Eletromagnetismo Avançado

1º ciclo
Aula de 25 de agosto

Leis de conservação

1. Carga elétrica

$$\vec{\boldsymbol{\nabla}} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

2. Energia

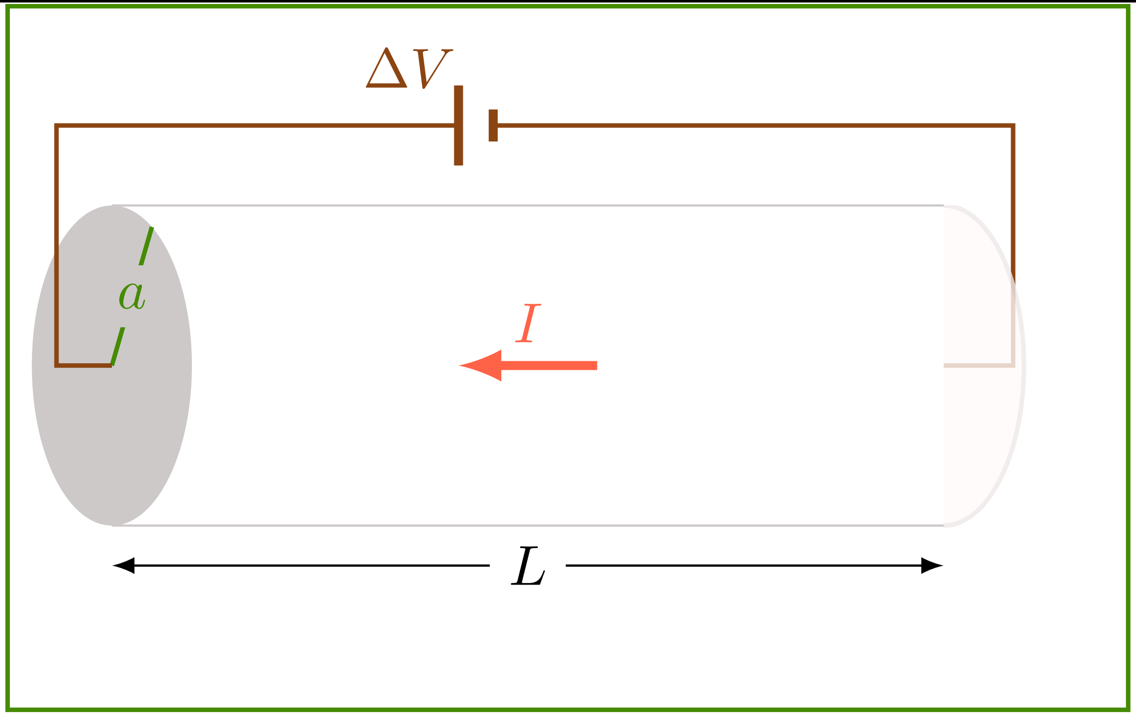
$$\frac{\partial}{\partial t}\left(u_{mec} + u_{em}\right) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$\vec{S} = ?$$

