PSI 5794 - MATRIX ANALYSIS   EXAM 1   NOVEMBER 21, 2021

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1. Exam starts at 14:00 h and finishes at 17:00 h. NO DELAYS!
2. Write your solutions in blank sheets (A4), number and name everything, then submit to the course Moodle in the proper link.

Name:____

1. Let $Ax = b$ be a system of linear equations where

$$A = \begin{bmatrix} 2 & 1 & 2 & 0 \\ -2 & -1 & 0 & 2 \\ 4 & 2 & 3 & 1 \\ -4 & -1 & -3 & 5 \end{bmatrix}.$$

   (a) Find the LU decomposition of A, showing your steps. Then show that matrix A can be retrieved from your decomposition;
   (b) Solve the linear system by blocks for $b^T = [2 \ -4 \quad 5 \quad -3]$. Show your steps.

2. Solve the matrix linear system $AX + XB = C$, knowing that the solution is a circulant matrix, when

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & 3 \\ 3 & 0 & 2 \\ 2 & 3 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 9 \\ 9 & 1 & 4 \\ 4 & 9 & 1 \end{bmatrix}.$$

3. Consider the vector space $V = \mathbb{R}^4$ and the subspace S formed by the vectors $s_1 = [1\ 0\ 2\ 0]^T$ and $s_2 = [0\ 2\ 0\ 1]^T$. Find an orthogonal complement subspace for S.

4. Consider two *square* matrices, T and C, not necessarily of the same size.
   (a) Matrix T must be built from the quantities $a, b, c, \ldots$, so that it is a 4×4 Toeplitz matrix;
   (b) Show how to augment your matrix T into a circulant matrix C.

5. Decompose the matrix A below into the product of three matrices, one of them being its rank normal form. You must obtain the three matrices explicitly, showing *all* the steps. Perform the product to show that the original matrix is recovered. What is the matrix rank? Hint: elementary matrices.

$$A = \begin{bmatrix} 1 & -2 & 0 & -1 \\ 0 & 2 & 1 & 3 \\ 1 & -4 & -1 & -4 \\ 1 & 0 & 1 & 2 \end{bmatrix}.$$