MONITORIA MACRO IV — 22/11 — LISTA 2

1) $U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t)$

RO C: $\sum_{t=0}^{\infty} q_t [c_t + k_{t+1} - (1-\delta)k_t] \leq \sum_{t=0}^{\infty} q_t [(1-\tau_{nt}) w_t n_t + (1-\tau_{kt}) r_t k_t]$

$\rightarrow F(k,n)$

RO G: $\sum_{t=0}^{\infty} q_t (w_t \tau_{nt} n_t + r_t \tau_{kt} k_t) = \sum_{t=0}^{\infty} q_t g_t$

RR: $c_t + k_{t+1} - (1-\delta)k_t + g_t = F(k_t, n_t)$

A) Problema do consumidor

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \quad \text{sa} \quad \sum_{t=0}^{\infty} q_t [c_t + k_{t+1} - (1-\delta)k_t] \leq \sum_{t=0}^{\infty} q_t [(1-\tau_{n_t}) w_t n_t + (1-\tau_{kt}) r_t k_t]$$

k_0, preços, impostos dados!

$$\mathcal{L}: \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) + \lambda \sum_{t=0}^{\infty} q_t \left[(1-\tau_{n_t}) w_t n_t + (1-\tau_{kt}) r_t k_t - c_t - k_{t+1} + (1-\delta) k_t \right]$$

CPO: $[c_t]$ $\beta^t u_{c,t} - \lambda q_t = 0$ (1)

$[n_t]$ $-\beta^t u_{l,t} + \lambda q_t (1-\tau_{n_t}) w_t = 0$ (2)

$[k_{t+1}]$ $-\lambda q_t + \lambda q_{t+1} [(1-\tau_{k,t+1}) r_{t+1} + (1-\delta)] = 0$ (3)

+ CT $\lim_{t \to \infty} q_t k_{t+1} = 0$ (4)

De (1) $\longrightarrow$ $\beta^t \mu_{c,t} = \lambda q_t$

$q_0 = 1 \Rightarrow \mu_{c,0} = \lambda \Rightarrow \boxed{q_t = \dfrac{\beta^t \mu_{c,t}}{\mu_{c,0}}}$ (5)

Fazendo (2)/(1) $\boxed{\dfrac{\mu_{\ell,t}}{\mu_{c,t}} = (1-\tau_{n,t}) w_t}$ (6)

- Substituindo (5) em (3)

$$\beta^t \frac{\mu_{c,t}}{\mu_{c,0}} = \beta^{t+1} \frac{\mu_{c,t+1}}{\mu_{c,0}} \left[(1-\tau_{k,t+1}) r_{t+1} + (1-\delta) \right]$$

$$\boxed{\mu_{c,t} = \beta \mu_{c,t+1} \left[(1-\tau_{k,t+1}) r_{t+1} + (1-\delta) \right]} \quad (7)$$

- (5) em (4) $\longrightarrow$ $\boxed{\lim_{t\to\infty} \beta^t \mu_{c,t} k_{t+1} = 0}$ (8)

B) Problema da Firma

$$\max_{\{k_t^d, n_t^d\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} q_t \left[F(k_t^d, n_t^d) - w_t n_t^d - r_t k_t^d \right]$$ dados os preços

CPO: $[k_t^d]$: $F_K(k_t^d, n_t^d) = r_t$

$[n_t^d]$: $F_N(k_t^d, n_t^d) = w_t$

c) Equilíbrio Competitivo

Para dada sequência de impostos e gastos, um eq. competitivo é:

(1) uma sequência de alocações $\{c_t, k_{t+1}, n_t ; k_t^d, n_t^d\}_{t=0}^{\infty}$

(2) " " " preços $\{q_t, w_t, r_t\}_{t=0}^{\infty}$

t.q:

- Dados k_0 e preços e impostos a alocação resolve o problema do consumidor e da firma
- Mercados estão em equilíbrio

$$k_t = k_t^d \ ; \ n_t = n_t^d \ , \ c_t + k_{t+1} - (1-\delta)k_t + g_t = F(k_t, n_t)$$