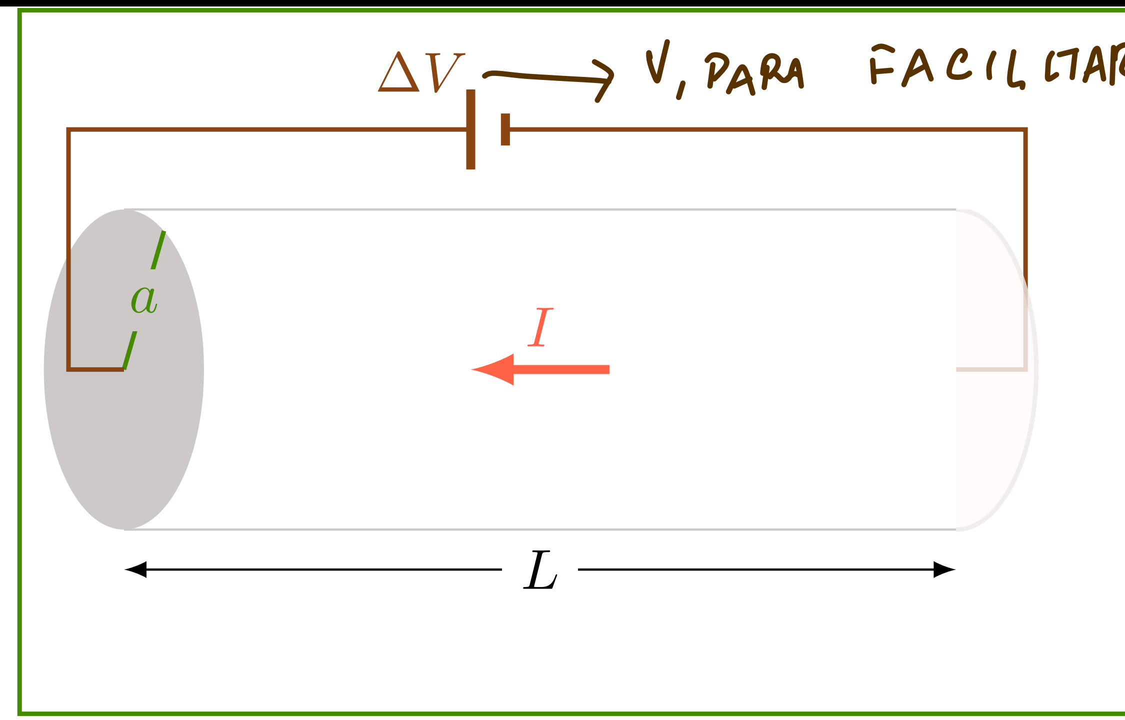


Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$E = \frac{V}{L}$$

↑ CAMPO UNIFORME

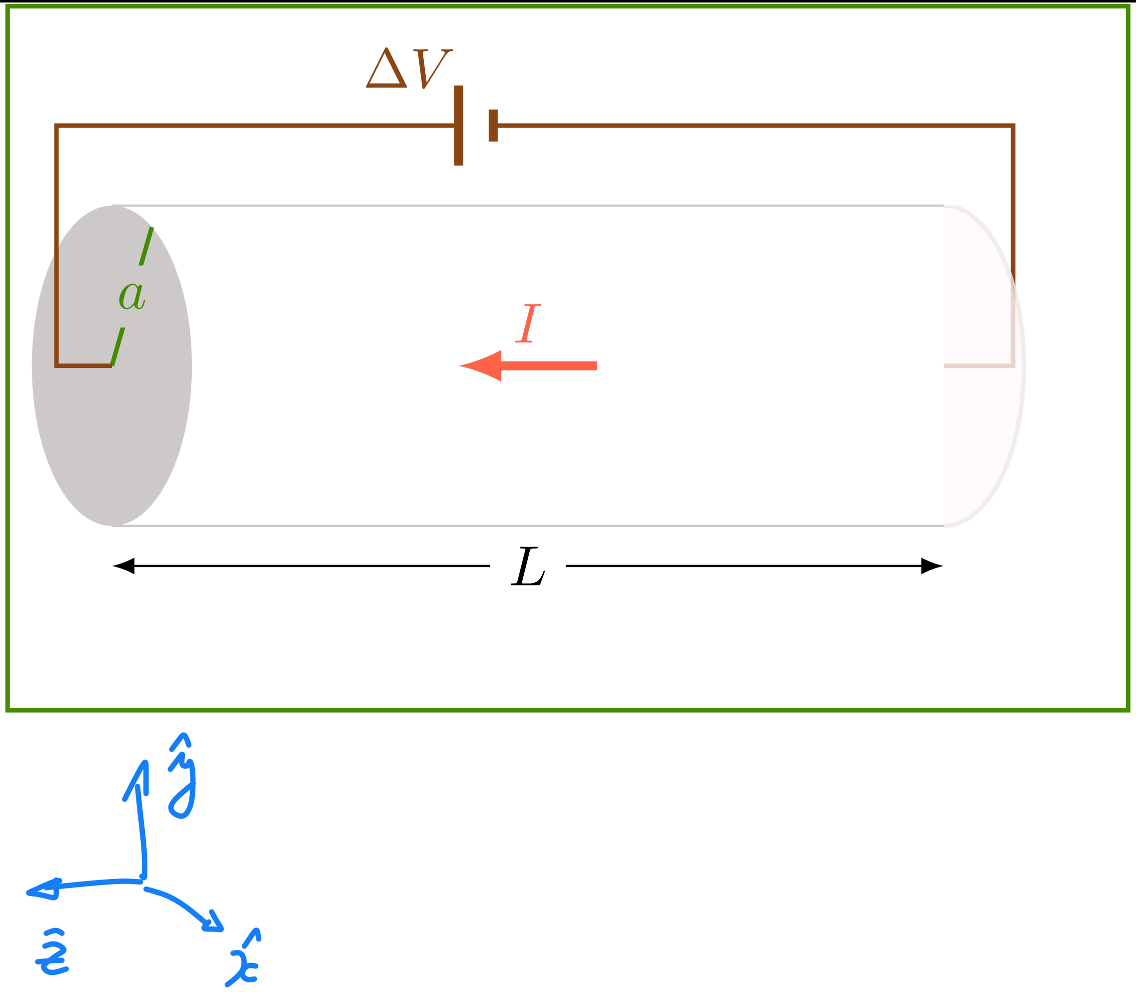


Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$E = \frac{V}{L}$$

$$\vec{E} = \frac{V}{L}\hat{z}$$



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$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

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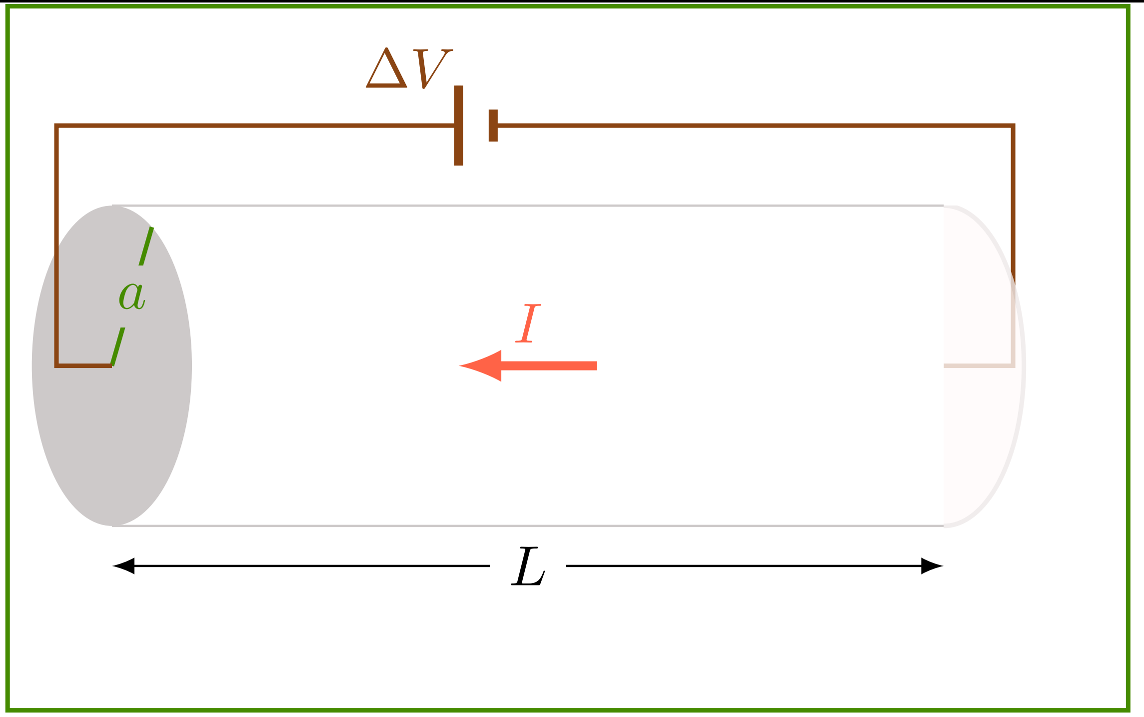
$$B = \frac{\mu_0}{2\pi a}I$$

