

Eletromagnetismo Avançado

3º ciclo
Aula de 10 de
novembro

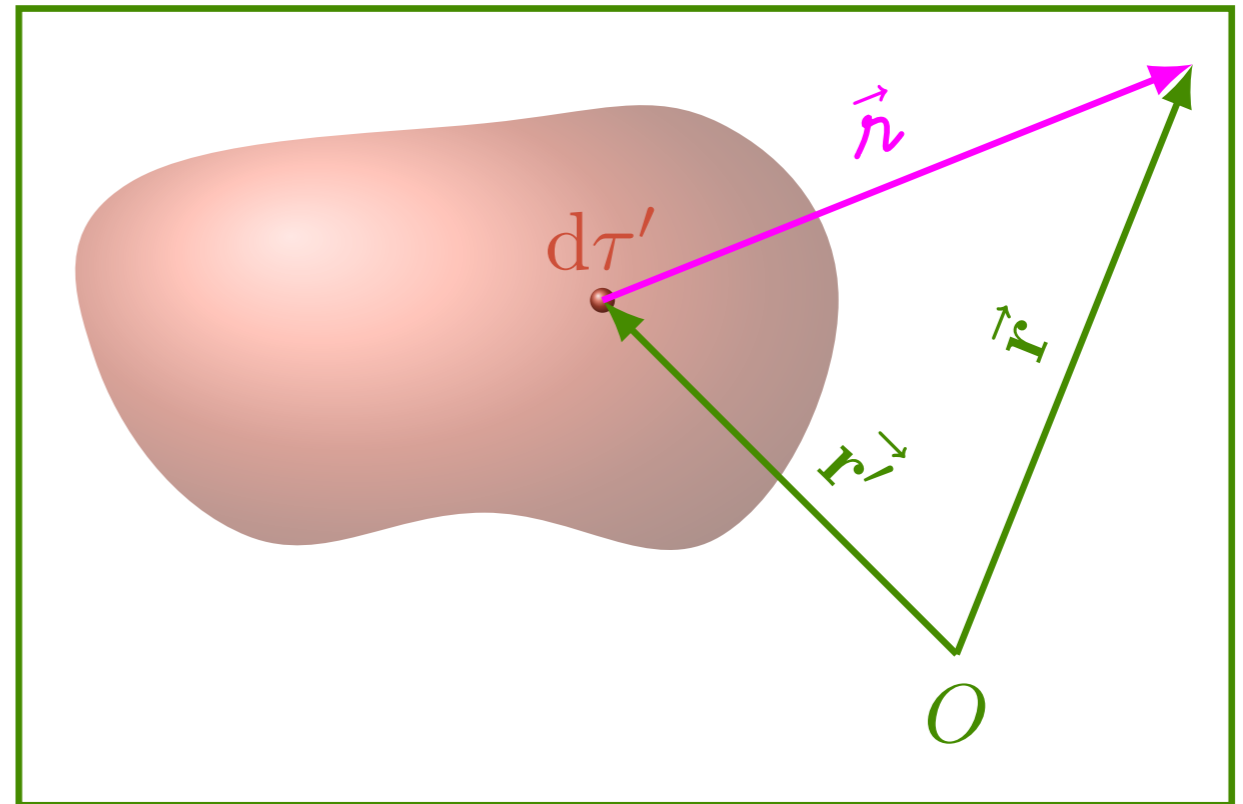
Potencial e potencial vetor

$$\square^2 V = -\frac{\rho}{\epsilon_0}$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{r} d\tau'$$

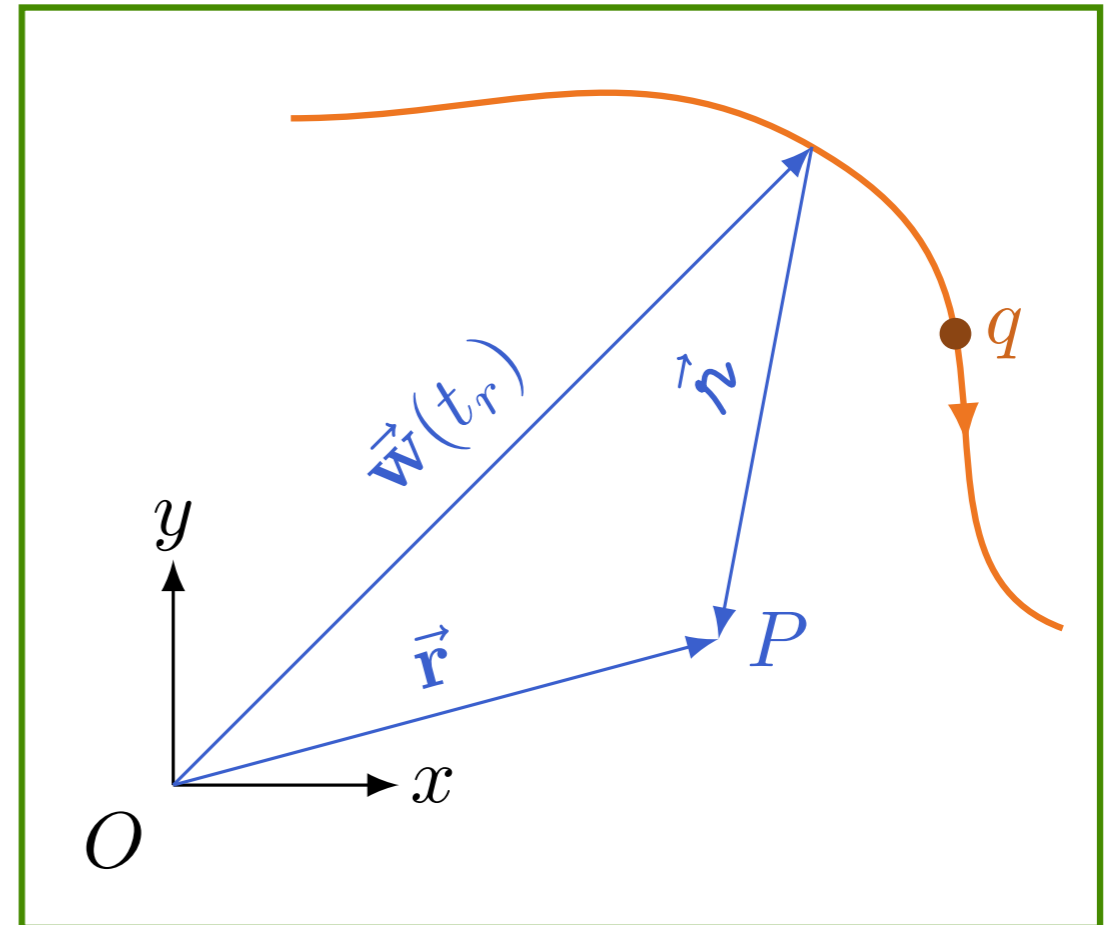
$$\square^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}', t_r)}{r} d\tau'$$



$$t_r \equiv t - \frac{r}{c}$$

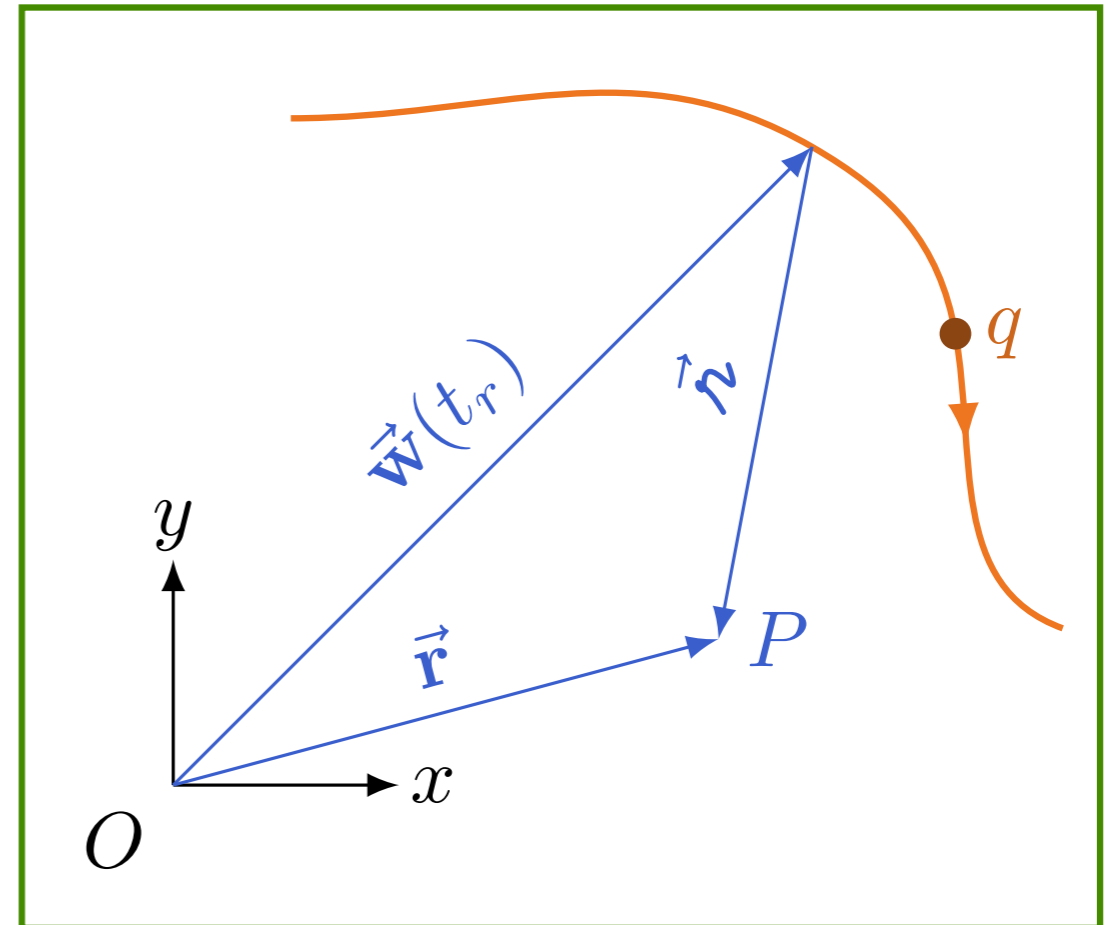
Potenciais de Liénard e Wiechert



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$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{r - \vec{r} \cdot \frac{\vec{v}}{c}}$$

