Eletromagnetismo Avançado

1º ciclo
Aula de 27 de agosto

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

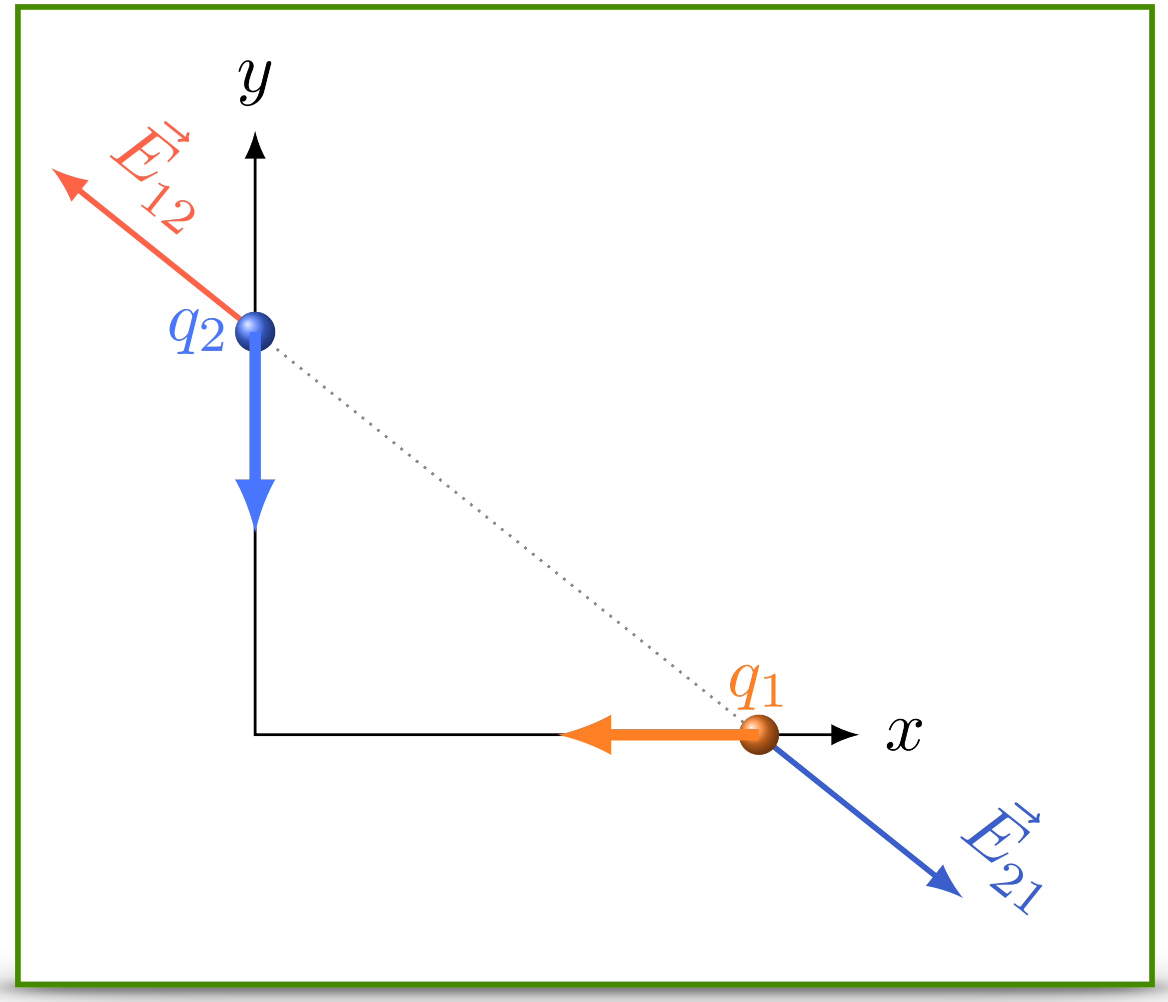
2. Energia

$$\frac{\partial}{\partial t}\left(u_{mec} + u_{em}\right) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Leis de conservação