LEI DE AMPÈRE



Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$E = \frac{V}{L}$$

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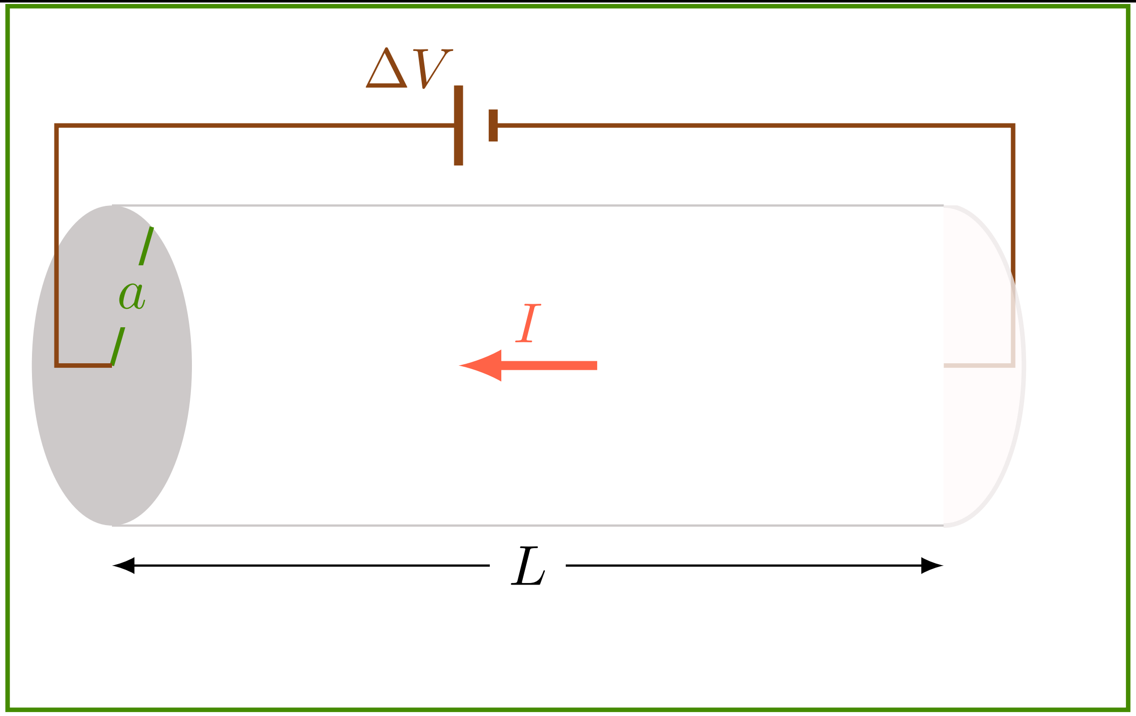
$$B = \frac{\mu_0}{2\pi a}I$$

$$\vec{B} = \frac{\mu_0}{2\pi a}I\hat{\phi}$$



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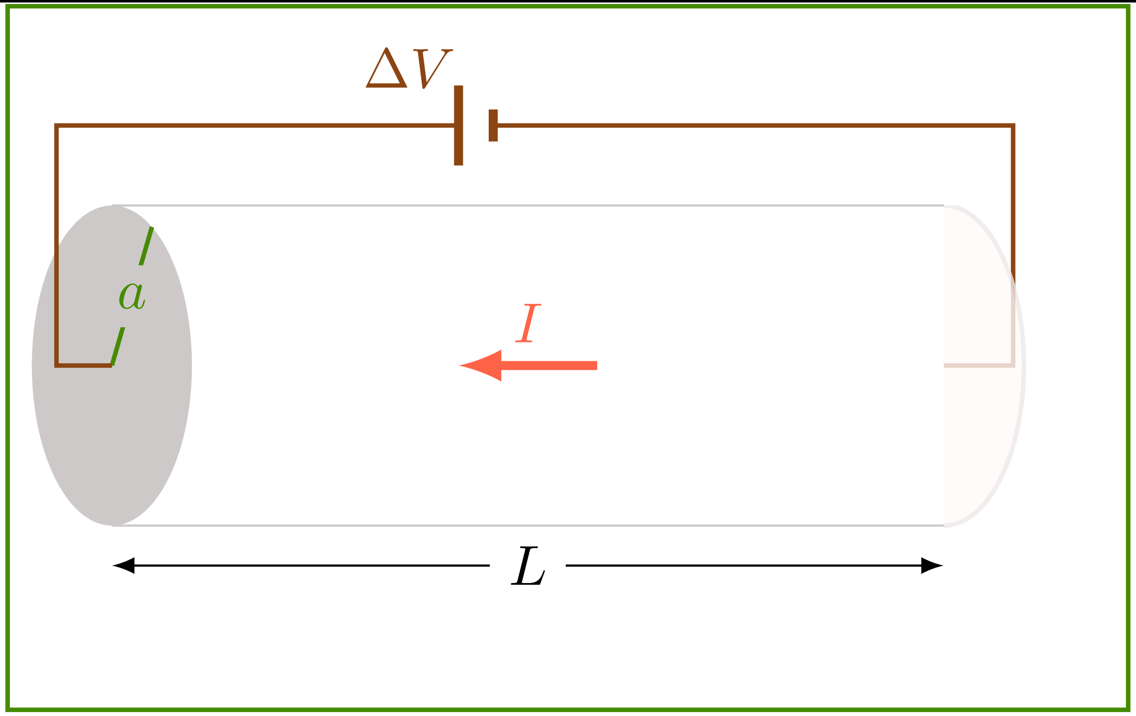

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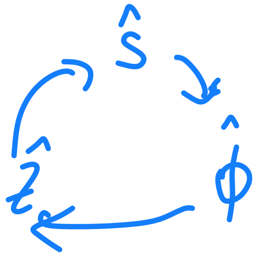

$$\vec{S} = \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0}{2\pi a} I \underbrace{\hat{z} \times \hat{\phi}}_{-\hat{s}}$$

$$\vec{S} = -\frac{VI}{\text{Área}}\hat{s}$$

Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

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$$\vec{B} = \frac{\mu_0}{2\pi a}I\hat{\phi}$$

$$\vec{S} = \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0}{2\pi a}I\,\hat{z} \times \hat{\phi}$$

