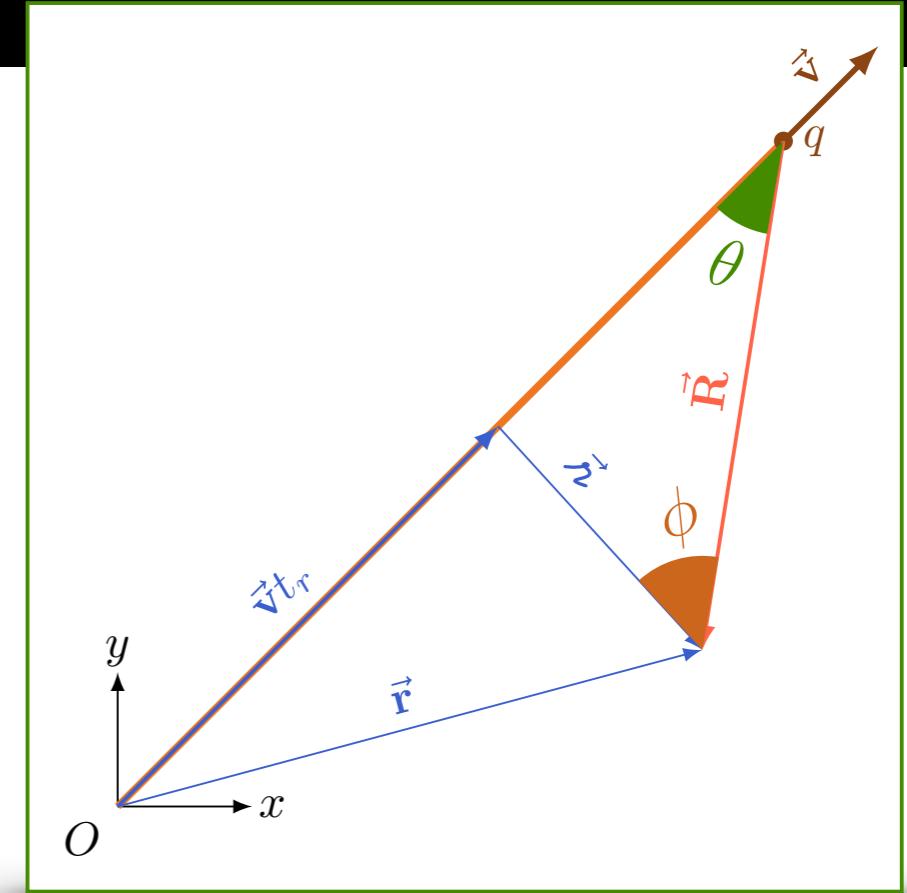
$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

$$\vec{B}(\vec{r}, t) = \frac{1}{\mu_0 c} (\hat{n} \times \vec{E})$$

$$\vec{n} = \vec{r} - \vec{v}t_r$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v}(\underbrace{t - t_r}_{= \frac{R}{c}})$$



# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\hat{n}}{(\vec{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{n} \times (\vec{u} \times \vec{a})]$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

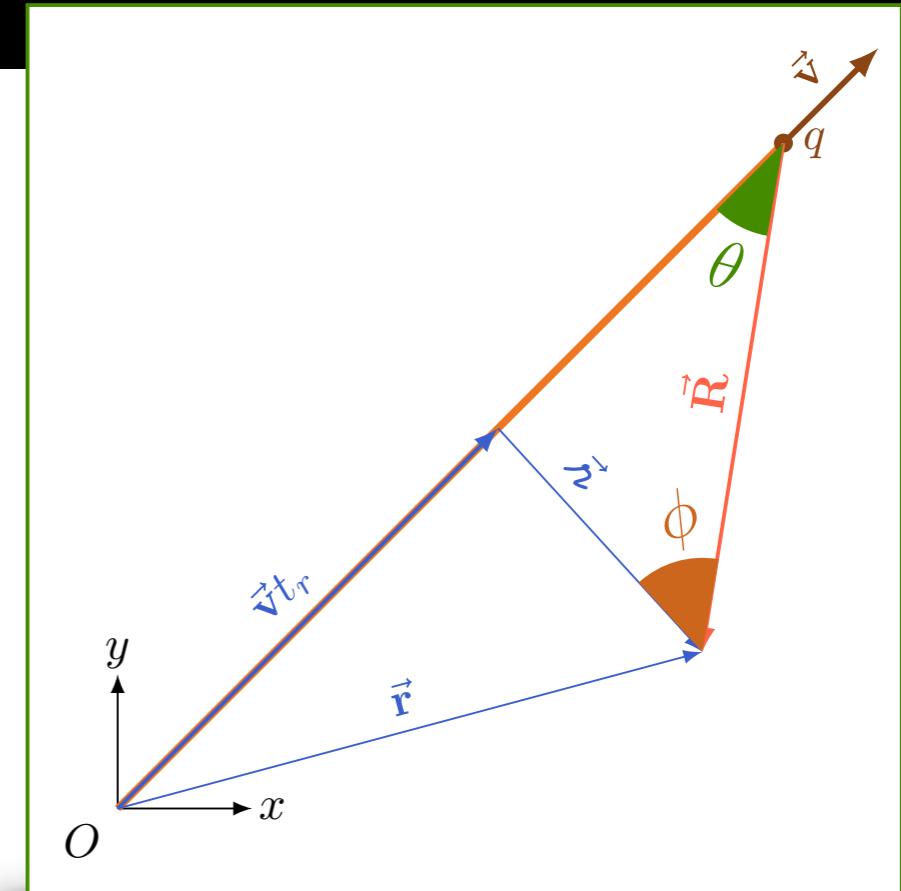
$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

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$$\vec{n} = \vec{r} - \vec{v}t_r$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v}(t - t_r)$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v} \frac{\vec{n}}{c}$$



# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\hat{n}}{(\vec{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{n} \times (\vec{u} \times \vec{a})]$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