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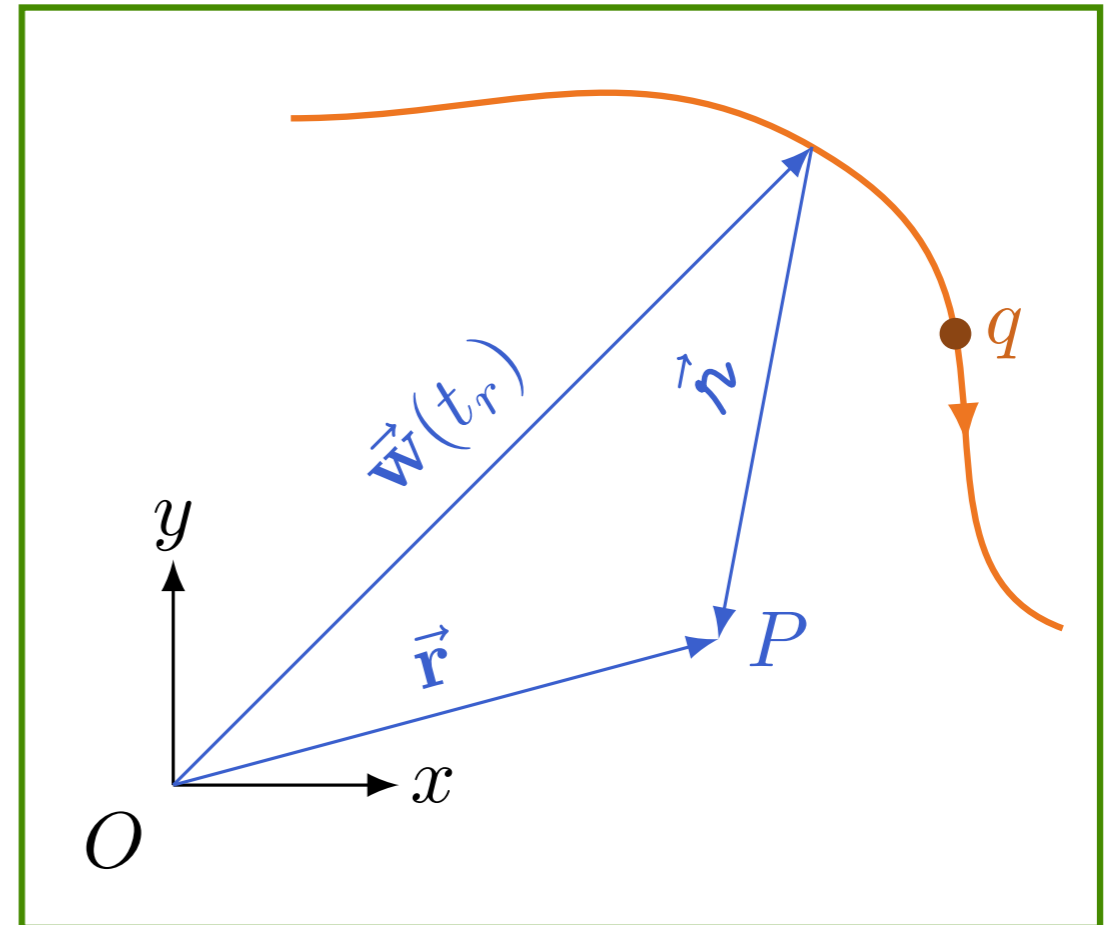


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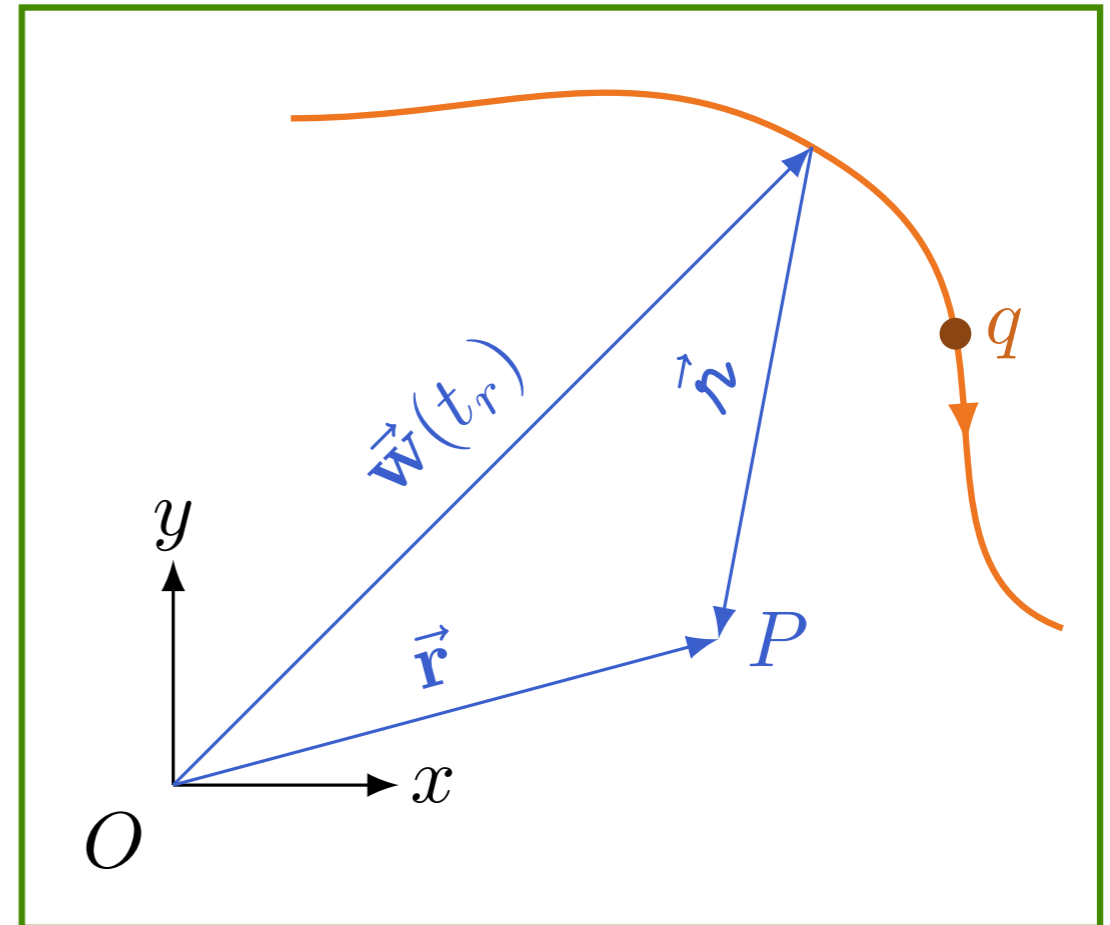
$$\vec{E}(\vec{r}, t) = ?$$



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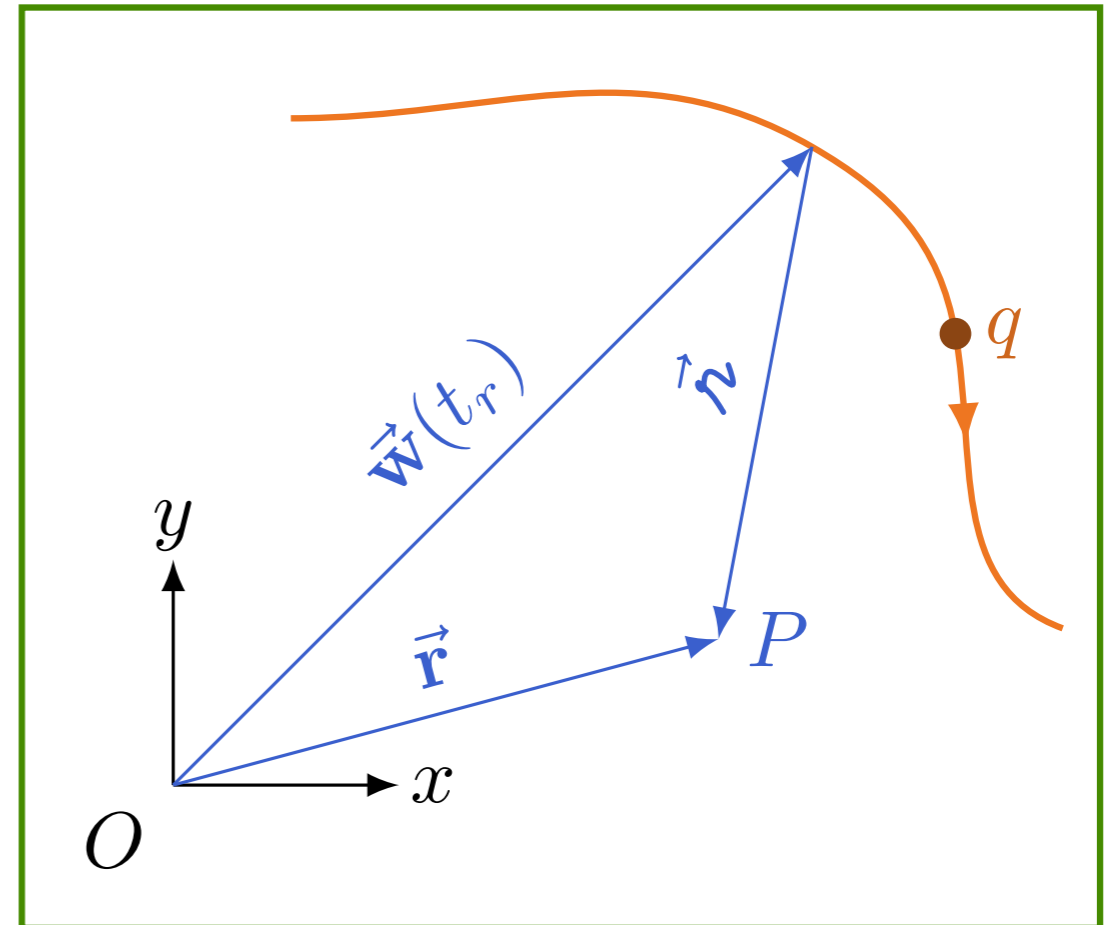


Potenciais de Liénard e Wiechert

$$V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{\hat{n} \cdot \vec{u}}$$

