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Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting

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For a wide class of infinitely lived agent models, Chamley has shown that the optimal capital income tax rate is zero in the long run. Lucas has argued that for the U.S. economy, there is a significant welfare gain from switching to this policy. This paper shows that for the Bewley class of models with incomplete insurance markets and borrowing constraints, the optimal tax rate on capital income is positive, even in the long run. Therefore, cutting the capital income tax to zero may well lead to welfare losses.

I. Introduction

For a wide class of infinitely lived agent models, Chamley (1986) has shown that the optimal capital income tax rate is zero in the long run. The capital income tax rate for the U.S. economy is quite far from zero: Lucas's (1990) number for the U.S. capital income tax rate is 36 percent. One possible response to this fact is to accept the prescription of theory and recommend a change in tax policy. Lucas has taken this route and, using a representative agent model, has argued that for the U.S. economy there is a significant welfare gain

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from switching to this policy. In Lucas's words, "supply-side economists . . . have delivered the largest genuinely free lunch I have seen in 25 years in this business" (p. 314). According to Lucas's estimates, eliminating the capital income tax can result in a welfare gain across steady states of over 5 percent of consumption and about 1 percent when transitional costs are taken into account. He suggests that this welfare gain is about twice as large as the gain from eliminating a 10 percent inflation rate, 20 times as large as the gain from eliminating postwar business cycles, and 10 times the gain from eliminating all product market monopolies as estimated by Harberger (1954).

One way to summarize this argument is to say that the capital stock in the U.S. economy is too low and ought to be higher and, further, that there are large welfare gains from making it so.

In this paper, I take the contrary view that perhaps there are good reasons why the capital income tax is what it is and hence that cutting it would lead to welfare losses rather than welfare gains. To put it another way, if the capital income tax were cut to zero, the capital stock would be too high, and it ought to be lower.

I present a class of environments together with a market structure such that the optimal capital income tax rate is not zero but strictly positive, even in the long run. Specifically, I show that for the Bewley (1986) class of models with incomplete insurance markets and borrowing constraints, the optimal tax rate on capital income is positive even in the long run.[1] If one regards such models as providing a good description of reality, then one needs to reassess the presumed welfare gains of reducing the capital income tax to zero. The presumed welfare gains may well turn into losses.

I should emphasize that the result in this paper is not just that the capital income tax rate is different from zero in the long run, but that it is always positive for the type of environment/market structure considered. In overlapping generations models with pure life cycle consumers, the long-run capital income tax is not generally zero; however, it may be positive or negative.

In the Bewley (1986) class of models considered in this paper, there is a continuum of infinitely lived agents subject to idiosyncratic shocks that are uninsured. Owing to the absence of insurance markets, agents become heterogeneous ex post.[2] Because of the idiosyncratic

[1] Jones, Manuelli, and Rossi (1993) give some examples of situations in which the capital income tax rate can be positive in the long run. Their examples essentially involve some restrictions on the tax structure. Thompson (1974) makes an argument based on national defense for capital taxation as part of an optimal tax structure.

[2] Presumably, private information and the resulting problems due to moral hazard and adverse selection have a lot to do with "missing markets" and incomplete risk sharing. While it would be desirable to take explicit account of these features of the

nature of the shocks, there is uncertainty at the individual level, but there is no aggregate uncertainty. Individuals can save and dissave via risk-free assets and are subject to borrowing constraints.

The intuition behind why the features above lead to a positive tax rate on capital income may be understood in a couple of ways. One is as follows. In the context of the Chamley (1986) model, Lucas (1990, p. 300) has suggested that one principle of Ramsey taxation is that "taxes should be spread evenly over similar goods. . . . Since capital taxation . . . involves taxing later consumption at heavier rates than early consumption, . . . capital is a bad thing to tax." However, the Bewley-type model resembles an overlapping generations model with finite-lived agents. Although agents live forever, sequences of bad shocks will periodically lead to binding borrowing constraints. An agent's infinite-horizon optimization problem is thus broken up into a sequence of finite-horizon problems, and the agent's effective horizon is shortened. Therefore, the principle suggested by Lucas does not apply.

Another way to understand the intuition behind a positive capital income tax rate is to note that because of incomplete insurance markets, there is a precautionary motive for accumulating capital. In addition, the possibility of being borrowing-constrained in some future periods leads agents to accumulate additional capital. These two features lead to increases in their saving and hence capital accumulation and thereby lower the return on capital below the rate of time preference (Bewley n.d.; Laitner 1979, 1992; Aiyagari 1994*b*). Therefore, the features above lead to excess (i.e., greater-than-the-optimal level of) capital.[3] As I shall show, a positive tax rate on capital income will be needed to reduce capital accumulation and bring the pretax return on capital to equality with the rate of time preference.

It is well known that, in overlapping generations models, competitive equilibria may be characterized by capital overaccumulation and that government debt (equivalently, interest-bearing money) can be used to soak up excess saving and reduce capital accumulation. This fact suggests the possibility that in the Bewley (1986) class of models also, government debt may serve to eliminate excess capital accumulation and bring the return on capital to equality with the rate of time preference without a tax on capital income. However, this idea turns out to be infeasible because of a crucial feature of this class of models.

environment, as in some recent literature (e.g., Atkeson and Lucas [1992] and references therein), this is beyond the scope of the present paper. Instead, I simply impose a particular market structure as in Bewley (1986).

[3] This idea should be distinguished from the standard notion of capital overaccumulation, which refers to an inefficiently high level of capital.