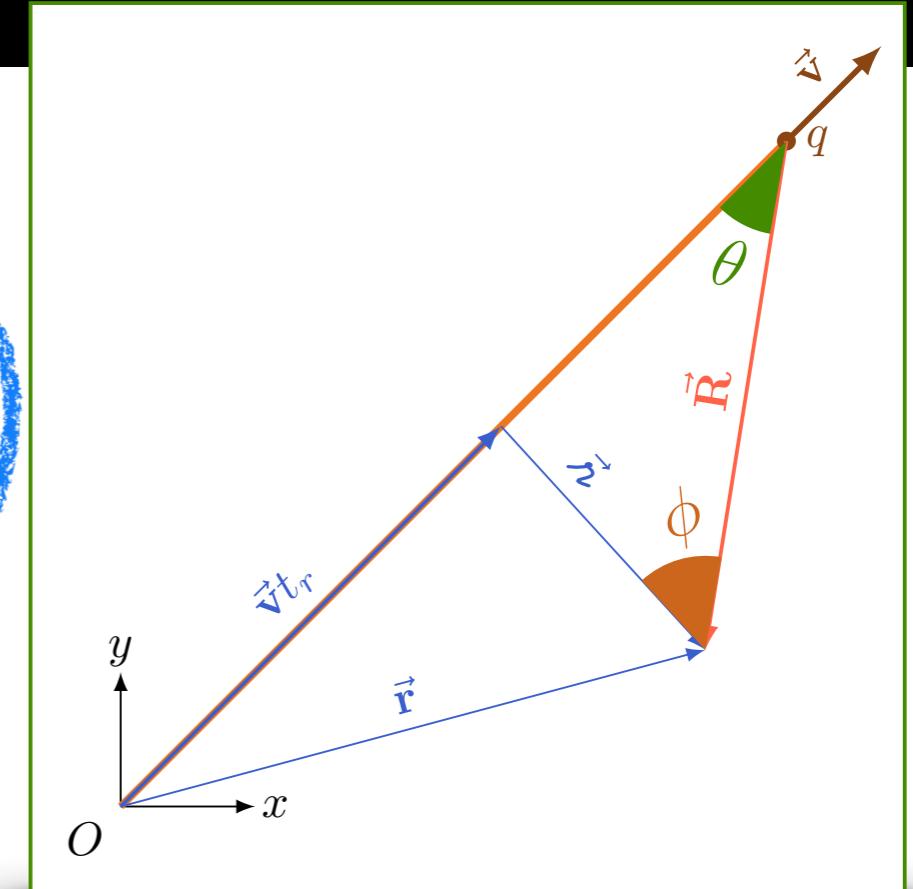
$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

$$\vec{B}(\vec{r}, t) = \frac{1}{n_c} (\hat{n} \times \vec{E})$$

$$\vec{n} = \vec{r} - \vec{v}t_r \quad \Rightarrow \quad \vec{R} \quad (\in \vec{R} \parallel \vec{E})$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v}(t - t_r)$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v} \frac{\vec{n}}{c} \quad \Rightarrow \quad \vec{B}(\vec{r}, t) = \frac{1}{n_c} \left( \vec{v} \frac{\vec{n}}{c} \times \vec{E} \right)$$



# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\hat{n}}{(\vec{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{n} \times (\vec{u} \times \vec{a})]$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

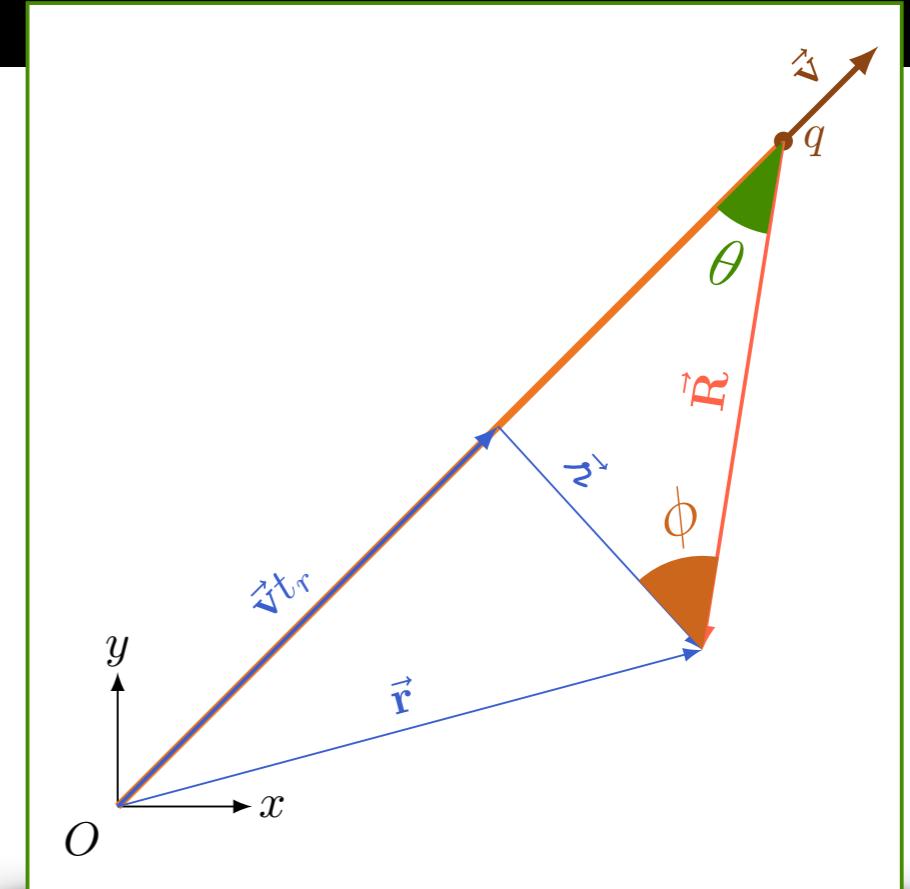
$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

$$\vec{B}(\vec{r}, t) = \frac{1}{\mu_0 c} (\hat{n} \times \vec{E})$$

$$\vec{n} = \vec{r} - \vec{v}t_r$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v}(t - t_r)$$

$$\vec{n} = \vec{r} - \vec{v}t + \vec{v} \frac{\vec{n}}{c} \Rightarrow \vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$