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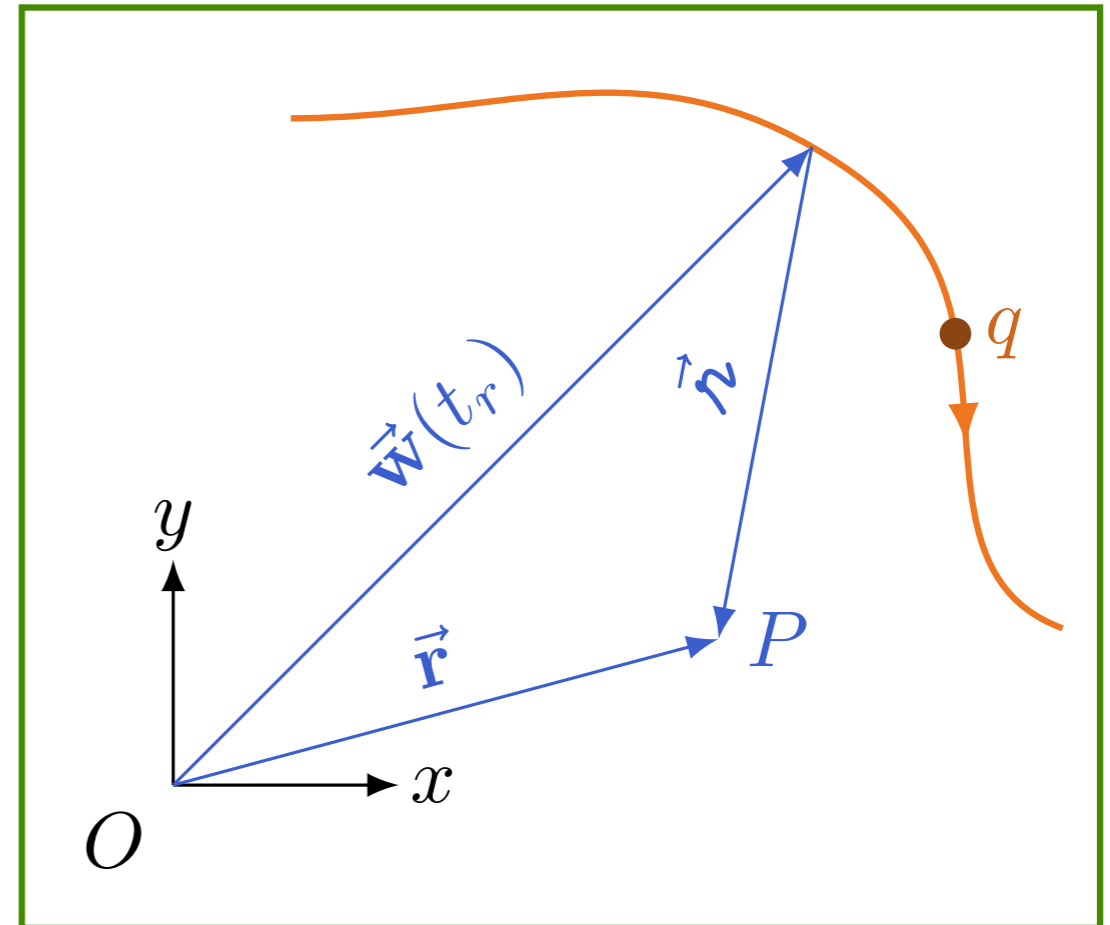
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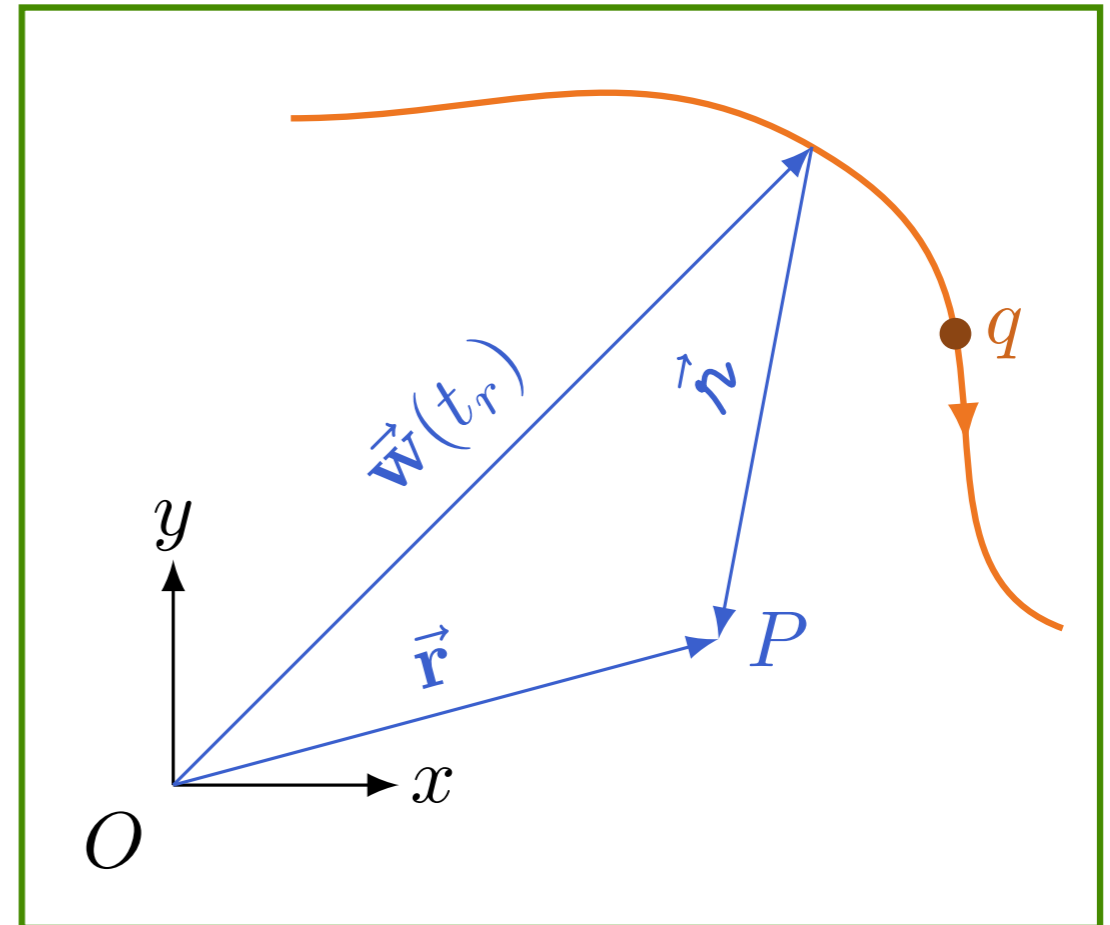
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$$\vec{\nabla}V = \frac{qc}{4\pi\epsilon_0} \frac{-1}{(\vec{r} \cdot \vec{u})^2} \vec{\nabla}(\vec{r} \cdot \vec{u})$$



