Eletromagnetismo Avançado

2º ciclo
Aula de 20 outubro

Potencial e potencial vetor

EQS. DE MAXWELL HOMOGÊNEAS,
ISTO É, QUE INDEPENDEM DE
ρ OU $\vec{J}$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0$$

Potencial e potencial vetor

$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

PERMITEM ← ESCREVER

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$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

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$$\nabla^2 V + \frac{\partial}{\partial t}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}}\right) = -\frac{\rho}{\epsilon_0}$$

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$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} + \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

Potencial e potencial vetor

$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\nabla^2 V + \frac{\partial}{\partial t}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}}\right) = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \vec{\mathbf{A}} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} - \vec{\boldsymbol{\nabla}}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \vec{\mathbf{J}}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}} = 0$$

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$$\nabla^2 V + \frac{\partial}{\partial t}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}}\right) = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \vec{\mathbf{A}} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} - \vec{\boldsymbol{\nabla}}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \vec{\mathbf{J}}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{A}} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t} = 0$$

Gauge de Lorenz

Potencial e potencial vetor

$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

DEFINIÇÃO

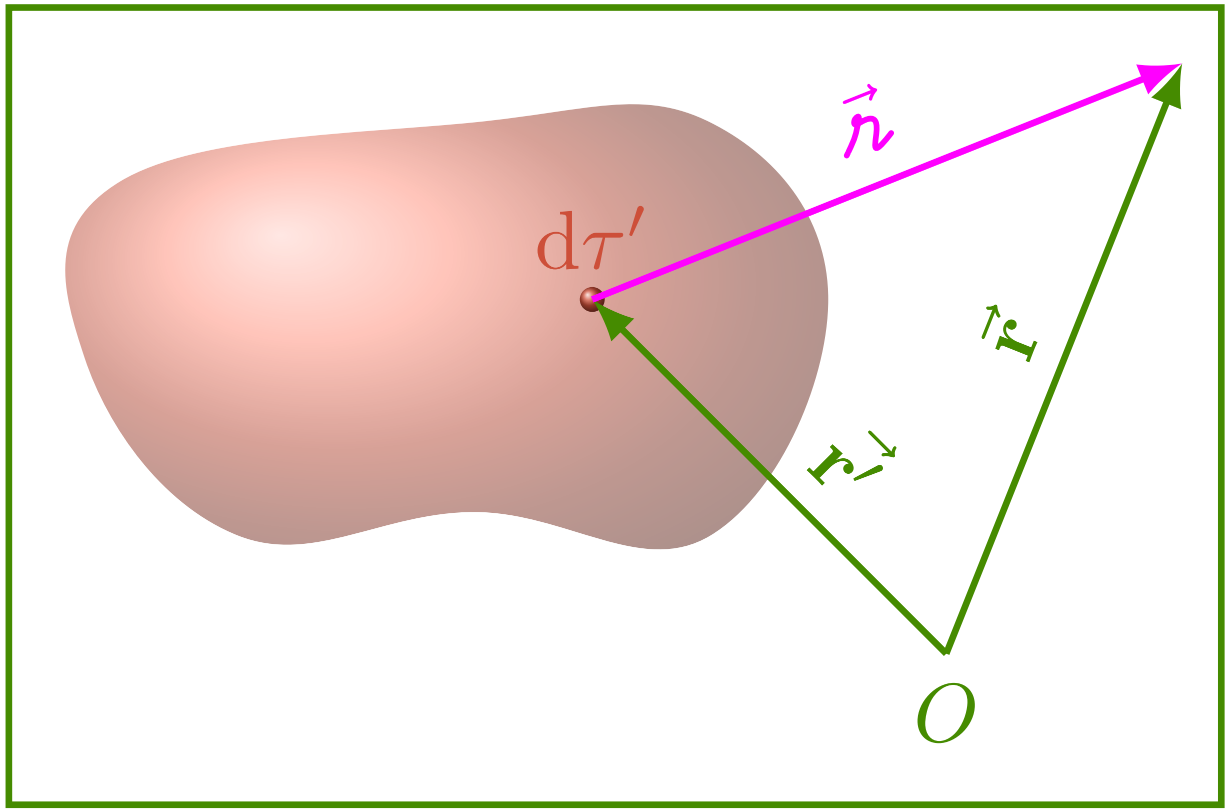
$$\Box^2 \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} = -\mu_0 \vec{\mathbf{J}}$$