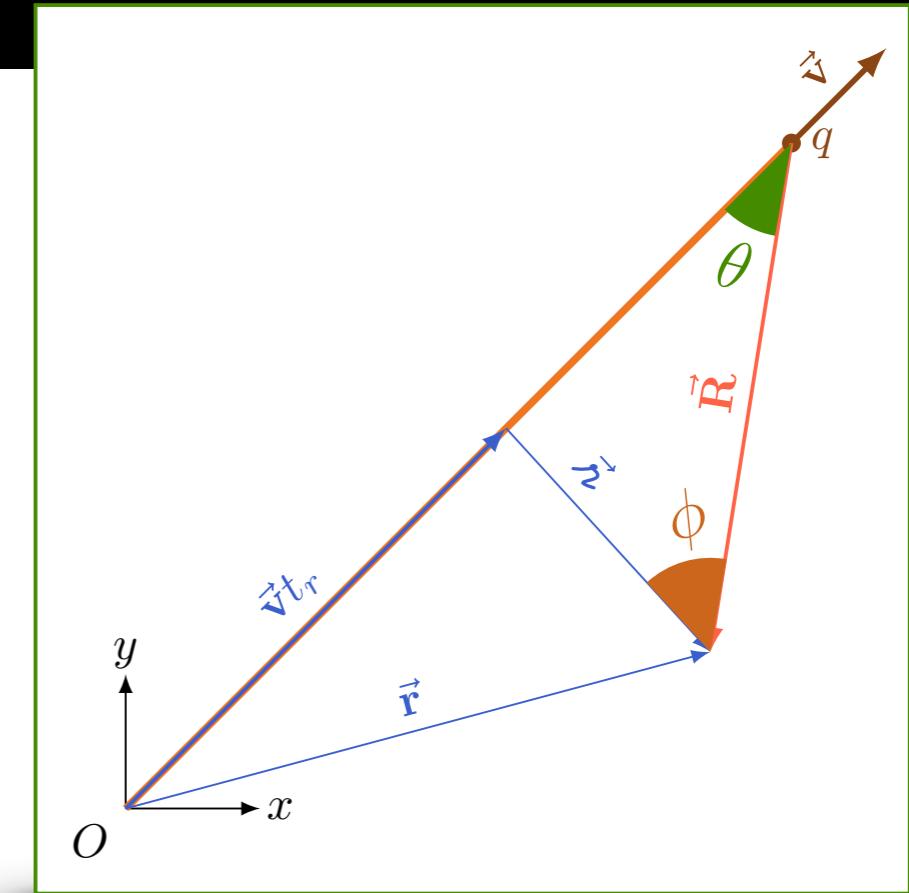
# Pratique o que aprendeu

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\vec{n}}{(\vec{n} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{n} \times (\vec{u} \times \vec{a})]$$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$

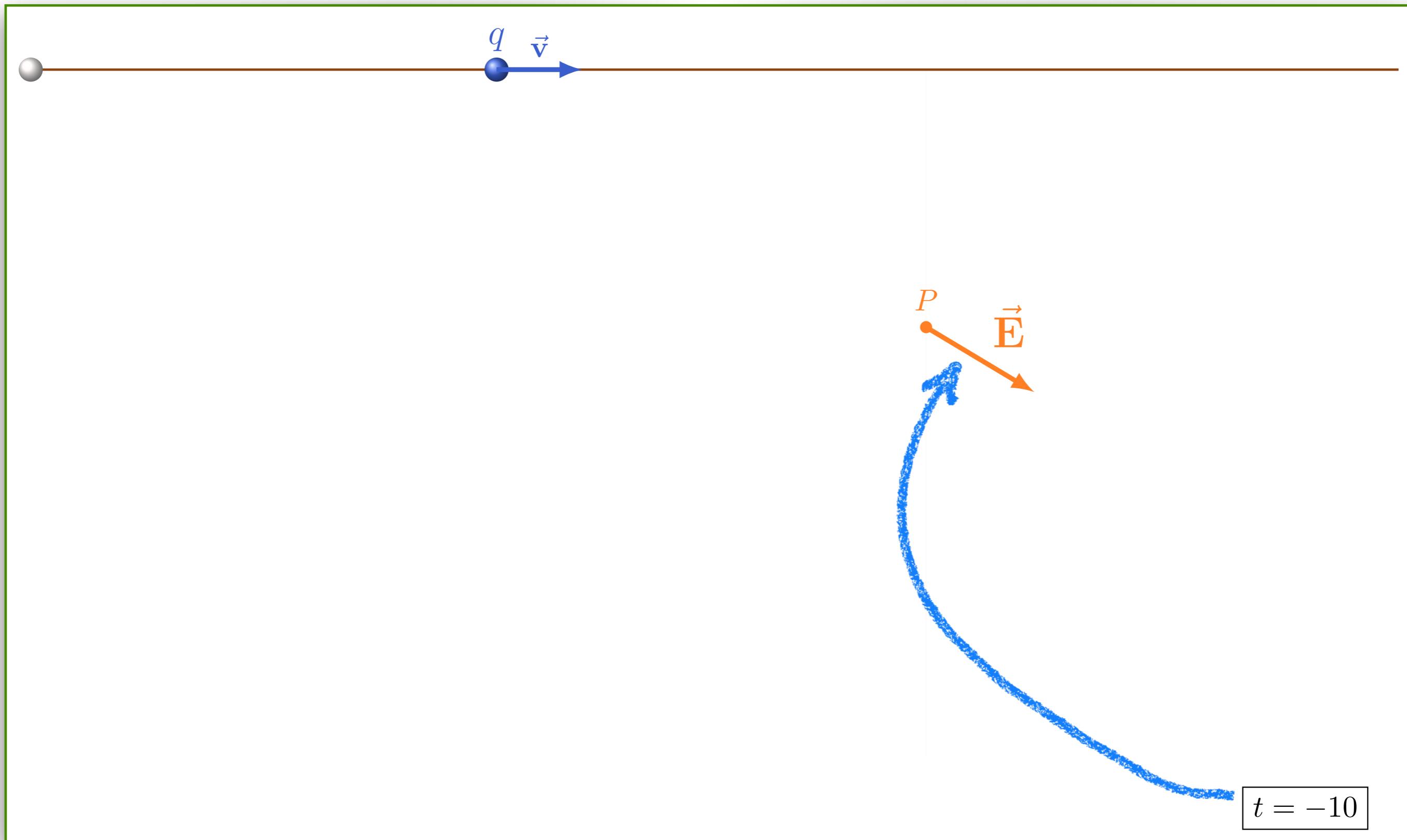


$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t)=\frac{q}{4\pi\epsilon_0}\frac{1-\left(\frac{v}{c}\right)^2}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t)=\frac{1}{c^2}\vec{\mathbf{v}}\times\vec{\mathbf{E}}$$