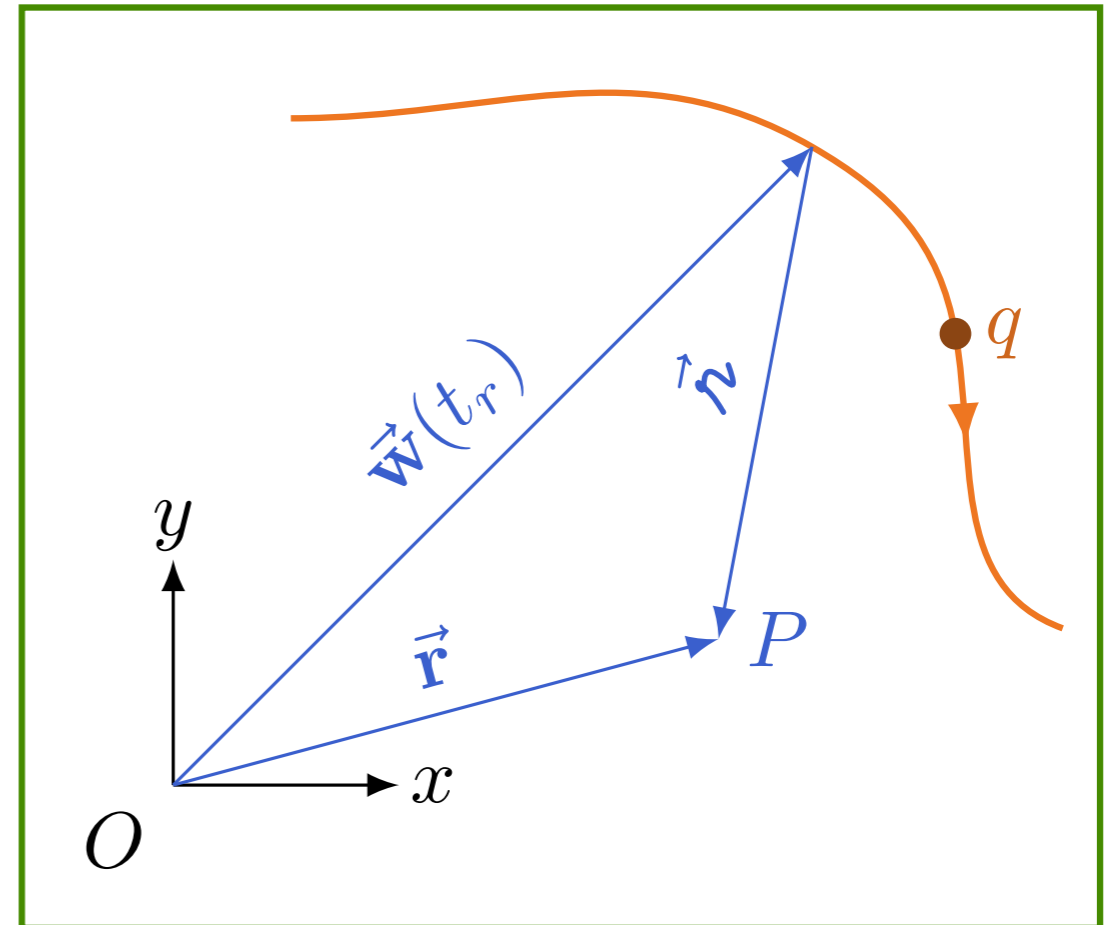
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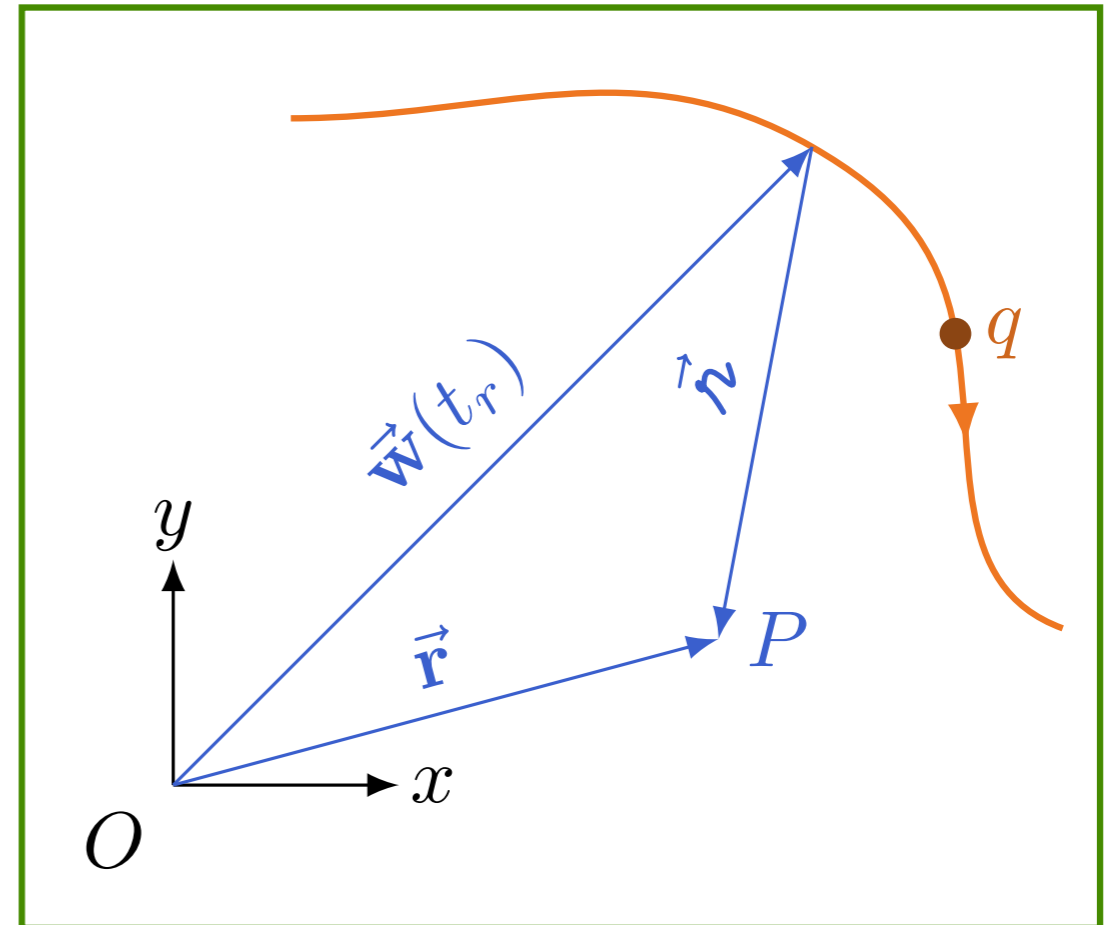


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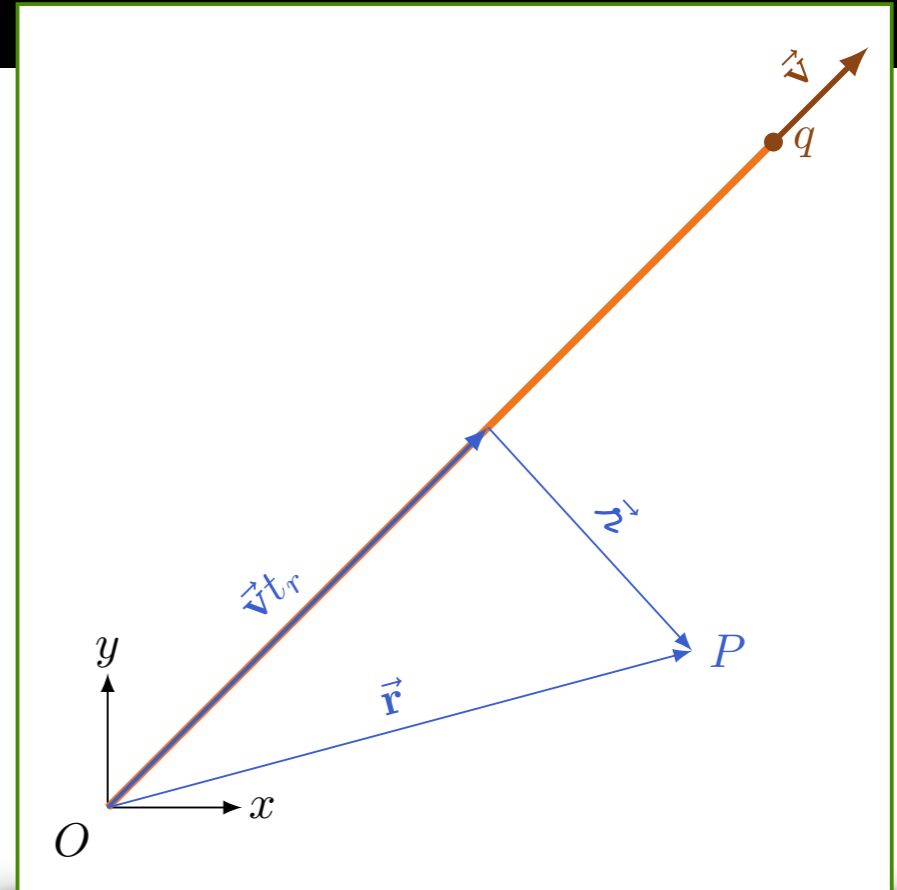
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$$\vec{B}(\vec{r}, t) = \frac{1}{c} \hat{r} \times \vec{E}(\vec{r}, t)$$

$$\vec{u} \equiv c\hat{n} - \vec{v}$$

Pratique o que aprendeu

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{r}{(\vec{r} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \vec{r} \times (\vec{u} \times \vec{a})]$$

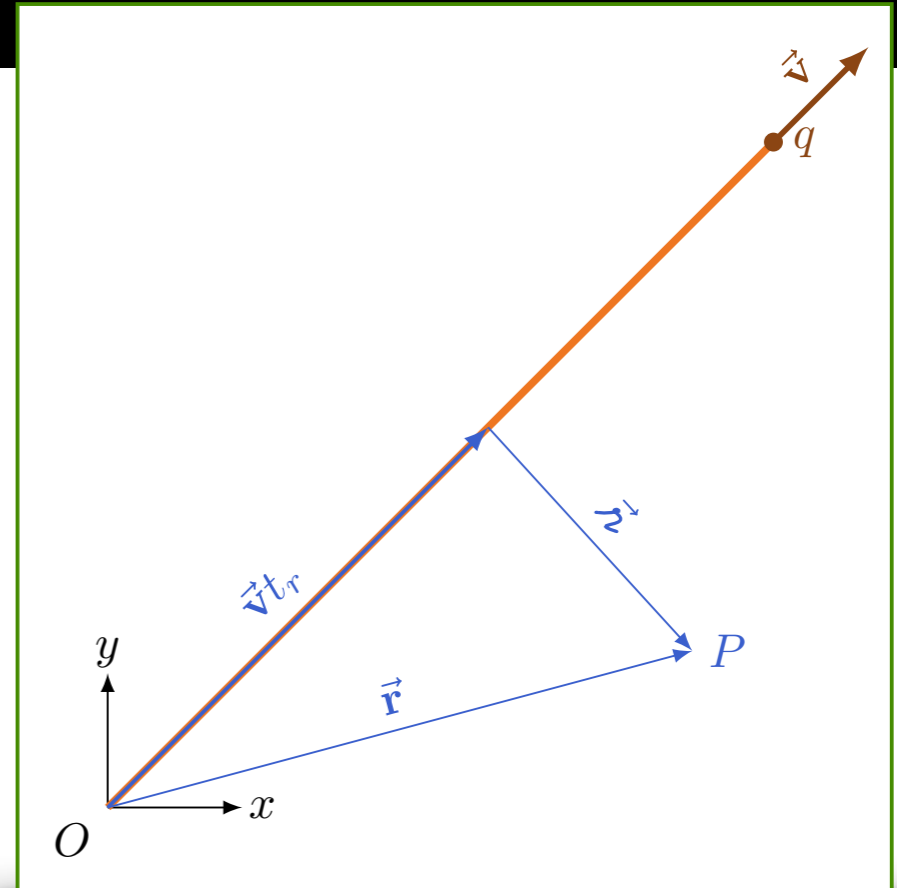


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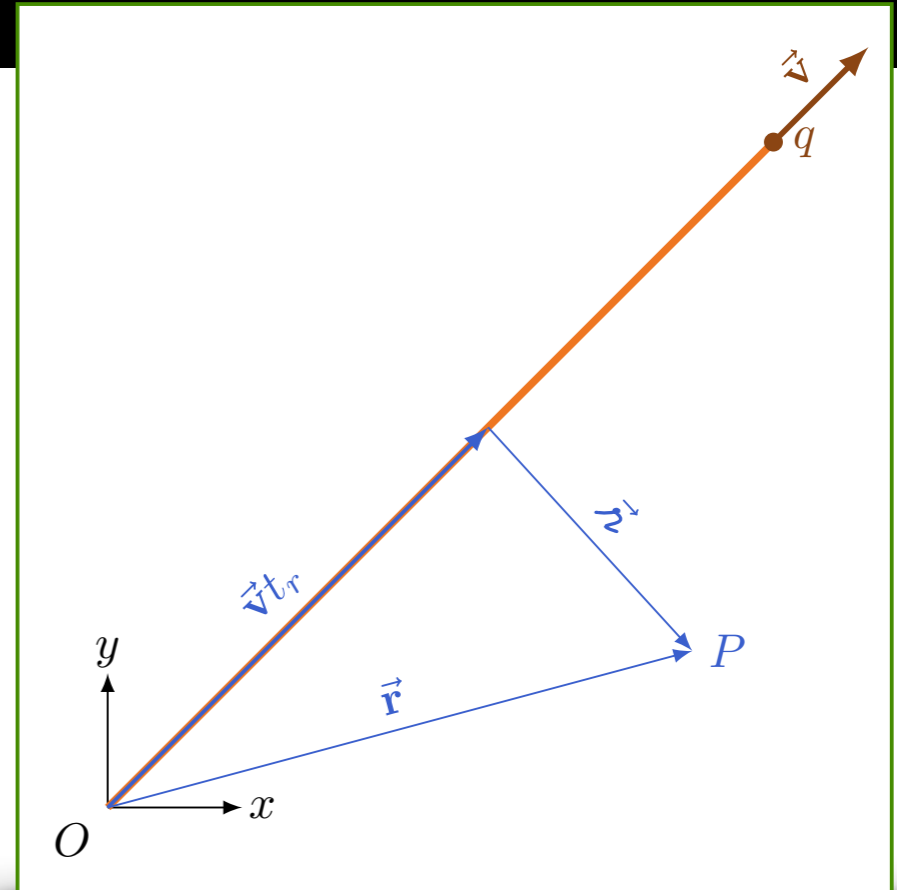
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Pratique o que aprendeu

