Eletromagnetismo Avançado

3º ciclo
Aula de 27 outubro

Potencial e potencial vetor

$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \frac{\partial \vec{\mathbf{A}}}{\partial t}$$

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

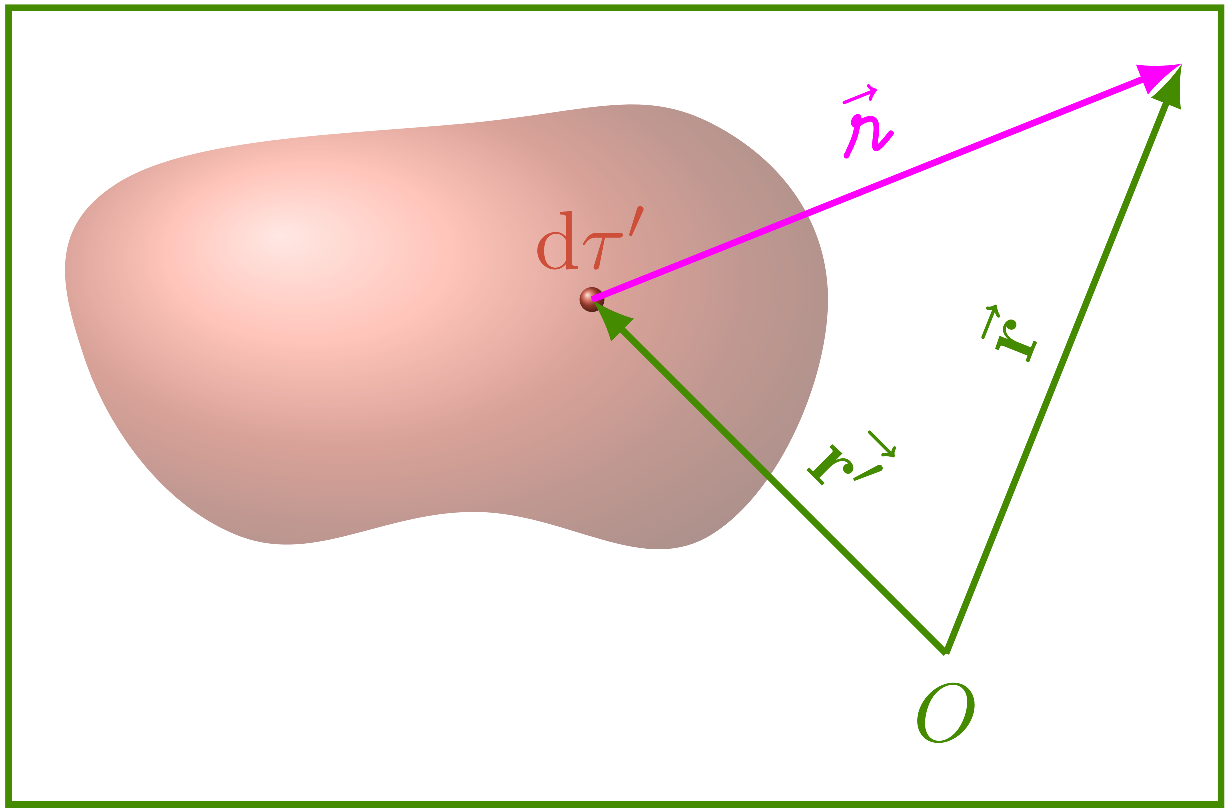
$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} = -\mu_0 \vec{\mathbf{J}}$$

$$\Box^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

Potencial e potencial vetor

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

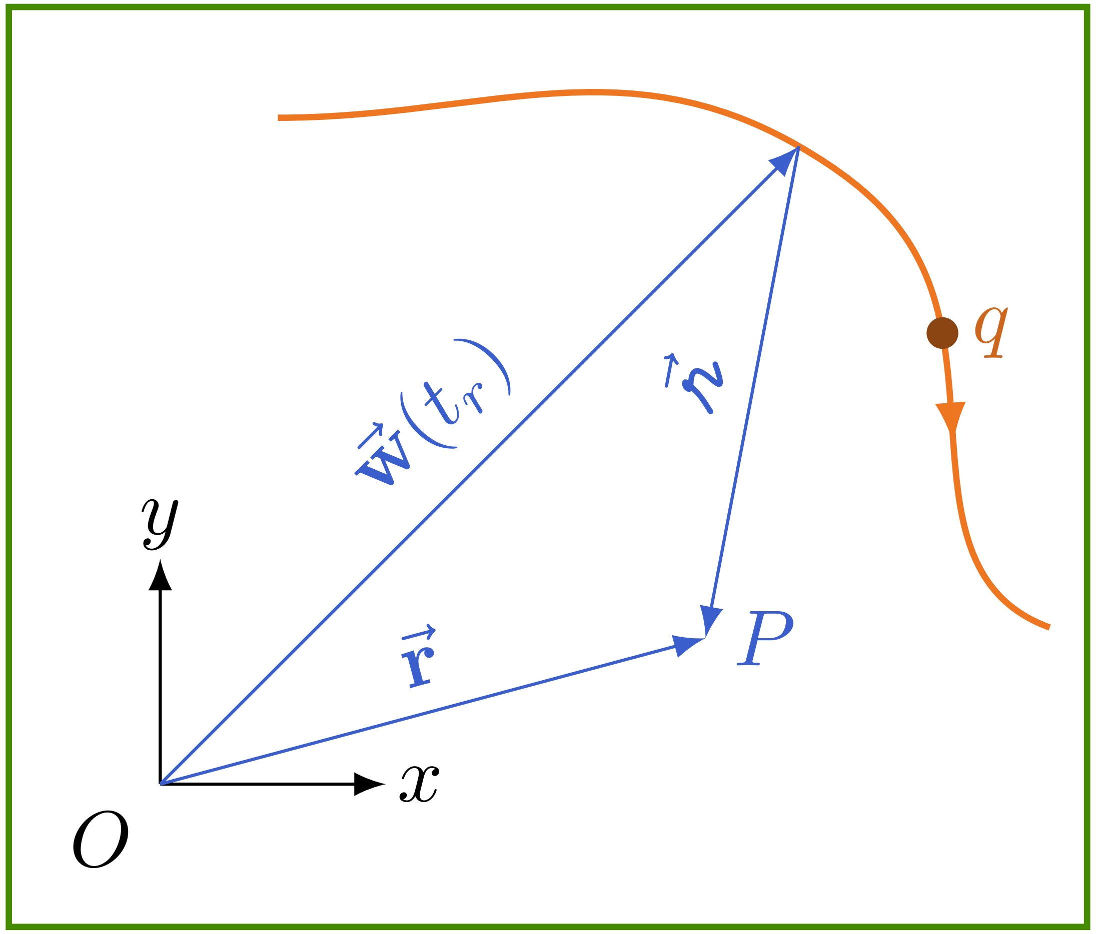


$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

$$\Box^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

Potenciais de Liénard e Wiechert



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$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}'}, t_r)}{\mathscr{r}} \, d\tau'$$

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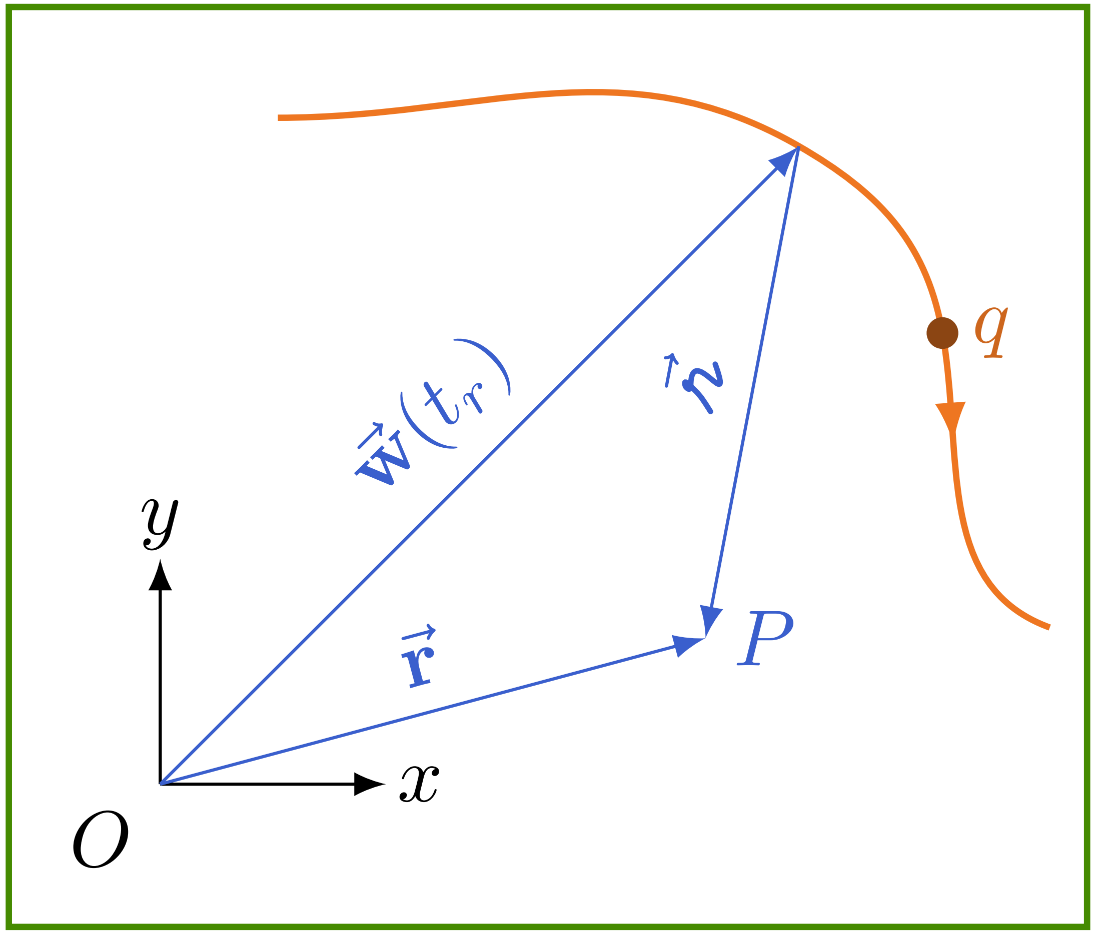

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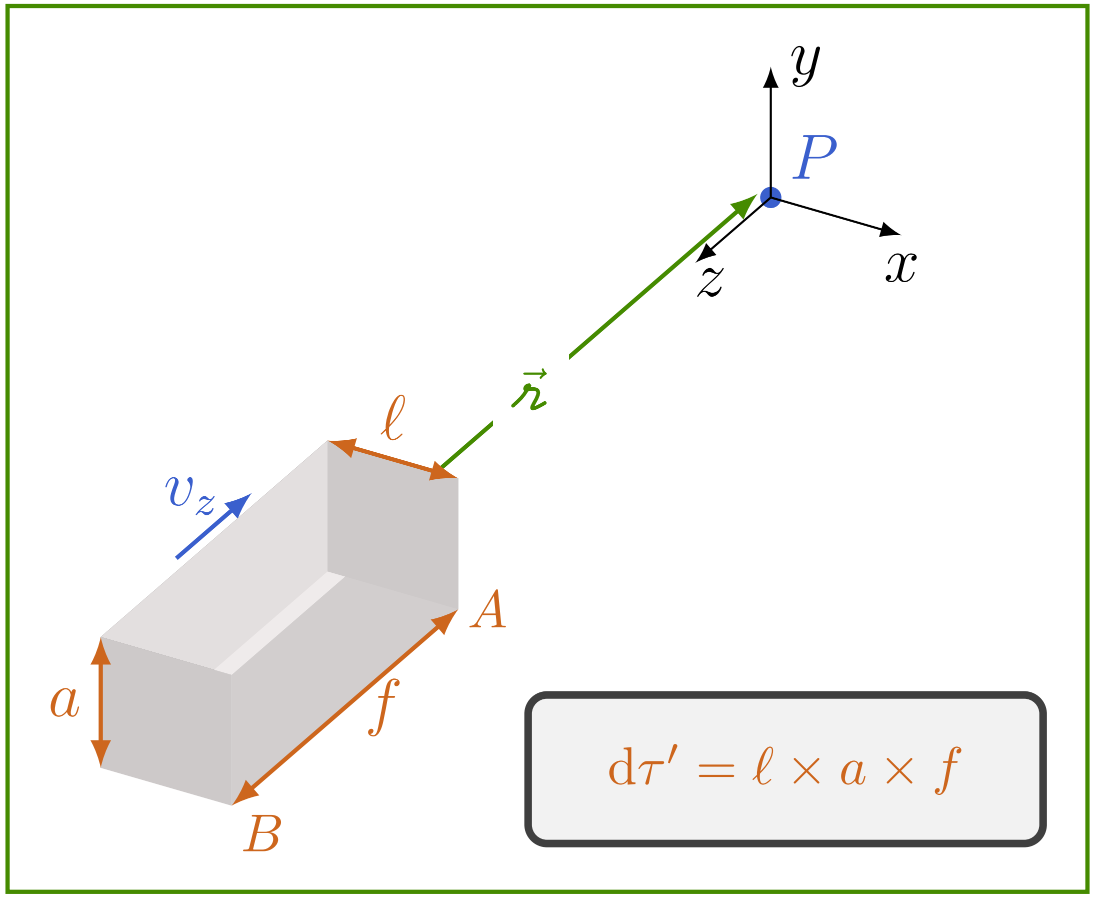

