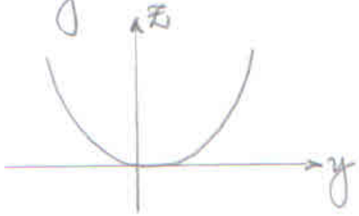
Exemplos: Esboce o gráfico das funções

Matéria referente a Funções de n variáveis

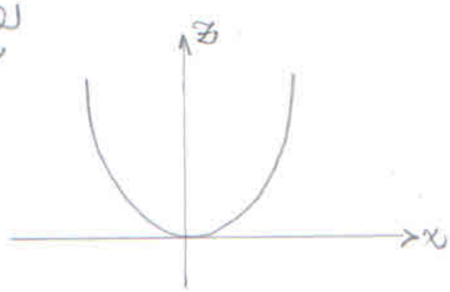
a) $z = f(x,y) = x^2 + y^2$

$z = x^2 + y^2$

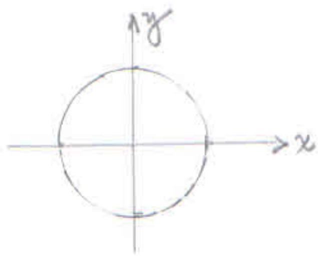
1º) $x = 0$; $z = y^2$

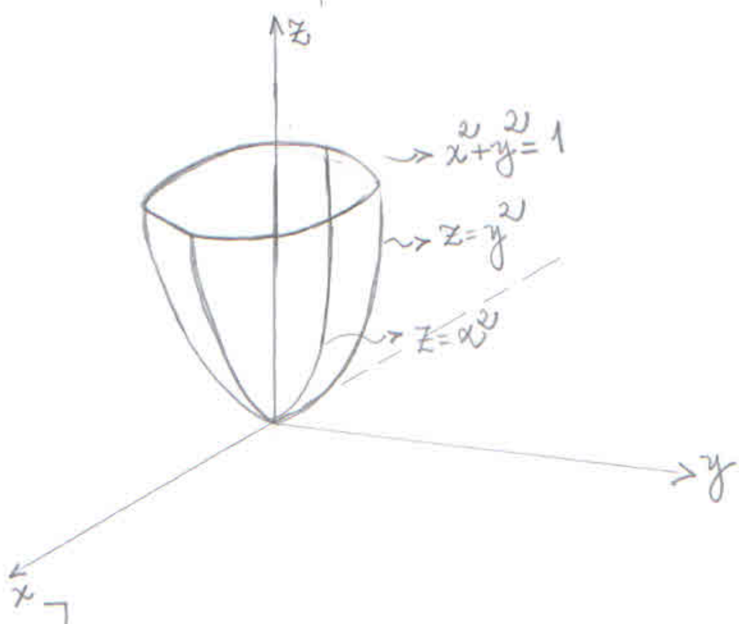


2º) $y = 0$; $z = x^2$



3º) $z = 1$; $x^2 + y^2 = 1$



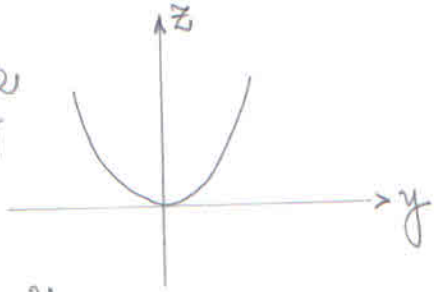


$\text{Im} f = [0, +\infty]$

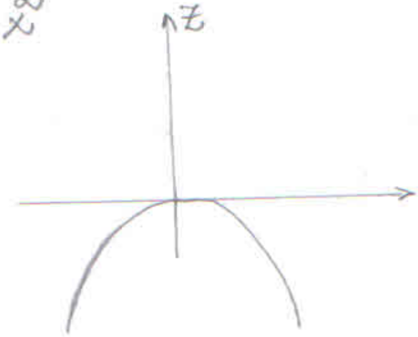