Obs: RCC + RR $\longrightarrow$ RCG

Equações que caracterizam Eq:

$$\text{EQC}\begin{cases} u_{c,t} = \beta u_{c,t+1}\left[(1-\tau_{k,t+1})F_{k,t+1} + (1-\delta)\right] \\ \dfrac{u_{l,t}}{u_{c,t}} = (1-\tau_{n,t})F_{n,t} \\ \lim\limits_{t\to\infty} \beta^t u_{c,t} k_{t+1} = 0 \\ c_t + k_{t+1} - (1-\delta)k_t + g_t = F(k_t, n_t) \\ \text{RCC, RCG} \end{cases}$$

$\longrightarrow$ Note que não temos impostos sobre o consumo ou lump-sum $\longrightarrow$ não implementamos a solução do P.C.

$\downarrow$

minimizar distorções!

D) Restrição de Implementabilidade

Partindo da ROC rearranjada:

$$\sum_{t=0}^{\infty} q_t\left[C_t-(1-\tau_{n,t})w_t\right] = \sum_{t=0}^{\infty} q_t\left\{\left[(1-\tau_{k,t})r_t+(1-\delta)\right]k_t - k_{t+1}\right\} =$$

SÓ LADO DIREITO

$$= q_0\left[(1-\tau_{k,0})r_0+(1-\delta)\right]k_0 - q_0 k_1 +$$
$$+ q_1\left[(1-\tau_{k,1})r_1+(1-\delta)\right]k_1 - q_1 k_2 +$$
$$+ q_2\left[(1-\tau_{k,2})r_2+(1-\delta)\right]k_2 - q_2 k_3 + \dots =$$

$$= q_0\left[(1-\tau_{k,0})r_0+(1-\delta)\right]k_0 +$$
$$+ \left\{q_1\left[(1-\tau_{k,1})r_1+(1-\delta)\right]-q_0\right\}k_1 +$$
$$+ \left\{q_2\left[(1-\tau_{k,2})r_2+(1-\delta)\right]-q_1\right\}k_2 + \dots + \lim_{t\to\infty} q_t k_{t+1}$$

* TERMOS ENTRE CHAVES = 0 (CPO)

* LIMITE É ZERO (CT)

$$\therefore \sum_{t=0}^{\infty} q_t\left[C_t-(1-\tau_{n,t})w_t n_t\right] = q_0\left[(1-\tau_{k,0})r_0+(1-\delta)\right]k_0 \iff$$

$$\sum_{t=0}^{\infty} q_t\left[C_t-(1-\tau_{n,t})w_t n_t\right] = \left[(1-\tau_{k,0})F_{k,0}+(1-\delta)\right]k_0 \iff$$

$$\sum_{t=0}^{\infty} \beta^t\frac{u_{c,t}}{u_{c,0}}\left[C_t-\frac{u_{l,t}\,n_t}{u_{c,t}}\right] = \left[(1-\tau_{k,0})F_{k,0}+(1-\delta)\right]k_0 \iff$$

$$\boxed{\sum_{t=0}^{\infty} \beta^t\left[u_{c,t}C_t-u_{l,t}\,n_t\right] = u_{c,0}\left[(1-\tau_{k,0})F_{k,0}+(1-\delta)\right]k_0}$$

C. Restrição de Implementabilidade

E) Problema de Ramsey

$$\max_{\substack{\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty} \\ \{g_t\}_{t=0}^{\infty}}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t, g_t) \quad \text{s.t.} \quad \begin{cases} \displaystyle\sum_{t=0}^{\infty} \beta^t \left[u_{c,t} c_t - u_{l,t} n_t\right] = u_{c,0}\left[(1-\tau_{k,0}) F_{k,0} + (1-\delta)\right] k_0 \\ c_t + k_{t+1} - (1-\delta) k_t + g_t = F(k_t, n_t) \quad \forall t \geq 0 \\ k_0 > 0, \ \tau_{k,0}, \quad \text{dados} \end{cases}$$

$$\mathcal{L}: \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, 1-n_t, g_t) + \lambda_t \left[F(k_t, n_t) - c_t - k_{t+1} + (1-\delta) k_t - g_t\right] \right\} +$$

$$+ \mu \left[\sum_{t=0}^{\infty} \beta^t \left(u_{c,t} c_t - u_{l,t} n_t\right) - u_{c,0}\left[(1-\tau_{k,0}) F_{k,0} + (1-\delta)\right] k_0 \right]$$