$$\Box^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

Potencial e potencial vetor

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

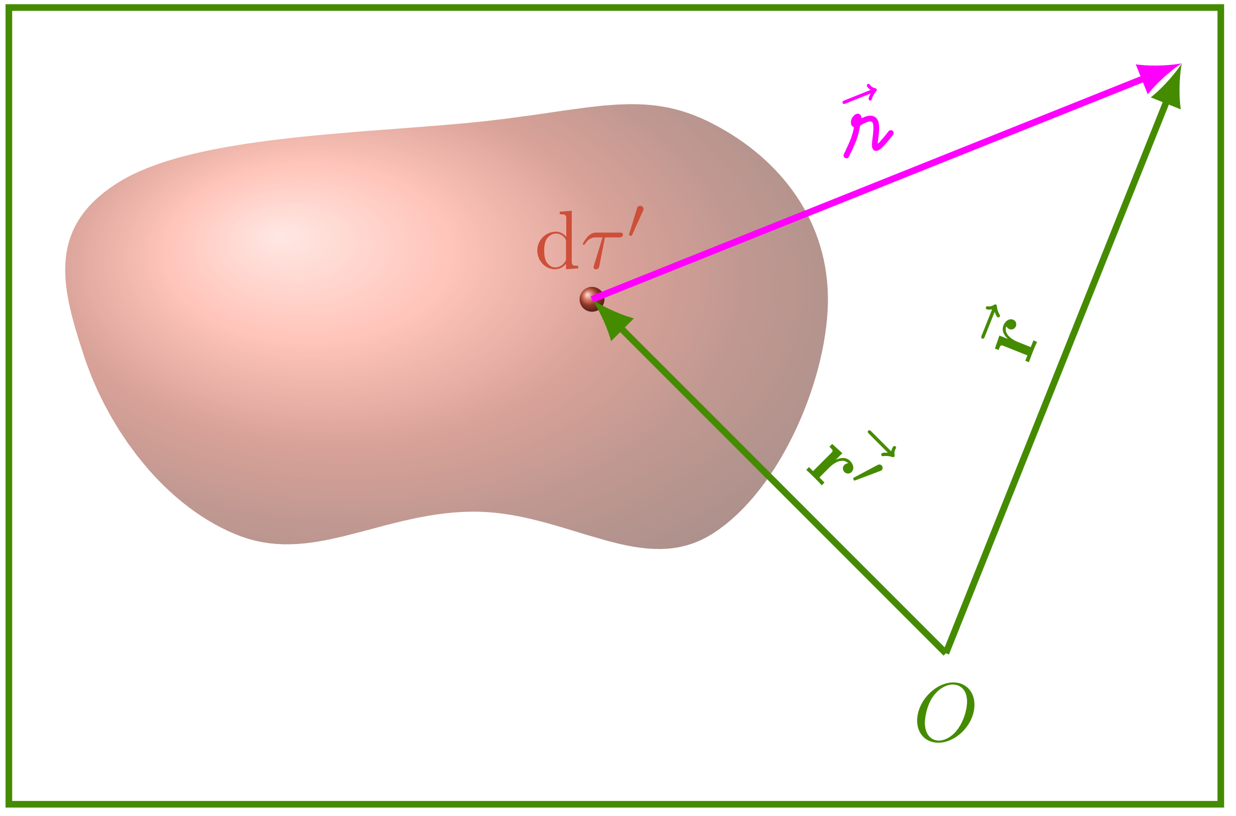


Potencial e potencial vetor

$\rho = \rho(\vec{\mathbf{r}}) \Rightarrow$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

INDEPENDENTE DE t

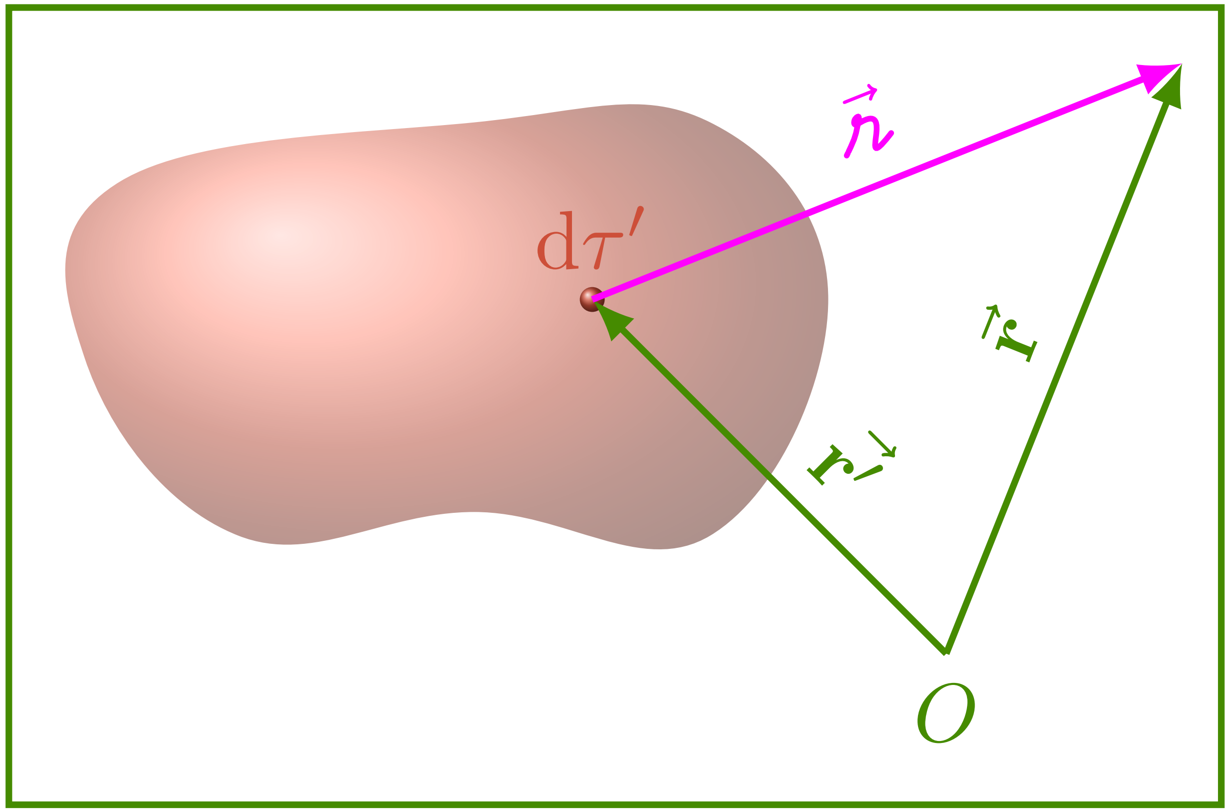


Potencial e potencial vetor

$$\rho = \rho(\vec{\mathbf{r}}) \Rightarrow \quad \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