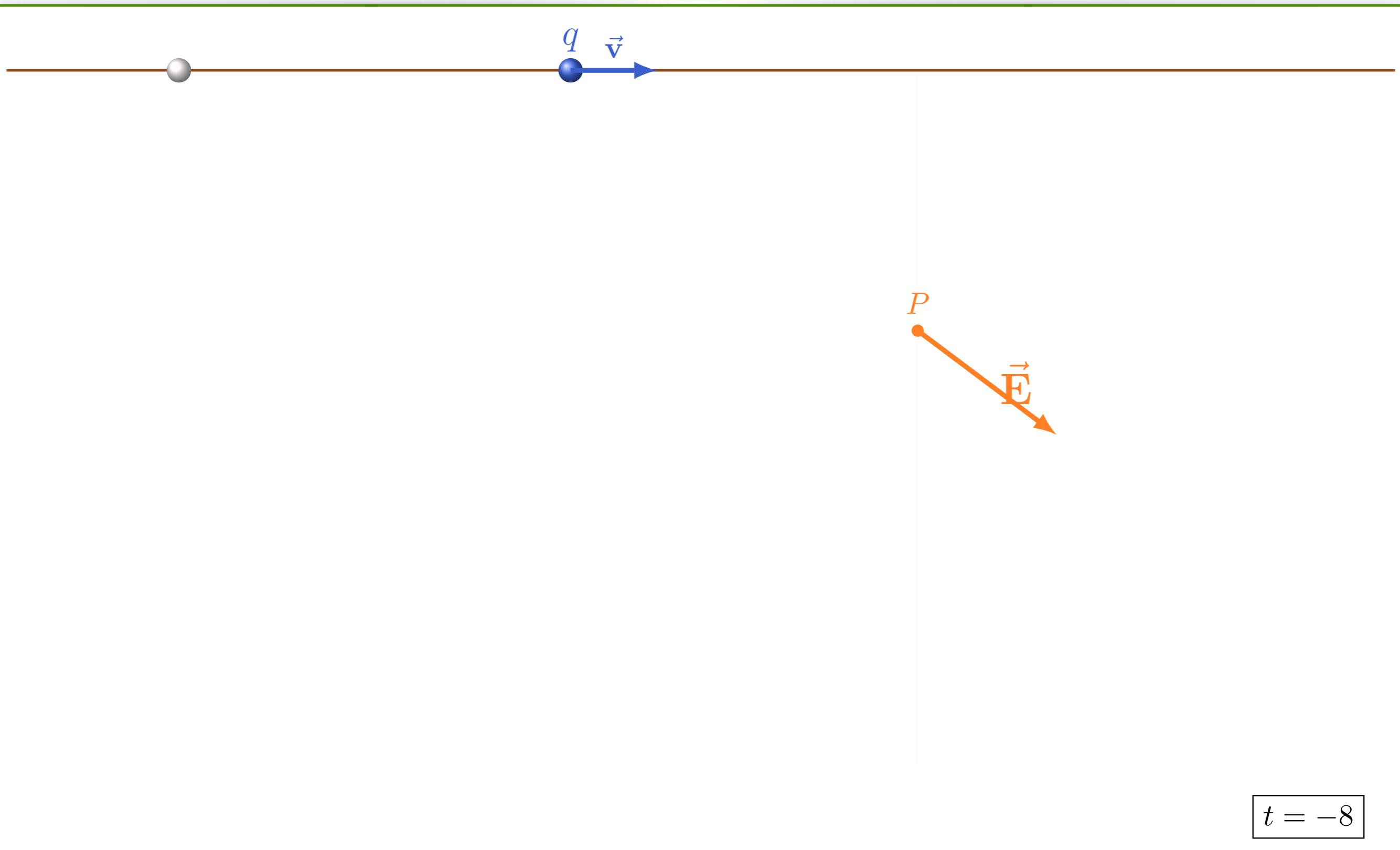
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

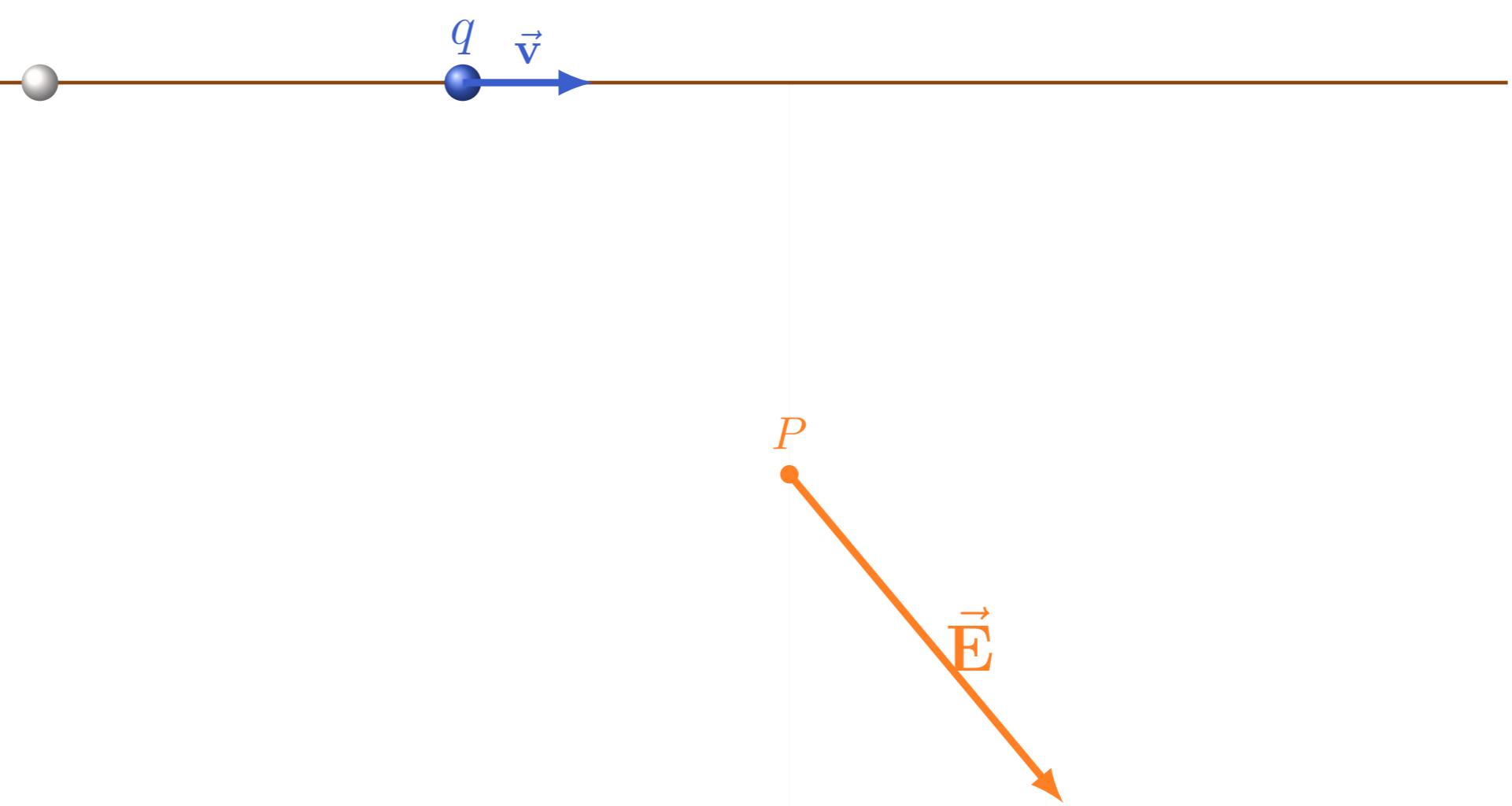
$$\vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$