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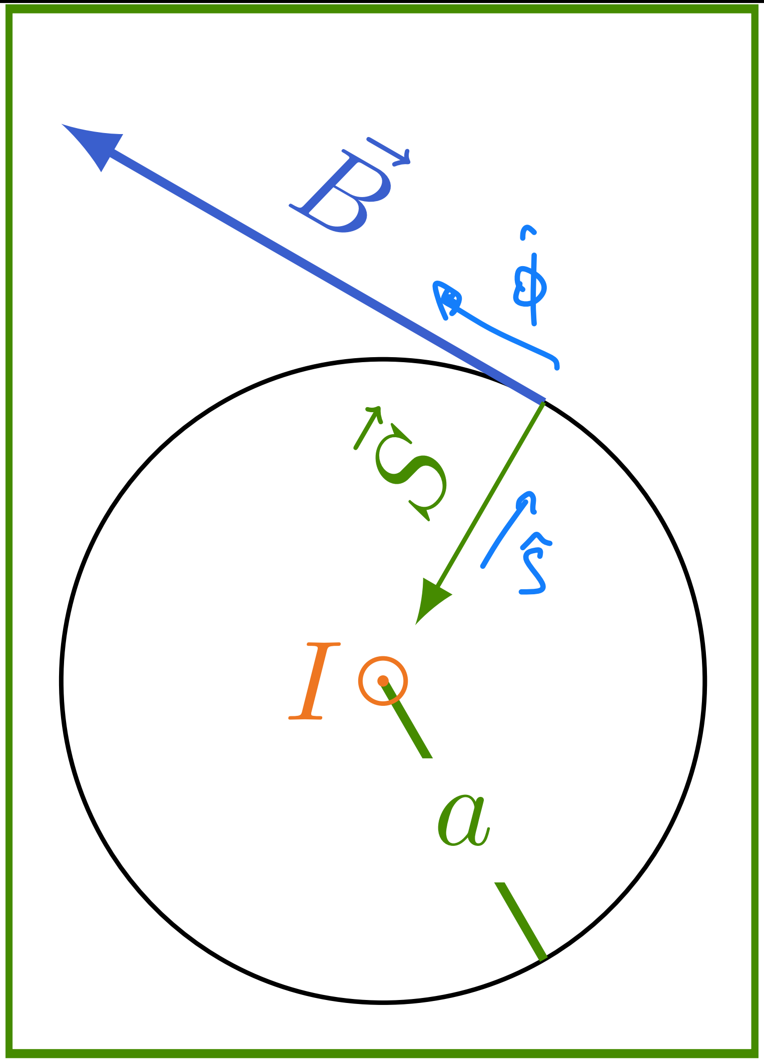
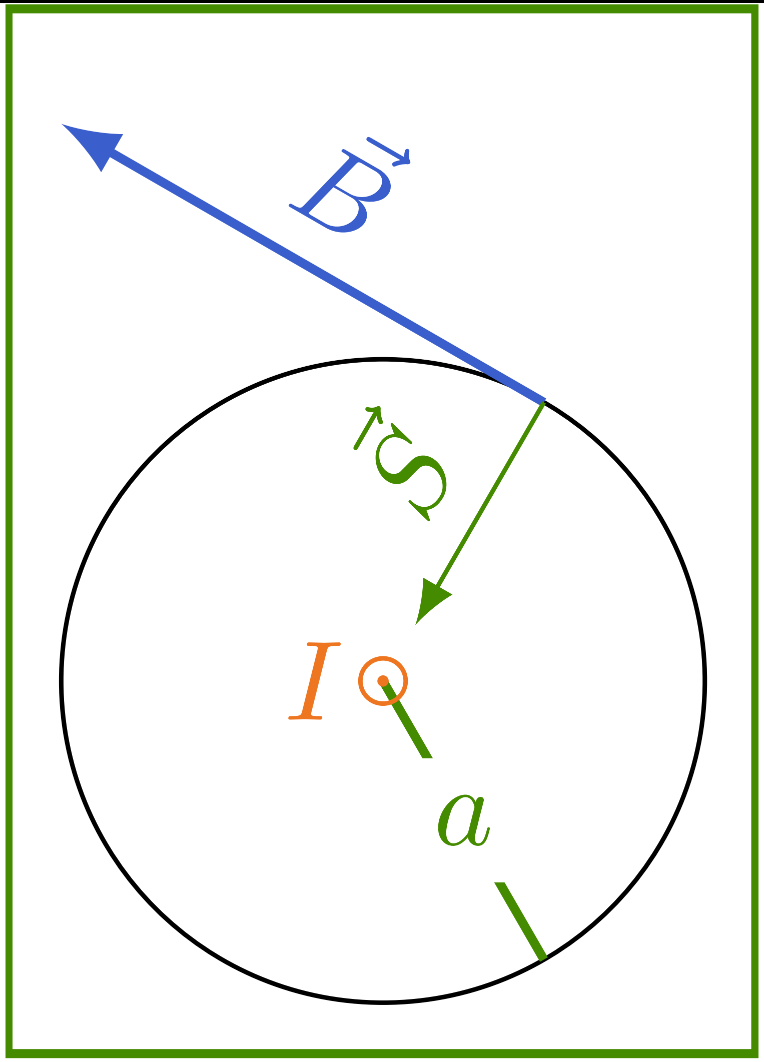


Pratique o que aprendeu

$$\overrightarrow{S} = \frac{1}{\mu_0}\overrightarrow{E} \times \overrightarrow{B}$$