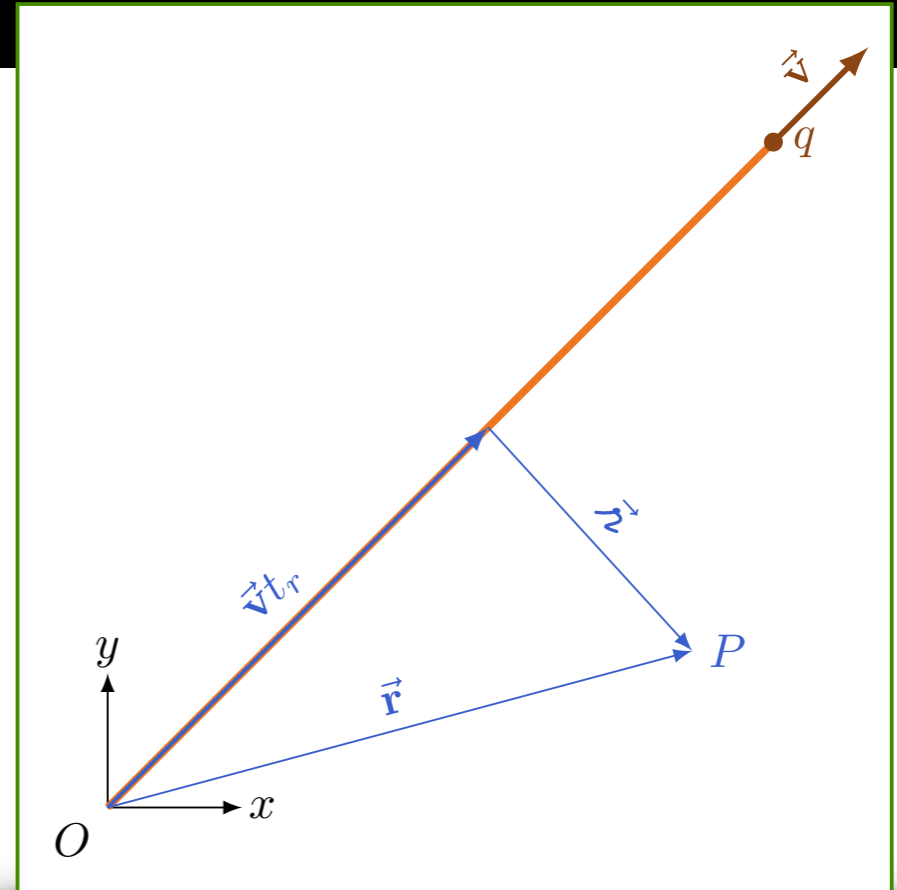
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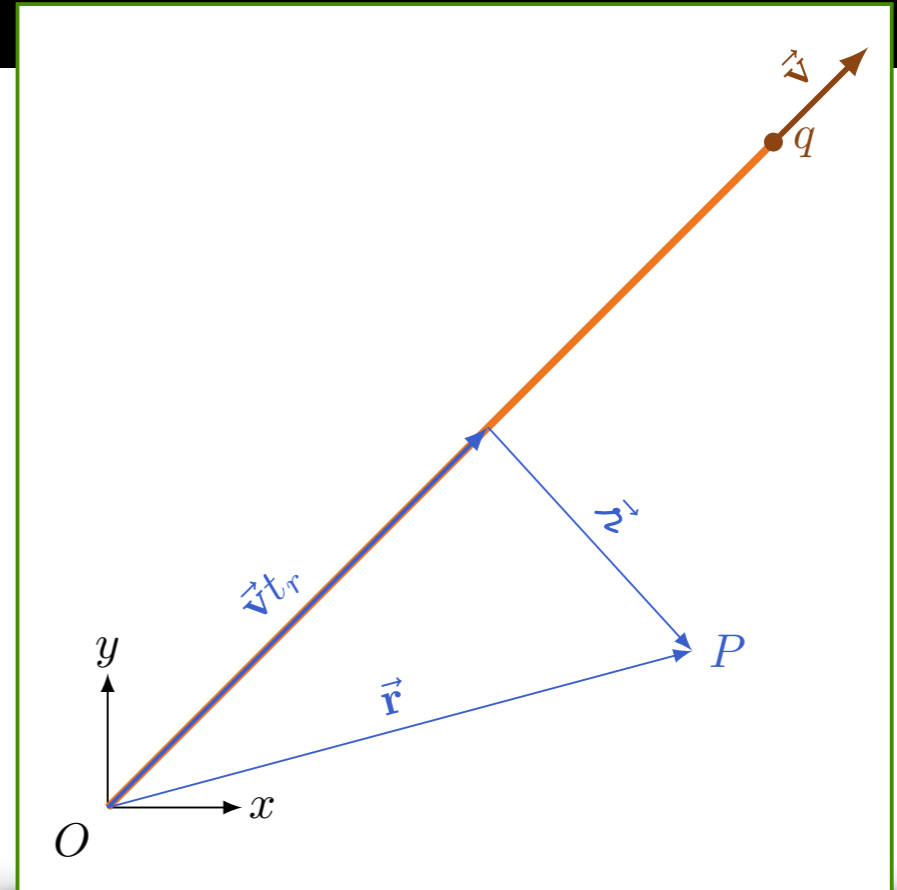
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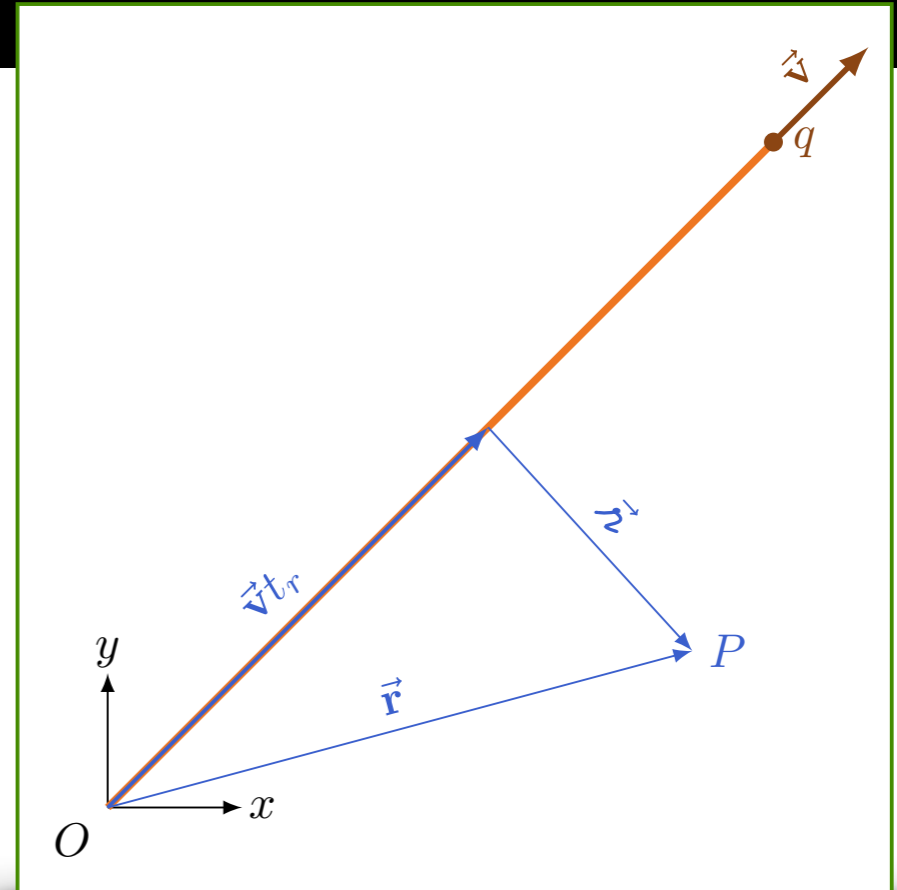
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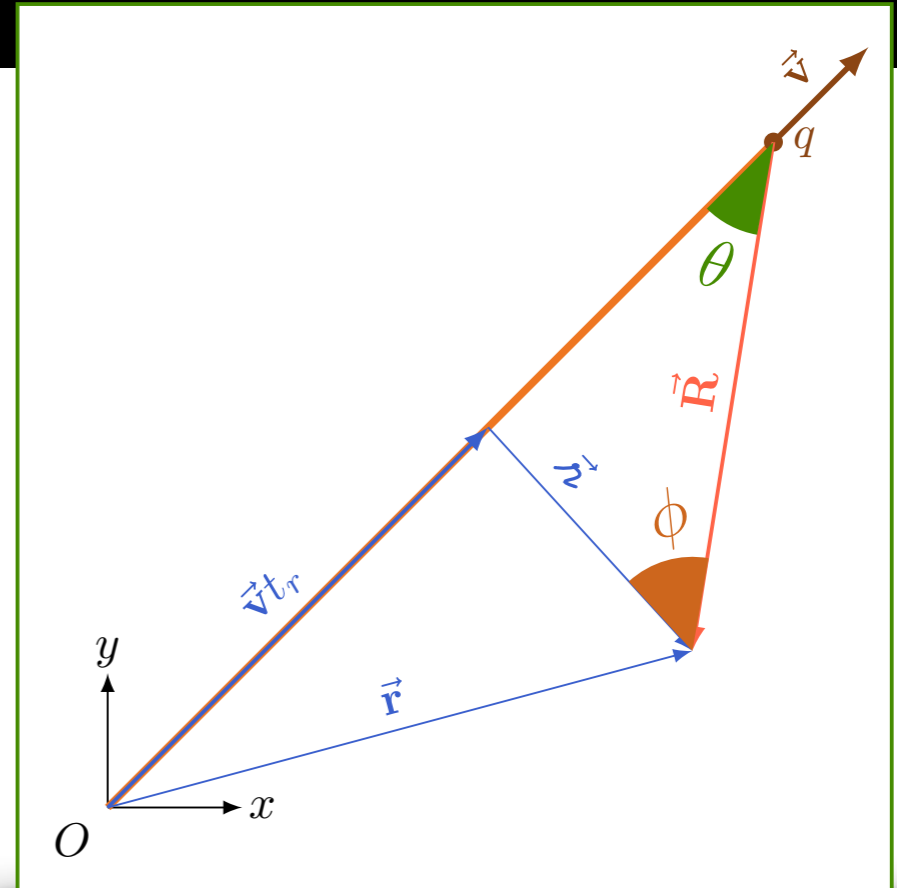
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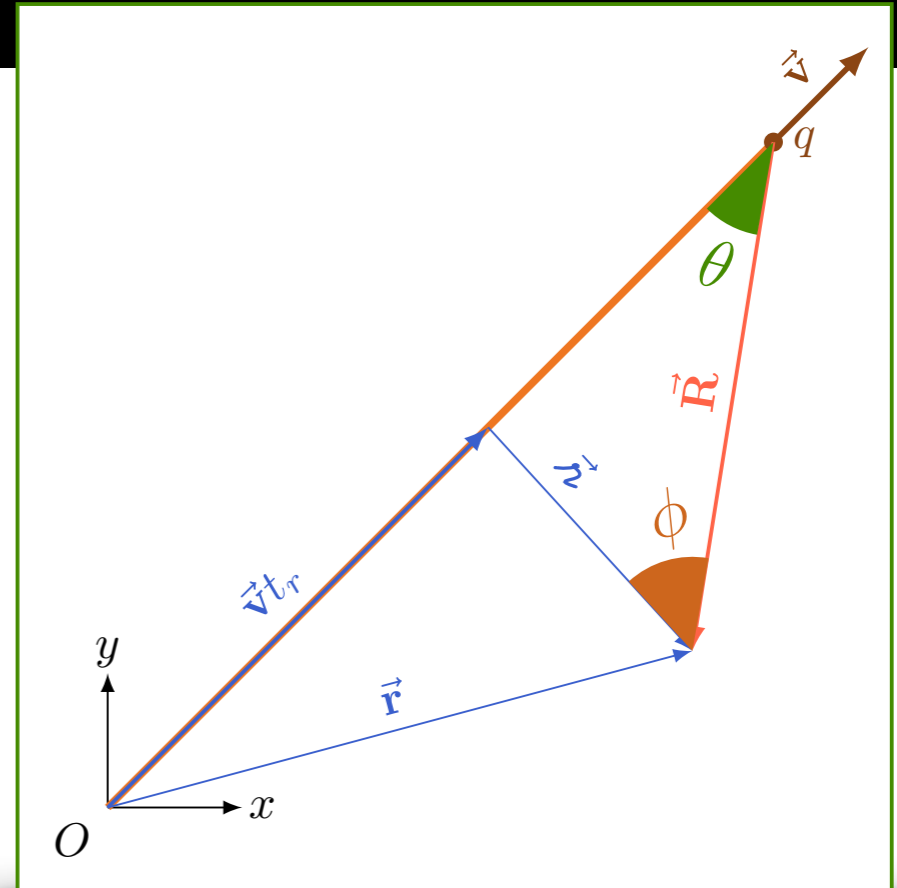
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Pratique o que aprendeu