This feature is that the demand for assets on the part of households for precautionary saving purposes tends to infinity as the return on the assets approaches the rate of time preference. However, the supply of capital is bounded because there is a maximum sustainable capital stock in the economy; further, the supply of government debt is bounded above because tax revenues from labor and capital are bounded above. Therefore, the supply of assets in the economy (capital plus debt) is bounded above. Consequently, it is not possible to support as an equilibrium an interest rate that is arbitrarily close to the time preference rate. Making this argument rigorous and showing that it implies that the capital income tax rate must be strictly positive even in the long run is the main goal of the paper.

A natural question is whether this result is quantitatively important, that is, whether a reasonably parameterized version of such a model can generate the observed capital income tax rate in the United States as being long-run optimal. Aiyagari (1994*a*, sec. 5) addresses this question and suggests that this is indeed possible.

The rest of this paper is organized as follows. In Section II, I describe the dynamic Ramsey optimal tax problem in a Bewley-type model with a continuum of agents subject to stochastic and idiosyncratic shocks and borrowing constraints. In Section III, I try to provide some intuition for the results by conducting steady-state analysis. In Section IV, I prove the result that the optimal capital income tax must be positive even in the long run. The Appendix contains many of the proofs.

II. A Bewley-Type Model

A. *The Environment*

Assume that there is a continuum of infinitely lived agents of size unity. Per capita variables (or averages across individuals) are distinguished from individual-specific variables by using uppercase letters for the former and lowercase letters for the latter.

1. Endowments and Technology

Agents are endowed with one unit of perfectly divisible labor each period that can be used either in the market sector or in the home sector. Let n_t and $1 - n_t$ be an agent's market work at time t and homework at time t, respectively. Home production is given by a production function $\theta_t H(1 - n_t)$, where H: $[0, 1] \rightarrow \mathbb{R}_+$ is bounded, continuously differentiable, strictly increasing, and strictly concave. In addition, $H(\cdot)$ satisfies $H(0) = 0$, $H'(0) = \infty$, and $H'(1) > 0$.

Here θ_t denotes an idiosyncratic shock to the home production of an agent in period t. Assume that θ_t is independently and identically distributed (i.i.d.) over time as well as across agents, so that there is no aggregate uncertainty.[4] Further, let $F(\theta) = \text{prob}(\theta_t \leq \theta)$ be the distribution function of θ, and assume that F has bounded support contained in $[\theta_{\min}, \theta_{\max}]$, with $\theta_{\min} > 0$.

In the market sector, production is governed by a neoclassical production function $f(K_t, N_t)$, where K_t is the per capita amount of capital in the economy, N_t is the per capita amount of market work, and $f(\cdot)$ is the per capita market output net of capital depreciation. Assume that $f(\cdot)$ is homogeneous of degree one and twice continuously differentiable. Further, $f(\cdot)$ satisfies (i) $f(0, N) = f(K, 0) = 0$; (ii) for $(K, N) \gg 0$, $f_{11} < 0$ and $f_{22} < 0$; (iii) for $K > 0$, $f_2 > 0$ and $\lim_{N\to 0} f_2 = \infty$; and (iv) for $N > 0$, $\lim_{K\to 0} f_1 = \infty$ and $\lim_{K\to\infty} f_1 = -\delta < 0$.

2. Preferences

An agent consumes the amount c_t of goods in period t, and the government consumes the amount G_t of goods (per capita) in period t. An agent's preferences are described by the following expected value of the sum of discounted utilities of private consumption and public consumption: $E_0\{\sum_{t=0}^{\infty} \beta^t[u(c_t) + U(G_t)]\}$, where $\beta \in (0, 1)$, $u(\cdot)$ is the utility from private consumption, and $U(\cdot)$ is the utility from public consumption.[5] The functions $u(\cdot)$ and $U(\cdot)$ are each assumed to be bounded, continuously differentiable, strictly increasing, and strictly concave.

Some remarks on my specification of preferences and technology (specifically, home production) may be helpful. The separability of utility in private and public consumption is convenient but not essential. It enables me to pose the consumer's optimization problem by simply ignoring the utility from public consumption. My specification of home production will lead to market labor supply's dependence only on the current market wage relative to home sector productivity; in particular, intertemporal substitution effects will be absent. Thus my specification is equivalent to assuming that there is no income

[4] The technical difficulties arising from a continuum of i.i.d. random variables (Feldman and Gilles 1985; Judd 1985) will be finessed in this paper in the same way as in Bewley (1986). The i.i.d. over time assumption is not essential to my result; see Aiyagari (1994*a*).

[5] It should be emphasized that Chamley's (1986) result that the capital income tax is zero in the long run holds for general recursive preferences, not just time-additive preferences, as is assumed here. However, the environment here, unlike Chamley's, contains uncertainty at the individual level, and results on the "income fluctuation problem" (Schechtman and Escudero 1977), which I shall rely on, are available only for time-additive preferences.

effect on nonmarket work (homework or leisure) in the more conventional specification of preferences in which nonmarket work is also an argument of the utility function $u(\cdot)$.[6] This feature is designed to turn the consumer's problem into a standard income fluctuation problem, which enables me to use results developed in that literature directly (see Bewley n.d., 1986; Laitner 1979, 1992; Chamberlain and Wilson 1984; Clarida 1987, 1990).

B. *Markets*

There are competitive markets in labor, capital services, the output good, and one-period consumption loans.

C. *Competitive Equilibrium*

1. Firms

Competition in product and factor markets and profit maximization on the part of firms imply that $w_t = f_2(K_t, N_t)$ and $r_t = f_1(K_t, N_t)$, where w_t denotes the pretax market real wage and r_t denotes the pretax real rental return on capital services.