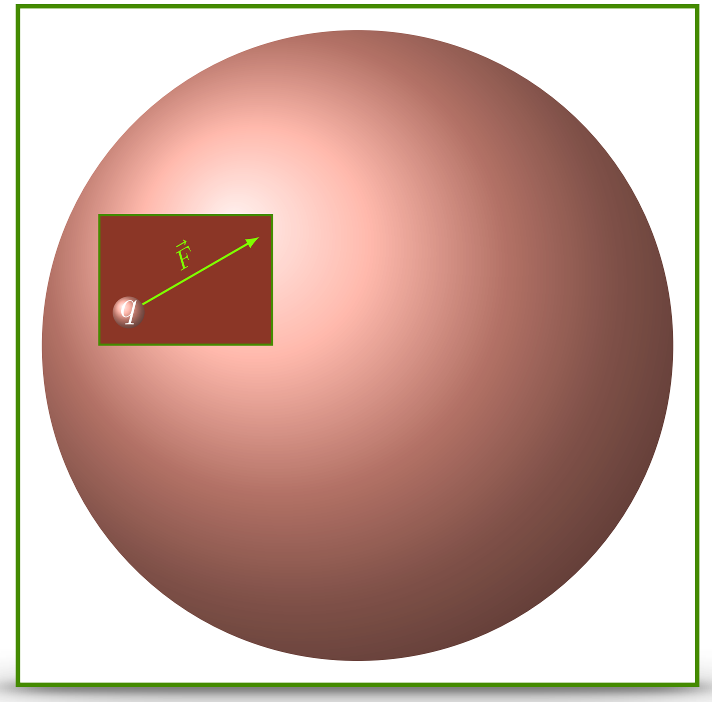
3. Momento



Leis de conservação

3. Momento

$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}} \; \mathrm{d}\tau$$

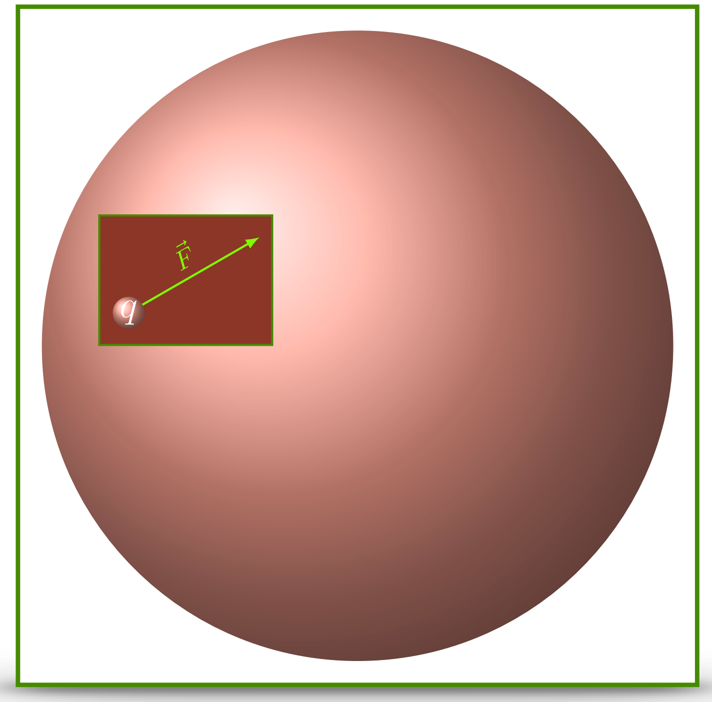


Leis de conservação

3. Momento

$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}\ \mathrm{d}\tau$$

$$\vec{\mathbf{f}} = \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}$$



Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}] - \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}] - \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

Leis de conservação

3. Momento

$$(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} = ?$$

COMPARE COM

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} E_x E_x & E_x E_y & E_x E_z \\ E_y E_x & E_y E_y & E_y E_z \\ E_z E_x & E_z E_y & E_z E_z \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Leis de conservação

3. Momento

$$(\vec{\nabla}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\nabla})\vec{\mathbf{E}} = ?$$

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} E_x E_x & E_x E_y & E_x E_z \\ E_y E_x & E_y E_y & E_y E_z \\ E_z E_x & E_z E_y & E_z E_z \end{bmatrix} = \begin{bmatrix} (\vec{\nabla}\cdot\vec{\mathbf{E}})E_x + \vec{\mathbf{E}}\cdot\vec{\nabla}E_x & \{\}E_y & \{\}E_z \end{bmatrix}$$

ANÁLOGOS, PARA E_y e E_z

Leis de conservação

3. Momento

$$(\vec{\nabla}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\nabla})\vec{\mathbf{E}} = ?$$

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \times \begin{bmatrix} E_x & E_y & E_z \end{bmatrix} = \text{TENSOR}$$

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} E_xE_x & E_xE_y & E_xE_z \\ E_yE_x & E_yE_y & E_yE_z \\ E_zE_x & E_zE_y & E_zE_z \end{bmatrix} = \begin{bmatrix} (\vec{\nabla}\cdot\vec{\mathbf{E}})E_x + \vec{\mathbf{E}}\cdot\vec{\nabla}E_x & \{\}E_y & \{\}E_z \end{bmatrix}$$