$$\int_{\mathcal{A}} \vec{S} \cdot \hat{n} = VI$$

Um pouco de física



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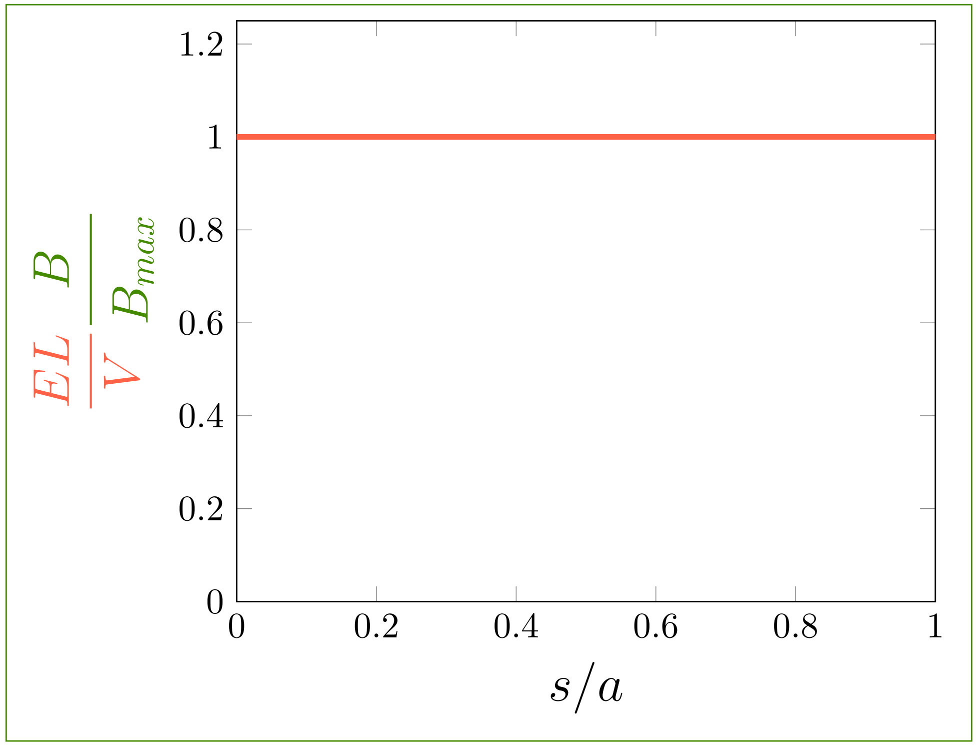
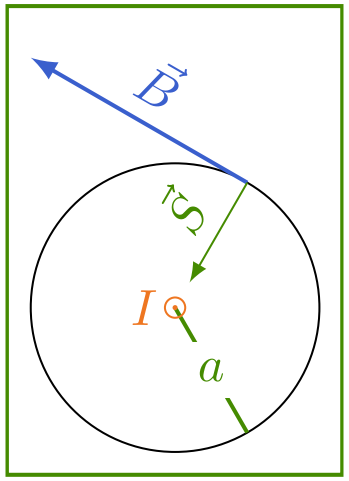
Um pouco de física





Pratique o que aprendeu

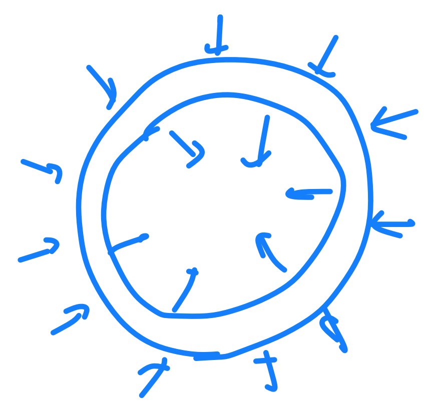
$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