2. Government

The government consumes the amount G_t (per capita) in period t; issues new debt in the (per capita) amount $B_{t+1} - B_t$, where B_t is the per capita debt outstanding at the beginning of period t; and taxes market labor income and interest income at the rates τ_{nt} and τ_{kt}, respectively.[7] Let $\overline{w}_t$ and $\overline{r}_t$ be the after-tax market real wage and the after-tax real rental return on capital services, respectively. Note that $\overline{w}_t = (1 - \tau_{nt})w_t$ and $\overline{r}_t = (1 - \tau_{kt})r_t$. Since there is no aggregate uncertainty, capital and consumption loans are perfect substitutes. Therefore, the pretax interest rate on one-period consumption loans (and government debt) must equal r_t.

The government budget constraint is

$$G_t + r_tB_t = B_{t+1} - B_t + \tau_{nt}w_tN_t + \tau_{kt}r_t(K_t + B_t). \tag{1}$$

[6] This may be seen by writing utility as $u(\tilde{c} + \theta H(1 - n))$, where $\tilde{c}$ is consumption of market-produced goods, i.e., consumption net of home production. Interpret $1 - n$ as leisure and θ as a taste for leisure shock. This form of the utility function implies a zero income effect on leisure and thereby leads to market labor supply's dependence only on the current market wage relative to θ. Further, a high θ shock is like a greater desire for leisure and reduces market labor supply.

[7] It is, of course, necessary to assume that while market work can be taxed, homework cannot be taxed. Otherwise, the government would effectively have access to a lump-sum tax.

Note that $\tau_{nt}w_t = w_t - \overline{w}_t = f_2(K_t, N_t) - \overline{w}_t$ and that $\tau_{kt}r_t = r_t - \overline{r}_t = f_1(K_t, N_t) - \overline{r}_t$. Making these substitutions into (1) and using the first-degree homogeneity of $f(\cdot)$, I can rewrite (1) in the following form:

$$G_t + \overline{r}_t B_t = B_{t+1} - B_t - \overline{w}_t N_t - \overline{r}_t K_t + f(K_t, N_t). \tag{2}$$

3. Consumers

An agent starts with some assets a_0 and a realized productivity shock θ_0 in period 0 and solves the following problem:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

subject to the sequence of budget constraints and borrowing constraints given by

$$c_t + a_{t+1} = \theta_t H(1 - n_t) + \overline{w}_t n_t + (1 + \overline{r}_t) a_t \tag{3}$$

and

$$0 \leq n_t \leq 1, \quad c_t \geq 0, \quad a_t \geq 0. \tag{4}$$

I shall now reformulate the consumer's optimization problem in a dynamic programming framework.

To simplify the reformulation, I start by noting that the solution to the labor allocation problem is obtained by maximizing $[\theta_t H(1 - n_t) + \overline{w}_t n_t]$ over $n_t \in [0, 1]$. This yields a supply function for market work denoted $n(\overline{w}_t/\theta_t)$. Using this, I can define an agent's total (market plus home) earnings function (denoted by $y(\theta_t, \overline{w}_t)$) as

$$y(\theta_t, \overline{w}_t) = \theta_t H\left[1 - n\left(\frac{\overline{w}_t}{\theta_t}\right)\right] + \overline{w}_t n\left(\frac{\overline{w}_t}{\theta_t}\right).$$

Note that $y(\theta_t, \overline{w}_t) \geq \theta_{\min} H(1) > 0$.

Now let

$$\overline{w}^t = \{\overline{w}_t, \overline{w}_{t+1}, \overline{w}_{t+2}, \ldots\}, \quad t \geq 0, \tag{5}$$

$$\overline{R}_t = 1 + \overline{r}_t, \quad t \geq 0, \tag{6}$$

and

$$\overline{R}^t = \{\overline{R}_t, \overline{R}_{t+1}, \overline{R}_{t+2}, \ldots\}, \quad t \geq 0. \tag{7}$$

An agent's decision problem can now be expressed in terms of the following Bellman equation, where v is the value function:

$$v(a_t, \theta_t, \overline{w}^t, \overline{R}^t) = \max\{u(c_t) + \beta E_t v(a_{t+1}, \theta_{t+1}, \overline{w}^{t+1}, \overline{R}^{t+1})\} \tag{8}$$

subject to

$$c_t + a_{t+1} = y(\theta_t, \overline{w}_t) + (1 + \overline{r}_t)a_t, \quad c_t \geq 0, a_t \geq 0, t \geq 0. \tag{9}$$

Note that in (8) the sequences $\overline{w}^t$ and $\overline{R}^t$ are deterministic.

4. Equilibrium

Let $J_t(a, \theta)$ be the cross-section cumulative distribution function of agents according to asset holdings and θ in period t, and let $J_0(a, \theta)$ be given as an initial condition. The evolution of $J_t(\cdot)$ over time will have to be determined as part of the equilibrium.

The solution to the consumer's problem in (8)–(9) will consist of the following decision rules:

$$c_t = c(a_t, \theta_t, \overline{w}^t, \overline{R}^t) \tag{10}$$

and

$$a_{t+1} = a(a_t, \theta_t, \overline{w}^t, \overline{R}^t). \tag{11}$$

Using (11) and the probability distribution function for θ, I can update the given initial distribution $J_0(a, \theta)$ to obtain $J_t(\cdot)$ for all t. Note that these distributions for $t \geq 1$ will depend on the sequence of after-tax prices. To make this dependence explicit, I shall denote them by $J_t(a, \theta, \overline{w}^0, \overline{R}^0)$. Per capita consumption C_t is then given by

$$C_t = \int c(a, \theta, \overline{w}^t, \overline{R}^t) dJ_t(a, \theta, \overline{w}^0, \overline{R}^0) \equiv \chi_t(\overline{w}^0, \overline{R}^0). \tag{12}$$

Per capita market work (denoted by N_t previously) is given by

$$N_t = \int n\left(\frac{\overline{w}_t}{\theta}\right) dF(\theta) \equiv \nu(\overline{w}_t). \tag{13}$$

I can also write per capita output of home-produced goods (denoted by H_t) as

$$H_t = \int \theta H\left[1 - n\left(\frac{\overline{w}_t}{\theta}\right)\right] dF(\theta) \equiv \eta(\overline{w}_t). \tag{14}$$

Note that $H_t = \eta(\overline{w}_t) \leq \theta_{\max} H(1) < \infty$.