SOLUÇÃO

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}'})}{\mathscr{r}}\,\mathrm{d}\tau'$$



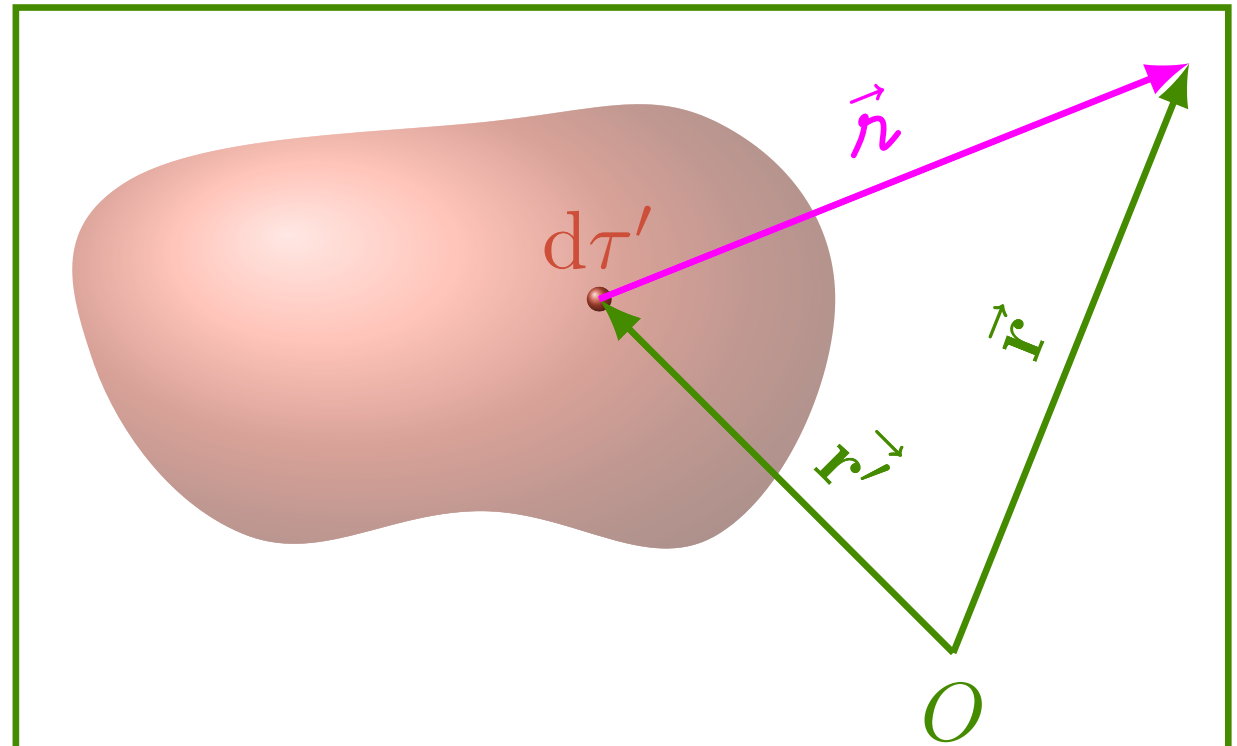
Potencial e potencial vetor

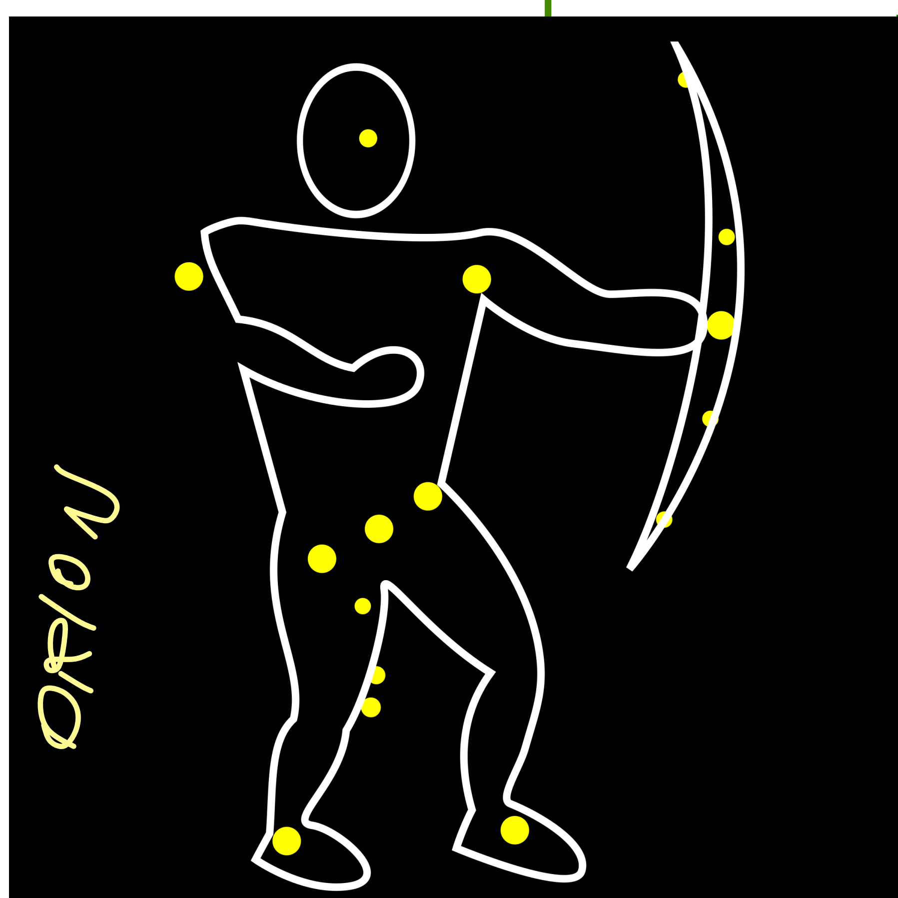
$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