c) $z = f(x,y) = y^2 - x^2$

$z = y^2 - x^2$

$x = 0, \quad z = y^2$

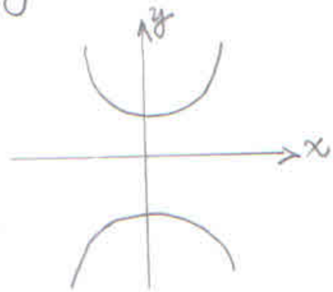


$y = 0 \; ; \quad z = -x^2$



$z = 1 \qquad y^2 - x^2 = 1$

$y = \pm \sqrt{x^2 + 1}$

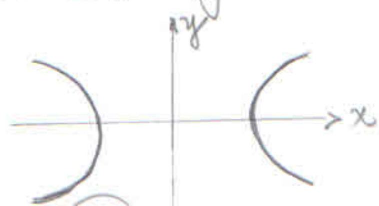


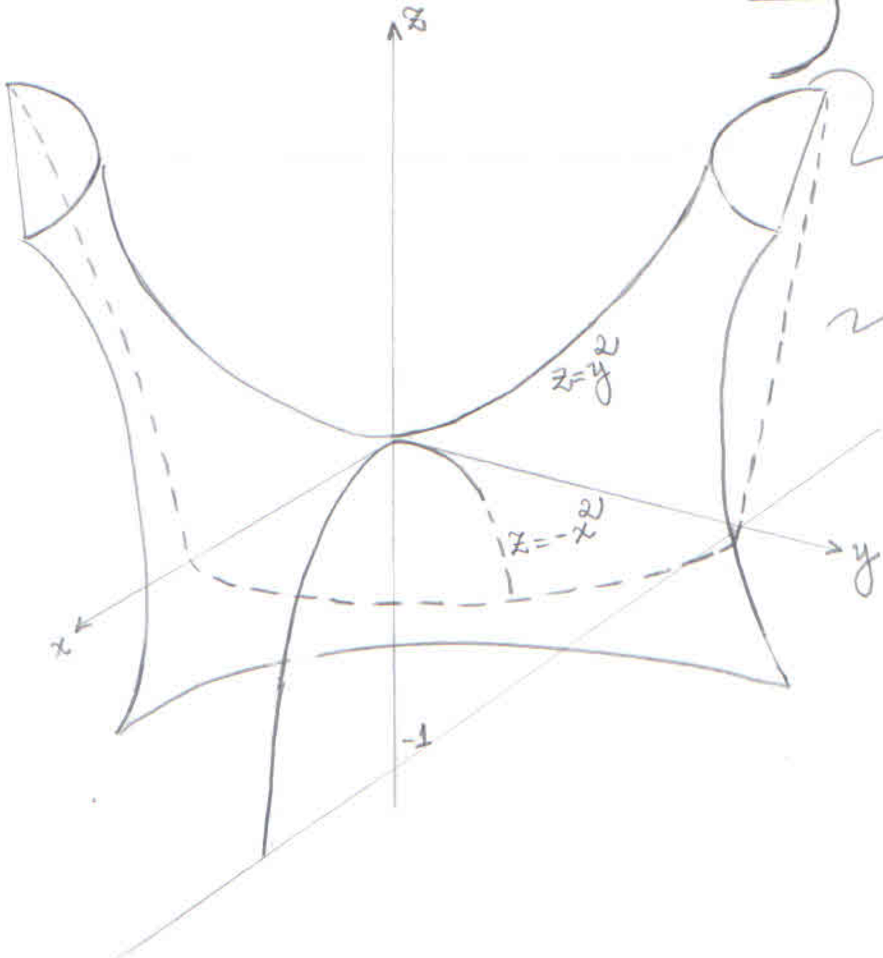
$z = -1$

$y^2 - x^2 = -1 \quad \times(-1)$

$x^2 - y^2 = 1$

$x = \pm \sqrt{1 + y^2}$





$y^2 - x^2 = 1$

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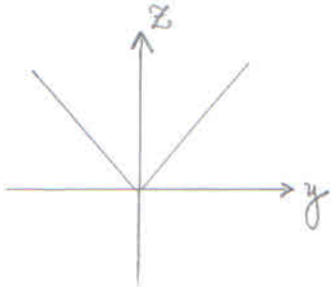
b) $z = \sqrt{x^2 + y^2}$