The resource constraint for this economy can now be written as

$$f(K_t, \nu(\overline{w}_t)) + \eta(\overline{w}_t) + K_t - K_{t+1} - G_t - \chi_t(\overline{w}^0, \overline{R}^0) \geq 0, \quad t \geq 0. \tag{15}$$

In (15), $\nu(\bar{w}_t)$ is per capita market work (from [13]), $\eta(\bar{w}_t)$ is per capita home production (from [14]), and $\chi_t(\bar{w}^0, \bar{R}^0)$ is per capita consumption (from [12]).

Given time paths for $\bar{w}_t$ and $\bar{r}_t$ and the stochastic process for θ_t, individuals choose processes for consumption and asset accumulation to solve the problem in (8)–(9). This results in a time path for per capita consumption and per capita assets. Together with a time path for G_t, the government budget constraint (2) then determines the time path for government debt, since K_t must equal per capita assets at time t (denoted A_t) minus B_t. The time paths for G_t, $\bar{w}_t$, and $\bar{r}_t$ are consistent with equilibrium if the resulting time paths for per capita capital and consumption clear the goods market in each period, that is, satisfy the resource constraint (15).

The description above is now formally summarized in the following definition of a competitive equilibrium.

DEFINITION. For given initial conditions K_0 and $J_0(\cdot)$ and time paths $\{G_t, \bar{w}_t, \bar{r}_t\}$, a *competitive equilibrium* consists of a value function $v(\cdot)$, consumer's decision rules $c(\cdot)$ and $a(\cdot)$, and sequences $\{J_t(\cdot), K_t\}$ such that the following conditions hold: (i) $v(\cdot)$ solves the Bellman equation (8), (ii) $c(\cdot)$ and $a(\cdot)$ attain $v(\cdot)$, (iii) $\{J_t(\cdot)\}$ is generated from $J_0(\cdot)$ and $a(\cdot)$, and (iv) $\{K_t\}$ satisfies (15).

D. *The Optimal Tax Problem*

The government's optimal tax problem is to choose time paths for G_t, $\bar{w}_t$, and $\bar{r}_t$ consistent with equilibrium such that the utilitarian social welfare, $\int v(a, \theta, \bar{w}^0, \bar{R}^0)dJ_0(a, \theta) + \Sigma\beta^t U(G_t)$, is maximized.

More formally, the government's optimal tax problem may be written as

$$\max \left[\int v(a, \theta, \bar{w}^0, \bar{R}^0)dJ_0(a, \theta) + \Sigma\beta^t U(G_t)\right] \tag{16}$$

subject to (15) and $(\bar{w}_t, \bar{R}_t, G_t, K_{t+1}) \geq 0$, $t \geq 0$, by choice of $\{\bar{w}_t, \bar{R}_t, G_t, K_{t+1}\}$ for $t \geq 0$.

Note that in the problem above, the only constraint (aside from nonnegativity constraints) is the resource constraint (15). The government budget constraint need not be included as an additional constraint since the individual decision rules automatically satisfy individual budget constraints, which together with the resource constraint imply the government budget constraint.

III. Steady-State Analysis

In this section, by analyzing steady states, I try to provide some intuition for the way these types of models work and why they necessarily imply a positive tax on capital income. The formal analysis of the Ramsey optimal tax problem is postponed to the next section.

It is convenient to index a steady state by τ_k (the capital income tax), τ_n (the wage tax), and government consumption G. Individual optimization and asset market clearing will then be used to determine $\bar{r}$ (the after-tax return to capital). I shall show that $\bar{r}$ is always less than ρ ($\equiv [1/\beta] - 1$, the time preference rate). In the next section, I shall show that r (the pretax return to capital) equals ρ. It then follows that the steady-state capital income tax is always positive.

I now describe how $\bar{r}$ is determined. The steady-state version of the government budget constraint (1) can be manipulated to express $K + B$ as a function of $\bar{r}$ (the after-tax return to capital) as follows:[8]

$$K + B = \frac{K}{1 - \tau_k} + \frac{\tau_n wN - G}{\bar{r}}. \tag{17}$$

For fixed values of τ_k and τ_n, note that K, w, and N are each decreasing functions of $\bar{r}$. The reason is that as $\bar{r}$ increases, so does r, which reduces w and hence $\bar{w}$ and thereby N (from [13]). Further, K/N is lower (since r is higher), and hence $K = N(K/N)$ is also lower. Furthermore, K, w, and $N \rightarrow 0$ as $\bar{r} \rightarrow \infty$, and K and $w \rightarrow \infty$ as $\bar{r} \rightarrow 0$. Therefore, the graph of $K + B$ versus $\bar{r}$ looks as shown in figure 1.[9]

For a given value of $\bar{r}$ (assumed less than ρ), the consumer's problem is a stationary problem in the steady state and is described as

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

subject to

$$c_t + a_{t+1} = y(\theta_t, \bar{w}) + (1 + \bar{r})a_t, \quad c_t \geq 0, a_t \geq 0, t \geq 0. \tag{18}$$

The solution to the consumer's decision problem yields a stationary decision rule for asset accumulation, $a_{t+1} = a(a_t, \theta_t; \bar{r})$. This decision rule together with the distribution of θ_t determines a Markov process for assets a_t. I make the following assumption, which ensures that the asset accumulation process for an individual remains bounded and that there exists a unique long-run distribution of assets that is stable; that is, starting from any initial distribution of assets, the sequence of

[8] Writing rB as $r(K + B) - rK$, I can rewrite the steady-state version of (1) as $\bar{r}(K + B) = rK + \tau_n wN - G$. Dividing through by $\bar{r}$ and noting that $r = \bar{r}/(1 - \tau_k)$ yields (17).