Reescrevendo:

$$\mathcal{L} \sum_{t=0}^{\infty} \beta^t \left\{ \overbrace{\left[u(c_t, l_t, g_t) + \mu u_{c,t} c_t - \mu u_{l,t} n_t\right]}^{W(c_t, n_t, \mu, g_t)} + \lambda_t \left[F(k_t, n_t) - c_t - k_{t+1} + (1-\delta) k_t - g_t\right] \right\}$$

$$- \mu u_{c,0}\left[(1-\tau_{k,0}) F_{k,0} + (1-\delta)\right] k_0$$

CPO (p/ $t \geq 1$)

$[c_t]$: $\beta^t \left[W_{c,t} - \lambda_t\right] = 0 \quad \Rightarrow \quad \boxed{W_{c,t} = \lambda_t}$

$[n_t]$: $\beta^t \left[W_{n,t} - F_{n,t} \lambda_t\right] = 0 \quad \Rightarrow \quad \boxed{W_{n,t} = W_{c,t} F_{n,t}}$

$[k_{t+1}]$: $-\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \left[F_{k,t+1} + (1-\delta)\right] = 0$

$$\Rightarrow \boxed{W_{c,t} = \beta W_{c,t+1} \left[F_{k,t+1} + (1-\delta)\right]}$$

$[g_t]$: $\beta^t \left[W_{g,t} - \lambda_t\right] = 0 \quad \Rightarrow \quad \boxed{W_{g_t} = \lambda_t}$

F) τ_K no longo prazo

Supondo que $\exists$ um EE $\Rightarrow$ $c_t = c$, $n_t = n$, $k_t = k$, $g_t = g$

$\therefore$ $w_{c,t} = w_c$

$\therefore$ $$1 = \beta\left[F_K + (1-\delta)\right]$$

A eq. Euler do E.C é $\rightarrow$ $u_{c,t} = \beta u_{c,t+1}\left[(1-\tau_{K,t+1})F_{K,t+1} + (1-\delta)\right]$

no E.E: $$1 = \beta\left[(1-\tau_K)F_K + (1-\delta)\right]$$

$\therefore$ $\boxed{\tau_K = 0}$ //

6) Caso particular

$$u(c,l,g) = \frac{c^{1-\gamma} - 1}{1-\gamma} + v(l,g) \Rightarrow$$

$$w(c,g,n,\mu) = u(c,l,g) + \mu u_c c - \mu u_l n$$

$$w(c,g,n,\mu) = \frac{c^{1-\gamma} - 1}{1-\gamma} + v(l,g) + \mu c^{-\gamma} c - \mu v_l n$$

$$= \left[\frac{1}{1-\gamma} + \mu\right] c^{1-\gamma} + v(l,g) - \mu v_l n - \frac{1}{1-\gamma}$$

$\therefore$ $w_c = \left[1 + \mu(1-\gamma)\right] c^{-\gamma}$

Solução de Ramsey:

$$w_{c,t} = \beta w_{c,t+1}\left[F_{K,t+1} + (1-\delta)\right] \quad \forall\, t \geq 1$$

$$[1+\mu(1-\gamma)]\,C_t^{-\gamma} = \beta[1+\mu(1-\gamma)]\,C_{t+1}^{-\gamma}\left[F_{K,t+1} + (1-\delta)\right] \quad \forall\, t \geq 1$$

$$C_t^{-\gamma} = \beta C_{t+1}^{-\gamma}\left[F_{K,t+1} + (1-\delta)\right] \quad \forall\, t \geq 1$$

E.O.C $\quad C_t^{-\gamma} = \beta C_{t+1}^{-\gamma}\left[(1-\tau_{K,t+1})F_{K,t+1} + (1-\delta)\right]$

$\therefore \quad \tau_{K,t+1} = 0 \;\; \forall\, t \geq 1 \quad$ ou $\quad \tau_{K,t} = 0 \;\; \forall\, t \geq 2$

é a política ótima

h) Ele escolherá o valor de $\tau_{K,0}$ para financiar todo seu plano de gastos de maneira que $\tau_{K,t} = 0$ já no período 1.