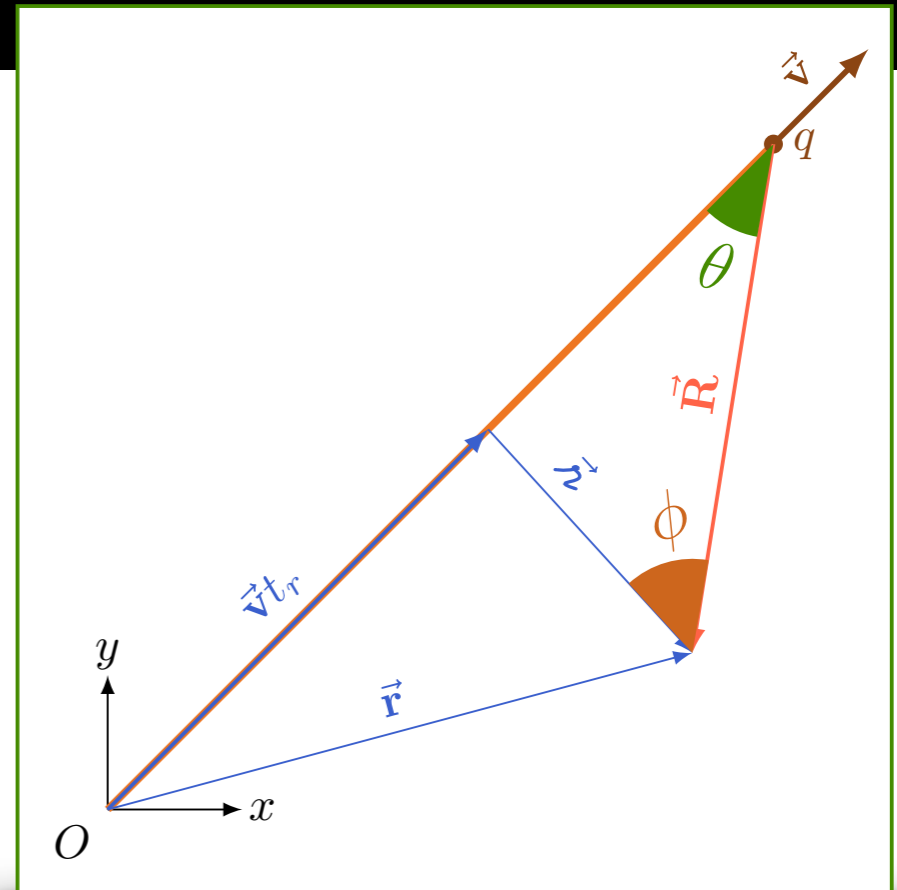
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$$\frac{v(t - t_r)}{\sin \phi} = \frac{c(t - t_r)}{\sin \theta}$$



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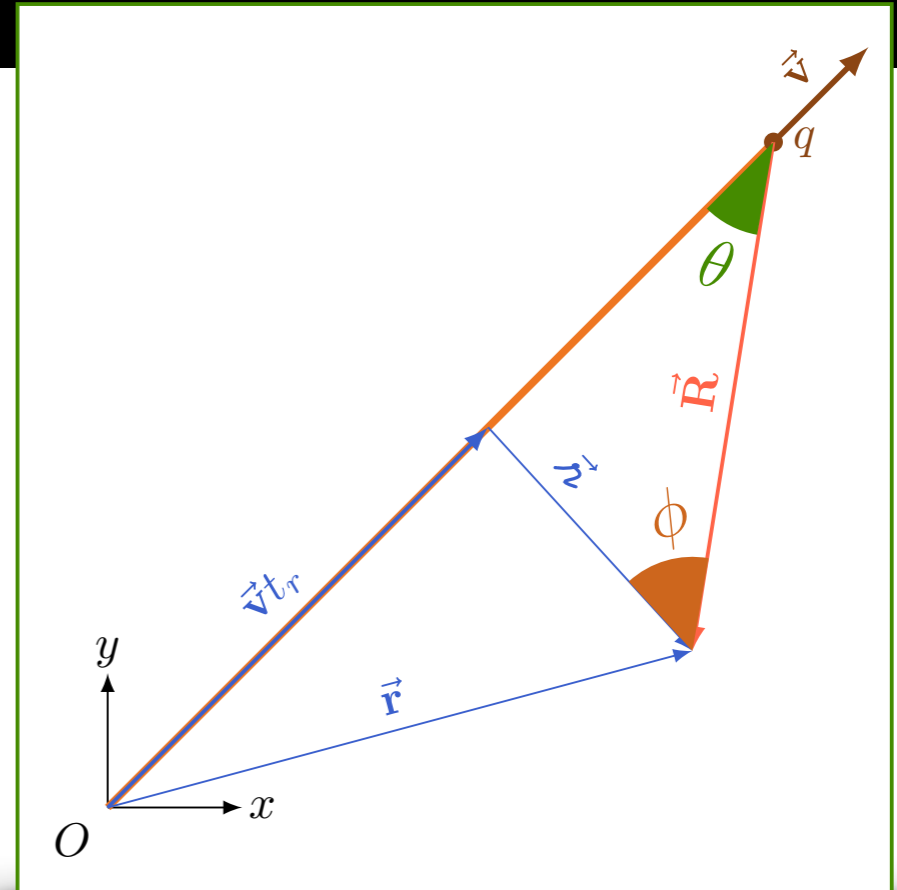
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$$v \sin \theta = c \sin \phi$$