[9] Note that $K + B$ need not be monotonically decreasing in $\bar{r}$ and may have a negative segment.

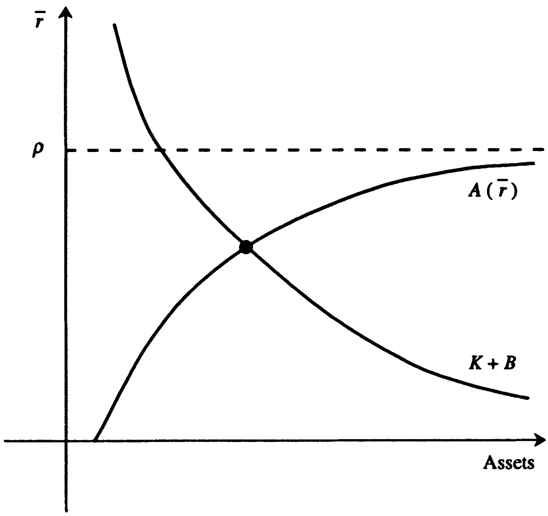

FIG. 1

distributions of assets converges to the unique long-run distribution (Schechtman and Escudero 1977; Clarida 1987, 1990).

ASSUMPTION 1. The function u is twice differentiable, and there exist positive numbers μ^* and c^* such that $-cu''/u' \leq \mu^*$ for all $c \geq c^*$.[10]

Let the unique stationary distribution be denoted by $J(a; \bar{r})$. Average asset holdings (denoted $A(\bar{r})$) are given by $A(\bar{r}) = \int a dJ(a; \bar{r})$. Here $A(\cdot)$ is a continuous function of $\bar{r}$ and tends to infinity as $\bar{r} \rightarrow \rho$ (Clarida 1990). A possible graph of $A(\bar{r})$ versus $\bar{r}$ is also shown in figure 1. The value of $\bar{r}$ is determined as the solution to the asset market equilibrium condition

$$K + B = A(\bar{r}), \tag{19}$$

that is, by the intersection of the two curves in figure 1. Note that by virtue of the properties of the two curves in figure 1, a solution is guaranteed to exist. Once $\bar{r}$ is known, r, w, $\bar{w}$, N, and K are known, and hence B can be found from (19).

I now give some intuition for why, with incomplete markets, $\bar{r} < \rho$ in the steady state. If there were no idiosyncratic shocks (equivalently, if markets were complete), then the consumer's asset demand func-

[10] Without this assumption, an individual's assets may go to infinity (almost surely), and no long-run distribution would exist. Schechtman and Escudero (1977) give such an example using a negative exponential utility function that violates assumption 1. As I shall note shortly, this assumption is not essential for my result, but simplifies the discussion of steady states.

tion $A(\cdot)$ would coincide with the vertical axis for $\bar{r} < \rho$ and would be perfectly elastic at $\bar{r} = \rho$. Therefore, $\bar{r} = \rho$ in a steady state, regardless of the values of τ_k, τ_n, or G. In particular, if there were no capital income tax, then $r = \rho$, which is the standard result that the capital stock satisfies the modified golden rule.

However, when there are idiosyncratic shocks (and markets are incomplete), the individual has a precautionary motive for accumulating assets and will hold positive amounts of assets on average, even when $\bar{r} < \rho$, in order to buffer earnings shocks and smooth consumption.[11] The borrowing constraint also plays a role since the possibility of being borrowing-constrained in future periods serves to enhance the individual's desire for current assets.[12]

More crucially, asset demand $A(\cdot) \to \infty$ as $\bar{r} \to \rho$ from below. The intuition is that when $\bar{r} = \rho$, the individual would like to maintain a smooth marginal utility of consumption profile. However, since there is some probability of receiving a sufficiently long string of bad θ's, the only way to maintain a smooth marginal utility of consumption profile is to have infinite assets.

It follows that with incomplete markets, the steady-state equilibrium value of $\bar{r}$ is always less than ρ, again regardless of the values of τ_k, τ_n, or G. As τ_k is varied, the curve marked $K + B$ in figure 1 shifts (since for a fixed $\bar{r}$, r varies and hence K, w, and N vary; see [17]) and leads to different steady-state values of $\bar{r}$, all of which will be less than ρ. Therefore, it must be the case that the return on capital r consistent with zero capital income tax is strictly less than ρ.[13] Consequently, under incomplete markets, there will always be capital overaccumulation if there is no tax on capital; that is, the capital stock will be higher than the modified golden rule level. The additional capital accumulation and the implied higher saving rate may be attributed to precautionary saving.

As I shall prove in the next section, the solution to the Ramsey optimal tax problem has the feature that (in the steady state) the modified golden rule holds; that is, the pretax return on capital

[11] If $\bar{r}$ is sufficiently low (close to negative unity, e.g.), then the individual will not ever hold any assets and will simply consume his earnings in each period.

[12] Even though I have ruled out borrowing, this is not essential to the analysis. If $\bar{r} > 0$, then the present value budget constraint and nonnegativity of consumption imply that $a_t \geq -y(\theta_{\min}, \bar{w})/\bar{r}$. That is, there is always a borrowing limit in this class of models. The intuition is that if ever $a_t < -y(\theta_{\min}, \bar{w})/\bar{r}$, then a sufficiently long series of bad θ's will force the consumer to increase his debt level to such an extent that, from then on, even if he received the best θ's forever, he would never be able to pay off his debt (see Aiyagari 1994*b*).