(2) Kydland & Prescott (1977); Barro & Gordon (1983)

P.C: $Y_t = Y_n + \alpha(\pi_t - \pi_t^e)$, $\alpha > 0$

$$L(\pi_t, Y_t) = \frac{1}{2}\lambda (Y_t - Y^*)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \quad , \lambda > 0$$

$Y^* > Y^n$

A) Se o governo pode se comprometer pode se comprometer c/ a inflação anunciada, a expectativa de inflação é a própria inflação. (depois que o governo anuncia).

Da Curva de Phillips:

$$Y_t = Y_n + \alpha \underbrace{(\pi_t - \pi_t^e)}_{=0} \Rightarrow \boxed{Y_t = Y_n}$$

, Y_t deixa de ser uma variável de escolha

Como o governo está preso pelo anúncio:

$$L(\pi_t, Y_t) = \frac{1}{2}\lambda (Y_n - Y^*)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \longrightarrow \text{faço min}$$

$$\frac{\partial L}{\partial \pi_t} = 0 \Rightarrow \pi_t - \pi^* = 0 \Rightarrow \boxed{\pi_t = \pi^*}$$

$$\boxed{L(\pi_t, Y_t) = \frac{1}{2}\lambda (Y_n - Y^*)^2}$$

B) Na situação de comprometimento ausente, o governo ~~toma~~ toma como dado a expectativa e resolve:

$$\min_{Y_t, \pi_t} \frac{1}{2}\lambda (Y_t - Y^*)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \quad \text{sa} \quad Y_t = Y_n + \alpha(\pi_t - \pi_t^e)$$

$$\min_{\pi_t} \frac{1}{2}\lambda \left[Y_n + \alpha(\pi_t - \pi_t^e) - Y^*\right]^2 + \frac{1}{2}(\pi_t - \pi^*)^2$$

CPO: $\frac{\partial \mathcal{L}}{\partial \pi_t} = 0 \Rightarrow \alpha\lambda\left[Y_n - Y^* + \alpha(\pi_t - \pi_t^e)\right] + \pi_t - \pi^* = 0 \quad (*)$

→ Supondo que o público tenha expectativas racionais, este sabe que ao "chutar" π_t^e, o governo escolhe π_t usando $(*)$

→ É common knowledge a expectativa de inflação.

→ Resolvendo de trás pra frente $\pi_t^e = \pi_t$

→ Plugando em $(*)$

$$\alpha\lambda(Y_n - Y^*) + \pi_t - \pi^* = 0$$

$$\Rightarrow \boxed{\pi_t = \pi^* + \alpha\lambda(Y^* - Y_n)} > \pi^*, \text{ pois } \begin{cases} Y^* > Y_n \\ \alpha, \lambda > 0 \end{cases}$$

Como $\pi_t^e = \pi_t \Rightarrow \boxed{Y_t = Y_n}$

A perda do governo é:

$$L = \frac{1}{2}\lambda(Y_n - Y^*)^2 + \frac{1}{2}(\alpha\lambda(Y^* - Y_n))^2 = \frac{1}{2}\lambda(Y_n - Y^*)^2 + \frac{\alpha^2\lambda^2}{2}(Y^* - Y_n)^2 > \frac{1}{2}\lambda(Y_n - Y^*)^2.$$

Logo, o governo tem perda maior neste caso — sem comprometimento.