$t = -8$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

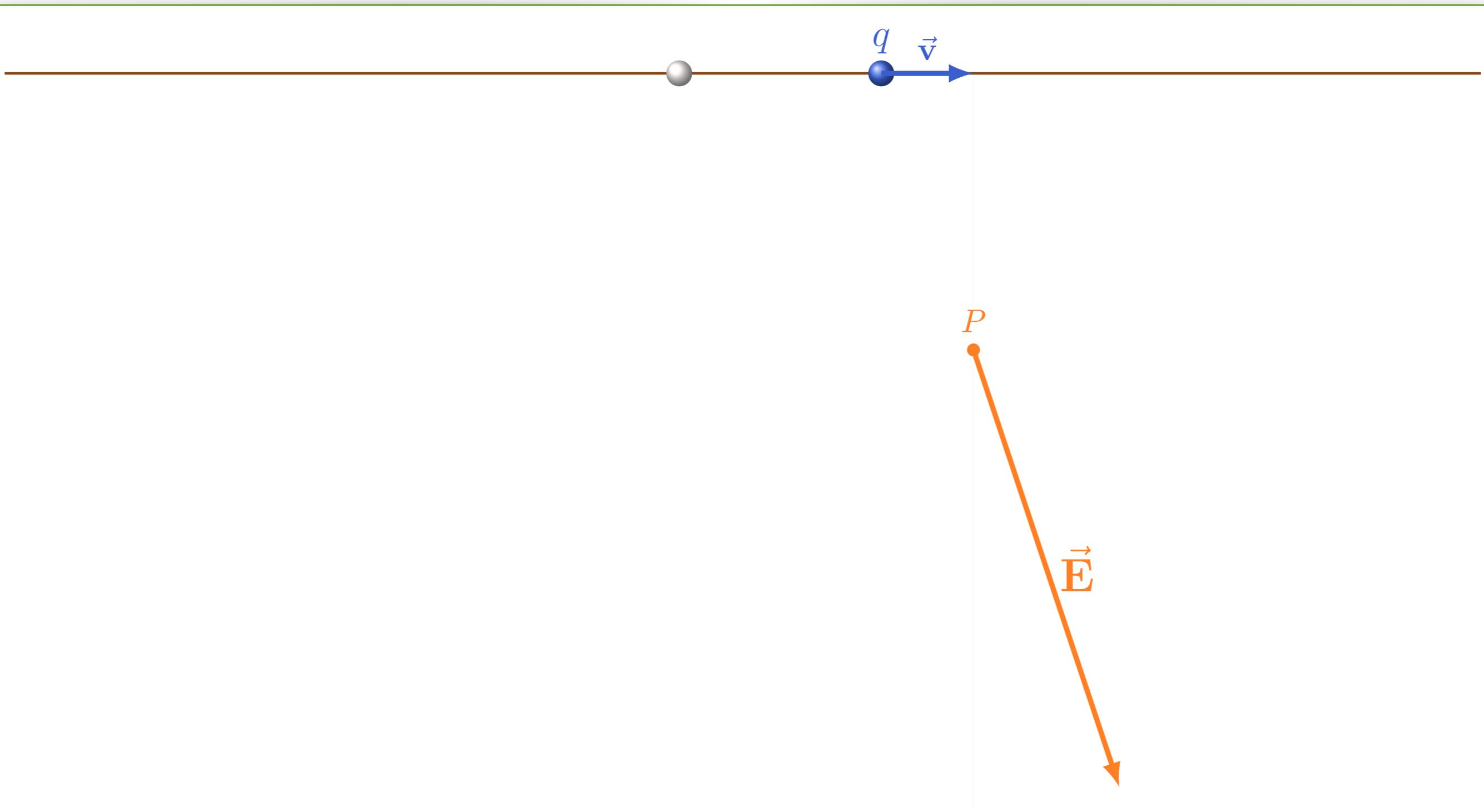
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$t = -5$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