$x = 0$

$z = \sqrt{y^2}$

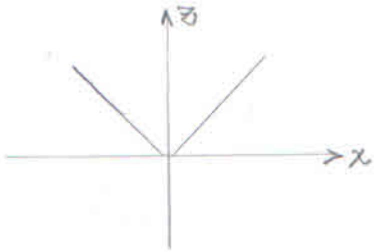
$z = |y|$



$y = 0$

$z = \sqrt{x^2}$

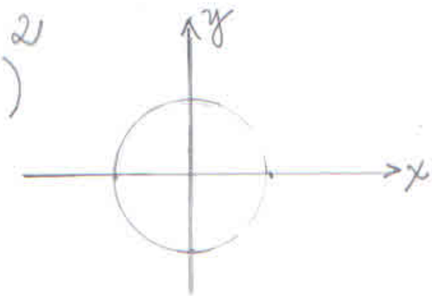
$z = |x|$

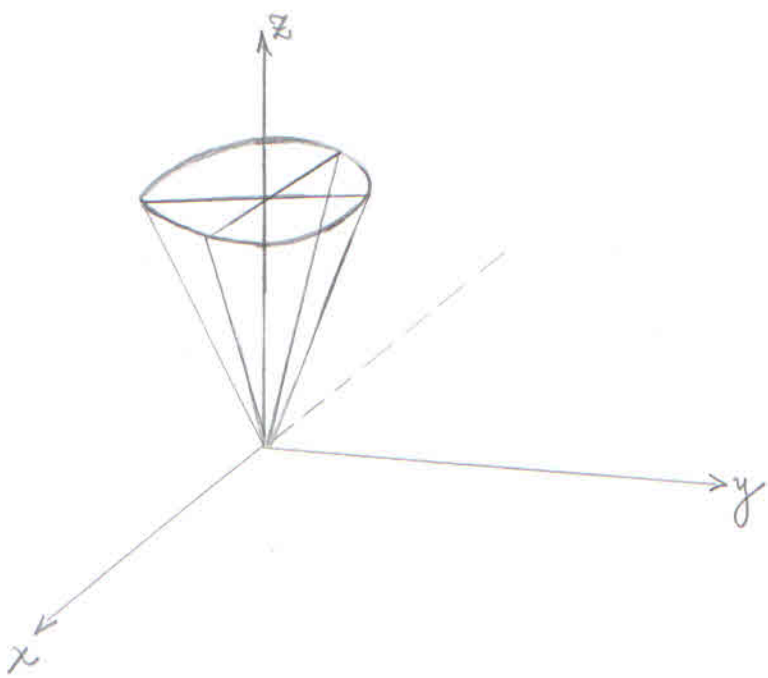


$z = 1$

$\sqrt{x^2 + y^2} = 1 \quad (\ )^2$

$x^2 + y^2 = 1$





d) $g(x,y) = \sqrt{36 - 9x^2 - 4y^2} \geq 0$

$$z = \sqrt{36 - 9x^2 - 4y^2}$$

$$36 - 9x^2 - 4y^2 \geq 0 \quad \text{ou} \quad 9x^2 + 4y^2 \leq 36$$

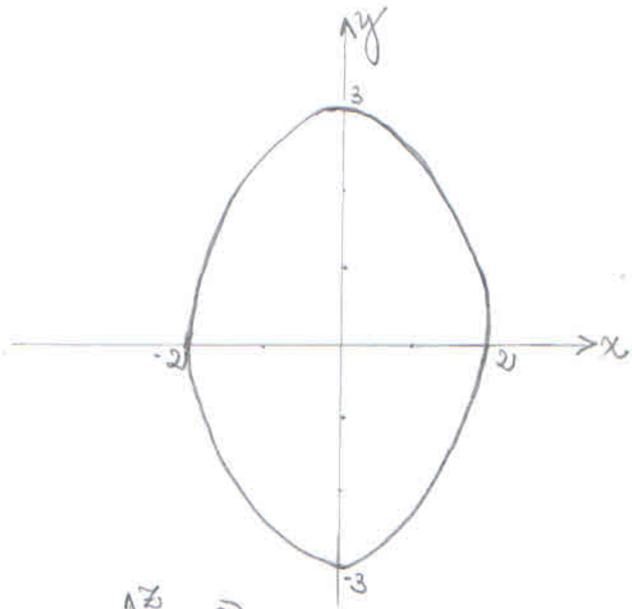
$$D = \{(x,y) / 36 - 9x^2 - 4y^2 \geq 0 \ \text{ou}\ 9x^2 + 4y^2 \leq 36\}$$