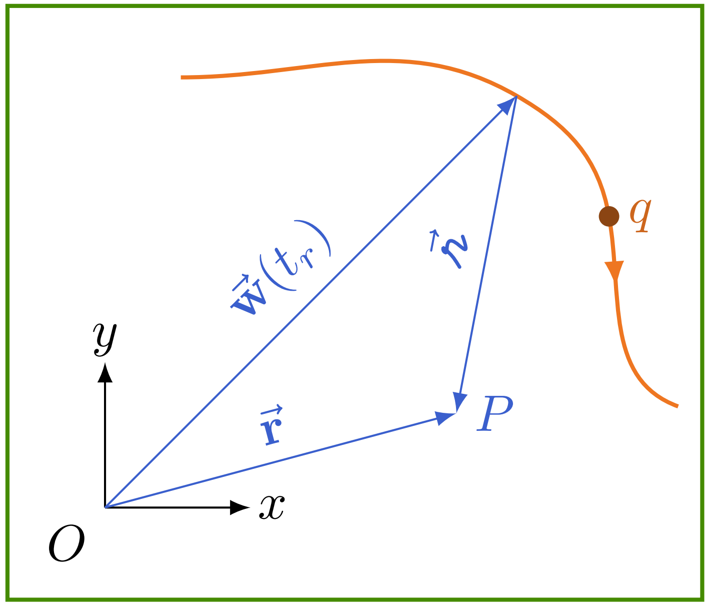

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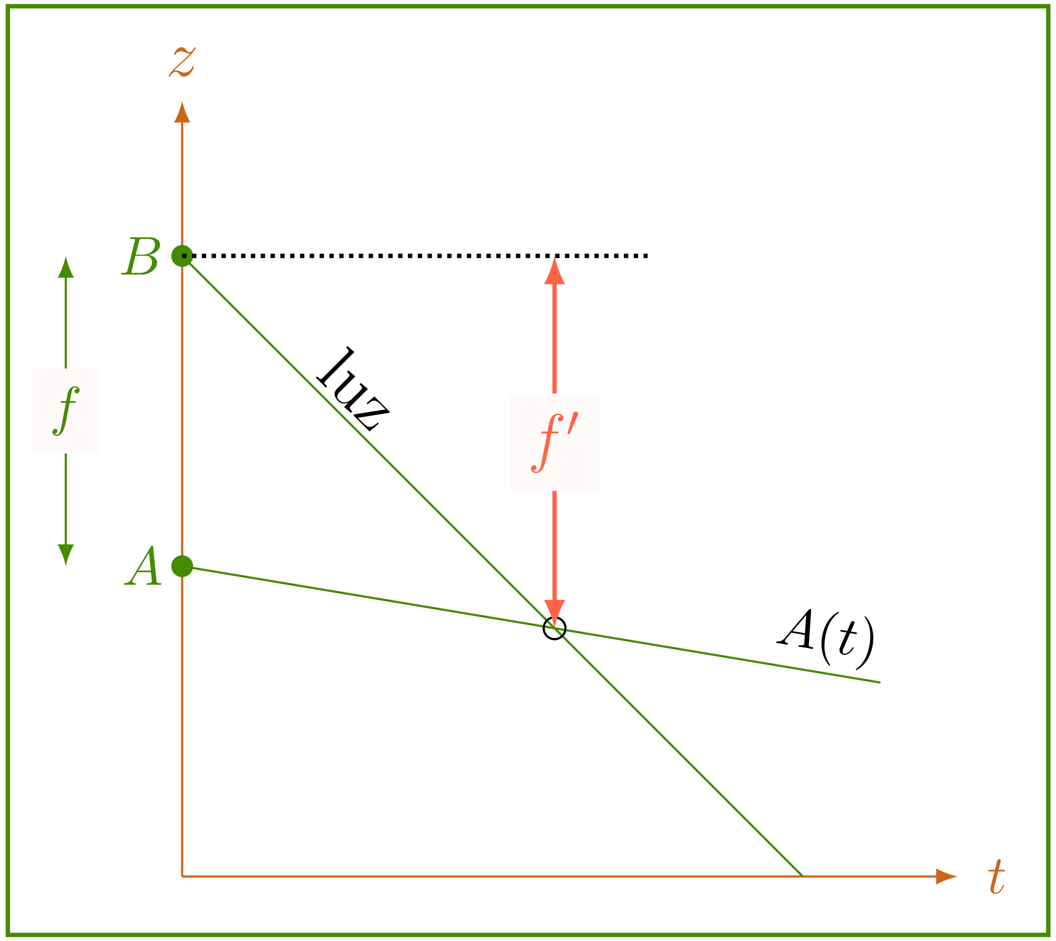

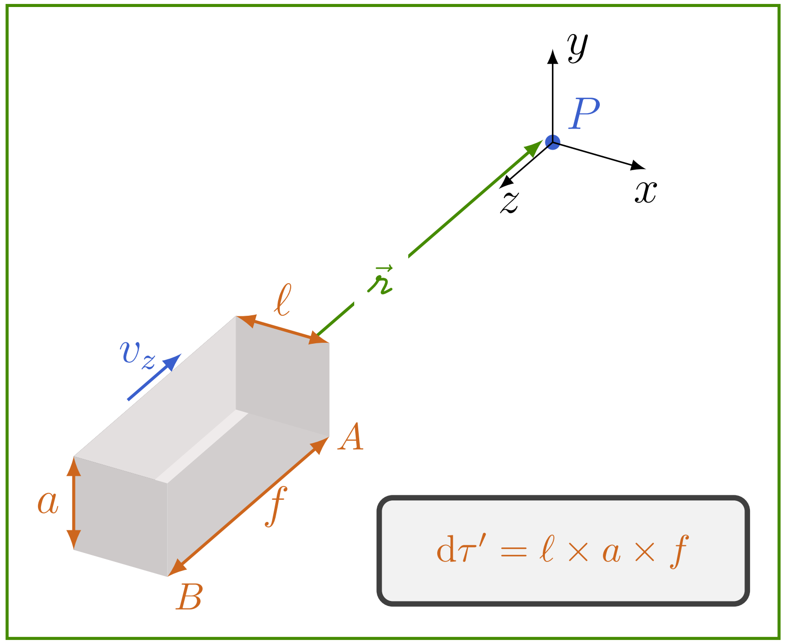

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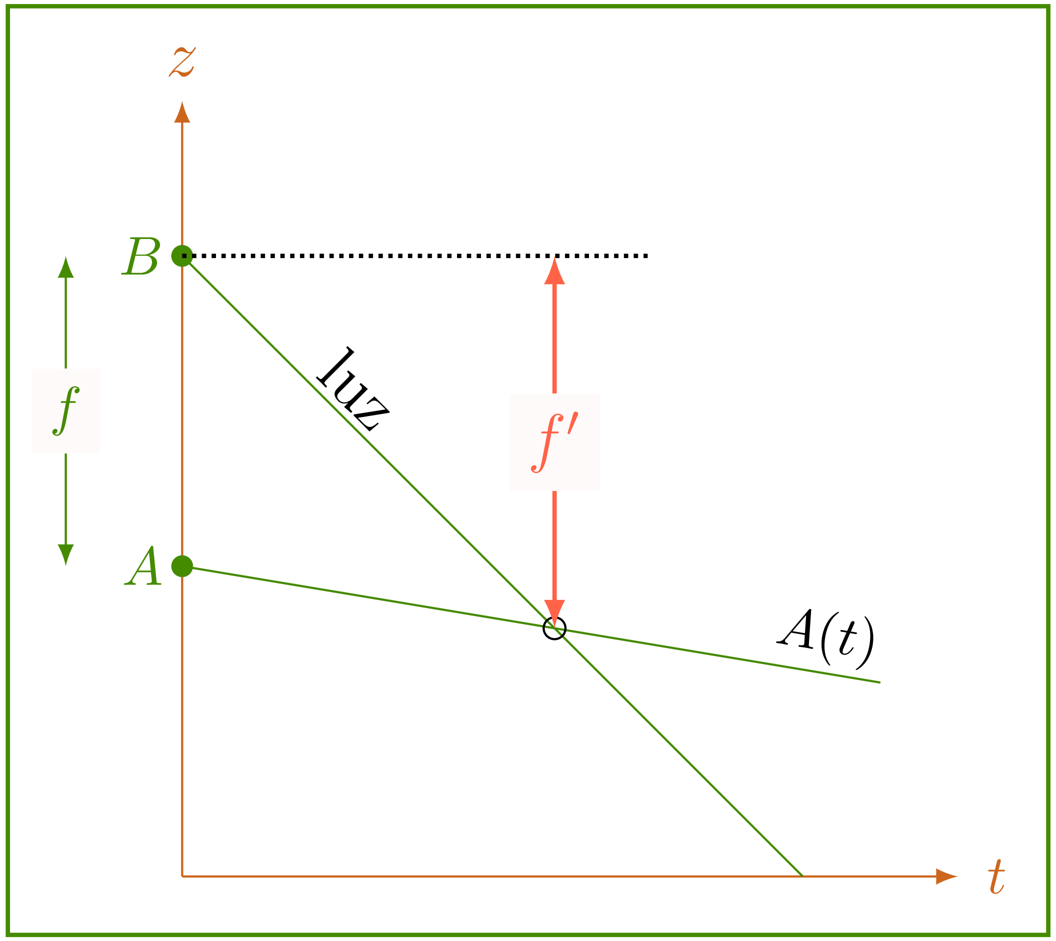

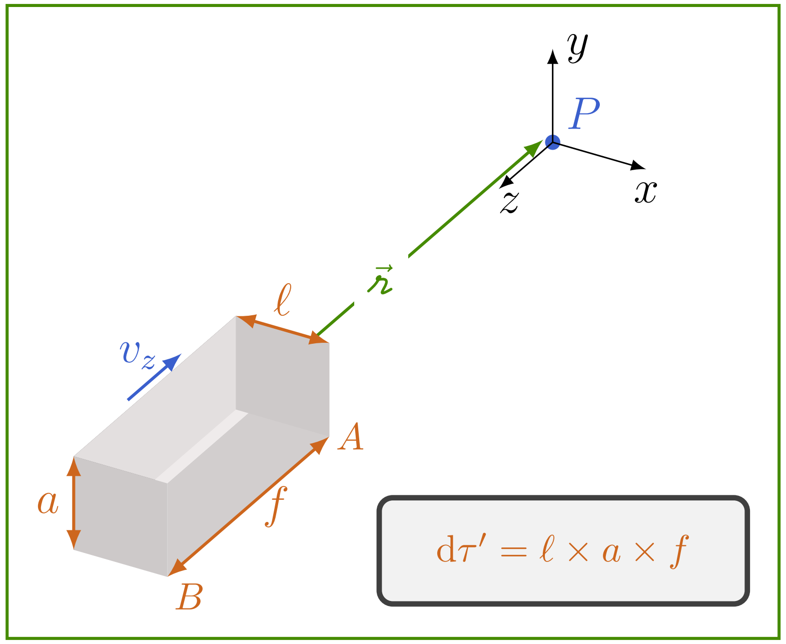

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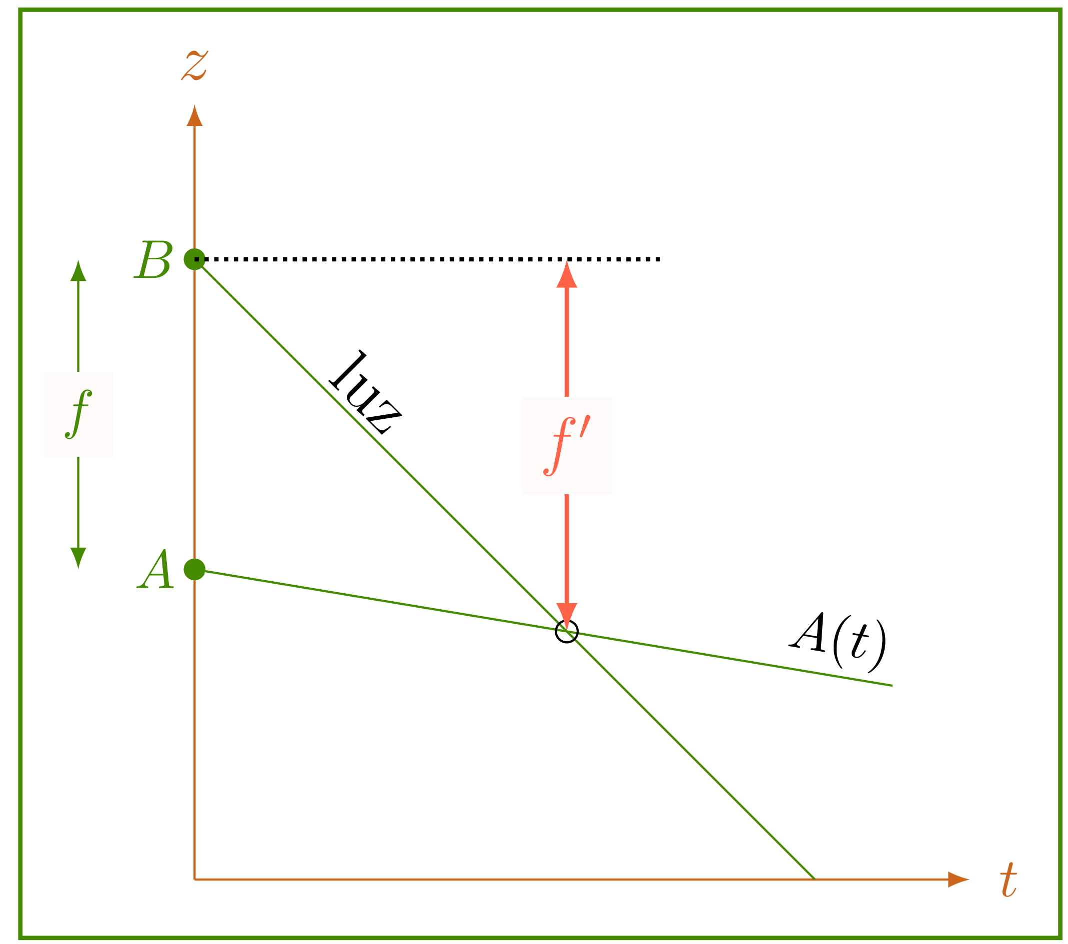