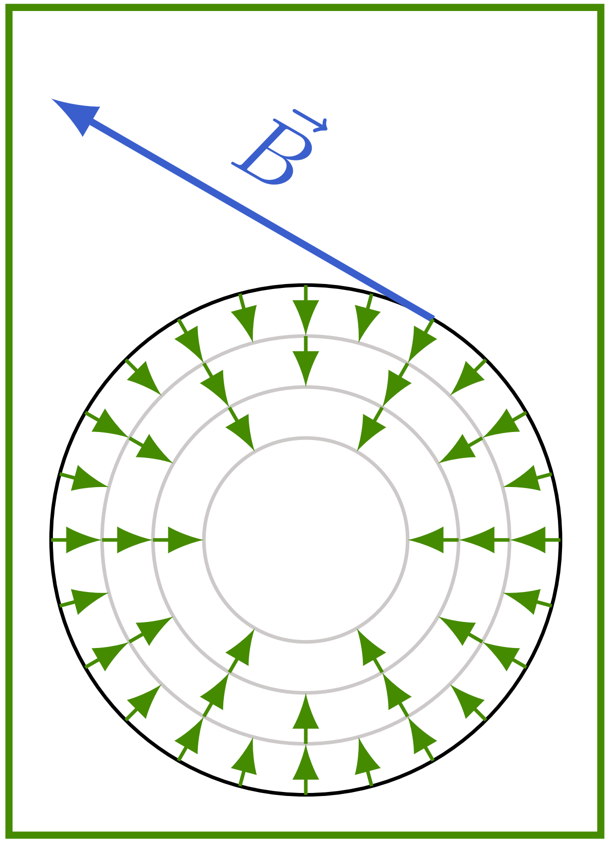
$$\int_A \vec{S} \cdot \hat{n} = VI$$

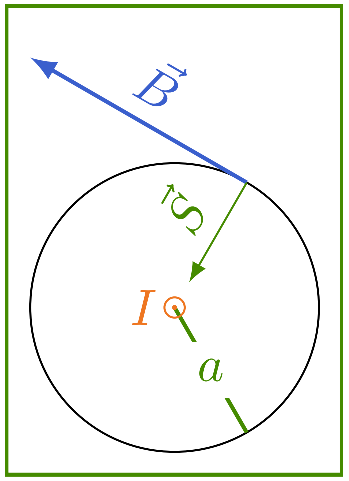
Um pouco de física

- EM CADA CASCA CILÍNDRICA, ENTRA MAIS ENERGIA DO QUE SAI

- SALDO AQUECE A CASCA $\Longleftrightarrow$ EFEITO JOULE

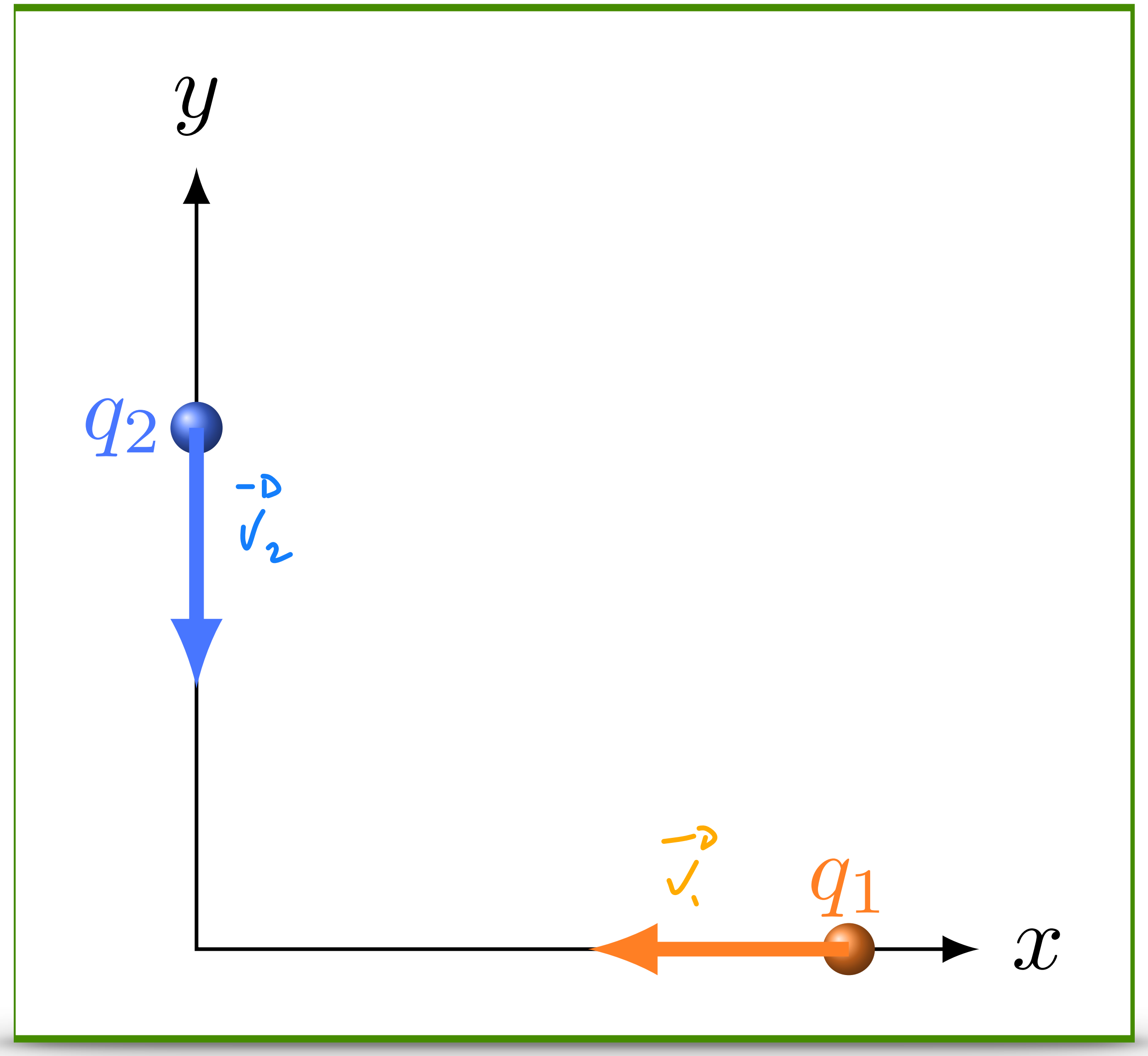






Leis de conservação

3. Momento



Leis de conservação

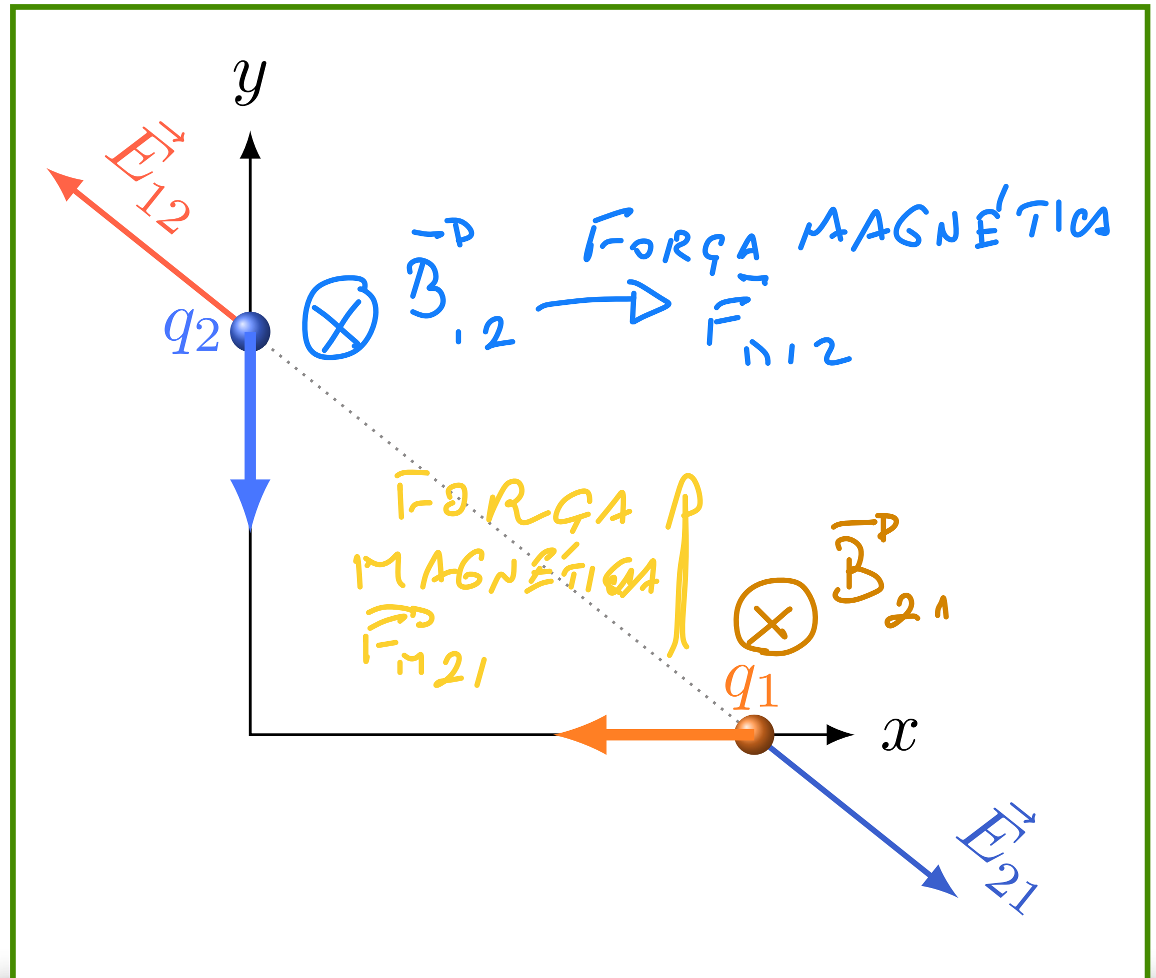
3. Momento

$$q_2\vec{E}_{12} = -q_1\vec{E}_{21}$$

→ Forças iguais e contrários

$$\vec{F}_{M12} \neq -\vec{F}_{M21}$$