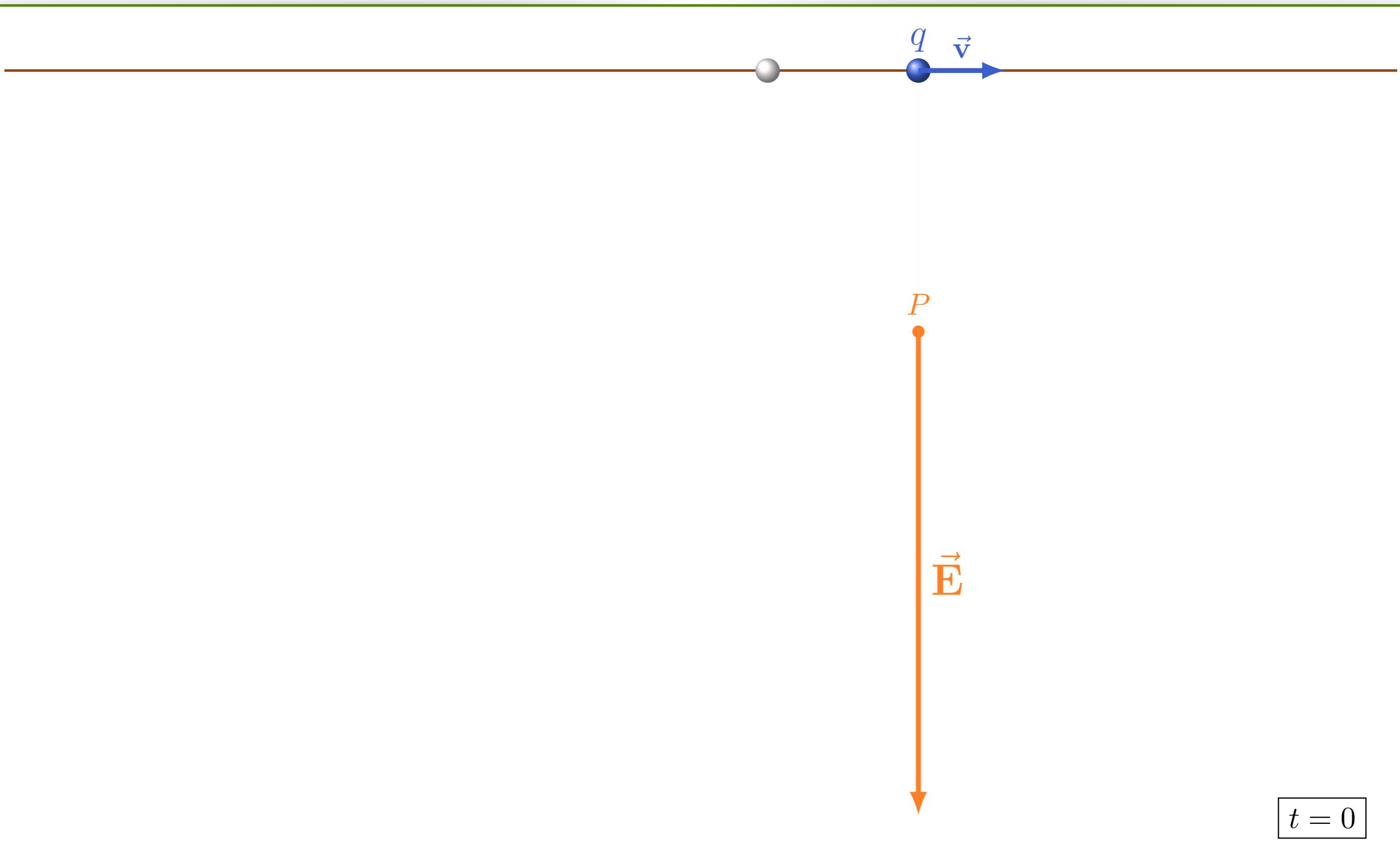
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$t = -2$

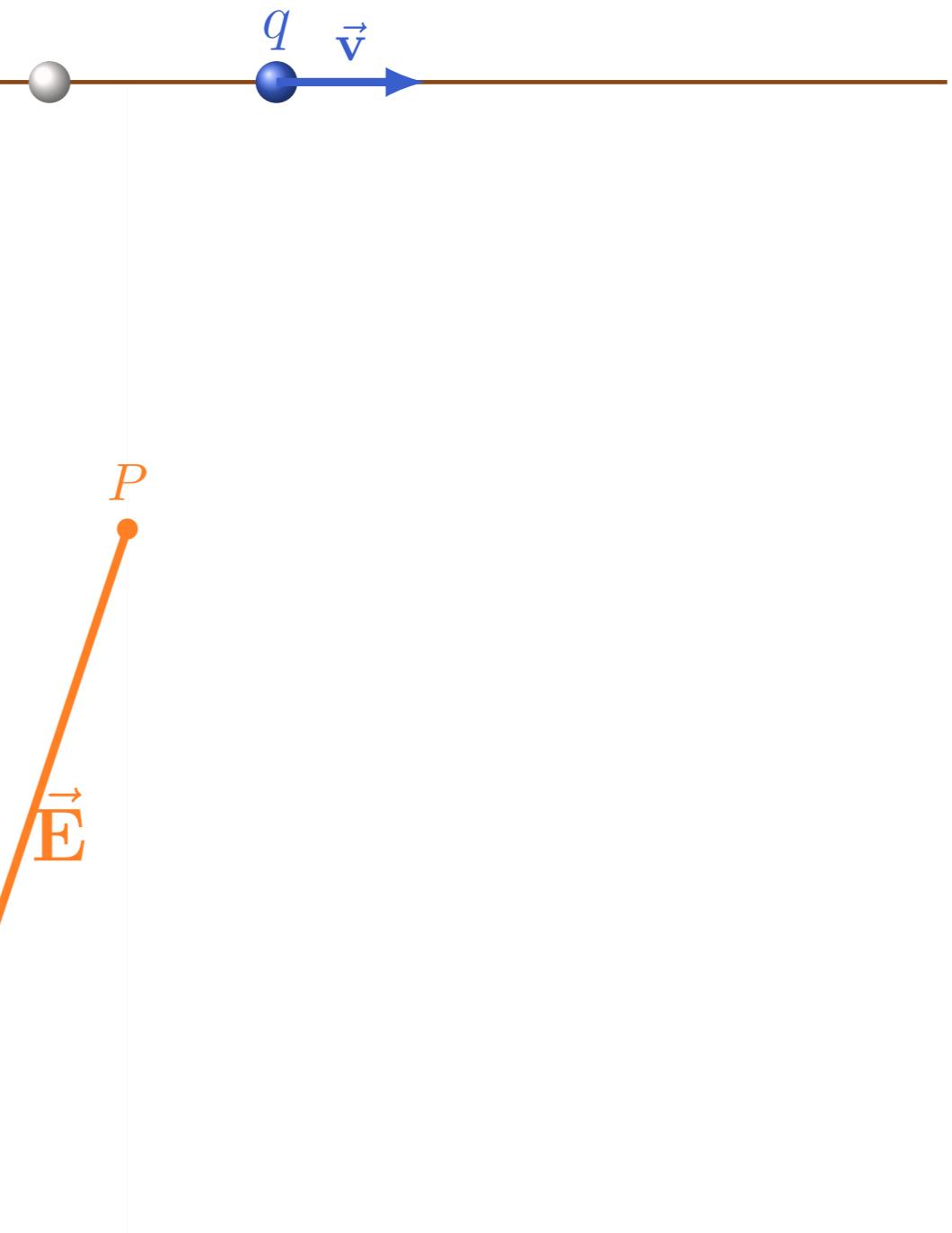
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