SOLUÇÃO

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$t_r \equiv t - \frac{\mathscr{r}}{c}$$



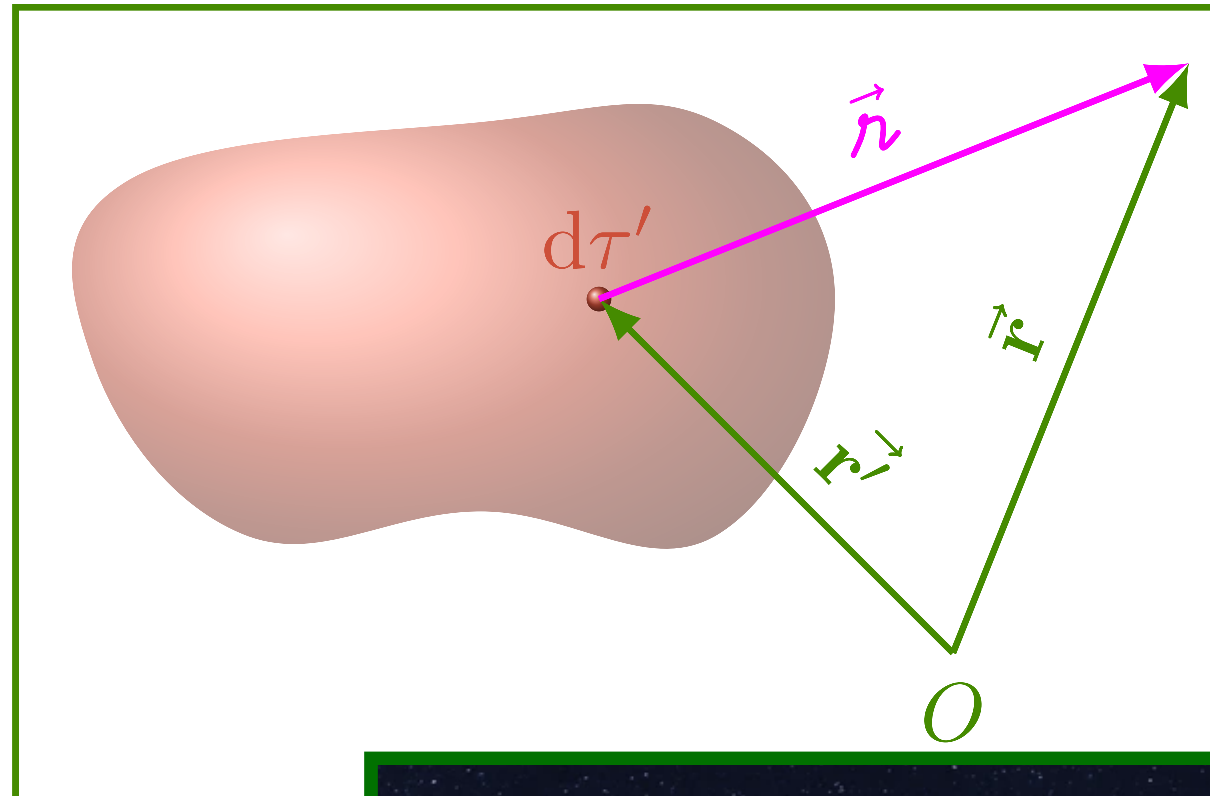


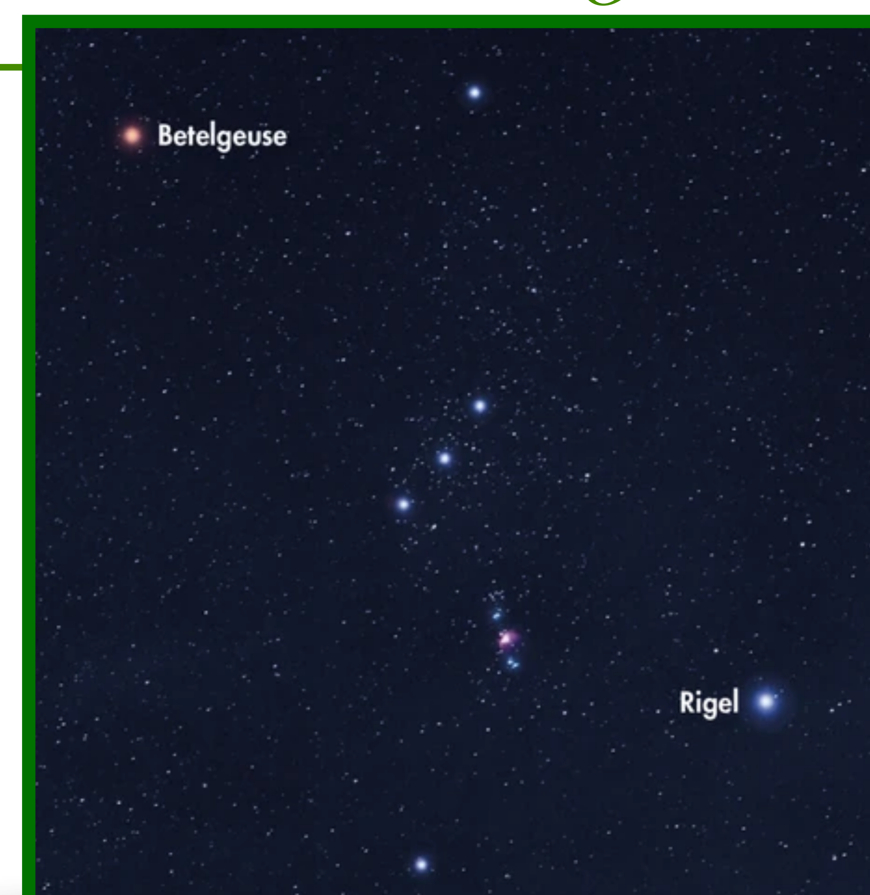
Potencial e potencial vetor

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, d\tau'$$

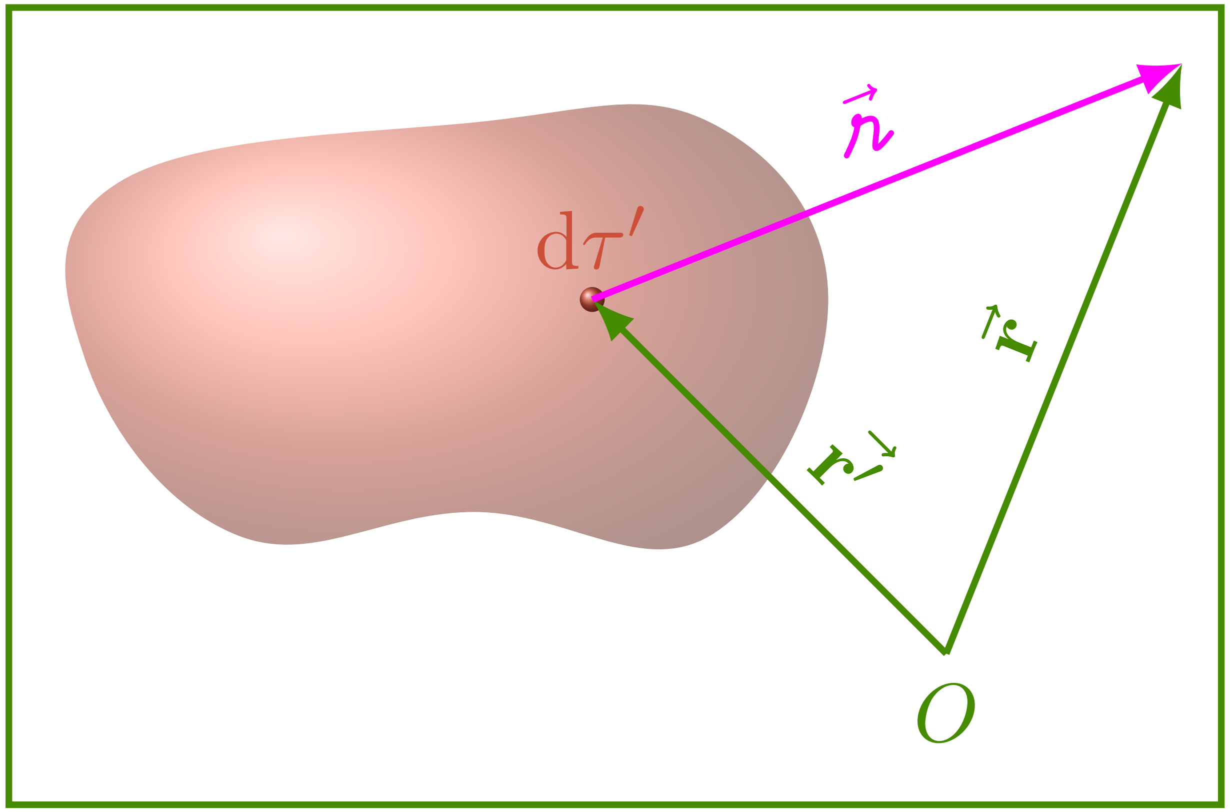
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$





Potencial e potencial vetor

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$



$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}}\, d\tau'$$

$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}}\, d\tau'$$

$$\Box^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, d\tau'$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, d\tau'$$