$$\cos \phi = \hat{R} \cdot \hat{n}$$



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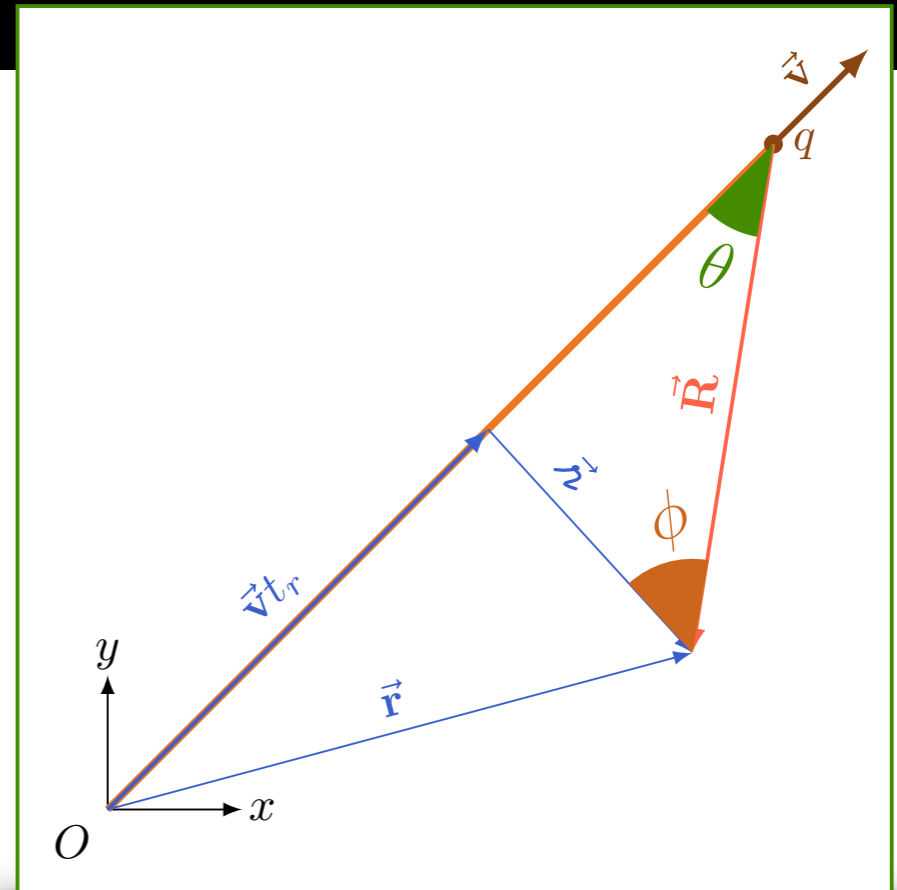
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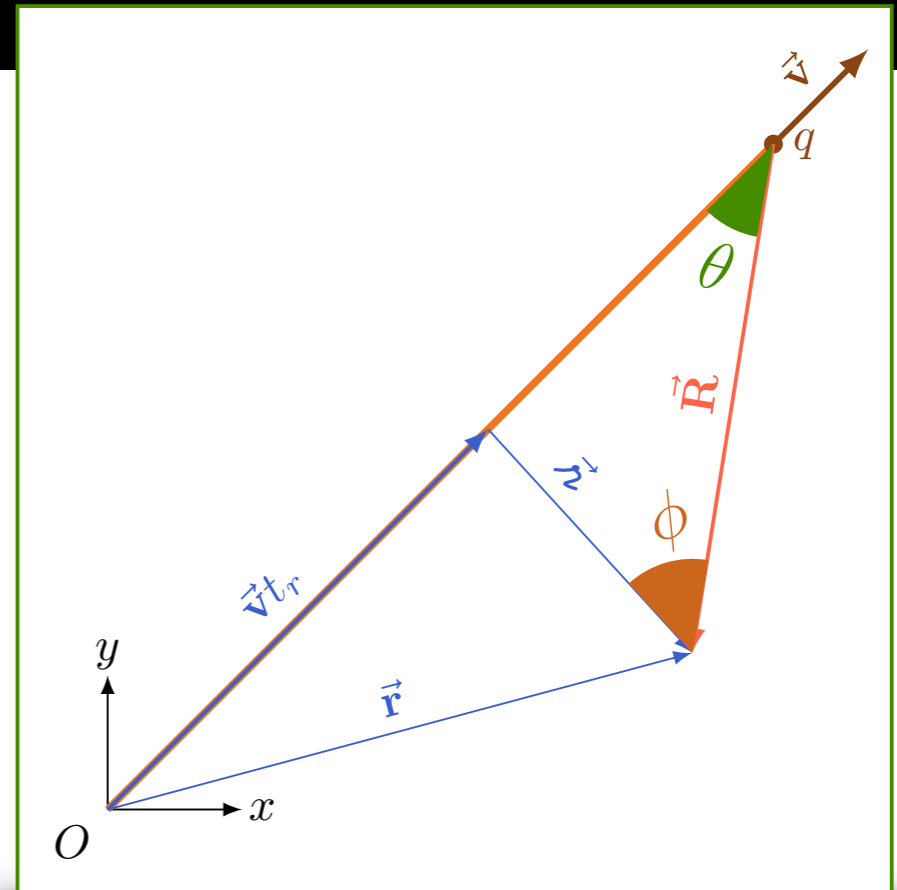
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Pratique o que aprendeu

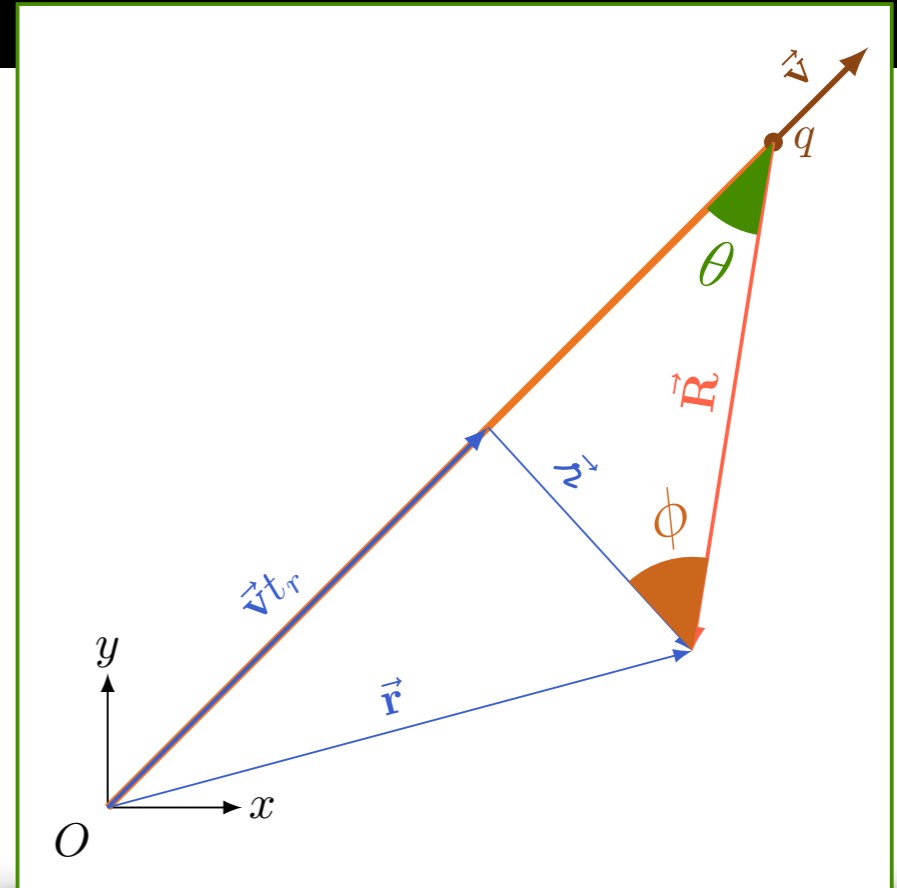
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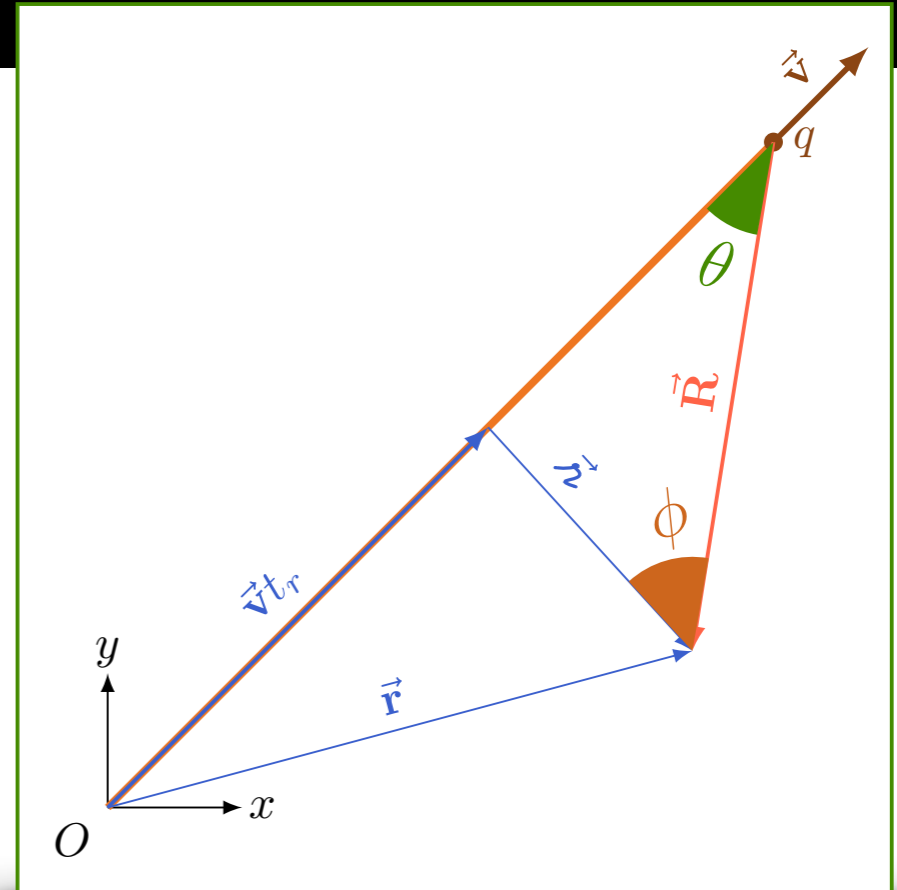
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$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{r}{(ru)^3 \left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} (c^2 - v^2)\vec{u}$$