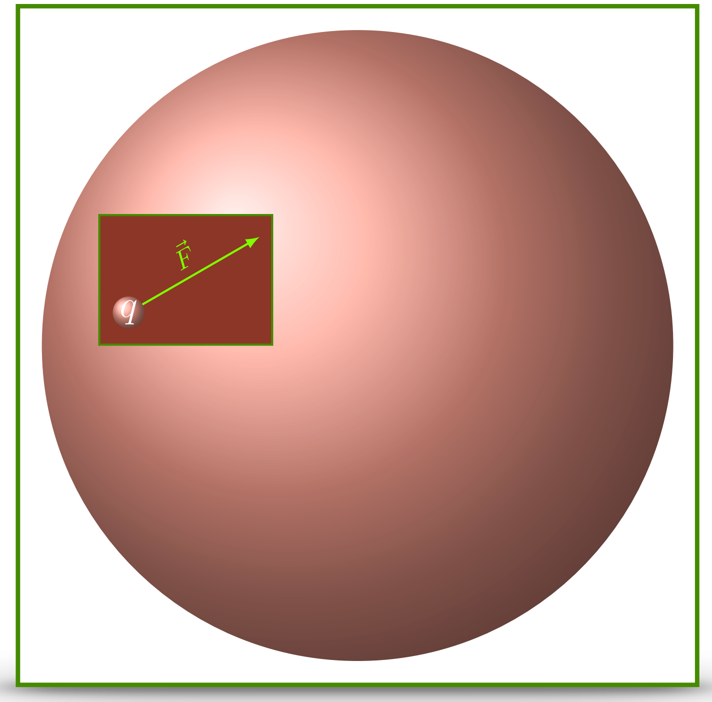
Violam 3ª lei de Newton



Leis de conservação

3. Momento

$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}} \; \mathrm{d}\tau$$



Leis de conservação

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$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}\ \mathrm{d}\tau$$

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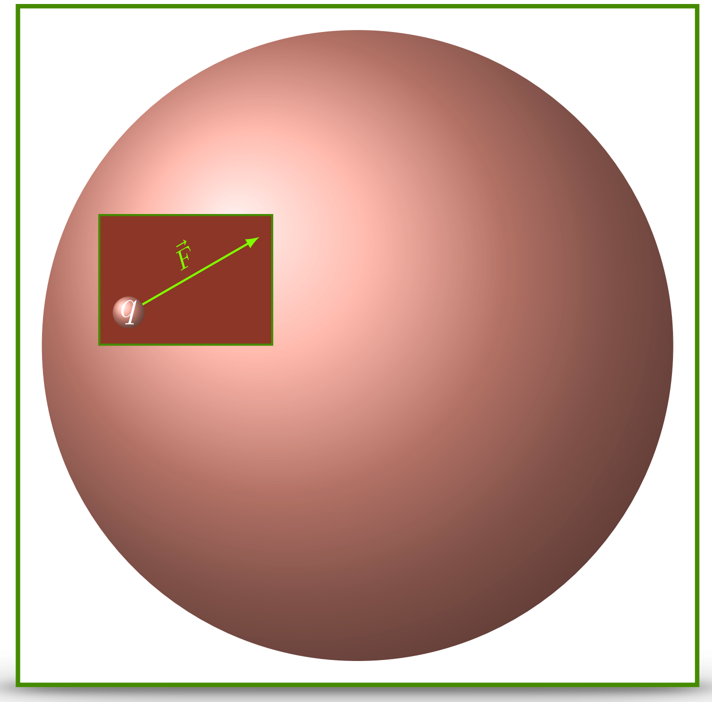

Leis de conservação

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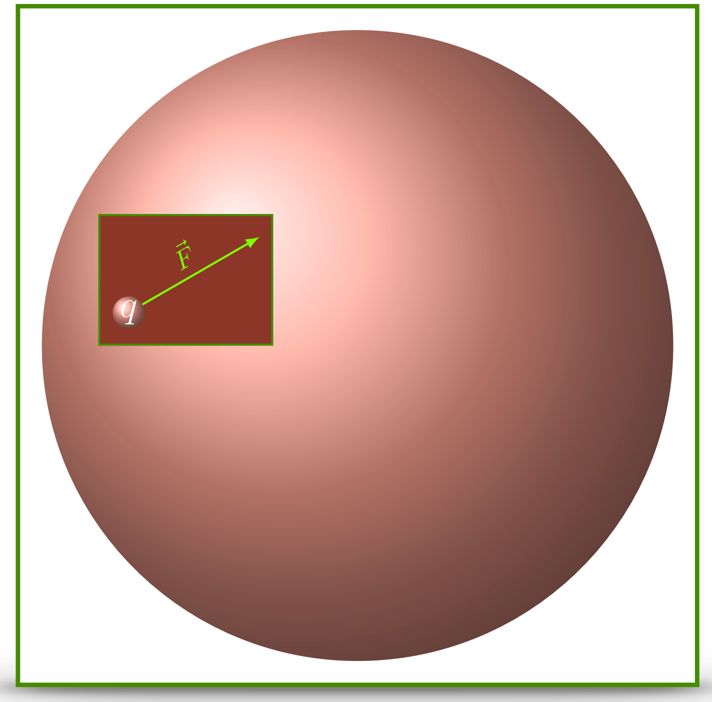
Poisson