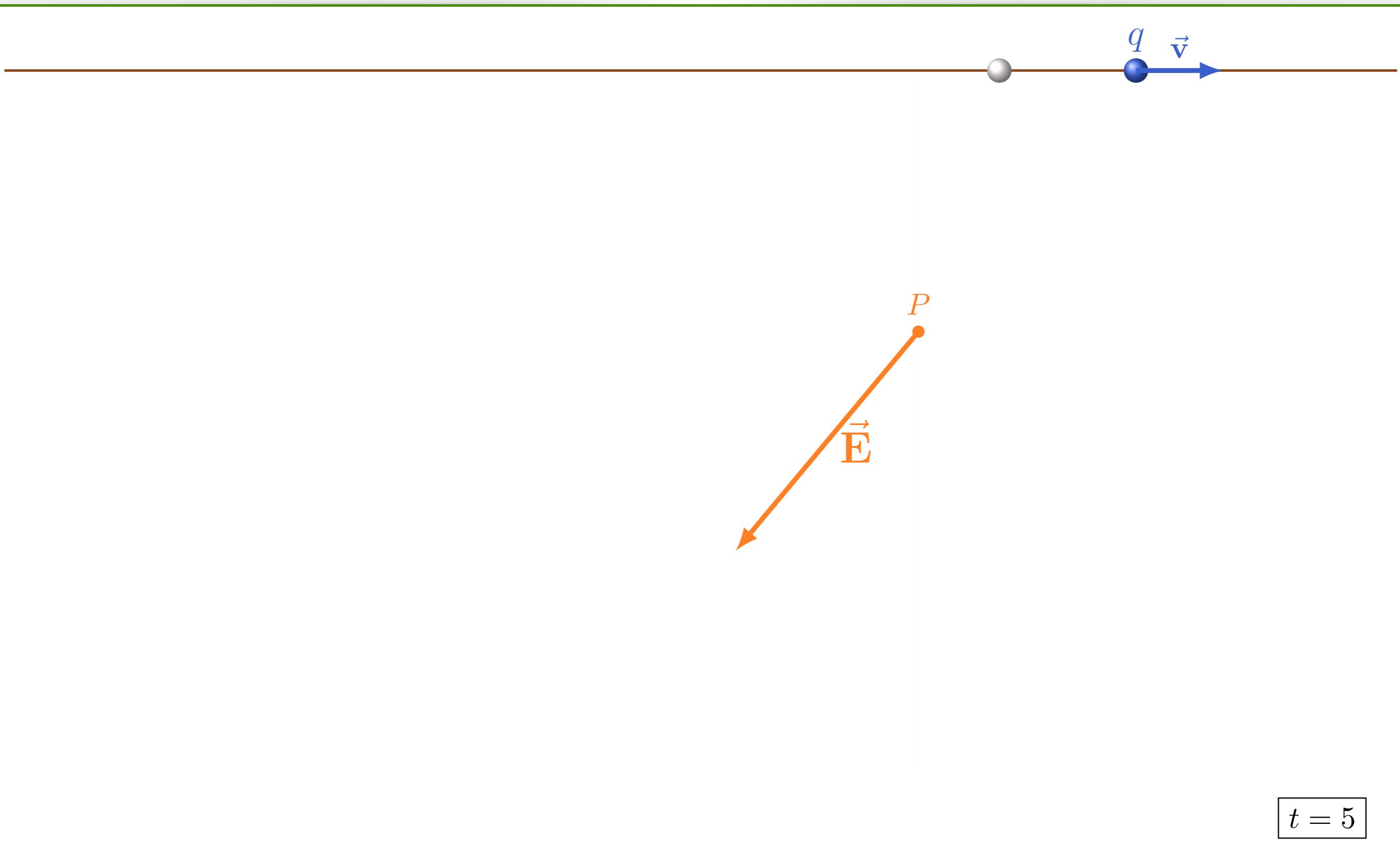
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$t = 2$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