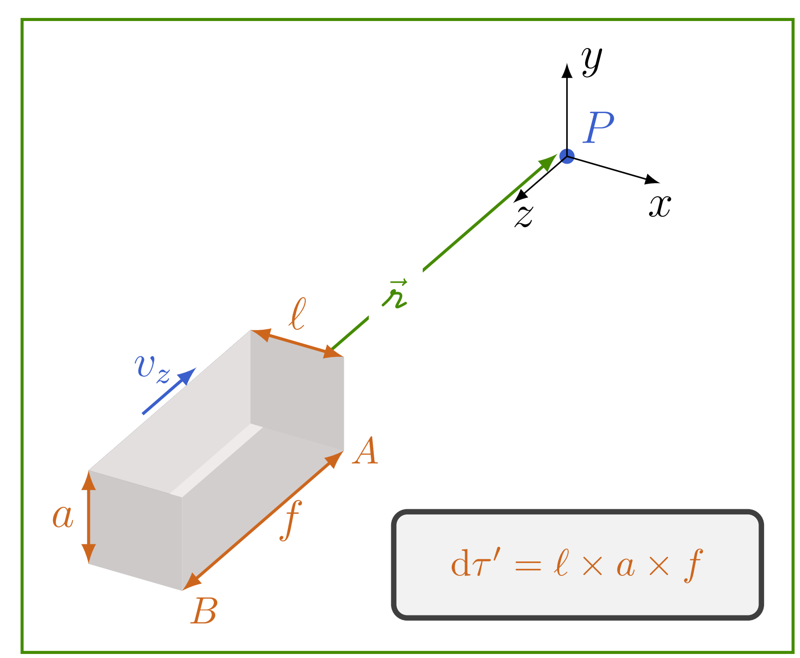

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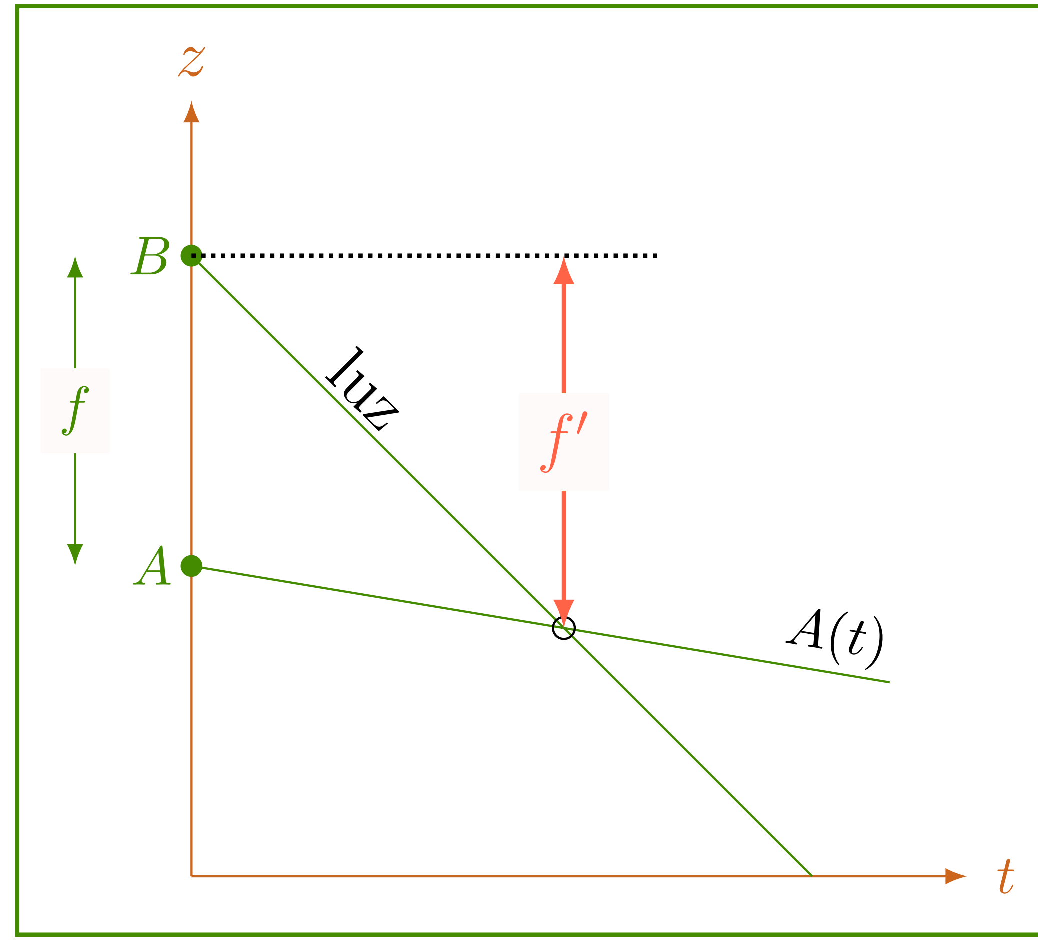

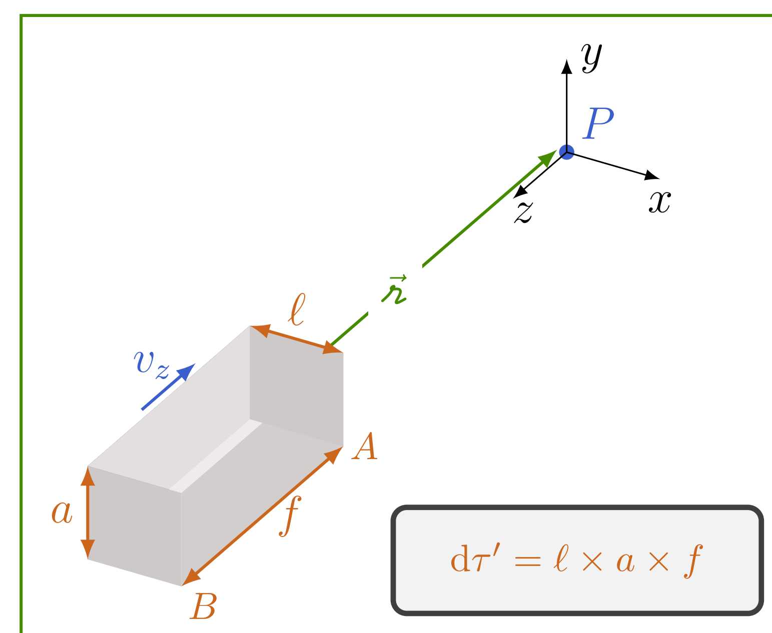

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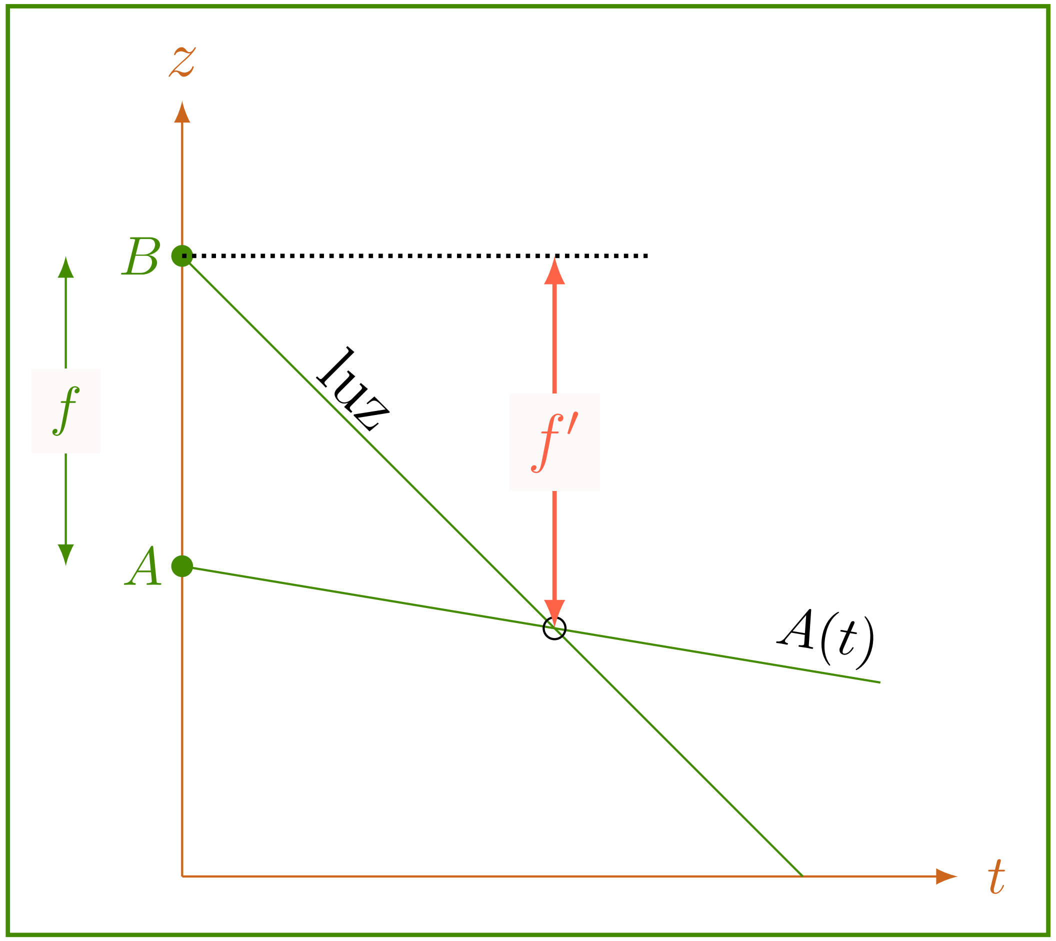

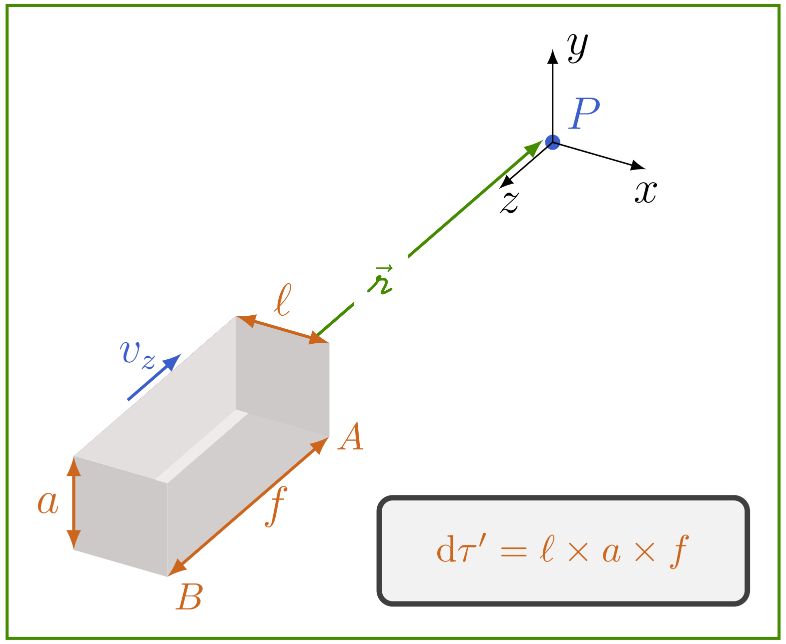

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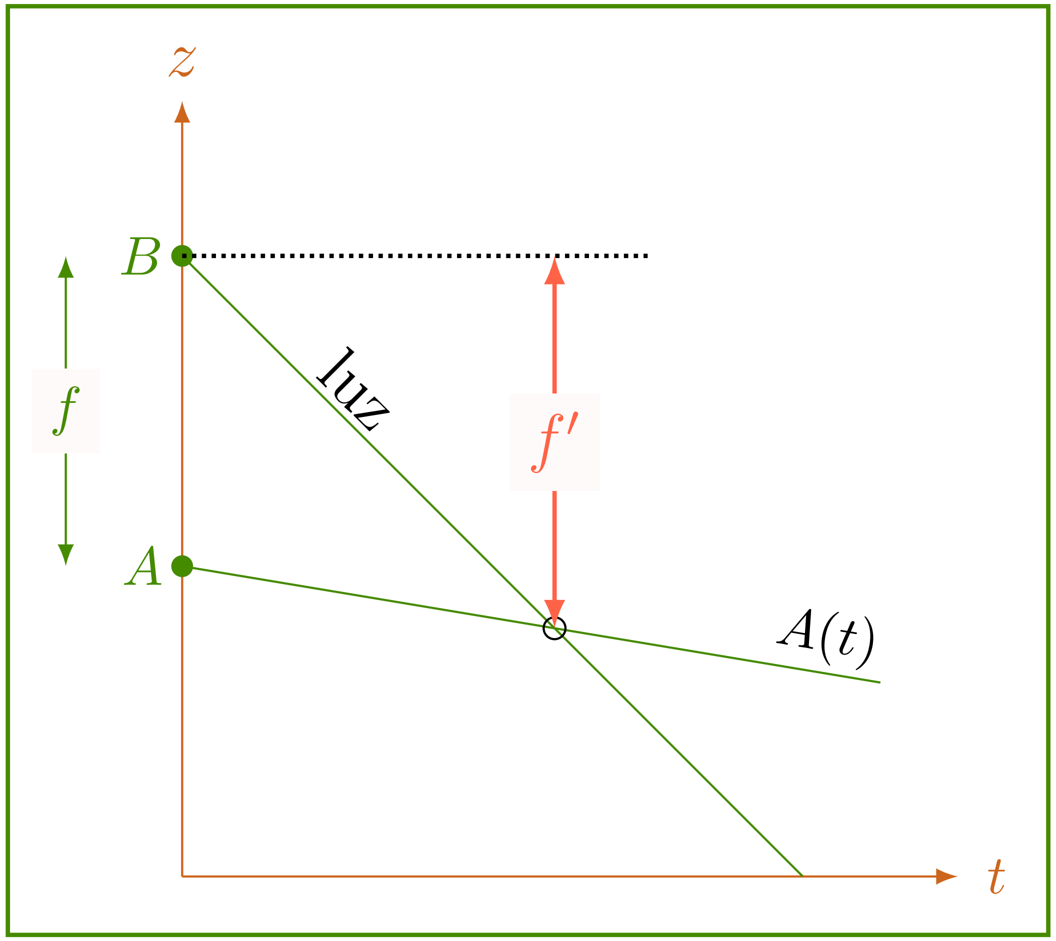