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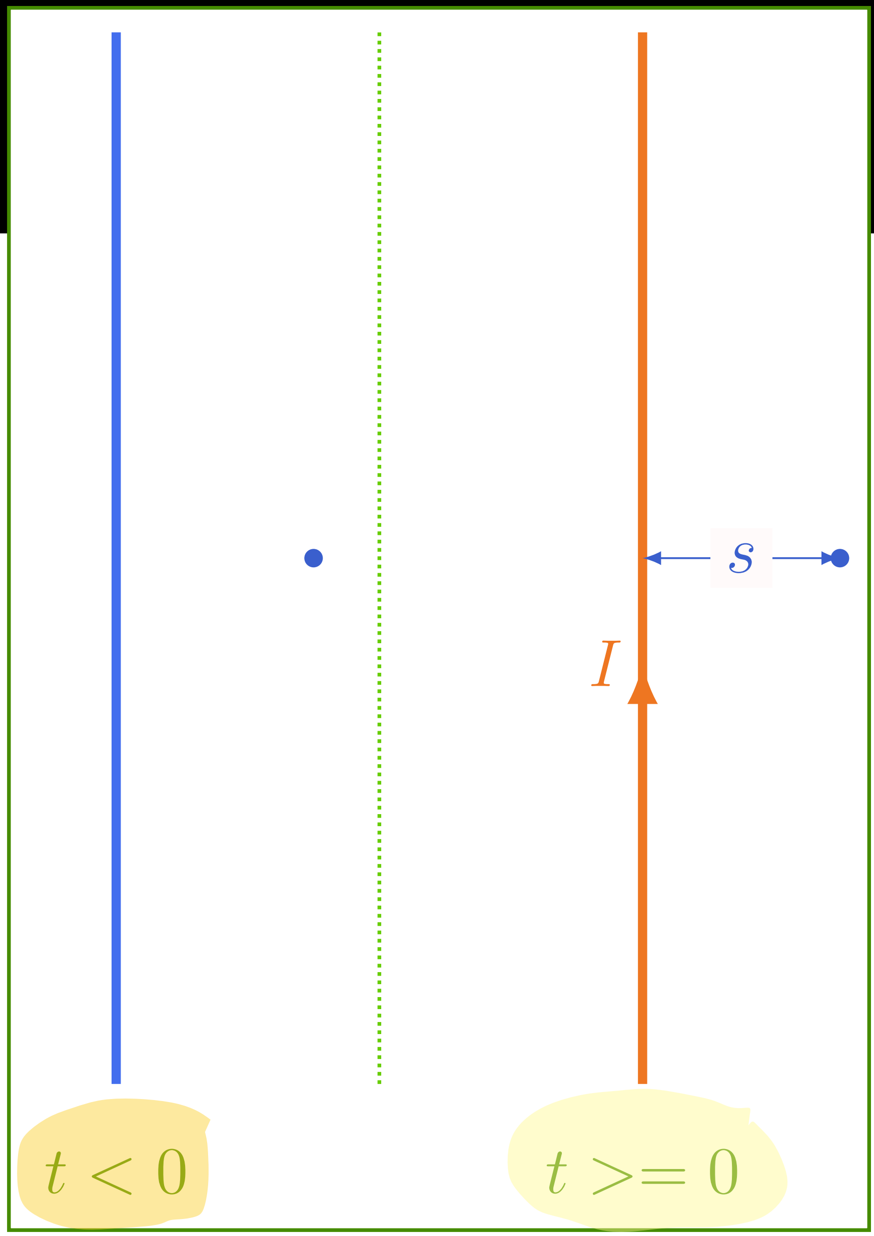

Pratique o que aprendeu

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathcal{r}} \, d\tau'$$

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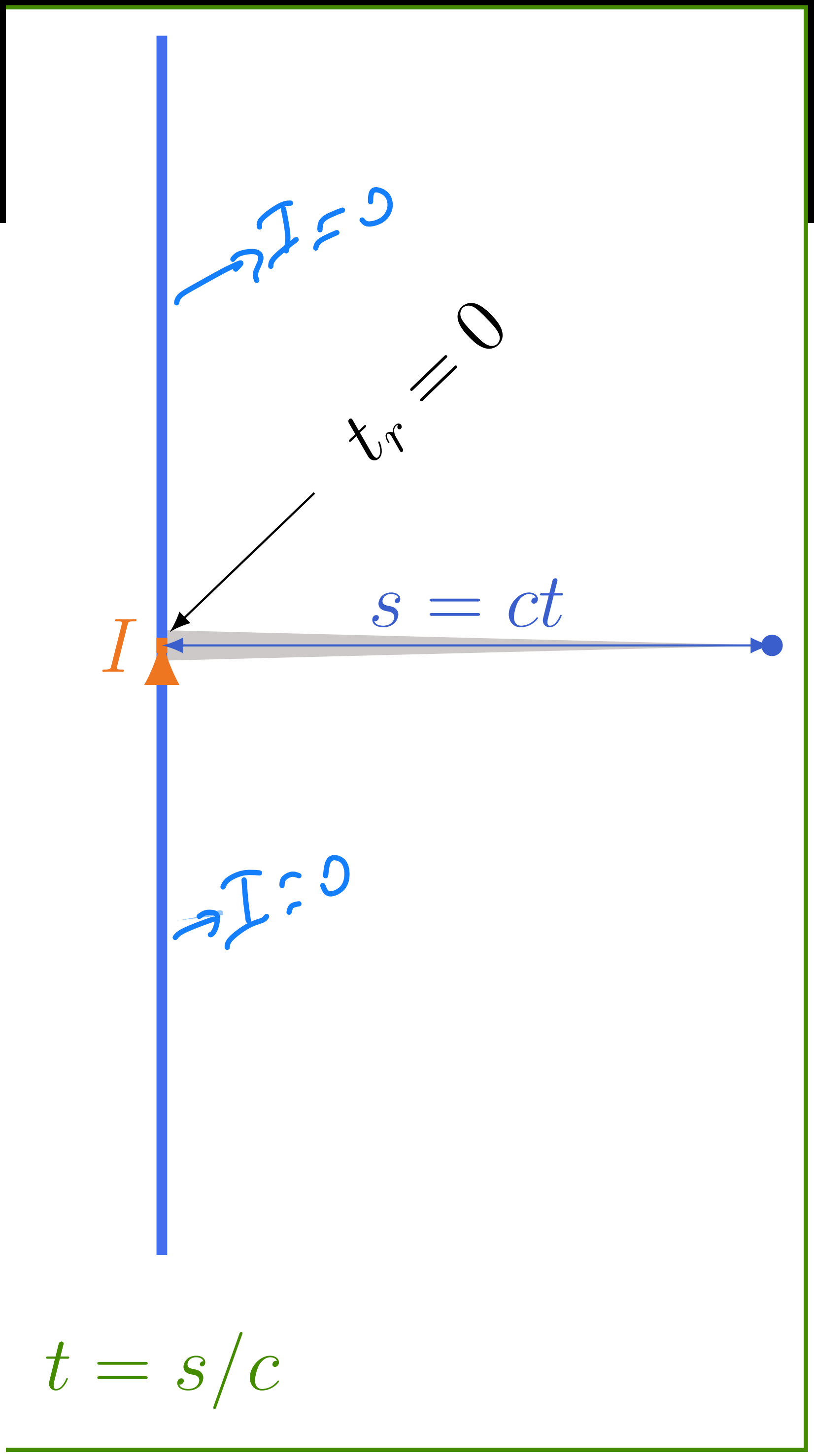