[13] It should be clear that this will continue to be the case even without assumption 1 since, in that case, the $A(\cdot)$ curve in fig. 1 may tend to infinity as $\bar{r}$ tends to some value less than ρ; see n. 10.

equals ρ (proposition 1). From the discussion above, this can be achieved only by having a positive tax on capital income, thereby eliminating capital overaccumulation.

IV. The Optimal Capital Income Tax in the Long Run

In this section, I return to the analysis of the Ramsey optimal tax problem formulated in Section II and show that the limiting pretax interest rate equals the time preference rate (proposition 1) and that the limiting after-tax interest rate is strictly less than the time preference rate (proposition 2). It follows that the optimal capital income tax rate must be positive even in the long run.

An existence result for the optimal tax problem is provided in Aiyagari (1994*a*). In what follows, it is assumed that a solution to the optimal tax problem converges to a steady state in which factor prices, per capita capital, per capita private and government consumption, per capita market work, and per capita homework converge to limiting values that are all strictly positive and finite.[14] This is formalized as assumption 2 below.

ASSUMPTION 2. The solution to the optimal tax problem (16) is such that $(\overline{R}_t, G_t, N_t, 1 - N_t) \rightarrow (1 + \bar{r}^*, G, N, 1 - N) \gg 0$ and is finite.

A. *The Long-Run Capital Income Tax*

PROPOSITION 1. $r_t \equiv f_1(K_t, \nu(\overline{w}_t)) \rightarrow \rho \equiv (1 - \beta)/\beta$.

Proof. The result follows directly from the steady-state version of the following Euler equation for government consumption in the planning problem (16):

$$-U'(G_t) + \beta U'(G_{t+1})[f_1(K_{t+1}, \nu(\overline{w}_{t+1})) + 1] = 0. \tag{20}$$

The nature of the variational experiment underlying the proof of proposition 1 is the following. Imagine that the government increases investment at time t by one unit (i.e., $\Delta K_{t+1} = 1$) and decreases government consumption by one unit ($\Delta G_t = -1$). The reduced public consumption is met by a reduction in new debt, also by one unit

[14] It seems quite difficult to guarantee that a solution to the optimal tax problem converges to a steady state. Even for the simpler version of the model without a government sector, results are available only for steady states with i.i.d. over time shocks (see Bewley n.d.; Laitner 1979, 1992; Clarida 1990). There is no existence result or convergence to a steady state result for an arbitrarily given initial condition—or even an example. The technical difficulty is that the distribution of assets across individuals is an aggregate state variable that is, in general, changing over time. In any case, this assumption is also made by Chamley (1986) and Lucas (1990). However, Chamley does provide an example that exhibits convergence to the unique steady state.

($\Delta B_{t+1} = -1$). As a consequence, the resource constraint and the government budget constraint continue to be satisfied at time t, and per capita assets do not change ($\Delta A_{t+1} = 0$). Further, individuals are unaffected by these changes, so that per capita consumption, per capita market work, per capita home production, and per capita desired assets do not change. At time $t + 1$, suppose that the government increases government consumption by the amount of the increment in output due to increased investment, that is, $\Delta G_{t+1} = f_1(K_{t+1}, \nu(\overline{w}_{t+1})) + 1$, and increases new debt issue so as to maintain B_{t+2} at the same level as before the experiment. It is easy to verify that the resource constraint and the budget constraint continue to be satisfied at time $t + 1$ as well. The first term in (20) measures the utility loss from reduced government consumption at time t, and the second term in (20) measures the utility gain from increased government consumption at time $t + 1$ discounted by β.[15]

Proposition 1 says that in the long run, the pretax return to capital must equal the rate of time preference. Therefore, to show that the capital income tax is strictly positive, even in the long run, I need to show that $\bar{r}^* \equiv \lim_{t\to\infty} \bar{r}_t < \rho$. This is shown in the Appendix via a series of claims. The proof is by contradiction; that is, I rule out $\bar{r}^* \geq \rho$ by showing that per capita assets go to infinity. Since per capita capital is bounded (there is a maximum sustainable capital stock) and per capita government debt is bounded (since tax revenues are bounded), the result follows.

PROPOSITION 2. $\bar{r}^* < \rho$.

Proof. See the Appendix.

It remains to show that in a complete markets version of this model, the capital income tax is zero in the long run, that is, $\bar{r}^* = r$. This follows because, under complete markets, the model in Section II is a special case of that in Chamley (1986). The complete markets case is equivalent to eliminating the idiosyncratic uncertainty, that is, setting $\theta_t = E(\theta)$, across agents as well as t. Assume, for simplicity, that initially all agents have the same assets; that is, the agents are identical. In this case, the intertemporal Euler condition for an agent is given by $u'(c_t) = \beta(1 + \bar{r}_{t+1})u'(c_{t+1})$. Therefore, in the steady state, $\bar{r}^* = \rho$. Proposition 1 continues to hold in the complete markets case, implying that $r = \rho$. Hence, it follows that with complete markets, the capital income tax is zero in the long run.

[15] The argument underlying the proof of proposition 1 is intuitively easier when government consumption is endogenous, but the result does not hinge on this modeling feature. If government consumption were exogenous, then $U'(G_t)$ in (20) would be replaced by the nonnegative multiplier on the constraint (15), and the result would still go through.

In contrast, with incomplete insurance markets and borrowing constraints, the capital income tax rate is positive, even in the long run.