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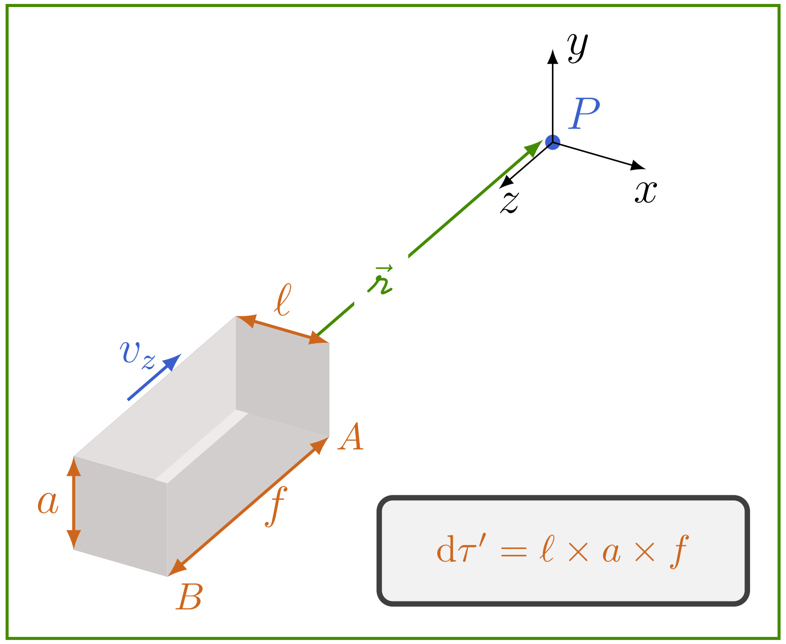

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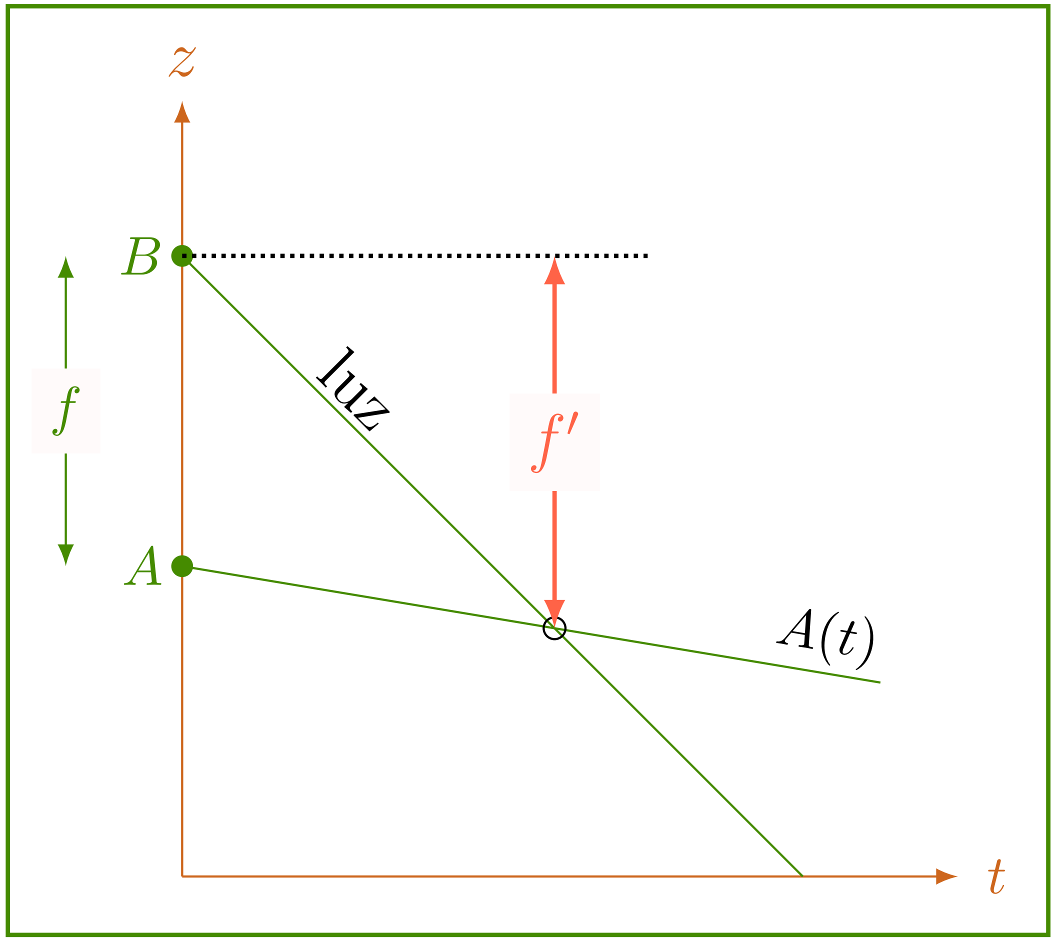

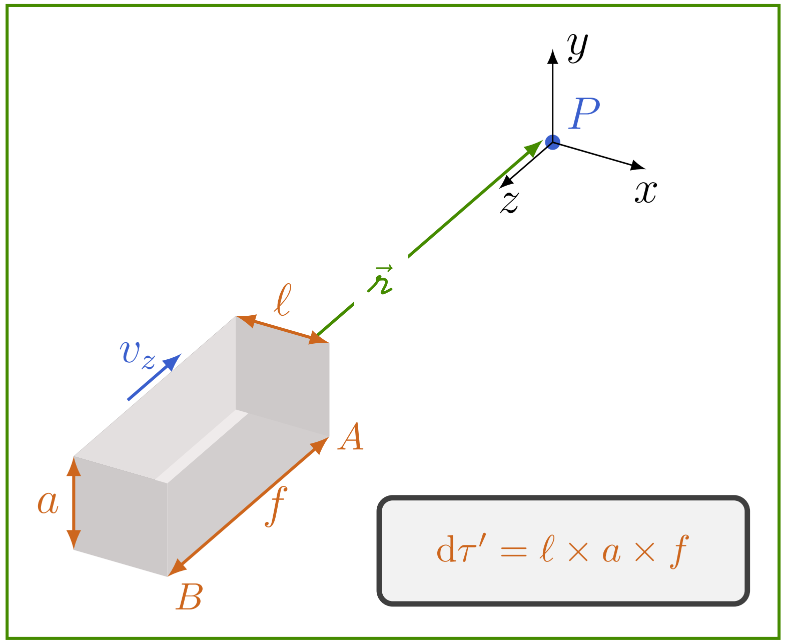

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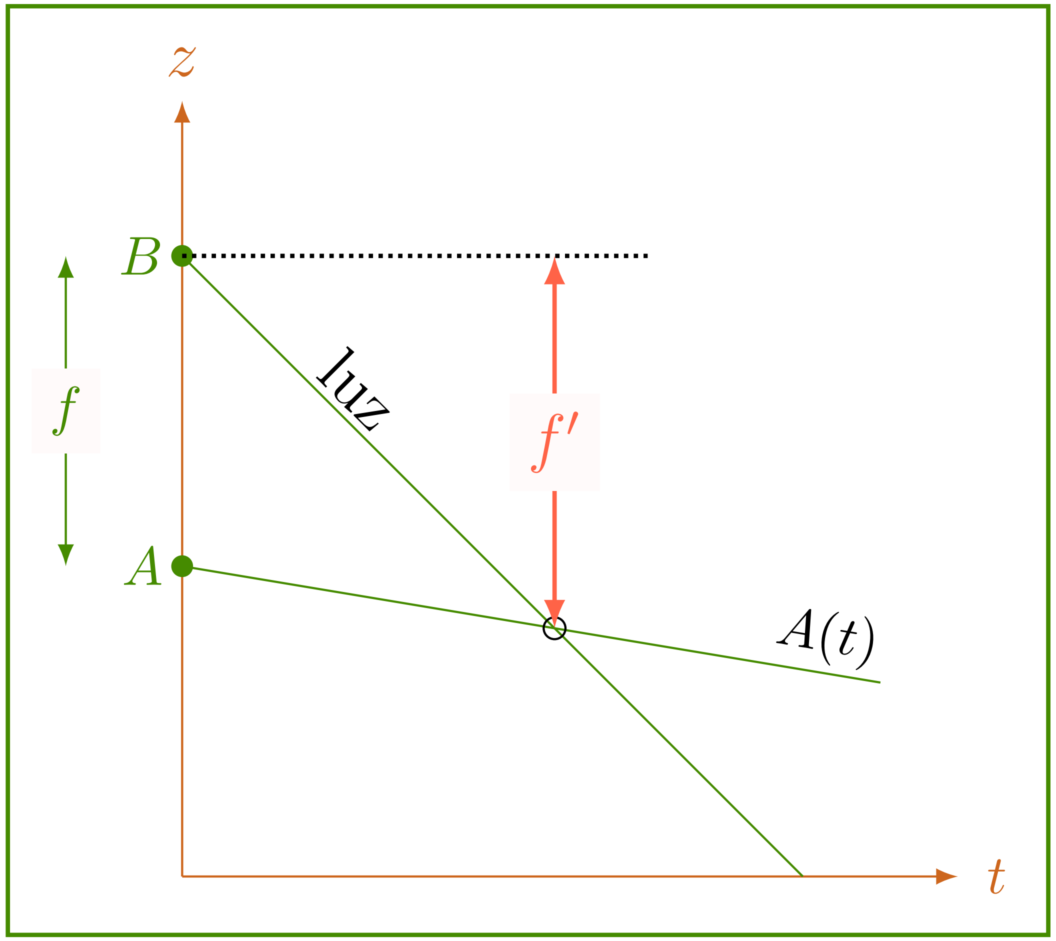

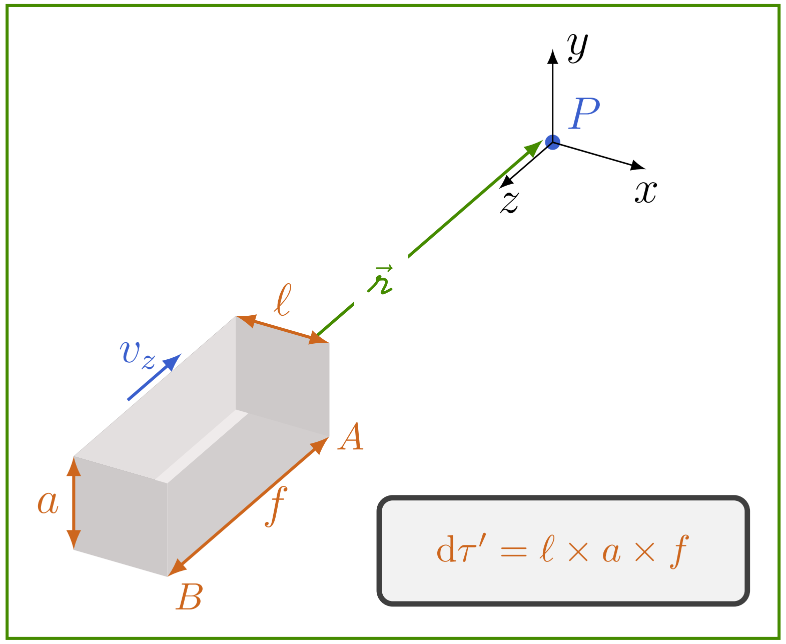

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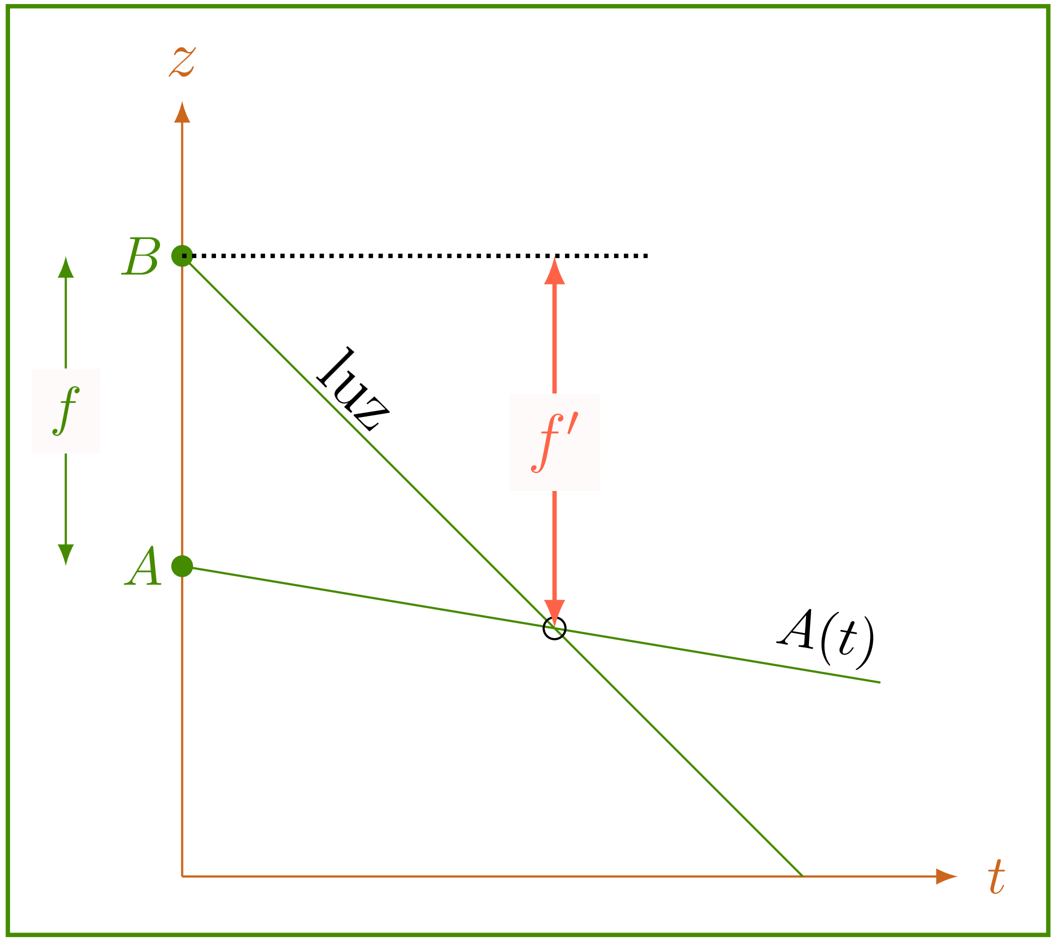
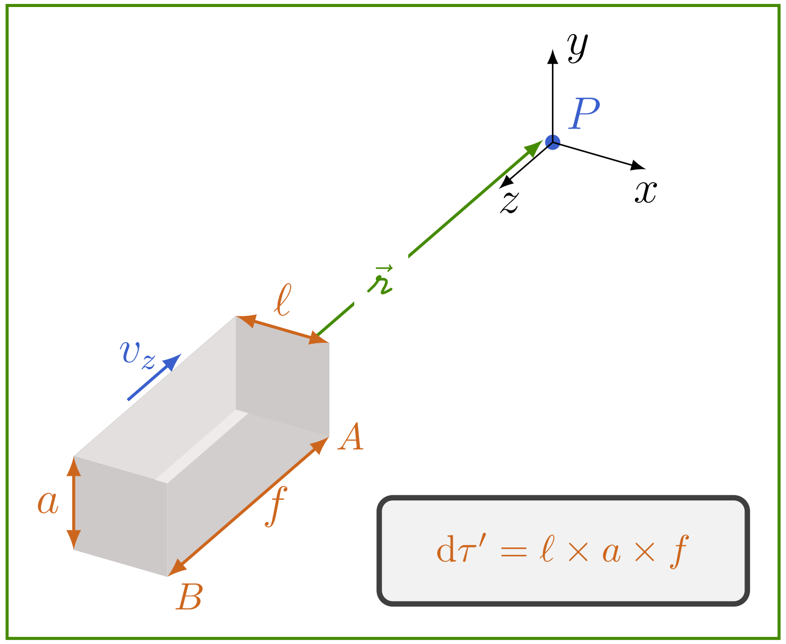
$$f' = z_B - z_A(t)$$