$$9x^2 + 4y^2 \leq 36$$

$$9x^2 + 4y^2 = 36 \ (\div 36)$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1} \ ; \ b = \pm 2 \ ; \ a = \pm 3$$



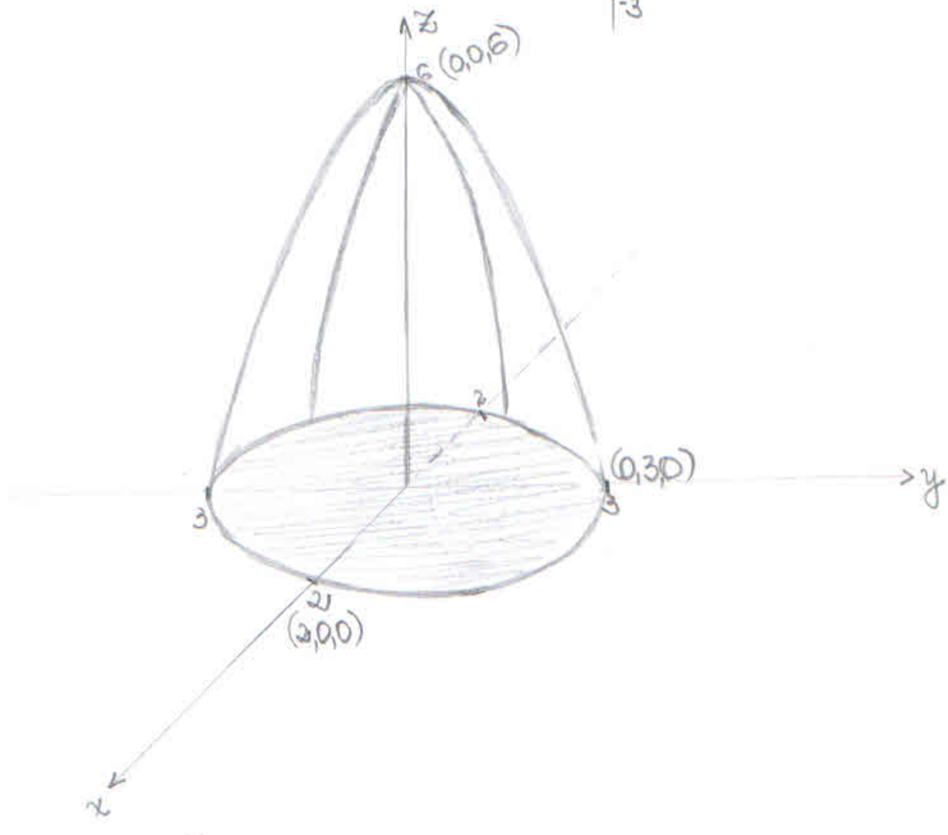
$$z = \sqrt{36 - 9x^2 - 4y^2} \quad (\ )^2$$

$$z^2 = 36 - 9x^2 - 4y^2$$

$$z^2 + 9x^2 + 4y^2 = 36$$

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{36} = 1$$

$$\begin{cases} x = y = 0 & z = \pm 6 \\ y = z = 0 & x = \pm 2 \\ x = z = 0 & y = \pm 3 \end{cases}$$



$$Im f = \{z / 0 \leq z \leq 6\}$$

Exemplos: Esboce o gráfico das curvas de nível

a) $f(x,y) = x^2 + y^2$

$f(x,y) = k$

$k=1$, a curva de nível $k=1$ é o conjunto dos pontos do Dom., tais que:

$f(x,y) = x^2 + y^2$ $\quad D = \{(x,y) \in \mathbb{R}^2 / x^2 + y^2 = 1\}$

$x^2 + y^2 = 1$

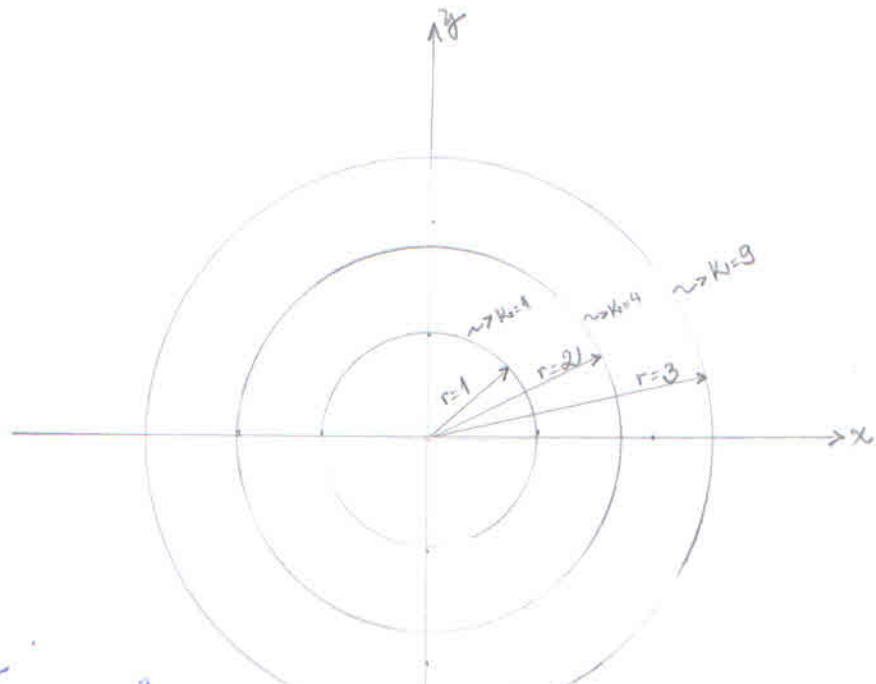
$k = 4$

$x^2 + y^2 = 4$; $r = 2$

$k = 9$

$x^2 + y^2 = 9$; $r = 3$

$k = 0 \Rightarrow$ ponto na origem $(0,0)$



1) Se k aumenta o raio aumenta, portanto, se k diminui o raio também diminui

2) Quanto mais longe da origem mais alto ficará o gráfico.

b) $f(x,y) = 2 - x - y$

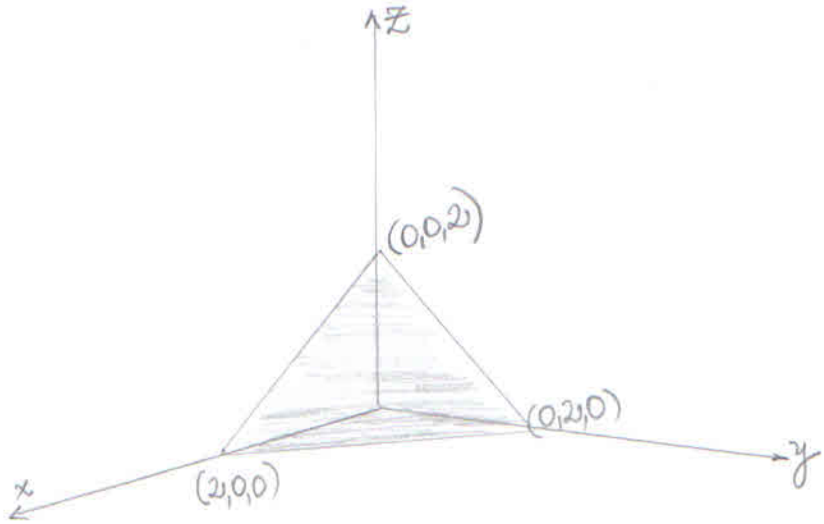
$f(x,y) = z$

$z = 2 - x - y$

$y, z = 0 \; ; \; x = 2$

$x, z = 0 \; ; \; y = 2$

$x, y = 0 \; ; \; z = 2$



c) $f(x,y) = 4$

$z = 4$ ou

$k = 4$