Appendix

Proof of Proposition 2

I start by stating some simple properties of the solution to the agent's optimization problem in (8)–(9) that will be needed later. Let l_∞ be the space of bounded sequences with the sup norm (denoted $|\cdot|_\infty$), and let l_∞^+ be the nonnegative orthant of l_∞. Let $\Theta \equiv [\theta_{\min}, \theta_{\max}]$. Let $C = \{v: \mathbb{R}_+ \times \Theta \times l_\infty^+ \times l_\infty^+ \rightarrow R \mid v \text{ continuous and bounded}\}$, and let the norm on C be the sup norm. The following proposition consists of easy extensions of standard results, and hence the proof is omitted (see, e.g., Stokey, Lucas, and Prescott 1989, chap. 9).

CLAIM 1. (i) There exists a unique $v \in C$ that solves the functional equation in (8)–(9); further, $v = \sup E_t\{\Sigma_{j=0}^{\infty} \beta^j u(c_{t+j})\}$ subject to (9). (ii) The function v is strictly increasing and strictly concave in a_t. (iii) There exist unique decision rules (10)–(11) that attain v. (iv) The decision rules (10)–(11) are continuous and nondecreasing in a_t. (v) The function v is continuously differentiable in a_t, and $v_1(a_t, \theta_t, \overline{w}^t, \overline{R}^t) = (1 + \overline{r}_t)u'(c_t)$. (vi) The solution to the maximization problem on the right side of (8)–(9) is characterized by $u'(c_t) \geq \beta E_t v_1(a_{t+1}, \theta_{t+1}, \overline{w}^{t+1}, \overline{R}^{t+1})$, with equality if $a_{t+1} > 0$, where E_t denotes expectation conditional on information at time t.

Now I show that $\overline{r}^* \leq \rho$. This result uses a special case of theorems 1 and 2 of Chamberlain and Wilson (1984, pp. 12, 15).

CLAIM 2. $\overline{r}^* \leq \rho$.

Proof. Suppose, if possible, that $\overline{r}^* > \rho$. Let $\zeta_t = \beta^t \Pi_{j=0}^{t} (1 + \overline{r}_j)$, and note that $\zeta_t \rightarrow \infty$.

From parts v and vi of claim 1, the following intertemporal Euler equation holds for a typical agent:

$$u'(c_t) \geq \beta(1 + \overline{r}_{t+1})E_t\{u'(c_{t+1})\}, \quad \text{with equality if } a_{t+1} > 0. \tag{A1}$$

By multiplying both sides of (A1) by ζ_t, I can rewrite it as

$$\zeta_t u'(c_t) \geq \zeta_{t+1} E_t\{u'(c_{t+1})\}, \quad \text{with equality if } a_{t+1} > 0. \tag{A2}$$

It follows that $\zeta_t u'(c_t)$ is a nonnegative supermartingale. Further, $y(\theta_t, \overline{w}_t) \geq \theta_{\min} H(1) > 0$ implies that $\zeta_0 u'(c_0) < \infty$. Therefore, $\zeta_t u'(c_t)$ converges with probability one to a finite random variable (Doob 1953, p. 324, theorem 4.1s). Since $\zeta_t \rightarrow \infty$, it follows that $u'(c_t) \rightarrow 0$ with probability one and hence that $c_t \rightarrow \infty$ with probability one. Since it must hold for all individuals, this implies that per capita consumption $C_t \rightarrow \infty$. However, assumption 2 and proposition 1 imply that $K_t \rightarrow K > 0$ and is finite. The resource constraint (15) then implies that $C_t \rightarrow C < \infty$, which is a contradiction. Therefore, $\overline{r}^* \leq \rho$. Q.E.D.

Now I rule out the possibility that $\overline{r}^* = \rho$. This is done by showing that when $\overline{r}^* = \rho$, per capita assets go to infinity. However, since per capita capital is bounded (there is a maximal sustainable capital stock) and per capita government debt is bounded above (because tax revenues are bounded above), this leads to a contradiction. Thus I establish that $\bar{r}^* \neq \rho$. Hence $\bar{r}^* < \rho$.

Consider the following stationary problem (denoted $P(S, \rho)$):

$$\max E_0\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\}$$

subject to

$$c_t + a_{t+1} = y(\theta_t, \bar{w}^*) + (1 + \rho)a_t, \quad c_t \geq 0, a_t \geq 0, t \geq 0. \tag{A3}$$

In contrast, the original problem with constraints (9) is nonstationary and is denoted $P(NS)$. Note that when $\bar{r}^* = \rho$, problem $P(S, \rho)$ is obtained by substituting the limiting values of $\bar{w}_t$ and $\bar{r}_t$ in problem $P(NS)$. I shall use the result that for problem $P(S, \rho)$, $E(a_t|a_0 = 0, \theta_0, P(S, \rho)) \to \infty$. Using this, I shall show that when $\bar{r}^* = \rho$, $E(a_t|a_0, \theta_0, P(NS)) \to \infty$. This implies that per capita assets for $P(NS)$ go to infinity because

$$A_t = \int a dJ_t(a, \theta, \bar{w}^0, \bar{R}^0) = \int E(a_t|a_0, \theta_0, P(NS)) dJ_0(a_0, \theta_0).$$

CLAIM 3. $E(a_t|a_0 = 0, \theta_0, P(S, \rho)) \to \infty$.

Proof. See corollary 2 of Chamberlain and Wilson (1984, p. 26). They show that $\text{prob}(\lim_{t\to\infty} c_t = \infty) = 1$. It follows that $\text{prob}(\lim_{t\to\infty} a_t = \infty) = 1$, since $c_t \leq y(\theta_t, \bar{w}^*) + (1 + \rho)a_t$. Therefore, $E(a_t|a_0 = 0, \theta_0, P(S, \rho)) \to \infty$. Q.E.D.

CLAIM 4. If $\bar{r}^* = \rho$, then $E(a_t|a_0, \theta_0, P(NS)) \to \infty$.