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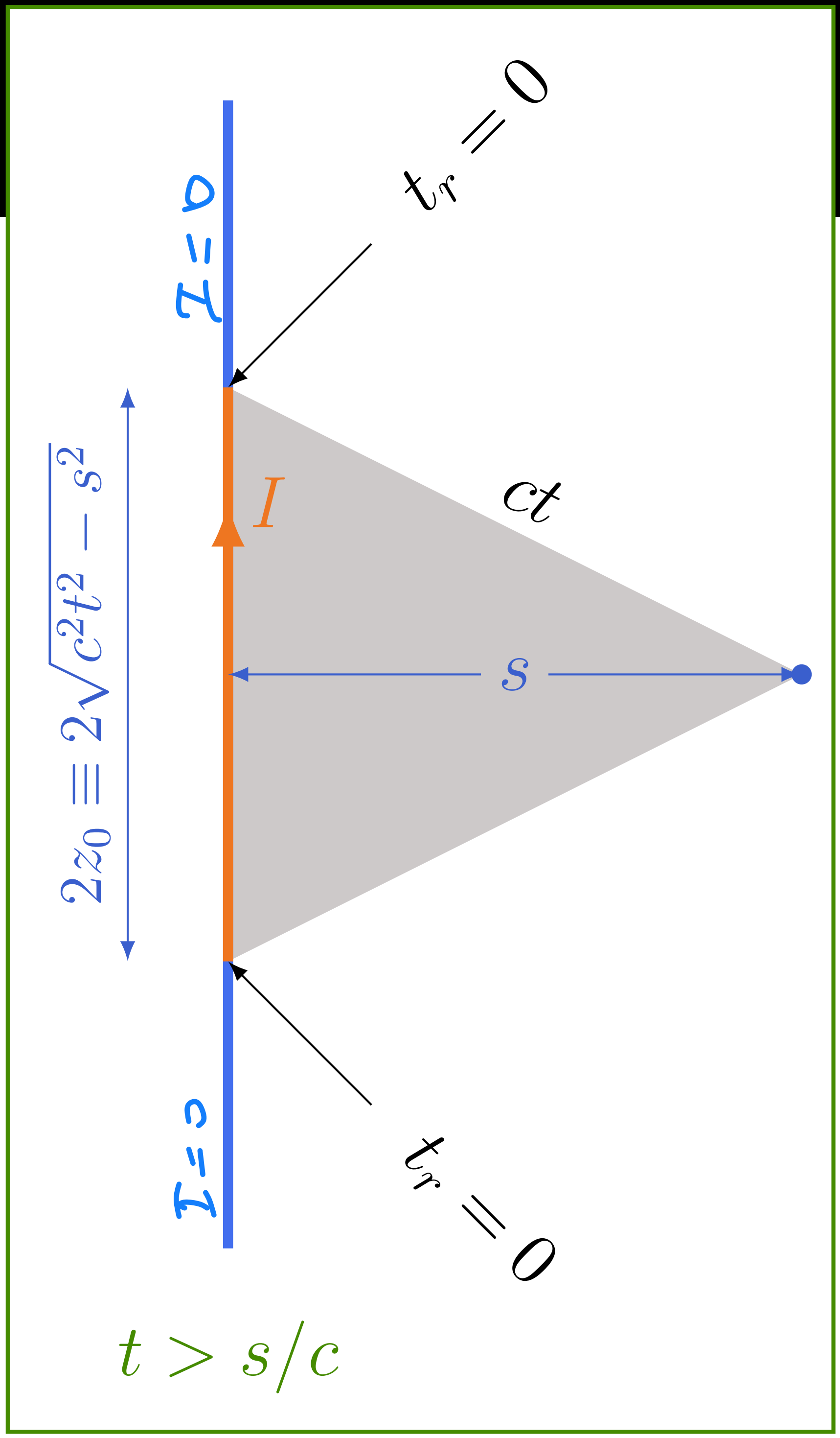
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{s}}, t) = \frac{\mu_0}{4\pi} \int_{-z_0}^{z_0} \frac{I}{\sqrt{z^2 + s^2}} \, dz \, \hat{\mathbf{z}}$$