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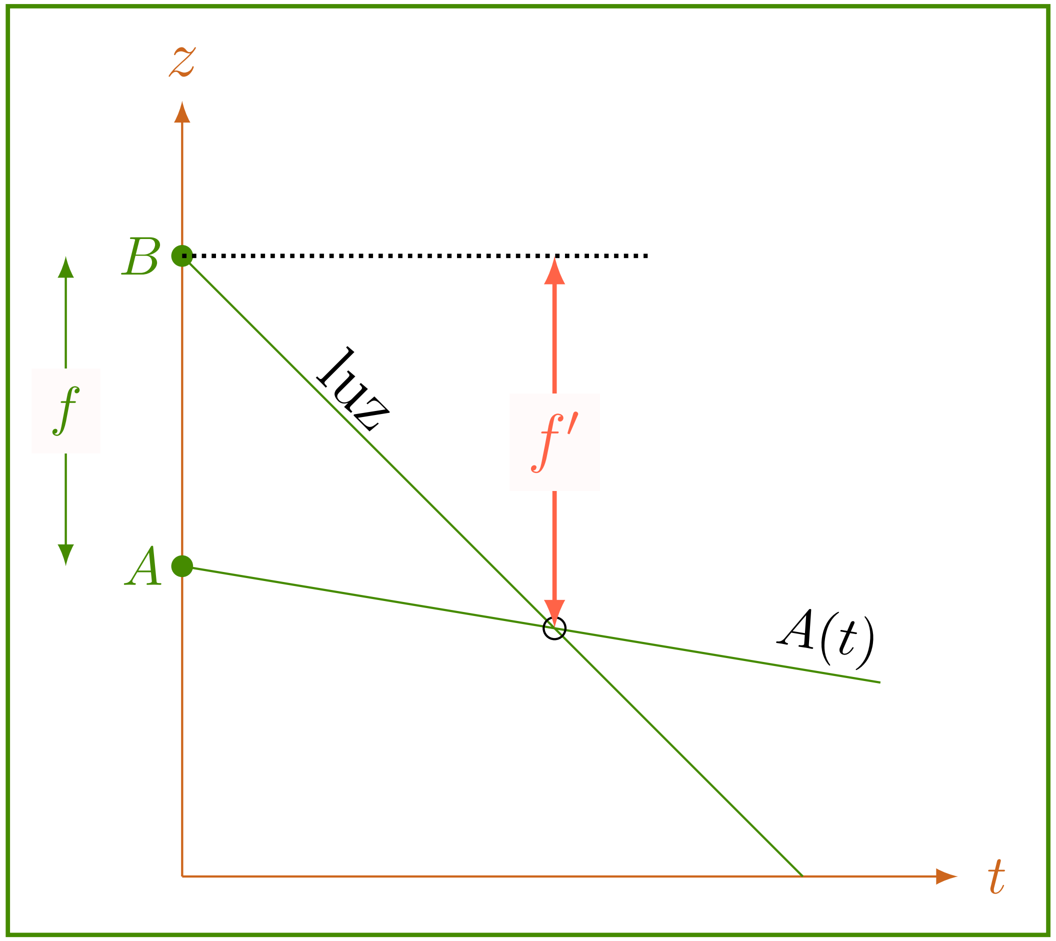

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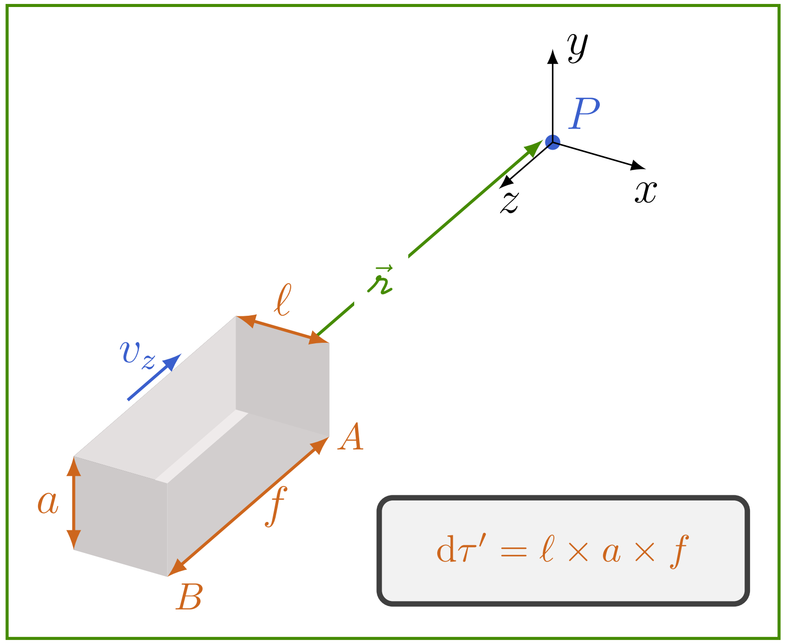

$$f' = z_B - z_A - v_z \frac{z_B - z_A}{c - v_z}$$

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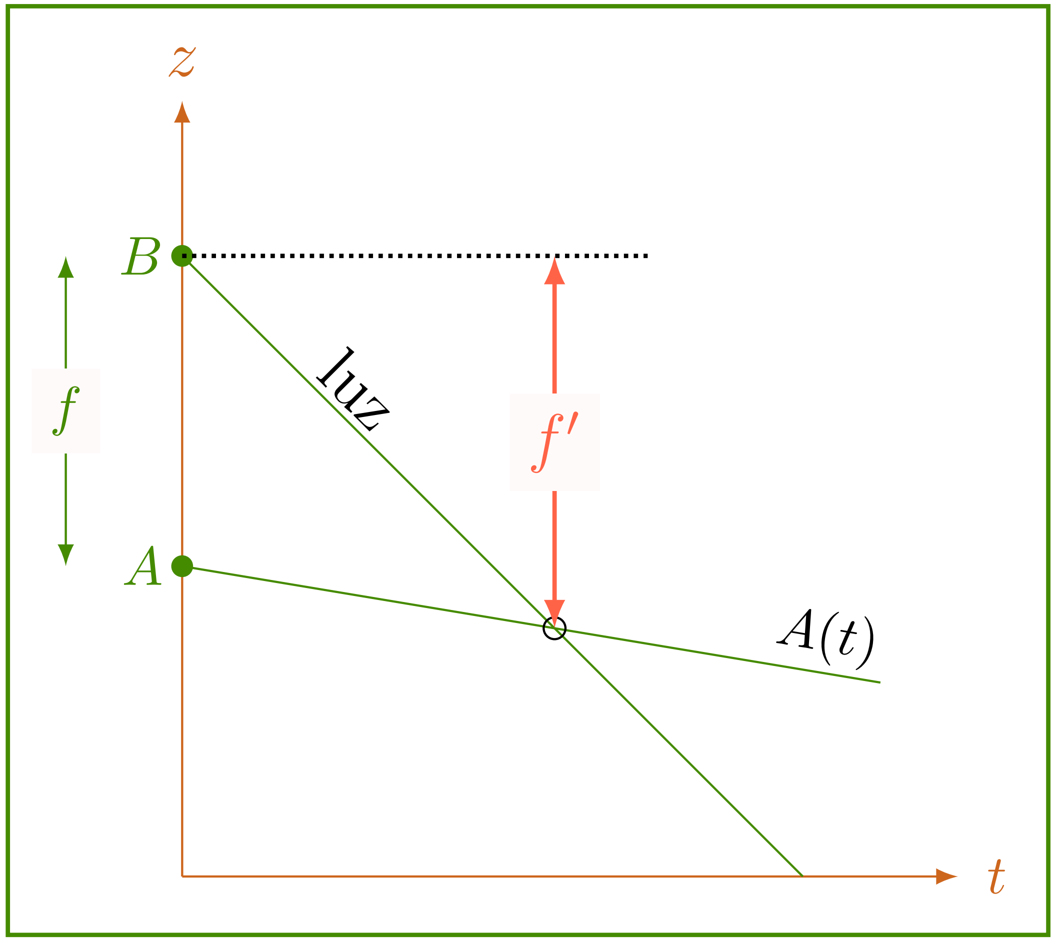

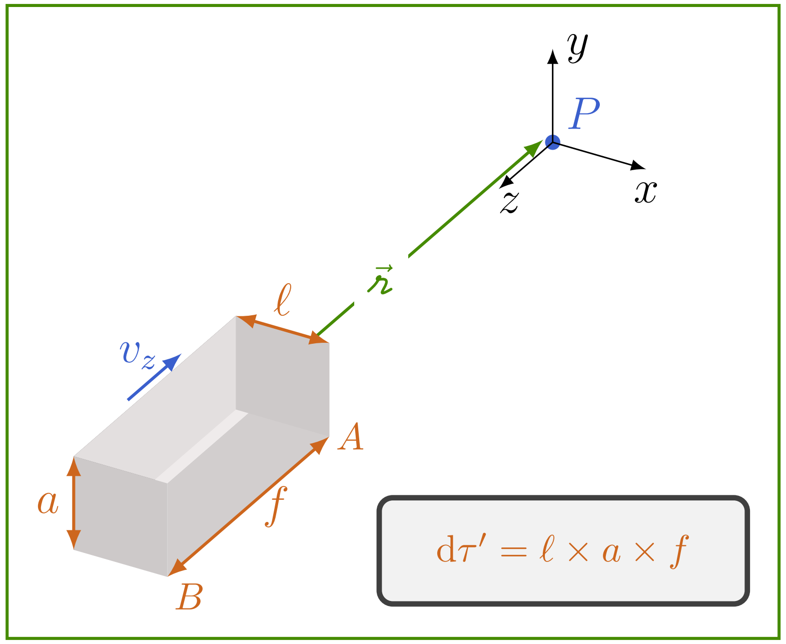

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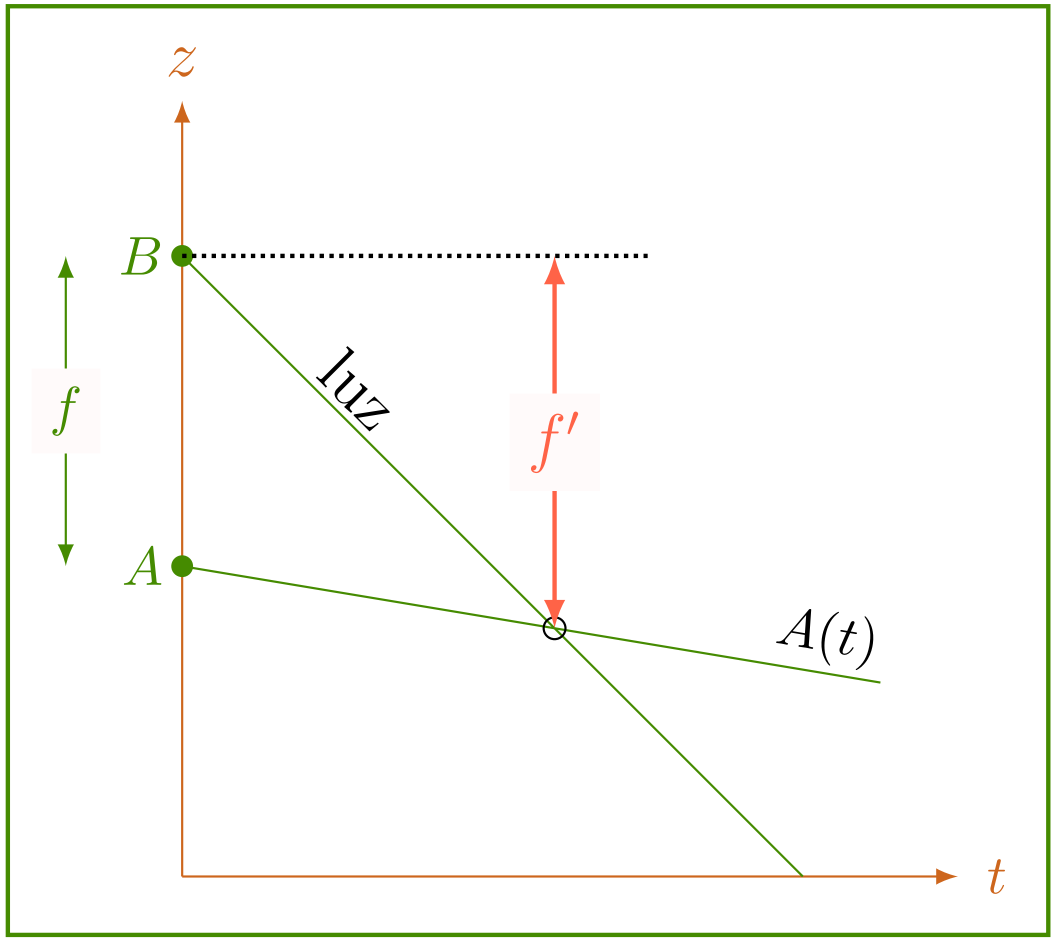