Proof. Since the asset accumulation decision rule (11) is nondecreasing in a_t (part iv of claim 1), it follows that $E(a_t|a_0, \theta_0, P(NS)) \geq E(a_t|a_0 = 0, \theta_0, P(NS))$. Therefore, it is sufficient to show that $E(a_t|a_0 = 0, \theta_0, P(NS)) \to \infty$. So suppose to the contrary that $E(a_t|a_0 = 0, \theta_0, P(NS)) \not\to \infty$. Then it must be true that, for any time τ, $E(a_{t+\tau}|a_\tau = 0, \theta_\tau, P(NS)) \not\to \infty$. The reason is that

$$E(a_{t+\tau}|a_0 = 0, \theta_0, P(NS)) = E(E(a_{t+\tau}|a_\tau, \theta_\tau, P(NS))|a_0 = 0, \theta_0, P(NS))$$

and

$$E(a_{t+\tau}|a_\tau, \theta_\tau, P(NS)) \geq E(a_{t+\tau}|a_\tau = 0, \theta_\tau, P(NS)),$$

again by part iv of claim 1. Therefore, there exists a subsequence of times $\{t_j + \tau\}$ and a number M such that

$$E(a_{t_j+\tau}|a_\tau = 0, \theta_\tau, P(NS)) < M < \infty, \quad \text{for all } t_j > 0. \tag{A4}$$

Since $E(a_t|a_0 = 0, \theta_0, P(S, \rho)) \to \infty$ (claim 3), there exists $T < \infty$ such that, for any τ,

$$E(a_{t+\tau}|a_\tau = 0, \theta_\tau, P(S, \rho)) > M + 1, \quad t \geq T. \tag{A5}$$

In view of (A4), I can choose T in such a way that

$$E(a_{T+\tau}|a_\tau = 0, \theta_\tau, P(NS)) < M < \infty. \tag{A6}$$

Let $\bar{w}(S) = (\bar{w}^*, \bar{w}^*, \bar{w}^*, \ldots)$ and $\bar{R}(S) = (1 + \rho, 1 + \rho, 1 + \rho, \ldots)$. Let A_T be an upper bound on asset holdings that can be attained in T periods starting from zero assets at any time τ in $P(NS)$. Such an upper bound exists (independently of the starting time τ) since $\{\overline{w}_t\}$, $\{\overline{R}_t\}$, and $y(\theta_t, \overline{w}_t)$ are bounded. It follows that $E\{a_{T+\tau}|a_\tau = 0, \theta_\tau, P(NS)\} \leq A_T$ for all τ. Now note that $E\{a_{T+\tau}|a_\tau = 0, \theta_\tau, P(NS)\}$ depends only on $\overline{w}^\tau$ and $\overline{R}^\tau$. Therefore, by making τ suitably large, I can make $|\overline{w}^\tau - \overline{w}(S)|_\infty$ and $|\overline{R}^\tau - \overline{R}(S)|_\infty$ as small as I like. Hence, by the continuity of the asset accumulation decision rule (11)—part iv of claim 1—I can choose a τ sufficiently large such that

$$|E(a_{T+\tau}|a_\tau = 0, \theta_\tau, P(NS)) - E(a_{T+\tau}|a_\tau = 0, \theta_\tau, P(S, \rho))| < 1 \qquad \text{(A7)}$$

holds. However, (A5), (A6), and (A7) are mutually contradictory. Therefore, $E\{a_t|a_0 = 0, \theta_0, P(NS)\} \rightarrow \infty$. Hence $E\{a_t|a_0, \theta_0, P(NS)\} \rightarrow \infty$. Q.E.D.

PROPOSITION 2. $\bar{r}^* < \rho$.

Proof. By claim 2, $\bar{r}^* \leq \rho$. So suppose, if possible, that $\bar{r}^* = \rho$.

First, I show that $\{K_t\}$ is bounded. From the resource constraint (15),

$$K_{t+1} \leq f(K_t, 1) + K_t + H_t \leq f(K_t, 1) + K_t + \theta_{\max} H(1).$$

Let K' satisfy $f(K', 1) + \theta_{\max} H(1) = 0$. Such a K' exists since $f(\cdot)$ is output net of depreciation and, by assumption, $\lim_{K\rightarrow\infty} f_1 < 0$. Define $K_{\max} = \max[K_0, K']$, where K_0 is the initial per capita capital. Then it is obvious that $K_t \leq K_{\max}$, for all t.

Next I show that $\{B_t\}$ is bounded. From the government budget constraint (2),

$$B_t \leq \frac{B_{t+1} + f(K_t, 1)}{1 + \bar{r}_t}. \qquad \text{(A8)}$$

Let $\overline{\gamma}_t = \Pi_{j=0}^{t} (1 + \bar{r}_j)^{-1}$. Since $\bar{r}_t \rightarrow \rho > 0$, consumer optimization implies that $\lim_{j\rightarrow\infty} \overline{\gamma}_{t+j} A_{t+j+1} = 0$ (almost surely) and hence that $\lim_{j\rightarrow\infty} \overline{\gamma}_{t+j} A_{t+j+1} = 0$. Since $B_t = A_t - K_t$ and $\{K_t\}$ is bounded, it follows that $\lim_{j\rightarrow\infty} \overline{\gamma}_{t+j} B_{t+j+1} = 0$. Using this in (A8) and noting that $\{f(K_t, 1)\}$ is bounded above, I can conclude that $\{B_t\}$ is bounded above. Therefore, $\{K_t + B_t\}$ and hence per capita assets are bounded above. This contradicts claim 4 and shows that $\bar{r}^* \neq \rho$. This fact together with claim 2 establishes that $\bar{r}^* < \rho$. Q.E.D.

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