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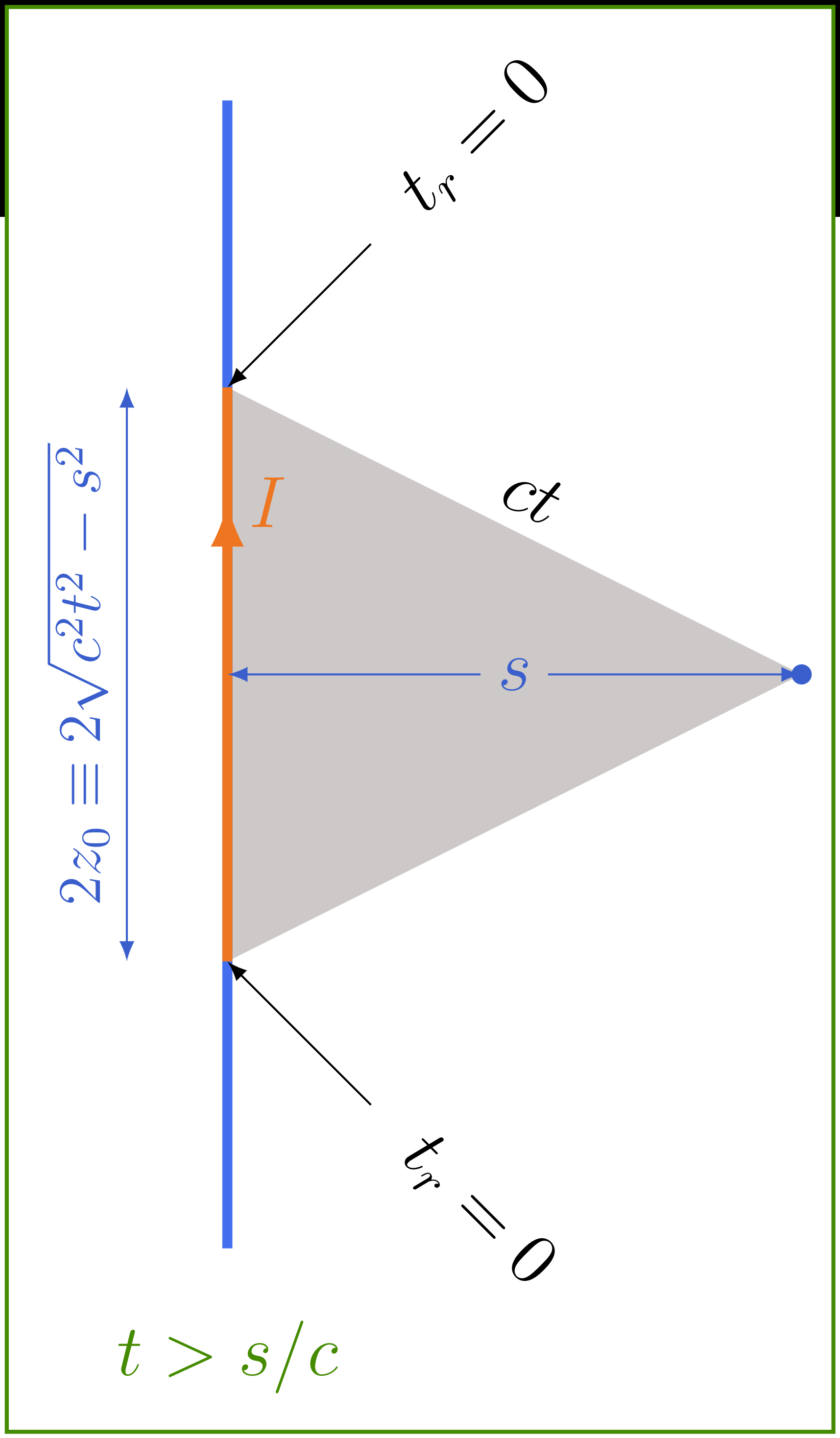
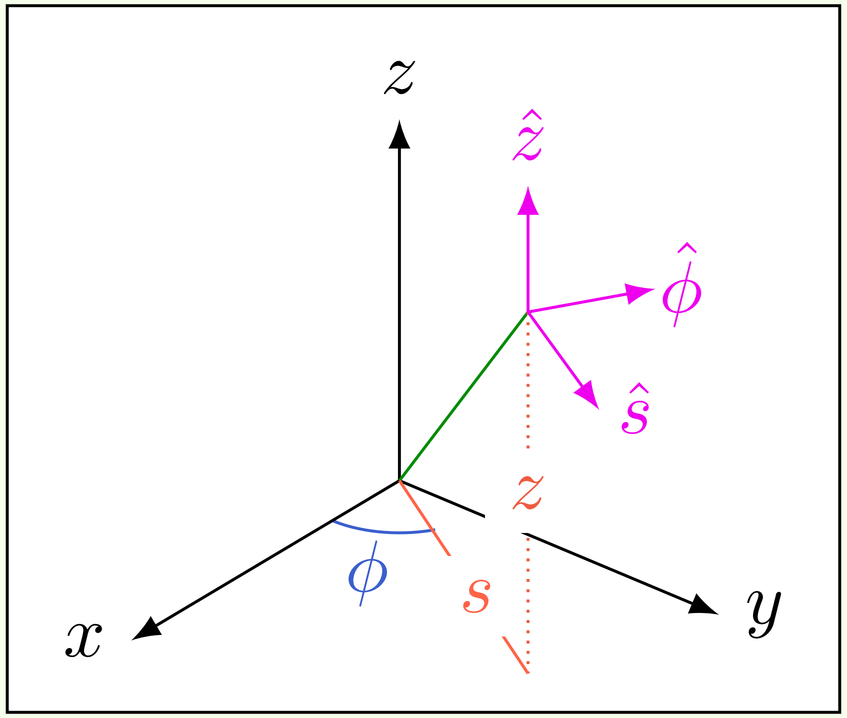
$\hookrightarrow z = s \, \mathrm{senh}\, u$

$$\vec{\mathbf{A}}(\vec{\mathbf{s}}, t) = \frac{\mu_0 I}{2\pi} \, \mathrm{asenh}\left(\frac{ct}{s}\right) \hat{\mathbf{z}}$$



Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Pratique o que aprendeu

$$\vec{\mathbf{A}}(\vec{\mathbf{s}}, t) = \frac{\mu_0 I}{2\pi}\,\mathrm{asenh}\left(\frac{ct}{s}\right)\hat{\mathbf{z}}$$

$$\vec{\mathbf{E}} = -\vec{\nabla} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I c t}{2\pi s \sqrt{(ct)^2 - s^2}}\hat{\phi}$$

$$ct \gg s \Rightarrow \vec{B} \longrightarrow \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

AMPÈRE