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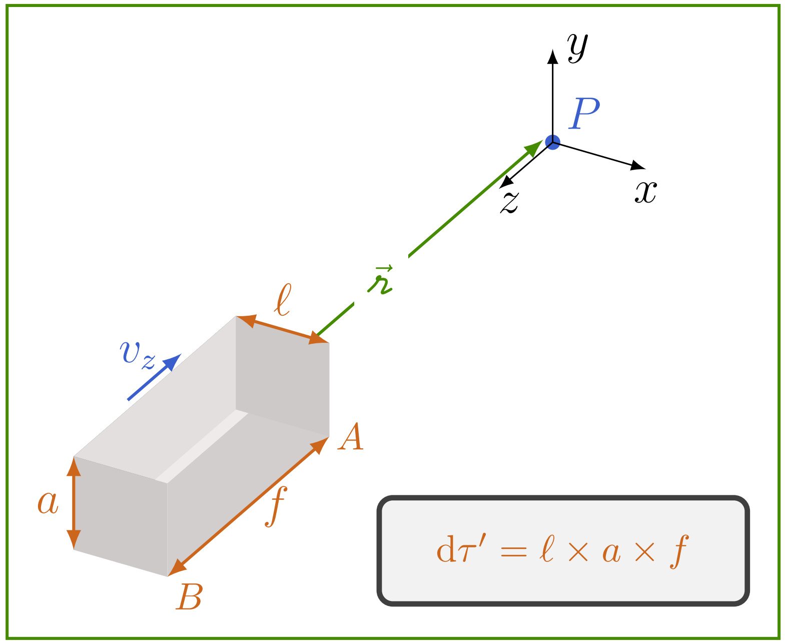
$$f' = f \frac{1}{1 - \frac{v_z}{c}}$$



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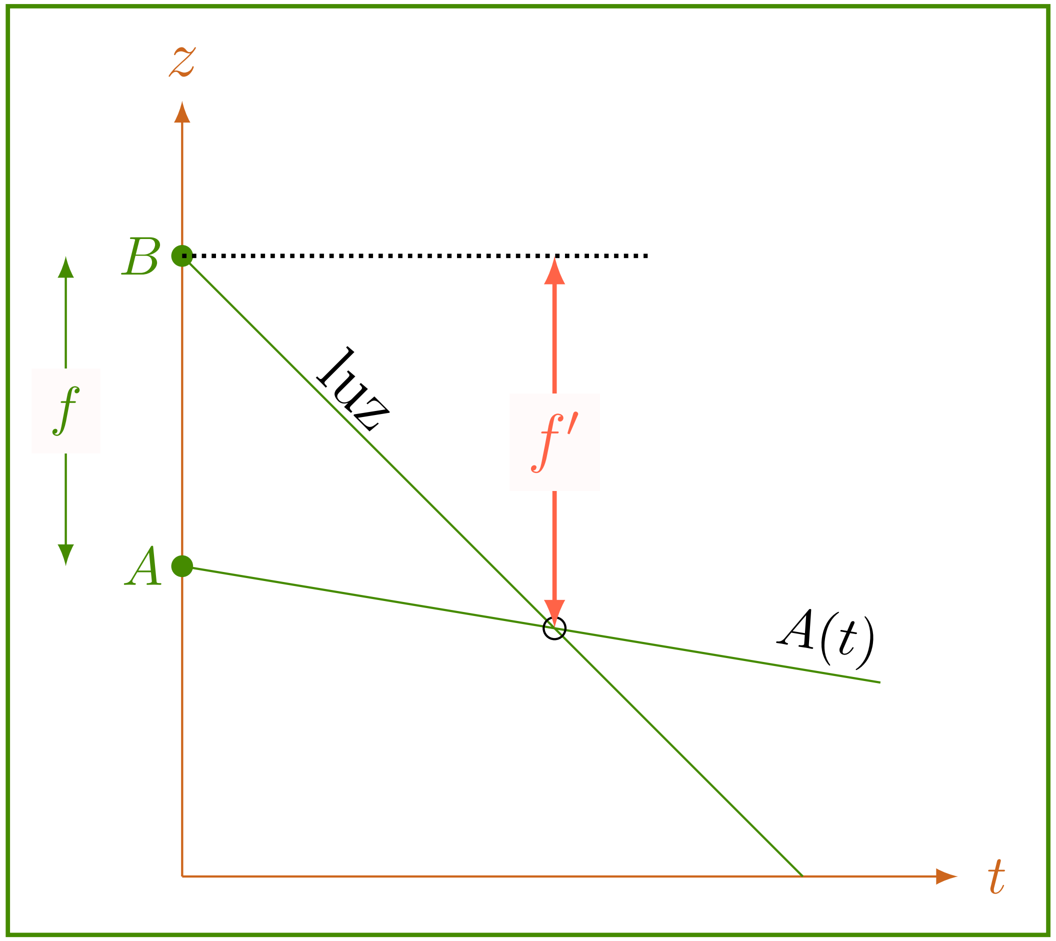

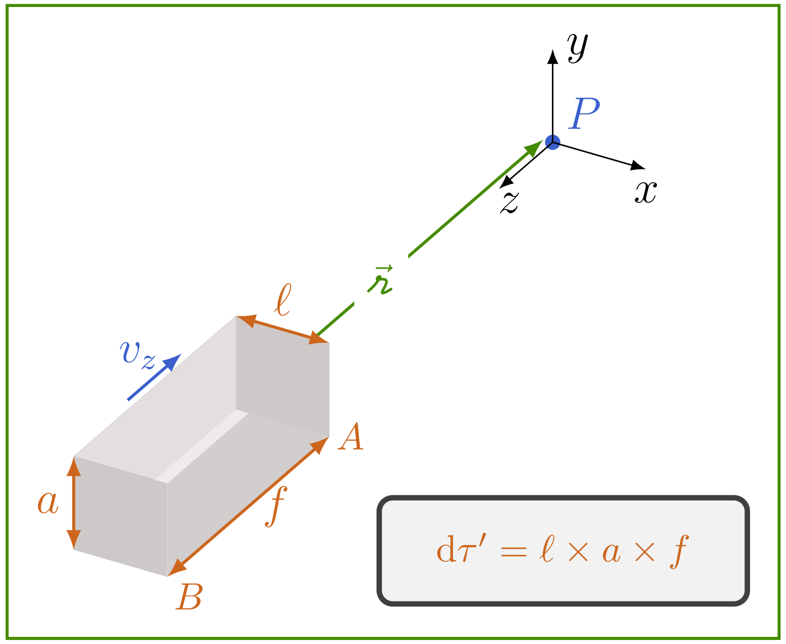

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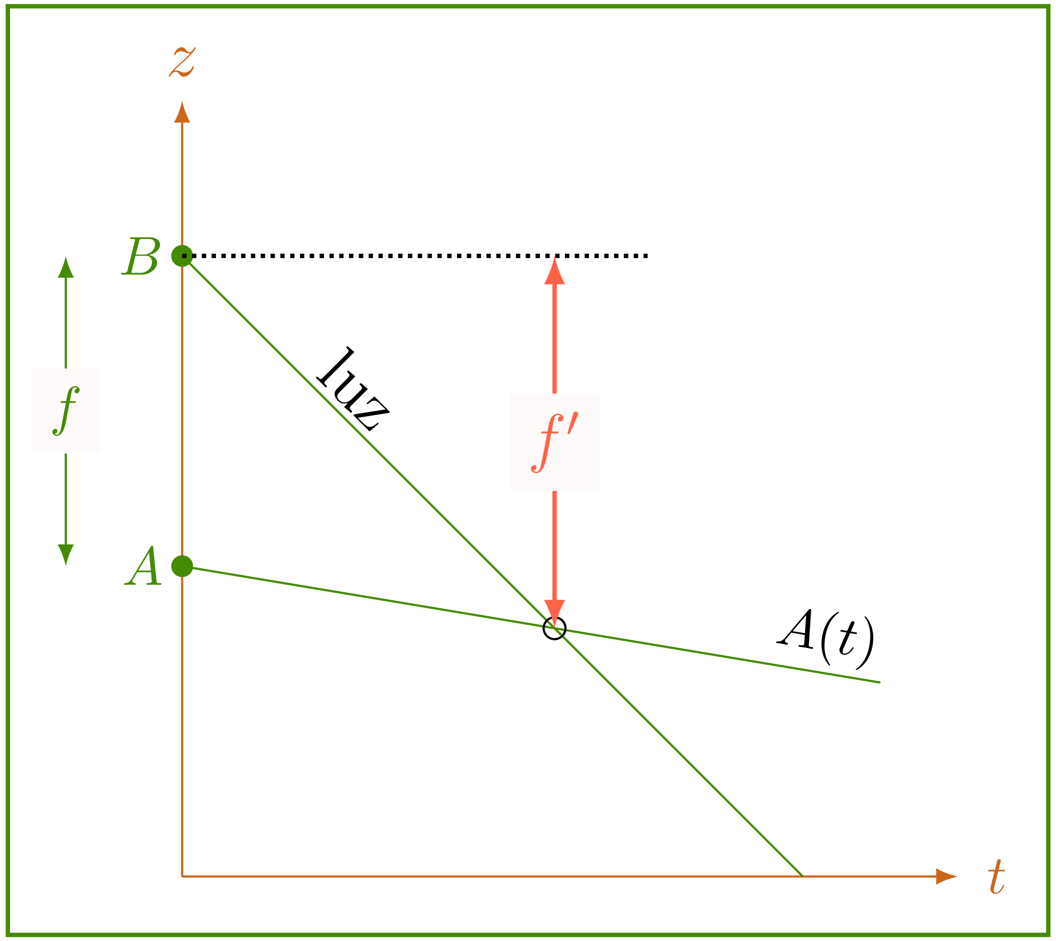
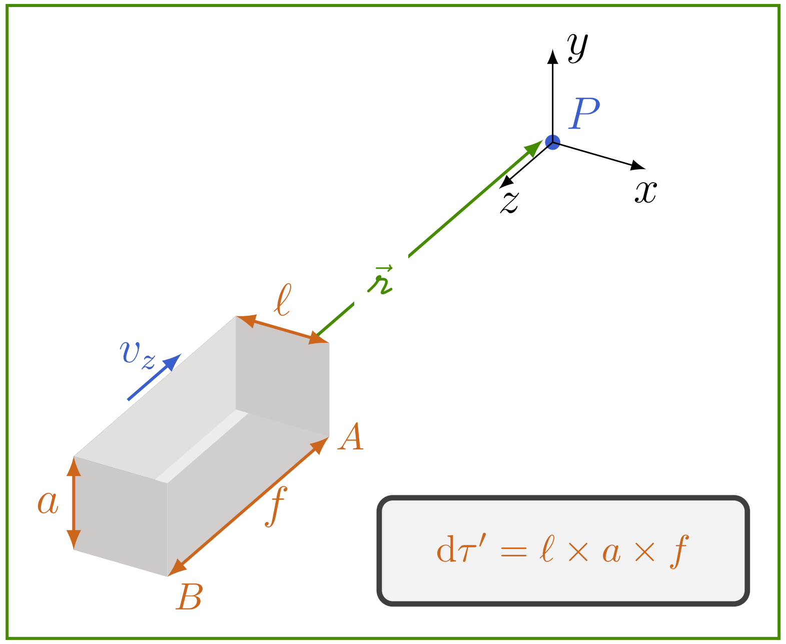
$$\mathrm{d}\tau'_{\mathrm{ret}} = \mathrm{d}\tau' \, \frac{1}{1 - \hat{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c}}$$