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\nabla}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\nabla})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\nabla}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\vec{\nabla})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\nabla}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

ESCREVER COMO DERIVADA DE TENSOR

Leis de conservação

3. Momento

$$(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} = ?$$

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}] - \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

TENSOR DE MAXWELL

$$\mathbb{T} \equiv \epsilon_0 \begin{bmatrix} E_xE_x & E_xE_y & E_xE_z \\ E_yE_x & E_yE_y & E_yE_z \\ E_zE_x & E_zE_y & E_zE_z \end{bmatrix} + \frac{1}{\mu_0} \begin{bmatrix} B_xB_x & B_xB_y & B_xB_z \\ B_yB_x & B_yB_y & B_yB_z \\ B_zB_x & B_zB_y & B_zB_z \end{bmatrix} - \frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\nabla}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\nabla})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\nabla}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\vec{\nabla})\vec{\mathbf{B}}]$$

$$-\frac{1}{2}\vec{\nabla}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0}B_i B_j - \frac{1}{2}\delta_{ij}(\epsilon_0 E_{ii}^2 + \frac{1}{\mu_0}B_{ii}^2)$$

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Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0}B_i B_j - \frac{1}{2}\delta_{ij}(\epsilon_0 E_{ii}^2 + \frac{1}{\mu_0}B_{ii}^2)$$

$$\vec{\mathbf{f}} = \vec{\boldsymbol{\nabla}}\cdot\mathbb{T} - \epsilon_0\mu_0\frac{\partial\vec{S}}{\partial t}$$

$$\vec{F} = \int_{\mathcal{V}}\vec{\boldsymbol{\nabla}}\cdot\mathbb{T}\,\mathrm{d}\tau - \epsilon_0\mu_0\int_{\mathcal{V}}\frac{\partial\vec{S}}{\partial t}\,\mathrm{d}\tau$$

Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$-\frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

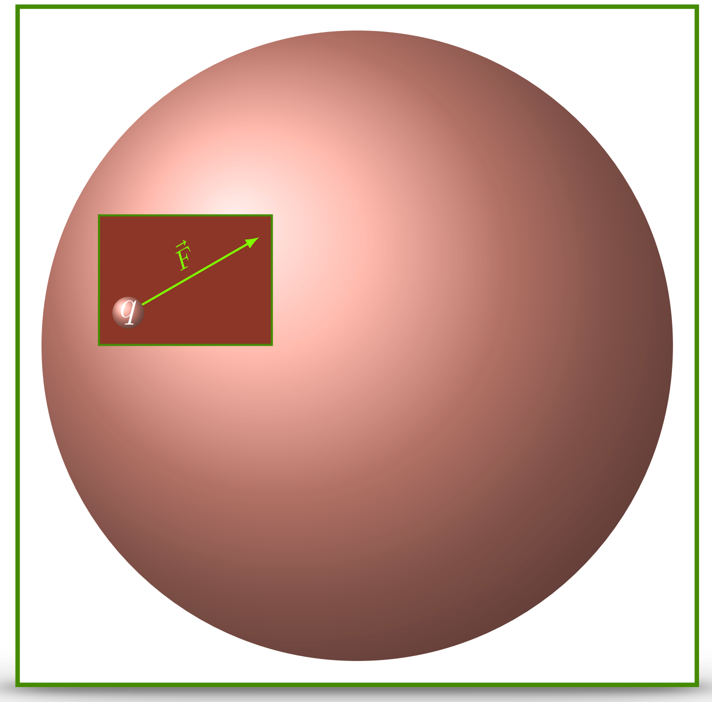
$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0}B_i B_j - \frac{1}{2}\delta_{ij}(\epsilon_0 E_{ii}^2 + \frac{1}{\mu_0}B_{ii}^2)$$

$$\vec{\mathbf{f}} = \vec{\boldsymbol{\nabla}}\cdot\mathbb{T} - \epsilon_0\mu_0\frac{\partial\vec{S}}{\partial t}$$

$$\vec{F} = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}}\cdot\mathbb{T}\,\mathrm{d}\tau - \epsilon_0\mu_0\int_{\mathcal{V}}\frac{\partial\vec{S}}{\partial t}\,\mathrm{d}\tau$$

TEOREMA DE GAUSS

$$\vec{F} = \int_{\mathcal{A}} \mathbb{T}\cdot\hat{\mathbf{n}}\,\mathrm{d}a - \epsilon_0\mu_0\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\vec{S}\,\mathrm{d}\tau$$