Resumindo: $\mathcal{L} = \frac{1}{2}\lambda(Y_n - Y^*)^2(1 + \alpha^2\lambda)$

c) Jogo Repetido infinitamente:

$$L: \sum_{t=0}^{\infty} \beta^t L(\pi_t, Y_t), \quad \beta \in (0,1)$$

→ Queremos que o preço de se comprometer ≤ preço de ñ se comprometer

1) Preço ao se comprometer:

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{2}\lambda (Y_n - Y^*)^2 = \frac{1}{1-\beta} \frac{\lambda}{2}(Y_n - Y^*)^2$$

2) Preço se se desviar:

→ O público espera comprometimento: $\pi_t^e = \pi^*$

O gov. então minimiza:

$$\min_{\pi_t} \frac{1}{2}\lambda\left(Y_n + \alpha(\pi_t - \pi^*) - Y^*\right)^2 + \frac{1}{2}(\pi_t - \pi^*)^2$$

CPO:

$$\alpha\lambda\left(Y_n - Y^* + \alpha(\pi_t - \pi^*)\right) + \pi_t - \pi^* = 0$$

$$(1 + \alpha^2\lambda)(\pi_t - \pi_t^*) = -\alpha\lambda(Y_n - Y^*)$$

$$\pi_t = \pi^* - \frac{\alpha\lambda(Y_n - Y^*)}{1 + \alpha^2\lambda}$$

$$\mathcal{L}.\ \frac{1}{2}\lambda\left[y_n - y^* - \frac{\alpha^2\lambda(y_n-y^*)}{1+\alpha^2\lambda}\right]^2 + \frac{1}{2}\left(\frac{-\alpha\lambda(y_n-y^*)}{1+\alpha^2\lambda}\right)^2 =$$

$$= \frac{1}{2}\lambda\,\frac{(y_n-y^*)^2}{(1+\alpha^2\lambda)^2} + \frac{1}{2}\,\frac{\alpha^2\lambda^2(y_n-y^*)^2}{(1+\alpha^2\lambda)^2} =$$

$$= \frac{1}{2}\lambda\,\frac{(1+\alpha^2\lambda)(y_n-y^*)^2}{(1+\alpha^2\lambda)^2} = \boxed{\frac{1}{2}\lambda\,\frac{(y_n-y^*)^2}{(1+\alpha^2\lambda)}} \longrightarrow$$

Payoff de desvio em um período

→ Se ele desviar em um período, só terá o payoff do não comprometimento nos seguintes.

$$\sum_{t=1}^{\infty}\beta^t\,\frac{1}{2}\lambda(y_n-y^*)^2(1+\alpha^2\lambda) = \frac{\beta}{1-\beta}\,\frac{1}{2}\lambda(y_n-y^*)^2(1+\alpha^2\lambda)$$

Assim, para a solução de comprometimento seja factível, temos que impor que:

$$\frac{1}{1-\beta}\,\lambda\,\frac{1}{2}(y_n-y^*) \le \frac{1}{2}\lambda\,\frac{(y_n-y^*)^2}{1+\alpha^2\lambda} + \frac{\beta}{1-\beta}\,\frac{1}{2}\lambda(y_n-y^*)^2(1+\alpha^2\lambda)$$

$$\frac{1}{1-\beta} \le \frac{1}{1+\alpha^2\lambda} + \frac{\beta}{1-\beta}(1+\alpha^2\lambda) \iff 1 \le \frac{(1-\beta)}{1+\alpha^2\lambda} + \beta(1+\alpha^2\lambda)$$

$$\iff 1+\alpha^2\lambda \le 1-\beta+\beta(1+\alpha^2\lambda)^2 \iff \alpha^2\lambda \le \beta\left[(1+\alpha^2\lambda)^2-1\right] \iff$$

$$\iff \beta \geq \frac{\alpha^2\lambda}{(1+\alpha^2\lambda)^2-1} = \frac{\alpha^2\lambda}{(1+\alpha^2\lambda+1)(1+\alpha^2\lambda-1)} = \frac{1}{2+\alpha^2\lambda}$$

$$\therefore \boxed{\beta \geq \frac{1}{2+\alpha^2\lambda}}$$