$$\rho = \epsilon_0 \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}$$



Leis de conservação

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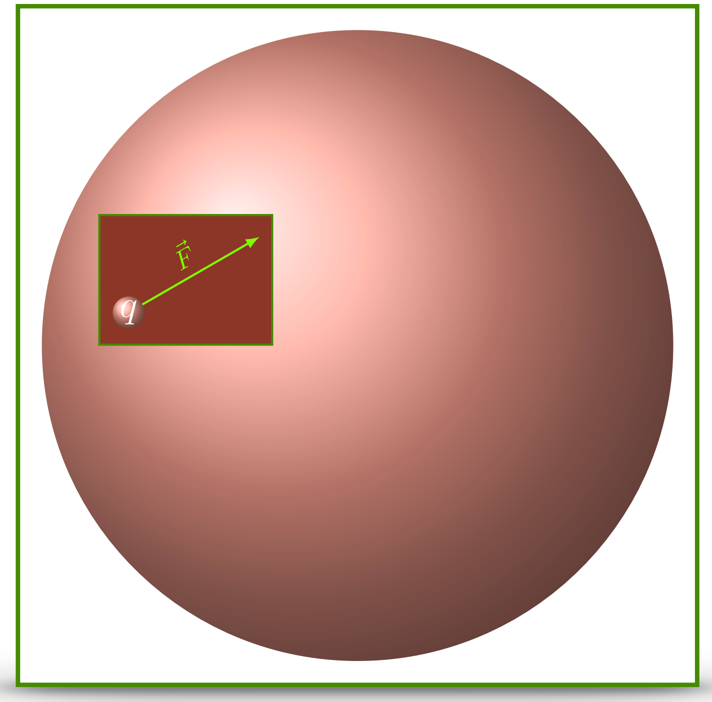

$$\vec{\mathbf{f}} = \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}$$

$$\rho = \epsilon_0 \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}$$

AMPÈRE + MAXWELL

$$\vec{\mathbf{J}} = \frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = ?$$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = ?$$

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = ?$$

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$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) + \vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}})$$

$$\vec{\mathbf{E}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right) = \frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}$$

ANALOGAMENTE, $\vec{B} \times (\vec{\nabla} \times \vec{B}) = \frac{1}{2} \vec{\nabla} B^2 - (\vec{B} \cdot \vec{\nabla}) \vec{B}$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = ?$$

$$\frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

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Leis de conservação

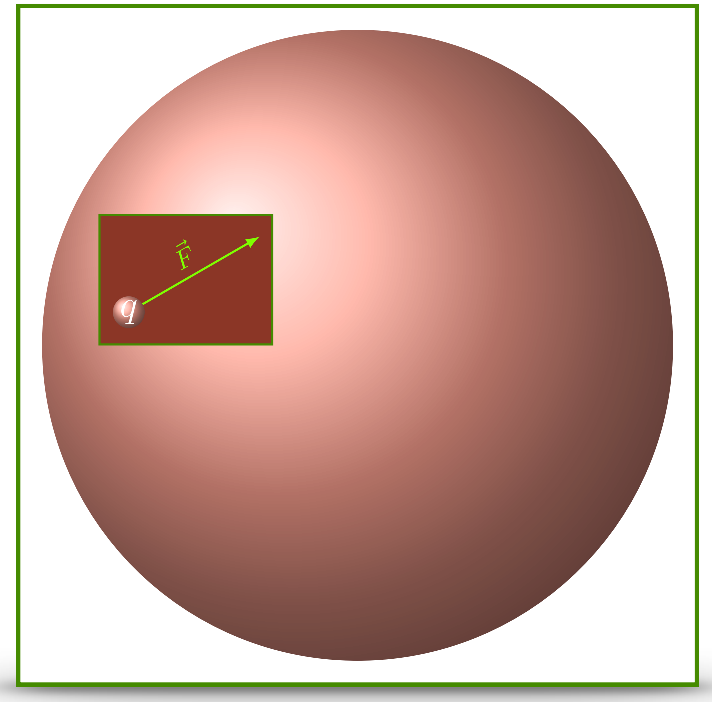
3. Momento

$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}\ \mathrm{d}\tau$$

$$\vec{\mathbf{f}} = \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}$$

$$\rho = \epsilon_0 \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}$$

$$\vec{\mathbf{J}} = \frac{1}{\mu_0}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$



$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) + \vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}})$$

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$$\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}$$

Leis de conservação

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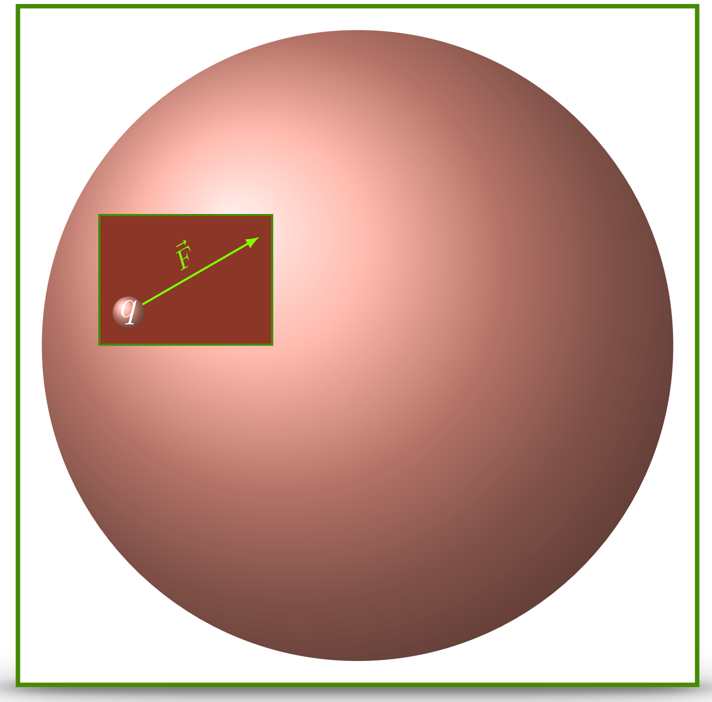

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$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) + \vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}})$$

$$\vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}$$

$$\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\frac{1}{\mu_0}\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) + \epsilon_0 \vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0 [(\vec{\mathbf{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\mathbf{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\mathbf{B}} \cdot \vec{\mathbf{\nabla}})\vec{\mathbf{B}}]$$

$$- \frac{1}{2}\vec{\mathbf{\nabla}}\underbrace{\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)}_{\text{DENSIDADE DE ENERGIA}} - \epsilon_0 \frac{\partial}{\partial t}\underbrace{\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)}_{\text{POYNTING}}$$

Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

PARA SIMETRIZAR,
JÁ QUE $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

Leis de conservação

Noether