Pratique o que aprendeu

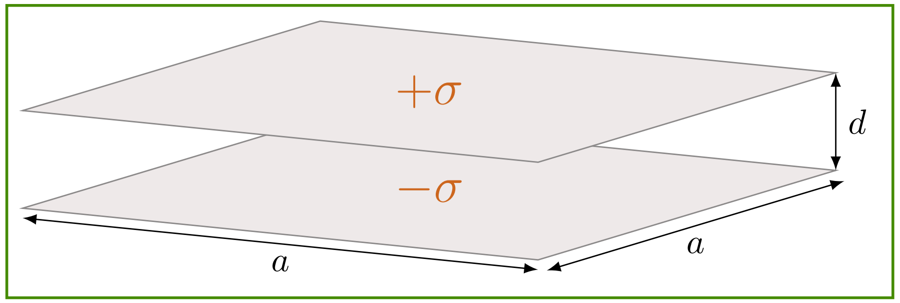
$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}}\, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S}\, \mathrm{d}\tau$$

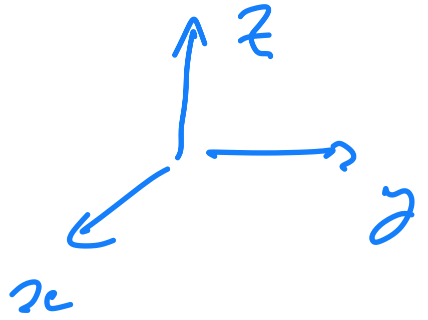
QUAL É A FORÇA SOBRE PLACA DE BAIXO?

$\vec{B} = 0$

$\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{z}$

TENSOR DE MAXWELL = ?

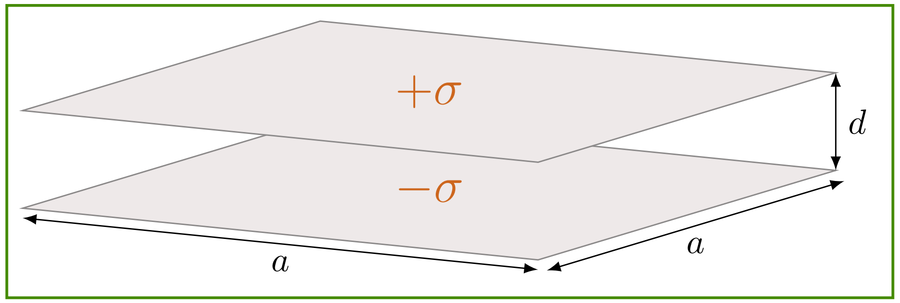




Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}}\, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S}\, \mathrm{d}\tau$$

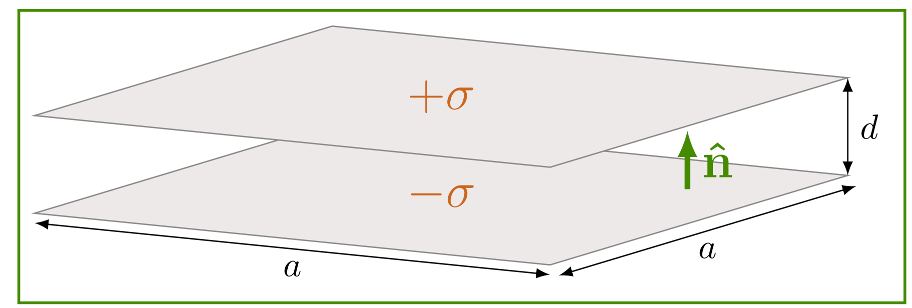
$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$

$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

PRODUTO MATRICIAL

REDUZ DIMENSÃO:

TENSOR 3X3

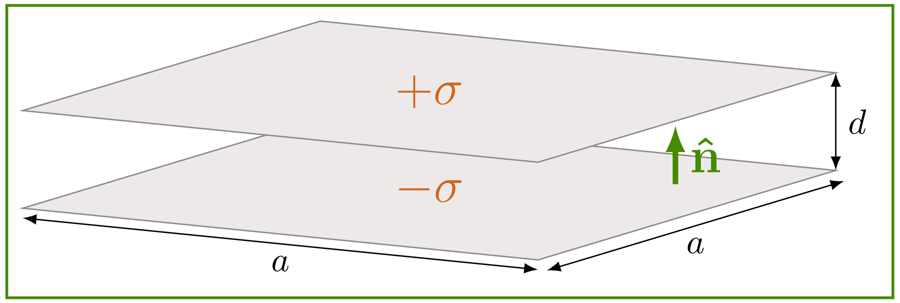
BAIXA PARA

VETOR 3X1

Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}}\, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S}\, \mathrm{d}\tau$$

$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sigma^2}{2\epsilon_0} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sigma^2}{2\epsilon_0} \hat{z}$$