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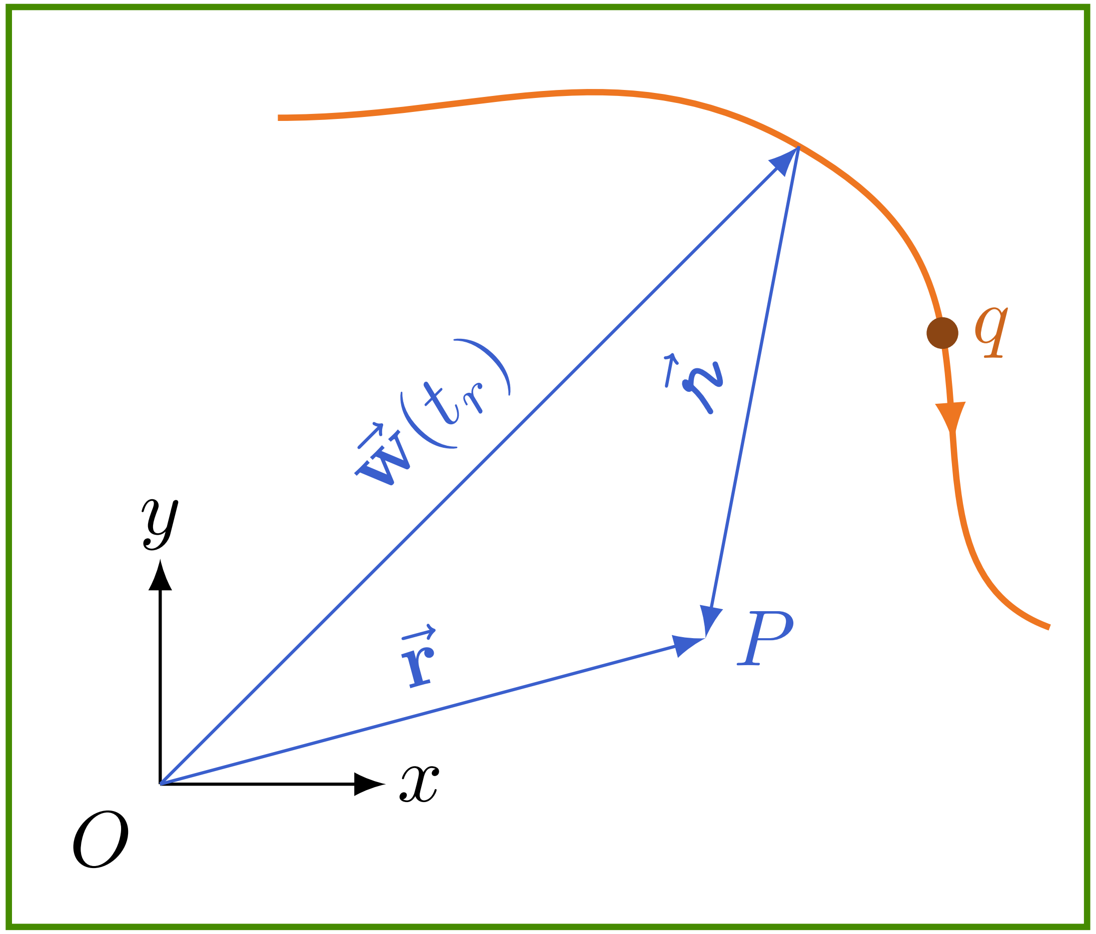

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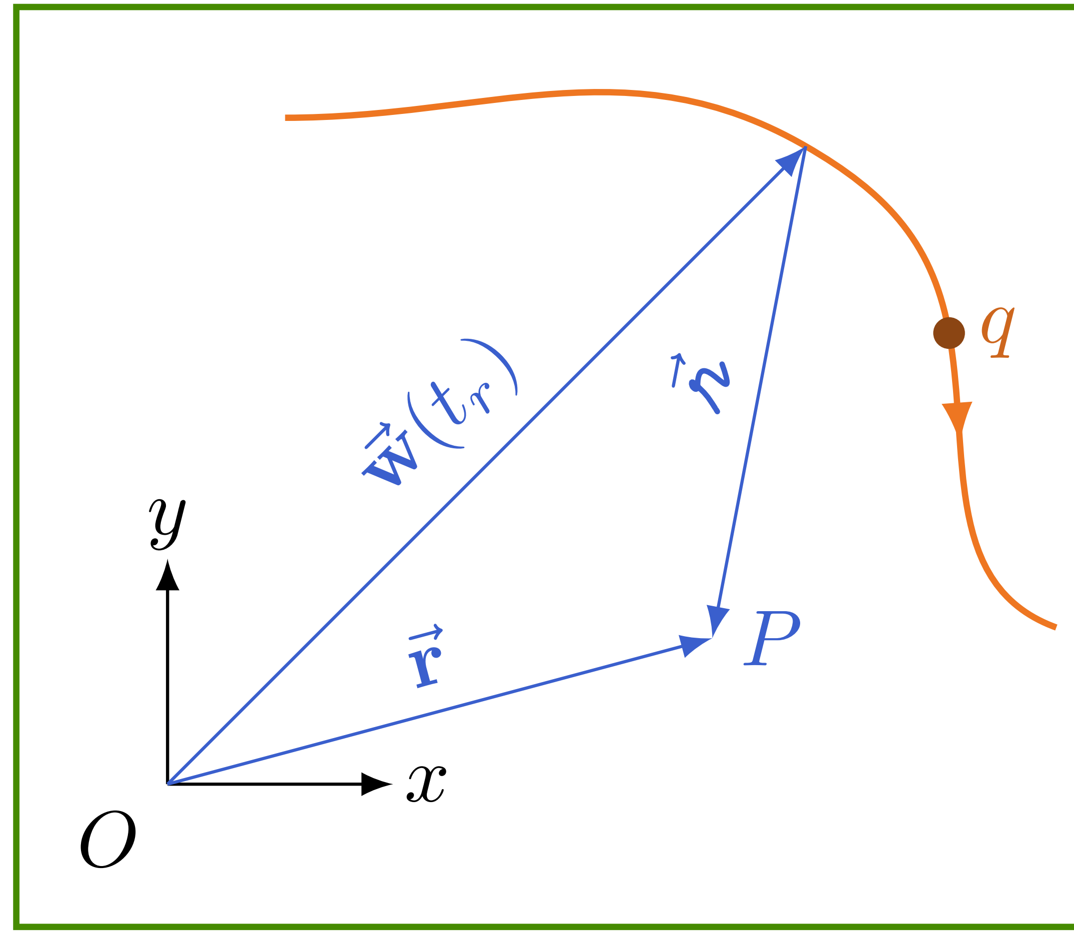

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$

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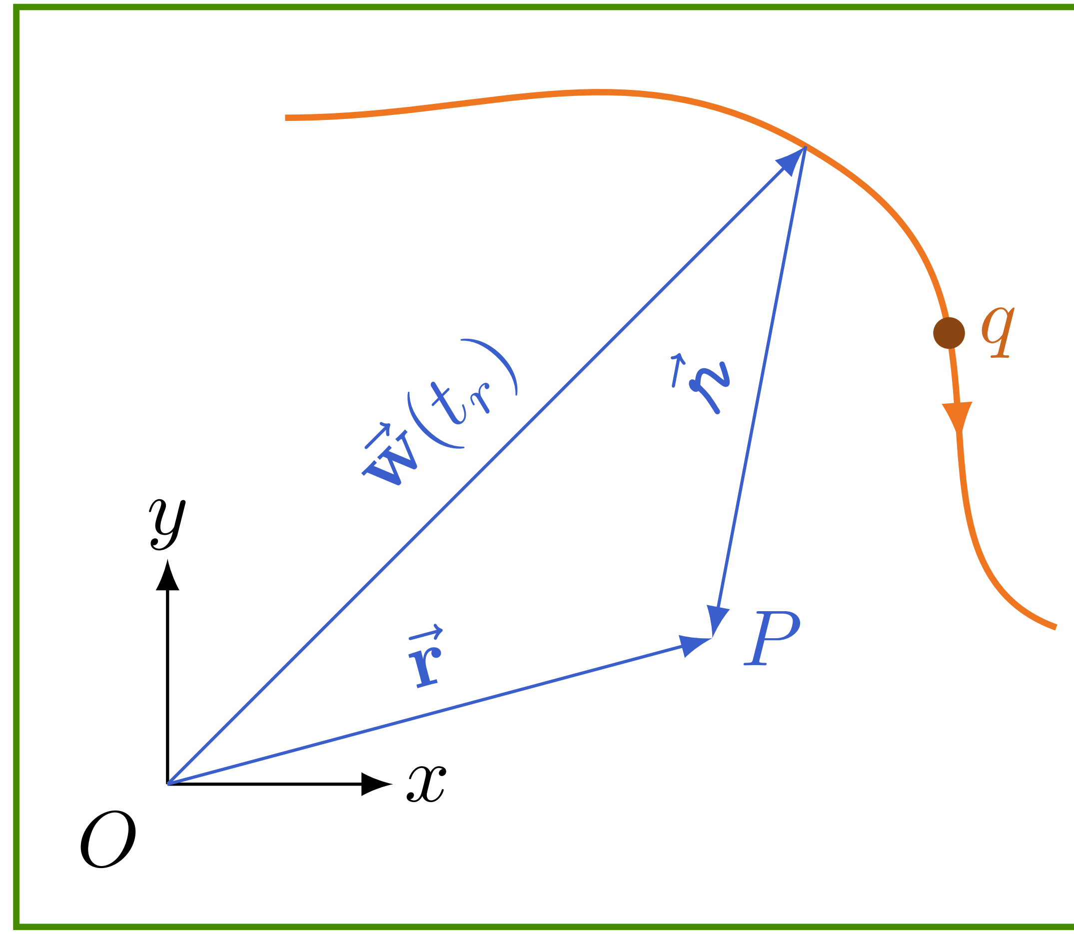

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$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{qc\vec{\mathbf{v}}}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$

Potenciais de Liénard e Wiechert

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r} - \vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}}$$

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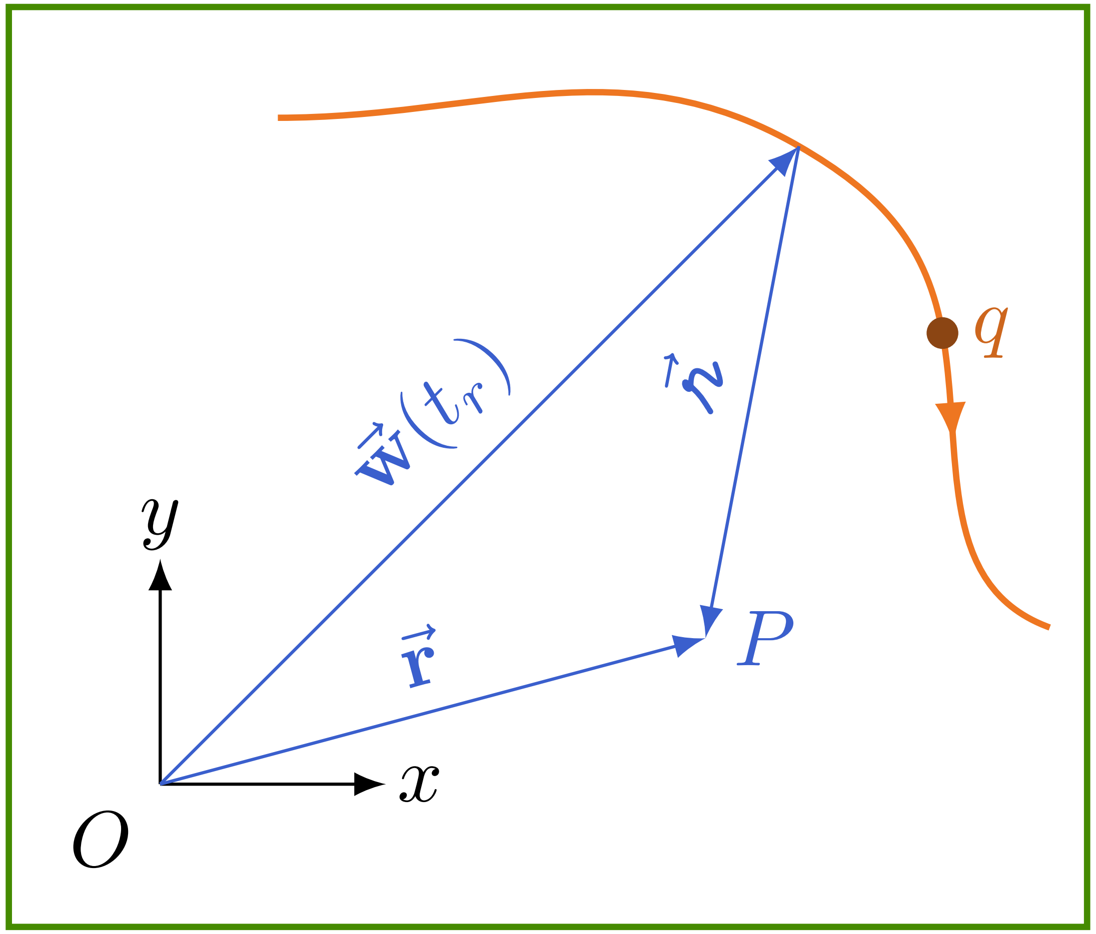
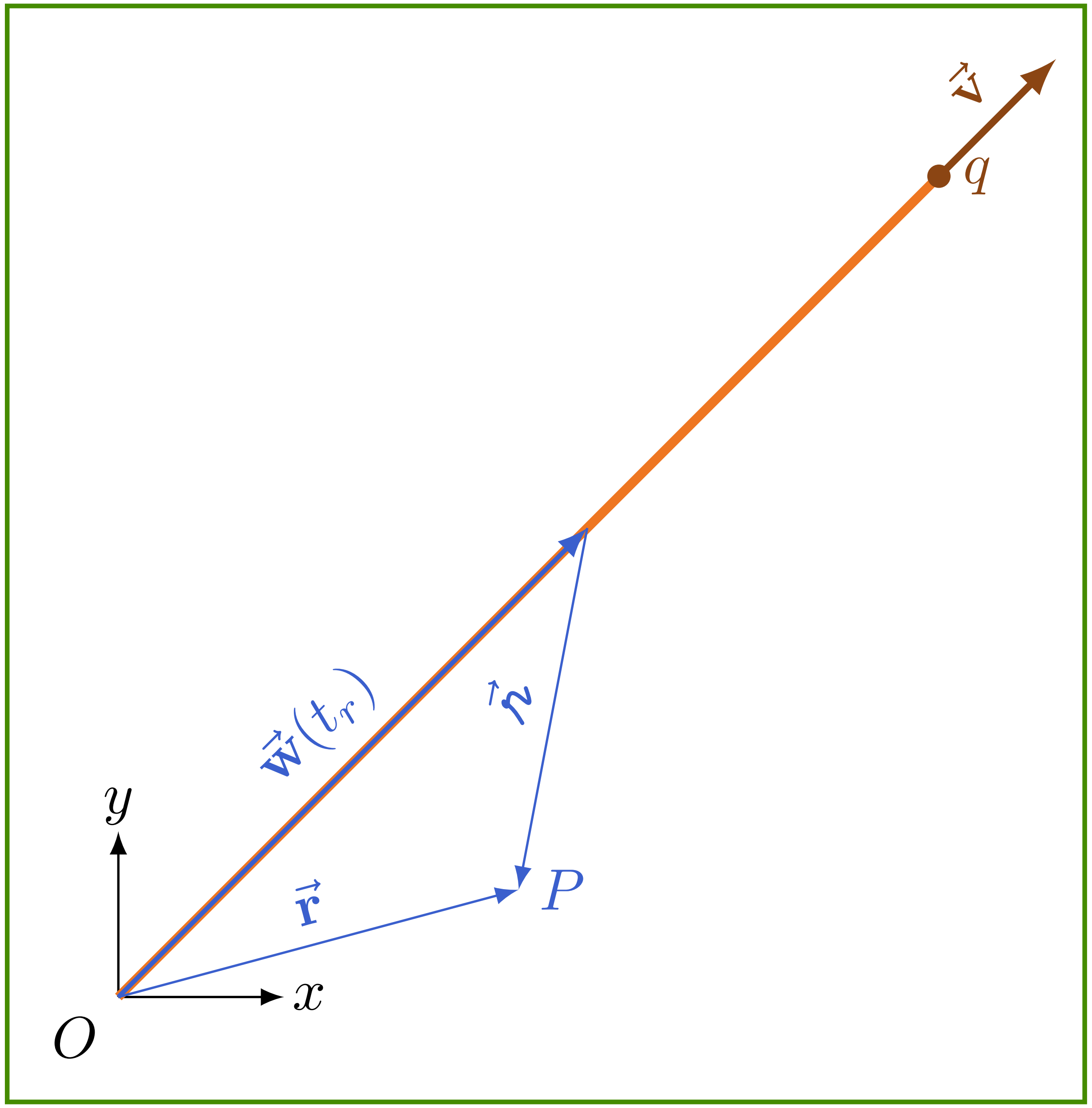