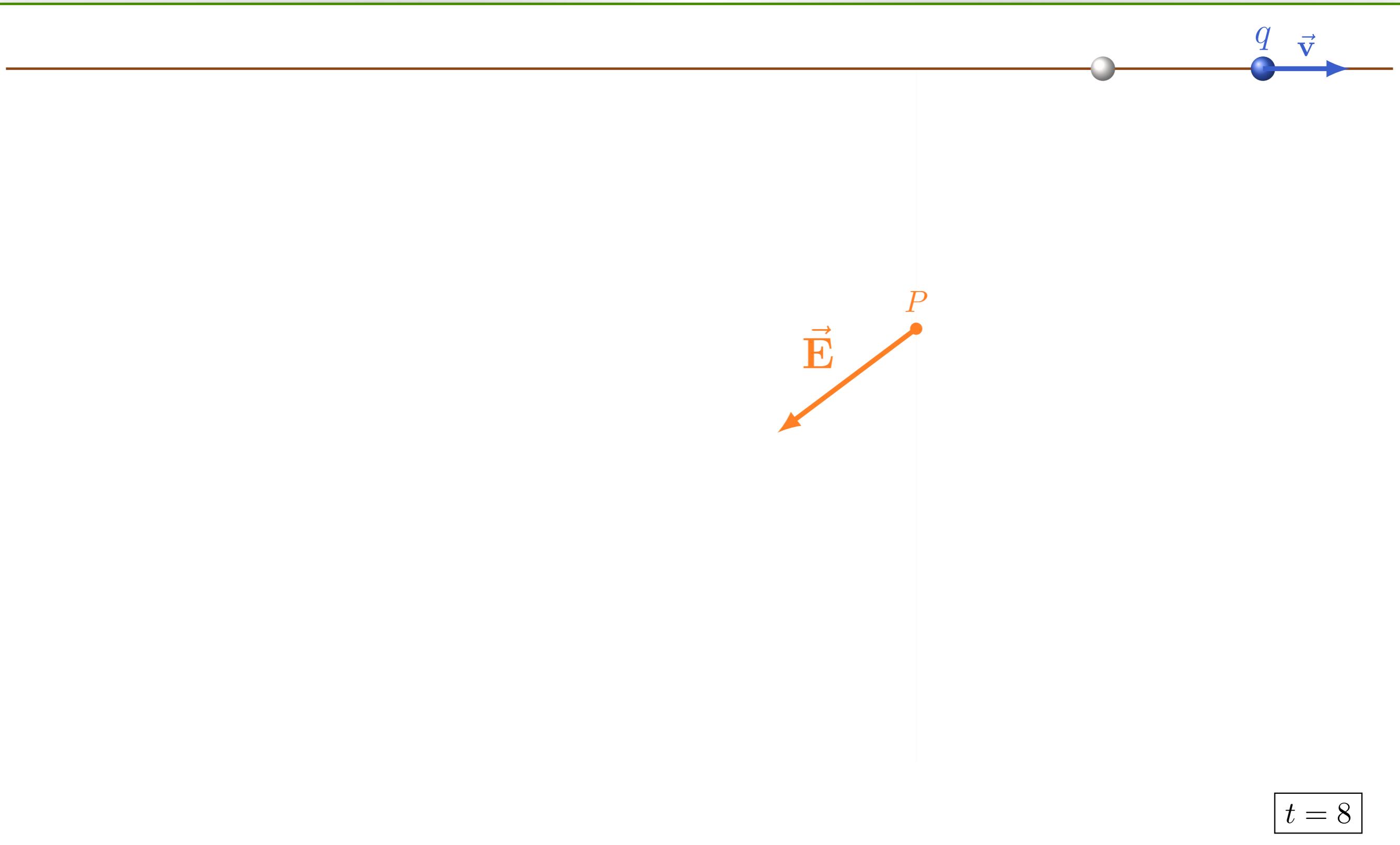
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$t = 5$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{1}{c^2} \vec{\mathbf{v}} \times \vec{\mathbf{E}}$$



$t = 8$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$

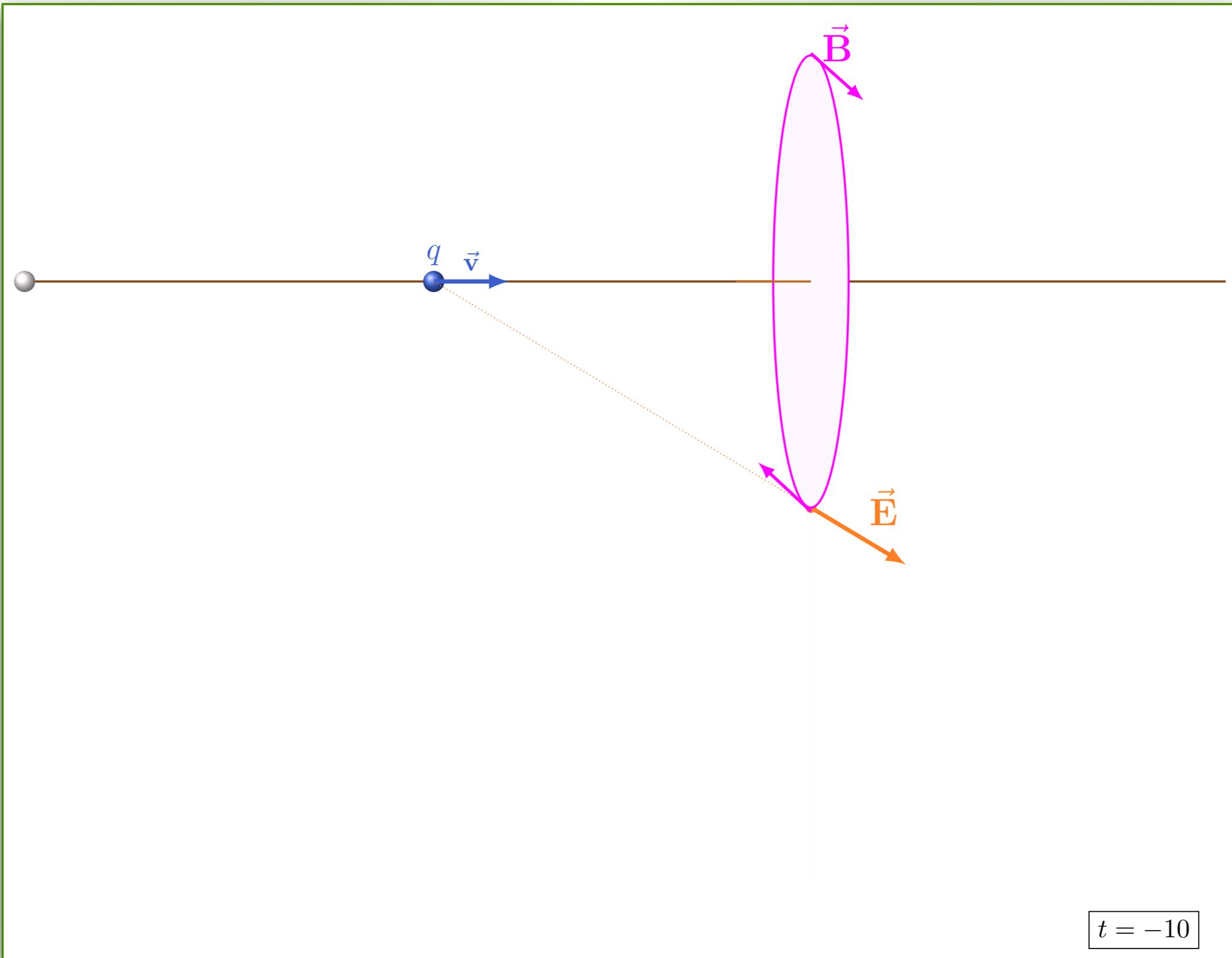


$E$  CRESCE,  $E'$  MÁXIMO  
QDO  $R$  É MÍNIMO,  
 $E$  DEPOIS DIMINUI

$t = 10$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$



$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c^2} \vec{v} \times \vec{E}$$