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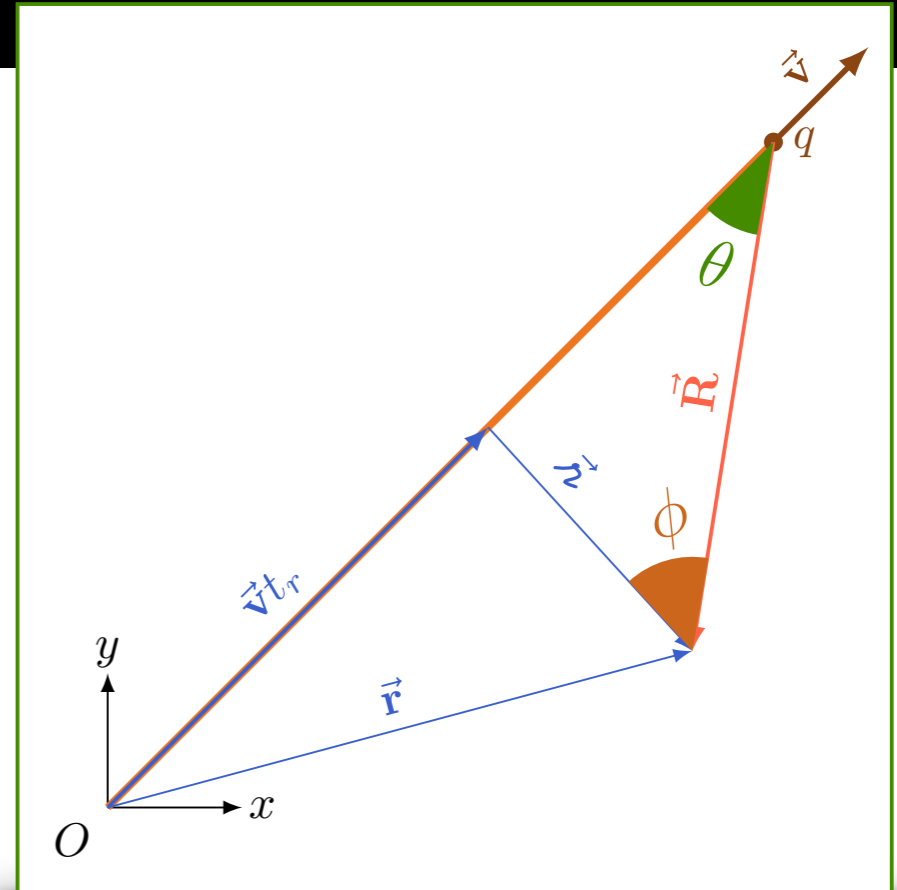
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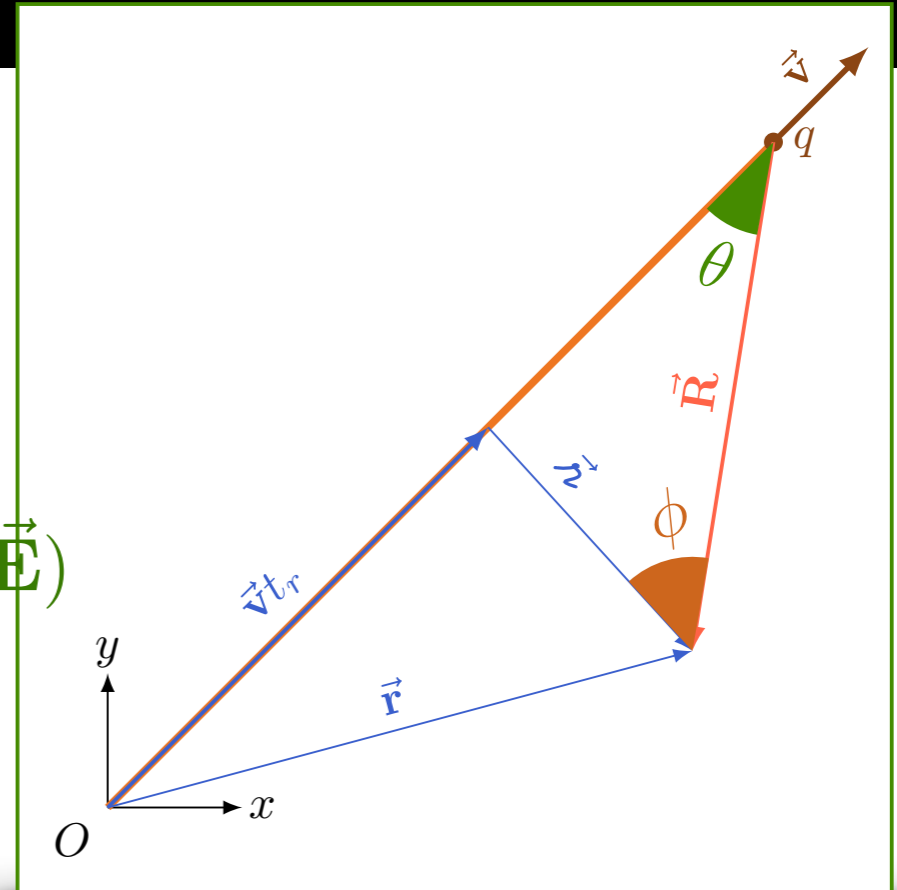
$$\vec{B}(\vec{r}, t) = \frac{1}{c} (\hat{n} \times \vec{E})$$

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$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$

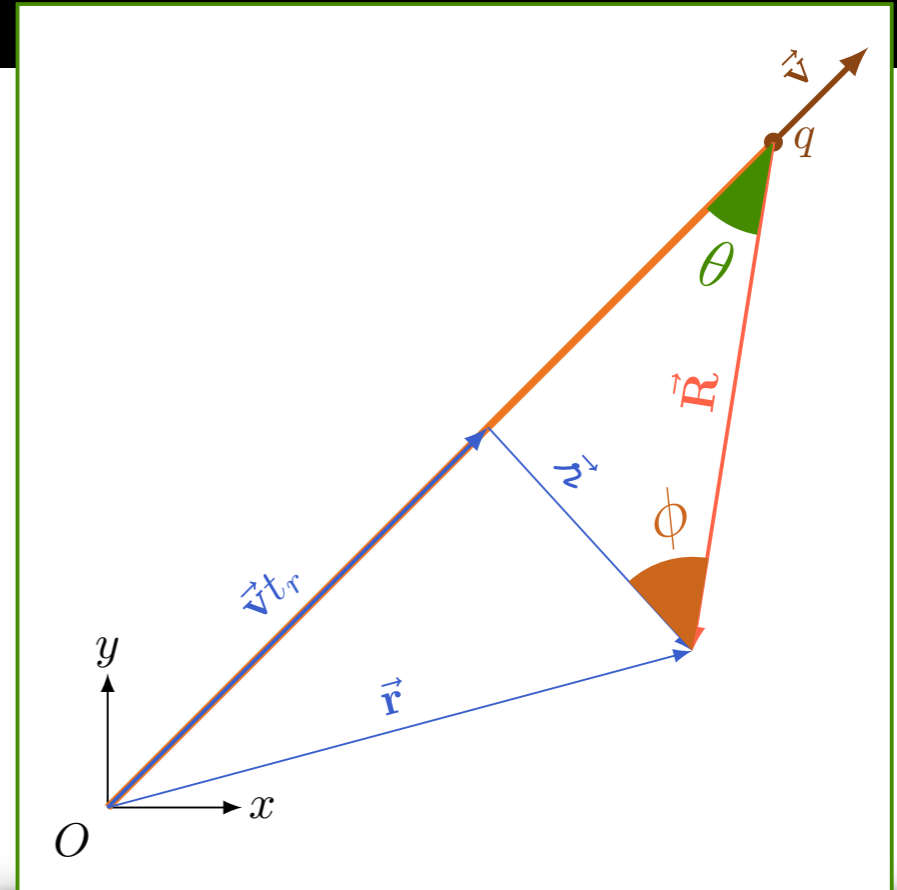


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$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - \left(\frac{v}{c}\right)^2}{\left(1 - \left(\frac{v}{c}\right)^2 \sin^2 \theta\right)^{3/2}} \frac{\hat{R}}{R^2}$$



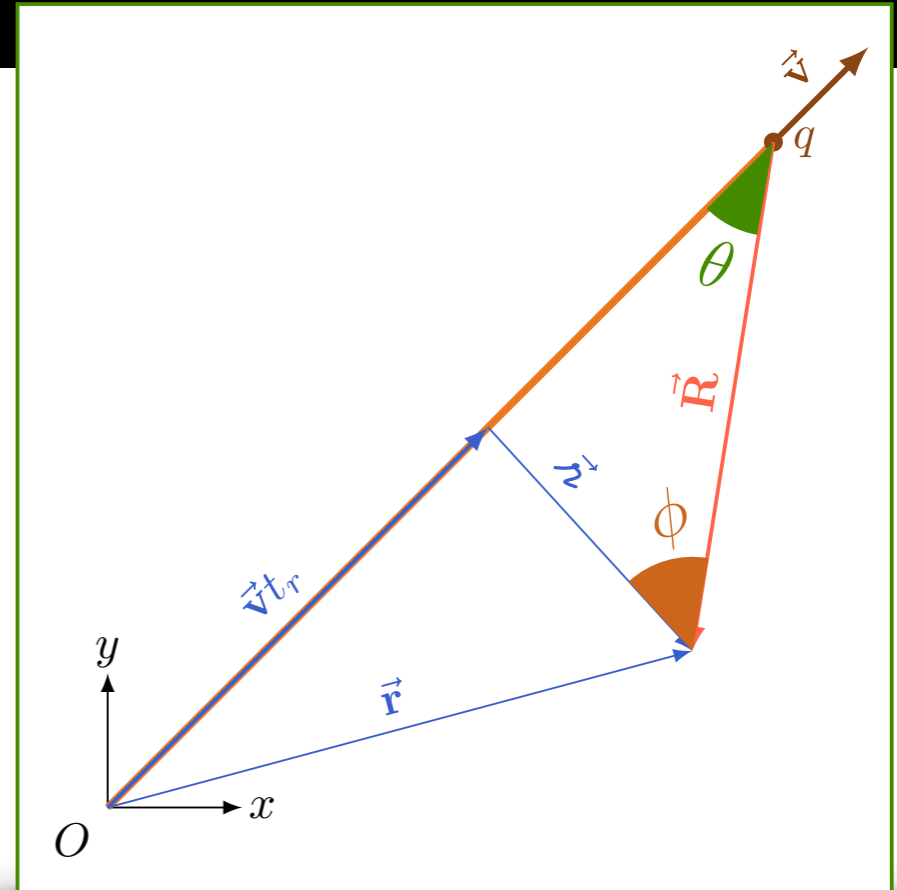
Pratique o que aprendeu

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