Pratique o que aprendeu

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$

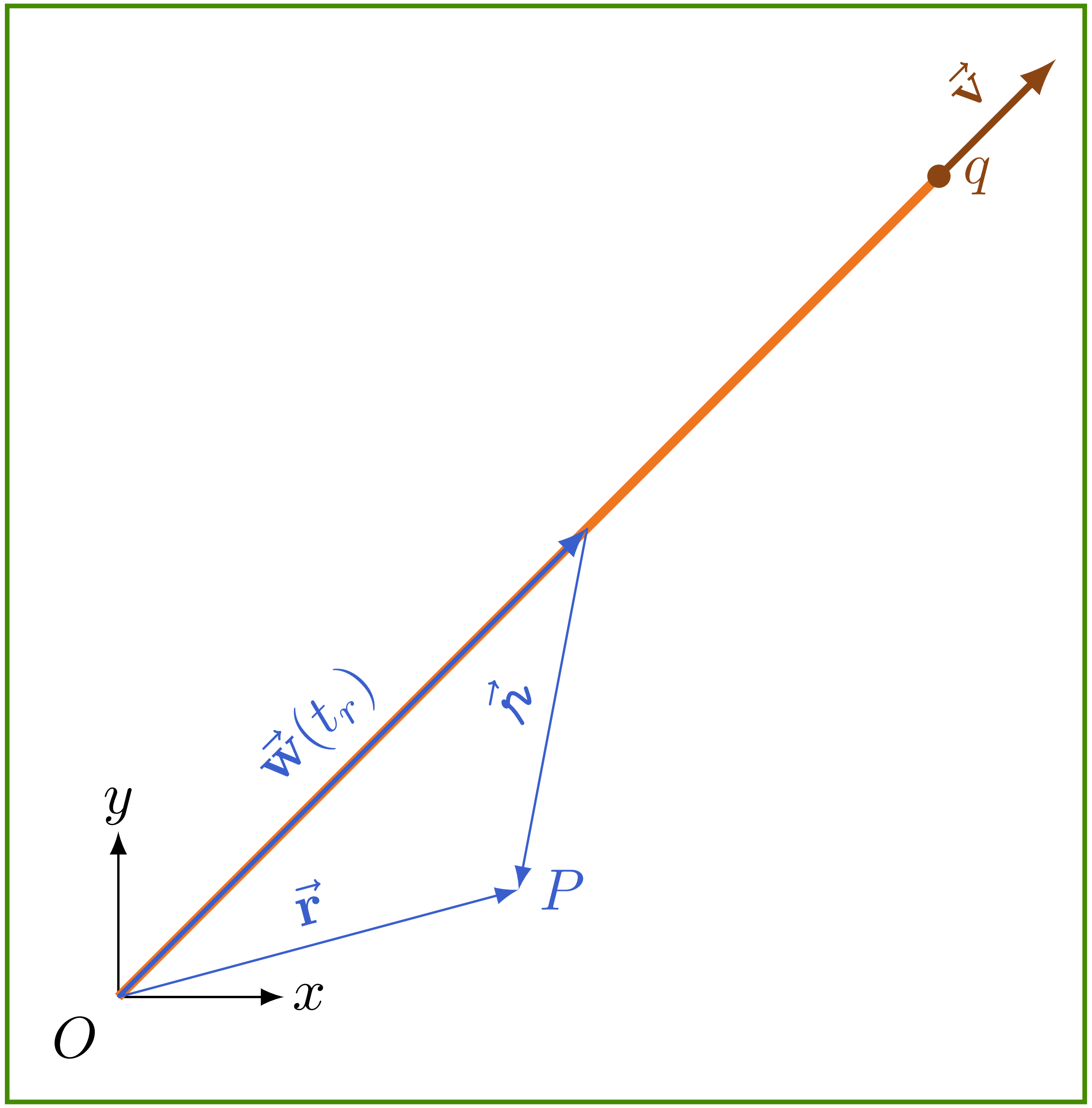
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{qc\vec{\mathbf{v}}}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$



Pratique o que aprendeu

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{c\mathscr{r} - \vec{\mathbf{v}}\cdot\vec{\mathscr{r}}}$$

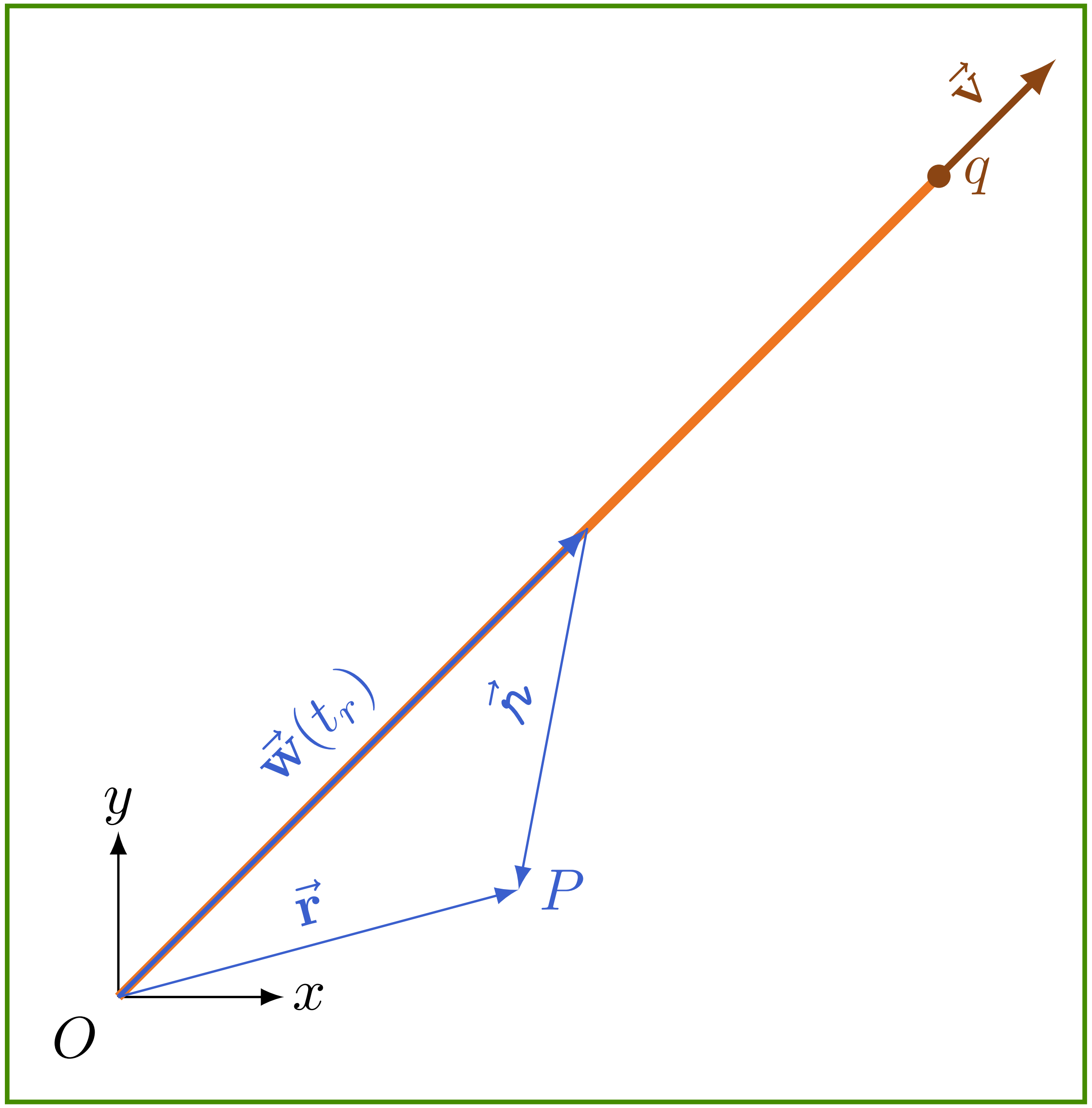
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi}\frac{qc\vec{\mathbf{v}}}{c\mathscr{r} - \vec{\mathbf{v}}\cdot\vec{\mathscr{r}}}$$

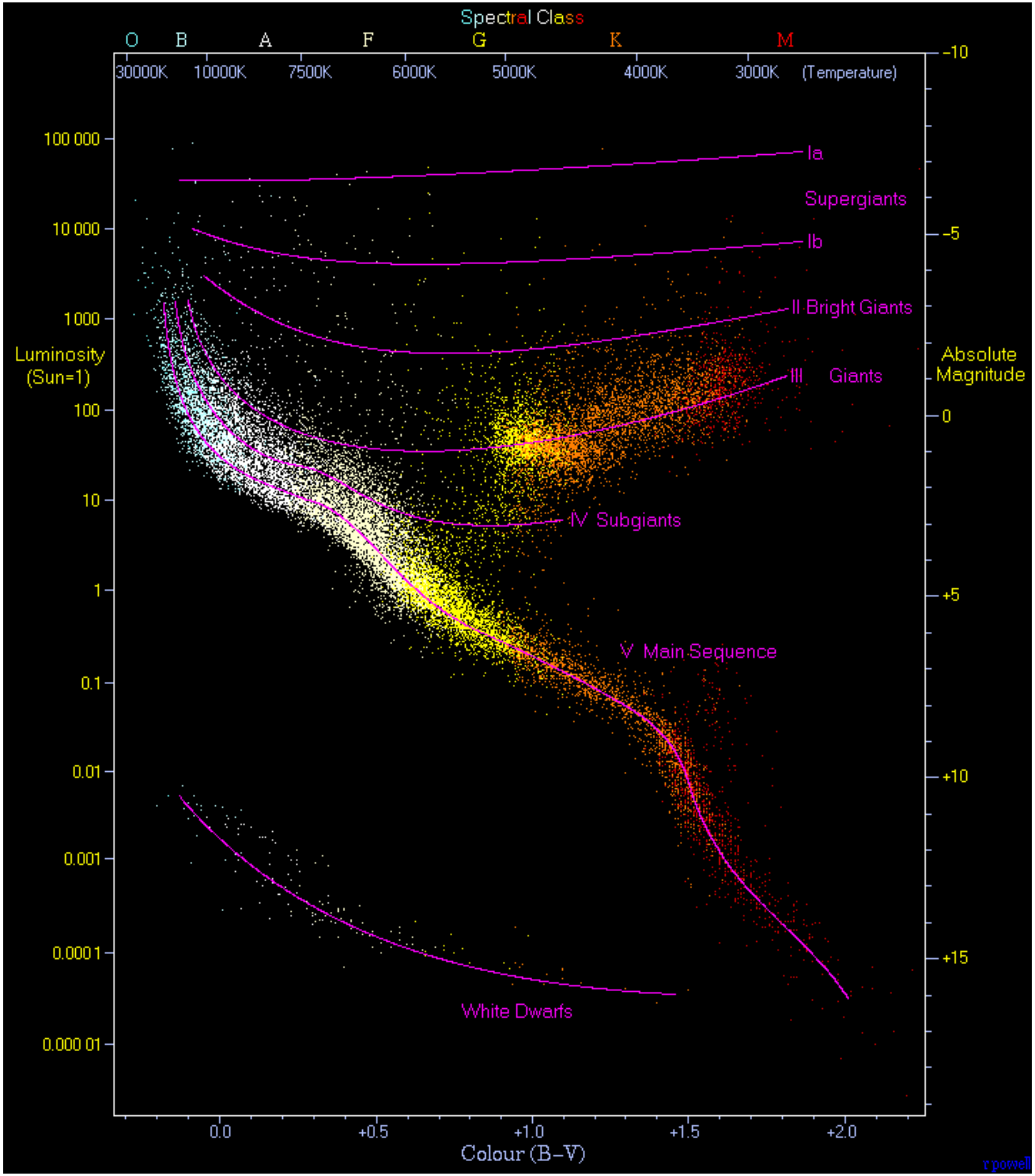


Pratique o que aprendeu

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{qc\vec{\mathbf{v}}}{c\mathscr{r} - \vec{\mathbf{v}} \cdot \vec{\mathscr{r}}}$$





Spectral Class
O
B
A
F
G
K
M
30000K
10000K
7500K
6000K
5000K
4000K
3000K
(Temperature)
100 000
10 000
1000
Luminosity
(Sun=1)
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
-10
-5
0
+5
+10
+15
Absolute
Magnitude
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B–V)
r.powell

Potenciais de Liénard e Wiechert

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r} - \vec{\mathscr{r}}\cdot\dfrac{\vec{\mathbf{v}}}{c}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}}{\mathscr{r} - \vec{\mathscr{r}}\cdot\dfrac{\vec{\mathbf{v}}}{c}}$$